

Bilateral Filtering

DS-GA 1013 Spring 2021

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NYU

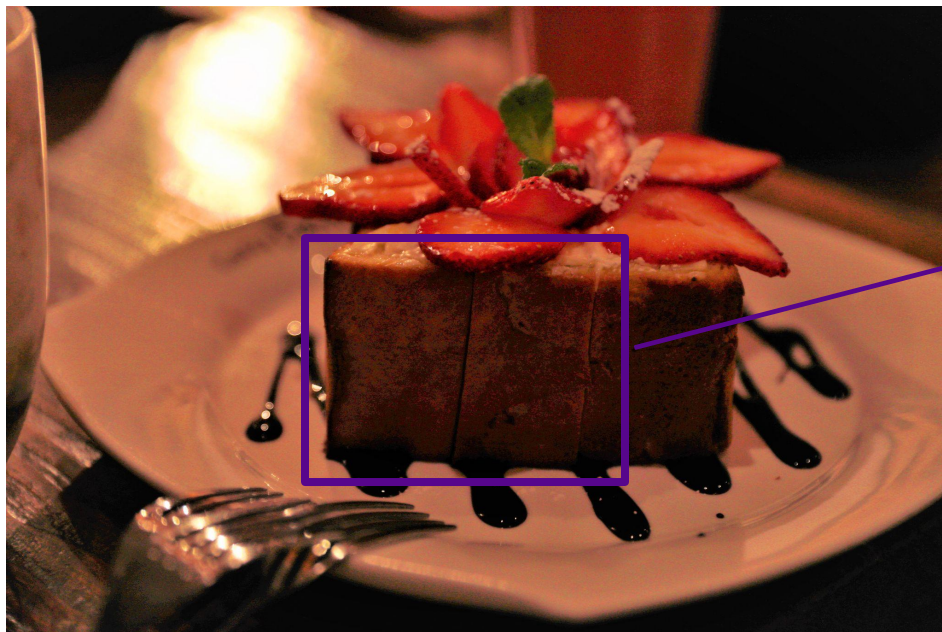
Outline

1. Motivation
2. Definition of Bilateral Filtering
3. Application
4. Hyperparameter Tuning

Why is image filtering important?

1. Produce stylish effects
2. Correct defects and smooth images

Why is image filtering important?



Denoising

Why is image filtering important?

