week1

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Clustering

Given a clustering $C = \{C_1, C_2, \dots, C_k\}$, we need some scoring function that evaluates its quality or goodness. This sum of squared errors scoring function is defined as:

$$W(C) = \frac{1}{2} \sum_{k=1}^K \sum_{i:C(i)=k} \|x_i - \bar{x}_k\|^2$$

The goal is to find the clustering that minimizes:

$$C^* = \arg\min_{C} \{W(c)\}$$

K-means employs a greedy iterative approach to find a clustering that minimizes loss function.

Note that the echo = FALSE parameter was added to the code chunk to prevent printing of the R code that generated the plot.

```
euclidean_dist <- function(point1, point2) {
    squared_diff <- (point1 - point2)^2
    sqrt(sum(squared_diff))
}

x <- y <- seq(-1, 1, length = 20)
grid <- expand.grid(x = x, y = y) # Create a grid of points
z <- matrix(0, nrow = length(x), ncol = length(y)) # Initialize the z matrix

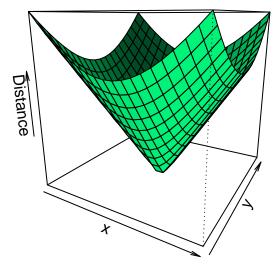
for (i in 1:length(x)) {
    for (j in 1:length(y)) {
        z[i, j] <- euclidean_dist(c(x[i], y[j]), c(0, 0))
    }
}
persp(x, y, z,</pre>
```

Algorithm 1: K-means Algorithm

```
Data: D, k, \varepsilon
  1 K-means(D, k, \varepsilon):
  2 t \leftarrow 0;
  3 Randomly initialize k centroids: \mu_1^t, \mu_2^t, \dots, \mu_n^t \in \mathbb{R}^d;
  4 repeat
               t \leftarrow t + 1;
  5
               \begin{split} C_i \leftarrow \emptyset \text{ for all } i = 1, \dots, k \\ /^* \text{ Cluster assignment step */} \end{split}
  7
               \begin{aligned} & \mathbf{for} \ x_j \in D \ \mathbf{do} \\ & | i^* \leftarrow \mathrm{argmin}_i \{ ||x_j - \mu_i^{t-1}||^2 \}; \\ & /^* \ \mathrm{assign} \ x_j \ \mathrm{to} \ \mathrm{closest} \ \mathrm{centroid} \ ^* / \end{aligned}
  8
   9
 10
                     C_{i^*} \leftarrow C_{i^*} \stackrel{\circ}{\cup} \{x_j\};
 11
12
                for i = 1,..,k do
13
                 16 until \sum_{i=1}^{k} ||\mu_i^t - \mu_i^{t-1}||^2 \le \varepsilon;
```

```
main = "3D Plot of Euclidean Distance",
zlab = "Distance",
theta = 30, phi = 15,
col = "springgreen", shade = 0.5)
```

3D Plot of Euclidean Distance

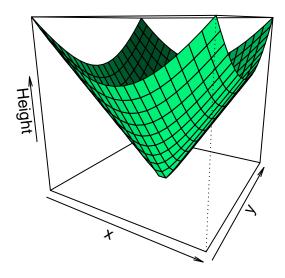


```
cone1 <- function(x, y){
sqrt(x^2+y^2)
}

x <- y <- seq(-1, 1, length= 20)
z <- outer(x, y, cone1)

persp(x, y, z,
main="Perspective Plot of a Cone",
zlab = "Height",
theta = 30, phi = 15,
col = "springgreen", shade = 0.5)</pre>
```

