Mathematics and Statistics for Data Science
Lecture 4
Hyperplanes and Exam Exercises

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Definition of a Hyperplane

A hyperplane $H \subset \mathbb{R}^n$ is the set of points $(x_1, x_2, ..., x_n)$ that satisfy a linear equation: $a_1x_1 + a_2x_2 + ... + a_nx_n = b$ with:

$$a_i \neq 0 \;\; \forall i$$
$$b \in \mathbb{R}$$

Remark

$$H \subset \mathbb{R}^1 \rightarrow a_1x_1 = b \rightarrow Point$$
$$H \subset \mathbb{R}^2 \rightarrow a_1x_1 + a_2x_2 = b \rightarrow Line$$

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1.)

Calculate the plane $H \subset \mathbb{R}^3$ such that point $P = (1, 2, 3)$ belongs to it and the plane is orthogonal to vector $v = (1, -3, 1)$.

1.) Solution

$$v \cdot (x, y, z) = 0 \rightarrow (1, -3, 1) \cdot (x, y, z) = 0$$

For the $\perp$ condition: $1 \cdot x - 3 \cdot y + 1 \cdot z = 0 \rightarrow x - 3y + z = 0$

$1 \cdot 1 - 3 \cdot 2 + 1 \cdot 3 = b$

Recall: $A \cdot B = \sum_{i=1}^{n} a_i \cdot b_i$ when $A = (a_1, a_2, ..., a_n)$; $B = (b_1, b_2, ..., b_n)$

Recall: $a_1 x + a_2 y + a_3 z = b$

2.)
Calculate the plane $H \subset \mathbb{R}^3$ generated by vectors $v_1 = (1, 2, 1)$ and $v_2 = (3, 1, 1)$ passing through point $P = (1, 1, 1)$.

2.) Solution

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + k \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad t, k \in \mathbb{R}$$

$$\begin{cases} x = t + 3k + 1 \\ y = 2t + k + 1 \\ z = t + k + 1 \end{cases}$$

$$\begin{aligned} & t = x - 3k - 1 \\ \rightarrow \quad & y = 2(x - 3k - 1) + k + 1 \\ & z = x - 3k - x + k + x \end{aligned}$$

$$\rightarrow k = \frac{x - z}{2}$$

$$y = 2x - 3z - 2 + \frac{x - z}{2} + 1 \rightarrow 2y - 5z + x = -2$$