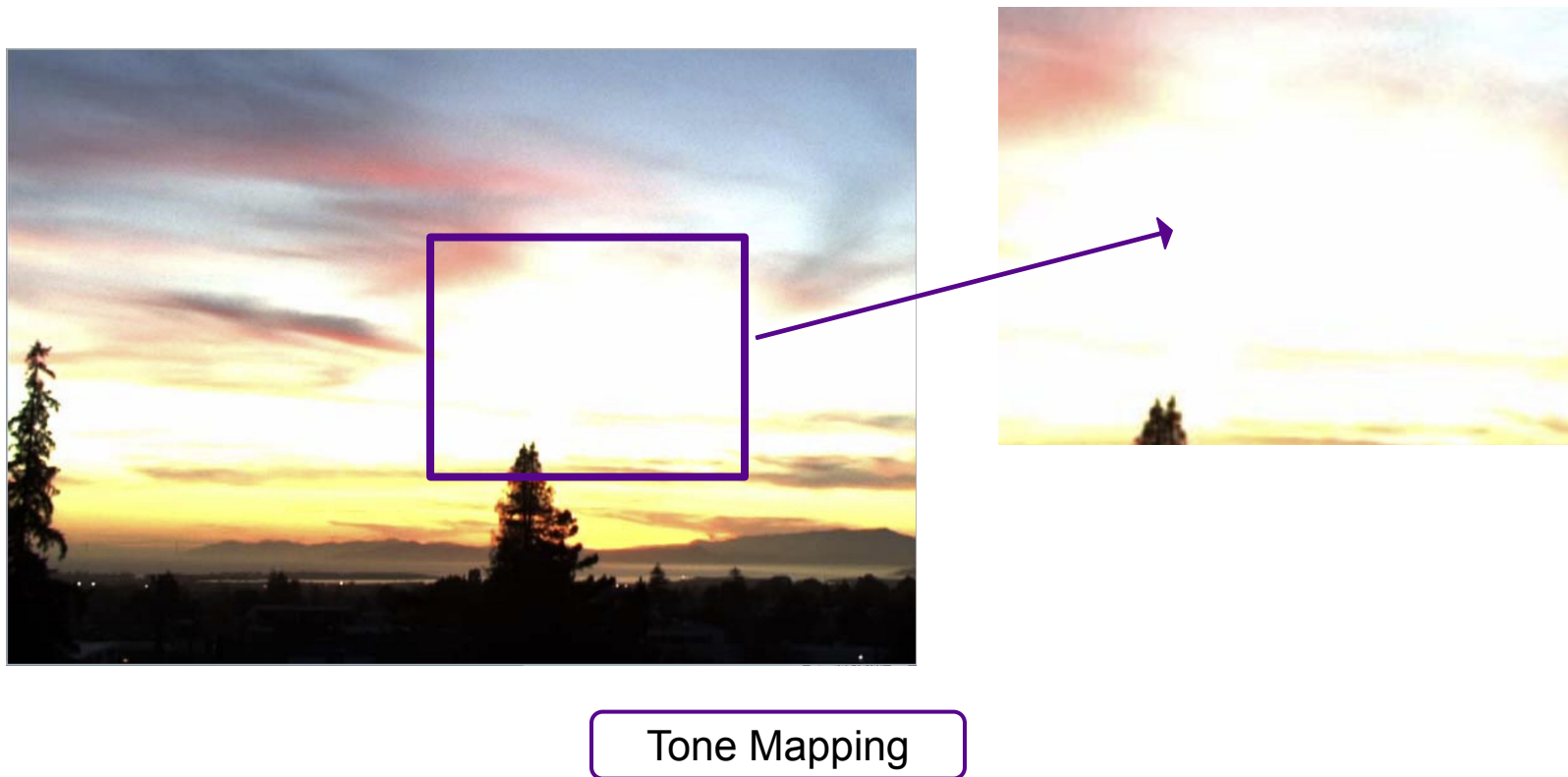
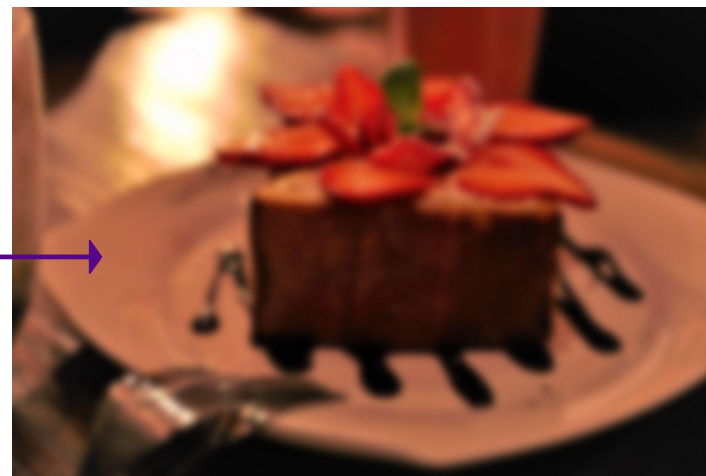


Figure from Durand 02

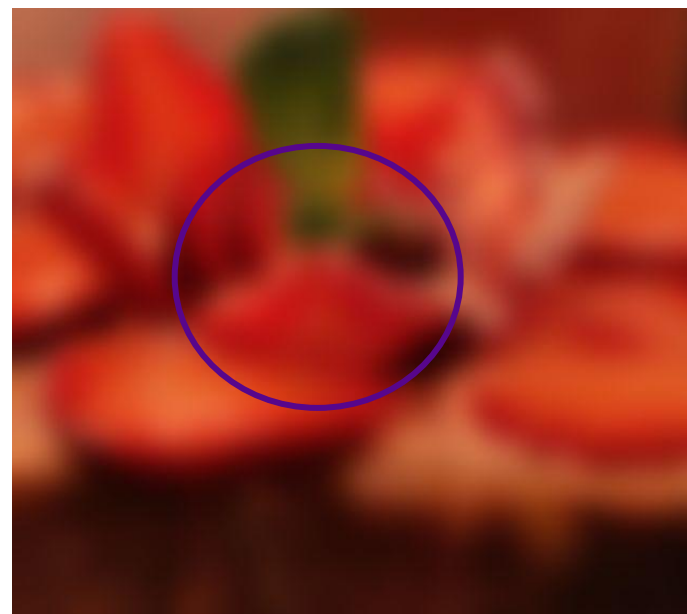
Where does bilateral filtering come from?