Perspective Plot of a Cone



```
euclidean_dist <- function(point1, point2) {
    squared_diff <- (point1 - point2)^2
    sqrt(sum(squared_diff))
}

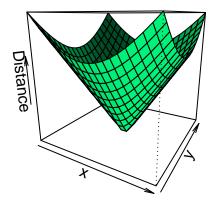
manhattan_distance <- function(point1, point2) {
    if (length(point1) != length(point2)) {
        stop("Both points should have the same number of dimensions.")
    }

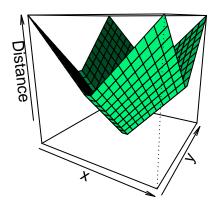
    abs_diff <- abs(point1 - point2)
    distance <- sum(abs_diff)
    return(distance)</pre>
```

```
x \leftarrow y \leftarrow seq(-1, 1, length = 20)
grid <- expand.grid(x = x, y = y) # Create a grid of points</pre>
z_euclidean <- matrix(0, nrow = length(x), ncol = length(y)) # Initialize the z matrix for Euclidean d</pre>
z_manhattan <- matrix(0, nrow = length(x), ncol = length(y)) # Initialize the z matrix for Manhattan</pre>
for (i in 1:length(x)) {
 for (j in 1:length(y)) {
    z_{euclidean[i, j]} \leftarrow euclidean_dist(c(x[i], y[j]), c(0, 0))
    z_manhattan[i, j] <- manhattan_distance(c(x[i], y[j]), c(0, 0))</pre>
  }
}
# Create a layout of subplots to show both Euclidean and Manhattan distances
par(mfrow = c(1, 2))
# Plot for Euclidean distance
persp(x, y, z_euclidean,
      main = "3D Plot of Euclidean Distance",
      zlab = "Distance",
      theta = 30, phi = 15,
      col = "springgreen", shade = 0.5)
# Plot for Manhattan distance
persp(x, y, z_manhattan,
      main = "3D Plot of Manhattan Distance",
      zlab = "Distance",
      theta = 30, phi = 15,
      col = "springgreen", shade = 0.5)
```

3D Plot of Euclidean Distance

3D Plot of Manhattan Distance

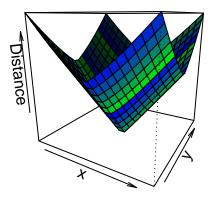




```
# Reset the layout
par(mfrow = c(1, 1))
euclidean_dist <- function(point1, point2) {</pre>
  squared_diff <- (point1 - point2)^2</pre>
  sqrt(sum(squared_diff))
manhattan_distance <- function(point1, point2) {</pre>
  if (length(point1) != length(point2)) {
    stop("Both points should have the same number of dimensions.")
  abs_diff <- abs(point1 - point2)</pre>
  distance <- sum(abs_diff)</pre>
  return(distance)
x \leftarrow y \leftarrow seq(-5, 5, length = 20)
grid <- expand.grid(x = x, y = y) # Create a grid of points</pre>
z_euclidean <- matrix(0, nrow = length(x), ncol = length(y)) # Initialize the z matrix for Euclidean d
z_manhattan <- matrix(0, nrow = length(x), ncol = length(y)) # Initialize the z matrix for Manhattan
for (i in 1:length(x)) {
for (j in 1:length(y)) {
```

```
z_euclidean[i, j] <- euclidean_dist(c(x[i], y[j]), c(0, 0))</pre>
    z_manhattan[i, j] <- manhattan_distance(c(x[i], y[j]), c(0, 0))</pre>
  }
}
# Combine the distances and choose different colors for each
combined_distances <- z_euclidean + z_manhattan</pre>
color_palette <- colorRampPalette(c("blue", "green"))(100) # Choose colors for mapping distances</pre>
# Create a layout of subplots
layout(matrix(c(1, 2), nrow = 1))
# Plot both distances on the same 3D plane with different colors
persp(x, y, combined_distances,
      main = "3D Plot of Combined Distances",
      zlab = "Distance",
      theta = 30, phi = 15,
      col = color_palette, shade = 0.5)
# Reset the layout
layout(1)
```

3D Plot of Combined Distances



```
# Include the script from the R directory
project_path <- here()
source(here("R", "utils.R"))</pre>
```

?entropy

No documentation for 'entropy' in specified packages and libraries:
you could try '??entropy'