# Exercises
# Leuphana University of Lueneburg

LaTeX in Collaboration

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## 1 Norms and Linear Spaces of Vectors and Functions (20 Credits)

- Please calculate:

  The $\mathcal{L}_1$, $\mathcal{L}_2$ and $\mathcal{L}_\infty$, Norm for the following vectors: $v_1 = (1, 1, 1)$; $v_2 = (1, 0, 1)$;

- Please discuss the linear dependence and linear independence of the following sets of vectors verifying if the null vector is generated in a trivial way:

$$S_1 = \{v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\} \qquad (1)$$

- Line and Hyperplane Find a line passing through point $P = (1, 0, 1)$ with direction $v = (-1, -2, 1)$.

- Find a line $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ passing through point $P = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ generated by vector $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

- Calculate the plane $H \subset \mathbb{R}^3$ such that point $P = (1, 0, 3)$ belongs to it and the plane is orthogonal to vector $v = (2, -1, 1)$.

- Calculate the plane $H \subset \mathbb{R}^3$ generated by vectors $v_1 = (3, 4, 1)$ and $v_2 = (1, 0, 1)$ passing through point $P = (1, 0, 0)$.

- Inner Product and Orthogonality: Given the following two vectors:

- $v_1 = (1, 5)$ and $v_2(1, 1)$, calculate the inner products and the projection of $v_1$ onto $v_2$ and viceversa and calculate the angle in between.

- $v_1 = (1, 0, 3)$ and $v_2(1, 1, 1)$, calculate the inner products and the projection of $v_1$ onto $v_2$ and viceversa and calculate the angle in between.
  Given the following set of independent vectors:
$$S_1 = \{v_1 = \begin{bmatrix} 1 \\ 1 \\ 0.1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\} \tag{2}$$
  calculating the inner products among them, indicate if the set is of vectors is linear depending.