Before bilateral filtering, we have Gaussian filtering:



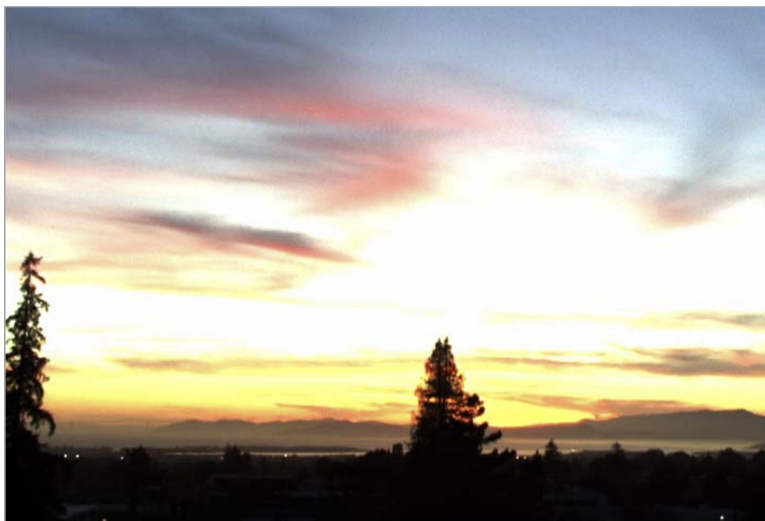
Averaging colors or intensities of nearby pixels in space

Why do we use bilateral filtering over others?



Blurred Edges from Gaussian Filter

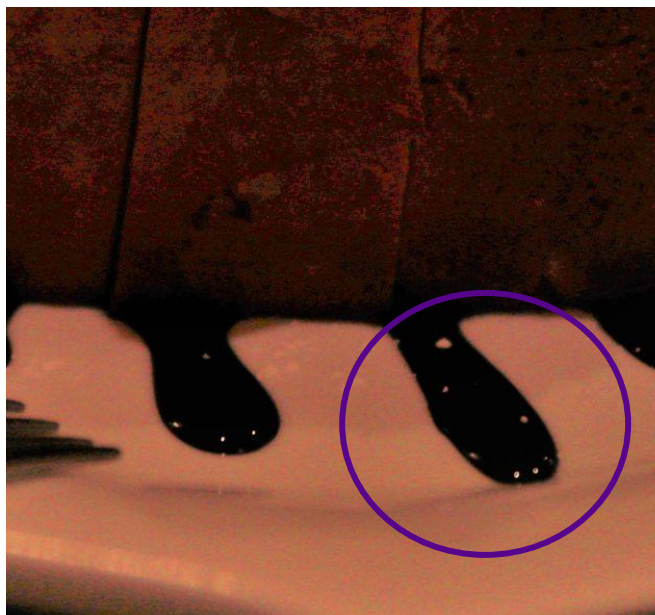
Why do we use bilateral filtering over others?



Halo Artifact from Gaussian Filter

Figure from Durand 02

Why do we use bilateral filtering over others?



