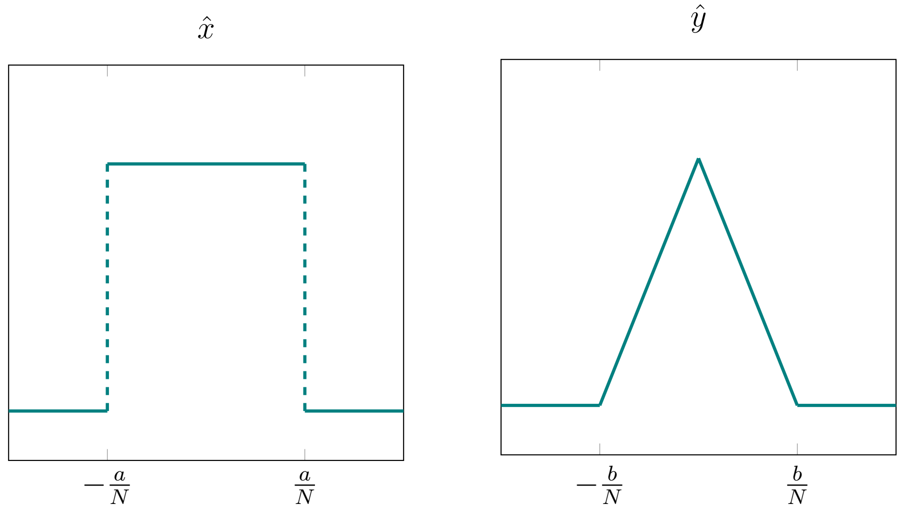
Homework 7

Due April 4 at 11 pm

1. (Radio signals) This problem illustrates a real-world technique for communicating radio signals (amplitude modulation or AM). A radio station wants to broadcast two different signals x and y at the same time (a news show and a cooking show). We model the signals as discrete for simplicity. Both are vectors of dimension N. Their DFTs are shown below (also for simplicity, we will assume they are real and even, so that we don't have to worry about the phase). The DFTs are indexed between $-\frac{N}{2}$ and $\frac{N}{2}$ and look continuous because N is large. Note that both signals are bandlimited.



   (a) The signals are multiplied by two different sinusoids with frequencies $\frac{C_1}{N}$ and $\frac{C_2}{N}$, and then added together. Assume that $C_1 > a$ and $C_2 > C_1 + a + b$. The resulting signal z is then broadcast. We have

$$z[j] := x[j]\cos\left(\frac{2\pi C_1 j}{N}\right) + y[j]\cos\left(\frac{2\pi C_2 j}{N}\right). \tag{1}$$

   Sketch the DFT of z. Annotate any points at which the DFT changes from being zero to nonzero or vice versa.

   (b) The signal z is decoded by radio receivers in the following way. It is multiplied by a sinusoid of frequency $\frac{R}{N}$,

$$\sigma[j] := 2\cos\left(\frac{2\pi R j}{N}\right), \tag{2}$$

   to obtain a signal $w = \sigma \circ z$. Then a low pass filter is applied to w, which preserves the DFT up to a certain cut-off frequency $\frac{B}{N}$, to generate the final received signal s. What should the values of R and B be so that $s = x$? Justify your answer by drawing a sketch of the DFT of w, annotating any points at which the DFT changes from being zero to nonzero or vice versa.

(c) What should the values of R and B be so that $s = y$? Justify your answer by drawing a sketch of the DFT of w, annotating any points at which the DFT changes from being zero to nonzero or vice versa.

(d) Does this work if $C_2 < 2a + b$? Why?

2. (Hann window) In this problem we analyze the Hann window in the frequency domain.

   (a) The Hann window $h \in \mathbb{C}^N$ of width $2w$ equals

$$h[j] := \begin{cases} \frac{1}{2}\left(1 + \cos\left(\frac{\pi j}{w}\right)\right) & \text{if } |j| \leq w, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

   Express the DFT of h in terms of three shifted, scaled copies of the DFT of a rectangular window of length $2w$.

   (b) Plot the DFT of h as well as the three different components that you derived in the previous question. Interpret what you see in terms of the desired properties of a windowing function.

3. (STFT inverse) In this problem we show a simple way to invert the STFT.

   (a) In the definition of the STFT set $w_{[\ell]}$ to be a rectangular window where all entries are equal to one, and let $\alpha_{\text{ov}} = 0.5$. Show that the STFT can be inverted using just two operations: applying the inverse DFT and subsampling.

   (b) What is the disadvantage of using this rectangular window?

4. (STFT of speech signal) In this question, we will analyze the effect of length and type of window while performing STFT. We will perform our analysis on speech signals using the support code provided in class.

   (a) Extend the analysis in the notebook to use Hann window of different sizes. Use atleast [100, 500, 1000, 3000, 5000, 20000]. Please look into `nfft` argument of `scipy.signal.stft()` as well.

   (b) Use a rectangular window with different lengths. Use atleast [500, 1000, 3000, 5000, 20000]. Again, please look into `nfft` argument of `scipy.signal.stft()` as well.

   (c) What do you observe when you change the length of the window?

   (d) What happens when you change from Hann window to rectangular window?