Halo Artifact from Gaussian Filter

Definition of Gaussian Filtering

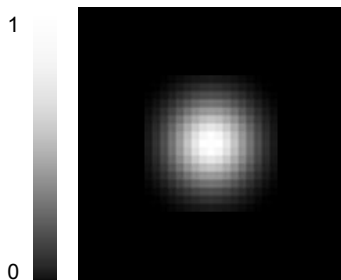
Image = 2D array of pixels (x, y)

Each pixel = intensity (scalar) in gray-level image or color (vector) in color image

$$GC[I]_p = \sum_{q \in S} G_{\sigma}(\|p - q\|) I_q$$

Diagram illustrating the Gaussian Filtering equation:

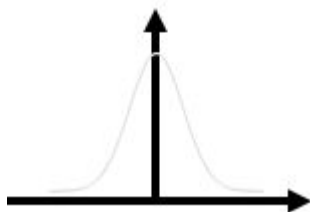
- $q \in S$: set of pixels
- p : pixel p
- q : pixel q
- I_q : intensity @ pixel q
- G_{σ} : Gaussian distribution (window size determined by σ)



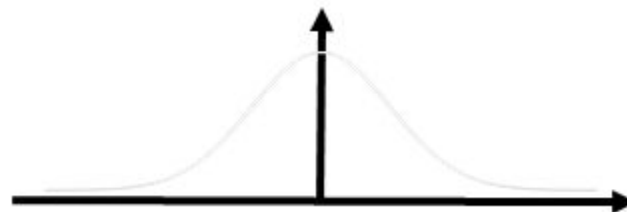
$$G_{\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-x^2}{2\sigma^2}\right)$$

Definition of Gaussian Filtering

$$GC[I]_p = \sum_{q \in S} G_{\sigma}(\|p - q\|) I_q$$



small σ



large σ



limited smoothing

strong smoothing



Definition of Bilateral Filtering

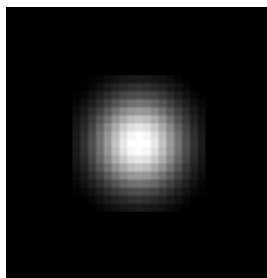
$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} \underbrace{G_{\sigma_S}(\|p - q\|)}_{\text{space difference}} \underbrace{G_{\sigma_r}(|I_p - I_q|)}_{\text{intensity range}} I_q$$

normalization factor

space difference

intensity range

$$W_p = \sum_{q \in S} G_{\sigma_S}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|)$$



Definition of Bilateral Filtering

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_S}(\|p - q\|) G_{\sigma_r}(|I_p - I_q|) I_q$$

σ_S = size of the neighborhood

σ_r = lower boundary of an edge intensity value

Definition of Bilateral Filtering

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} \underbrace{G_{\sigma_S}(\|p - q\|)}_{\text{spatial}} \underbrace{G_{\sigma_r}(|I_p - I_q|)}_{\text{range}} I_q$$

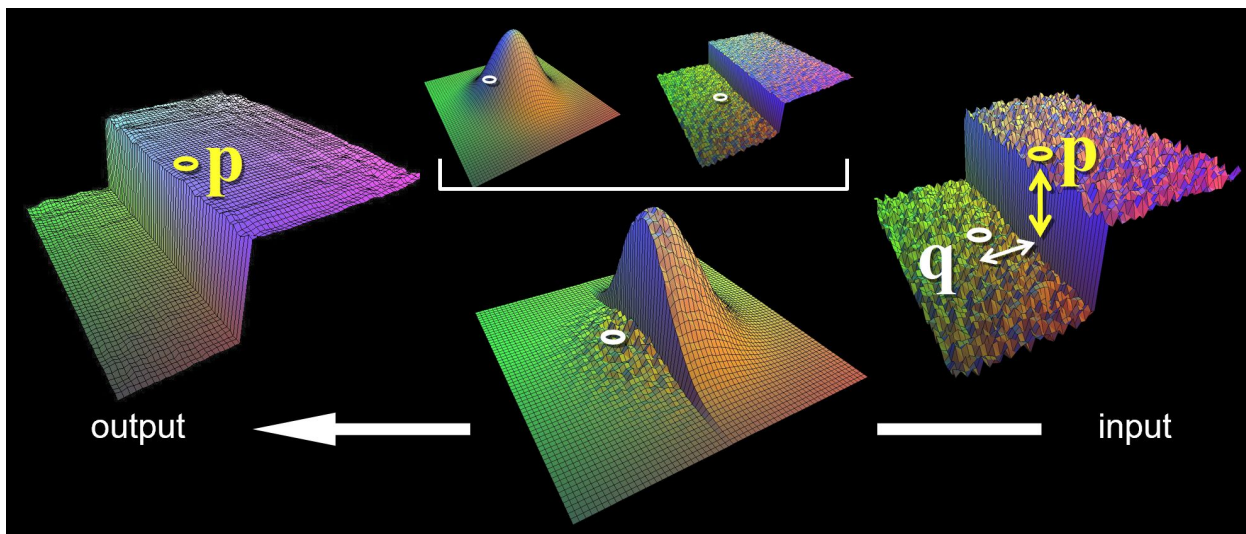


Figure from Durand 02

Definition of Bilateral Filtering

From gray-level image to color image:

Gray-level:
$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_S}(\|p - q\|) G_{\sigma_r}(\|I_p - I_q\|) \underline{I_q}$$

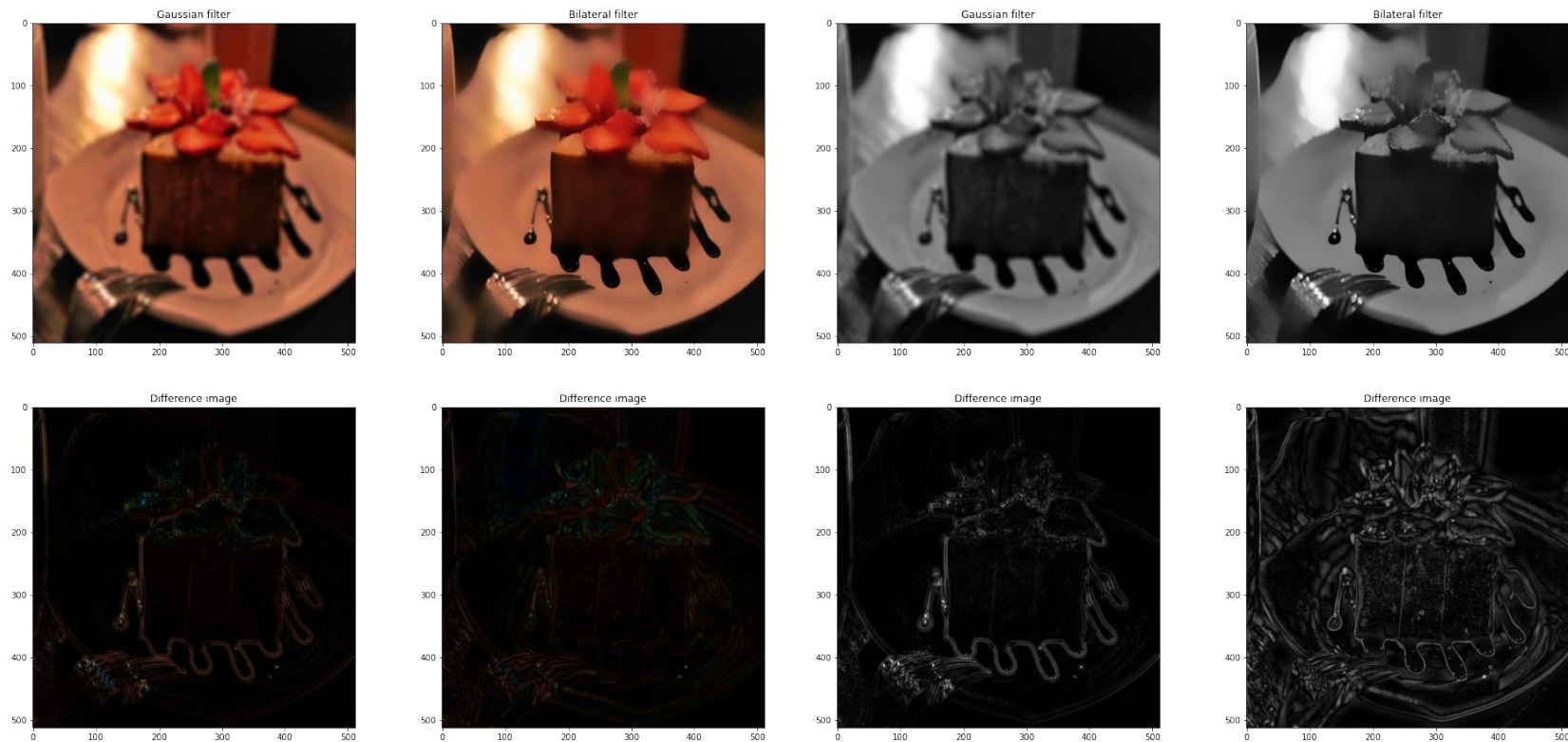
intensity (scalar)

Color:
$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_S}(\|p - q\|) G_{\sigma_r}(\|C_p - C_q\|) \underline{C_q}$$

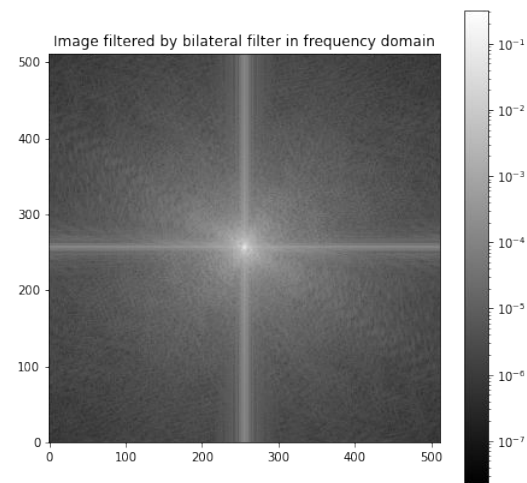
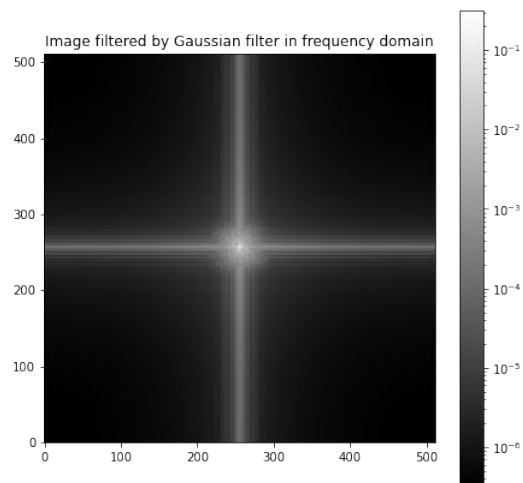
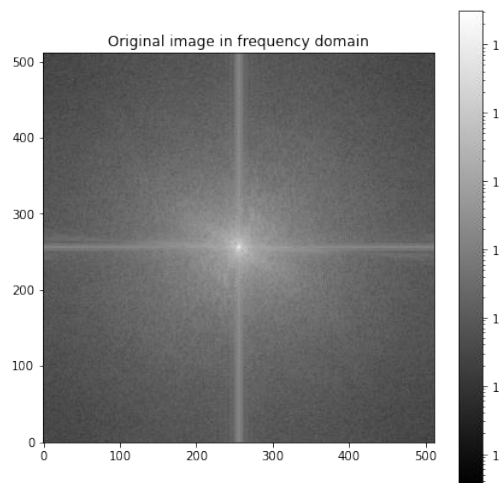
color, e.g. RGB (vector)



Gaussian vs Bilateral

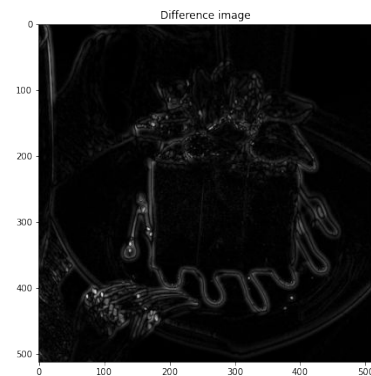
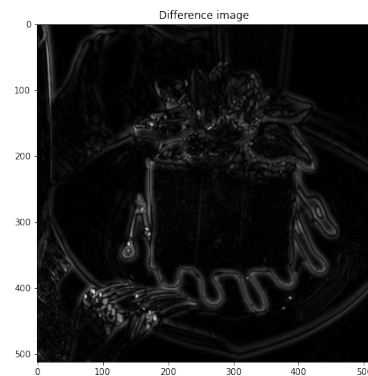
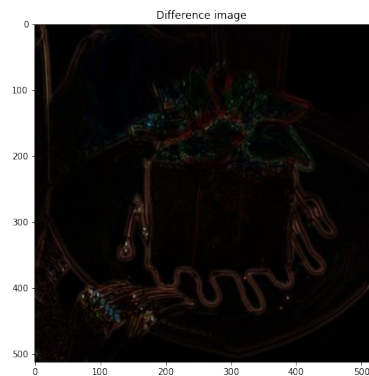
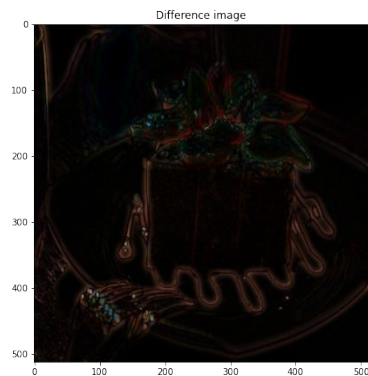
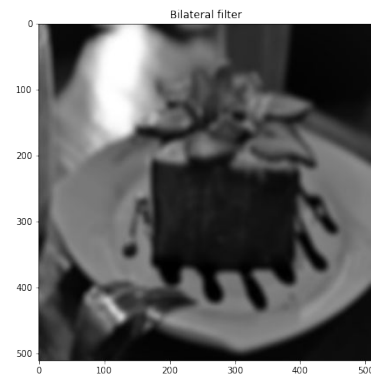
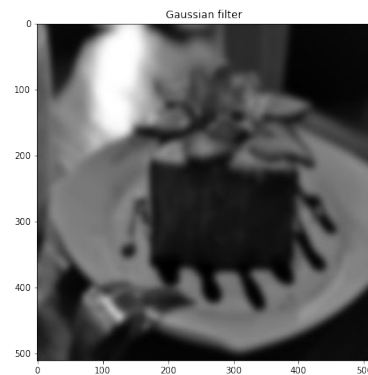
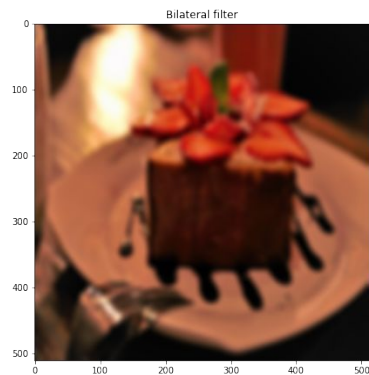
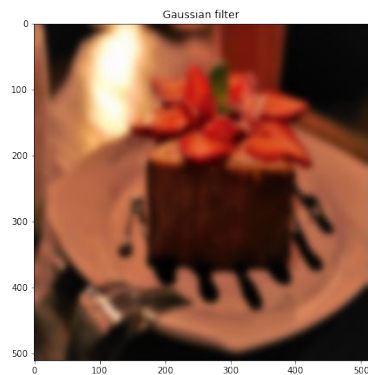


Gaussian vs Bilateral (frequency)



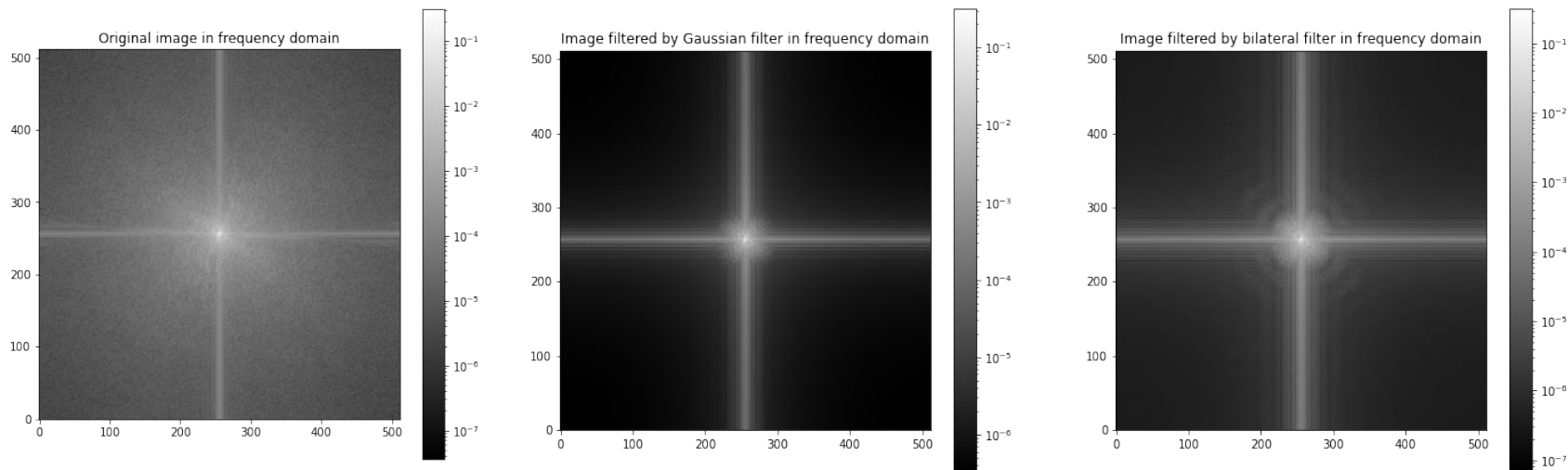


Gaussian vs Bilateral (large σ_r)



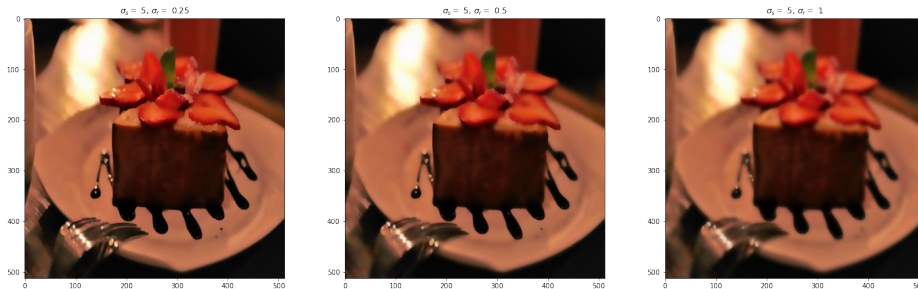


Gaussian vs Bilateral (large σ_r , frequency)

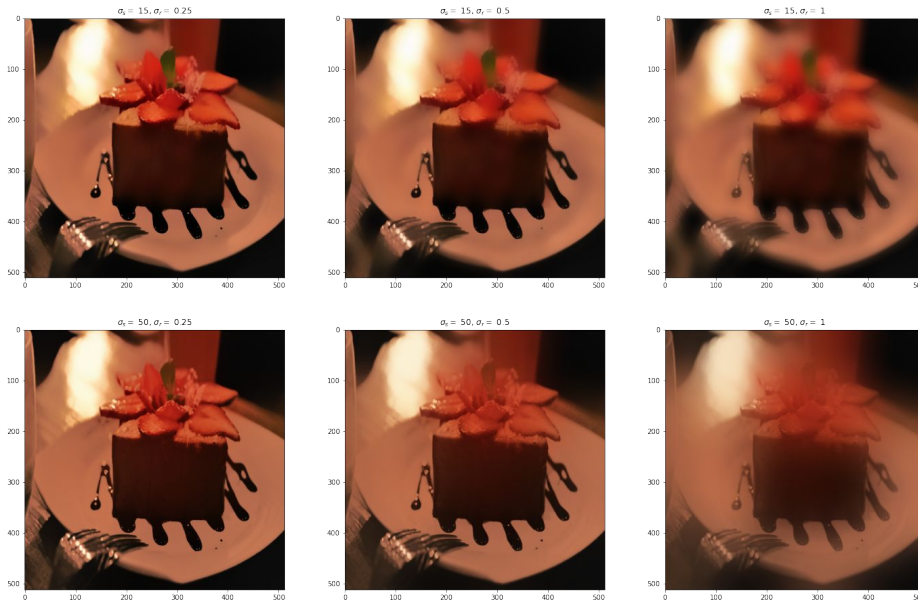


Hyperparameter Tuning

increasing σ_r →

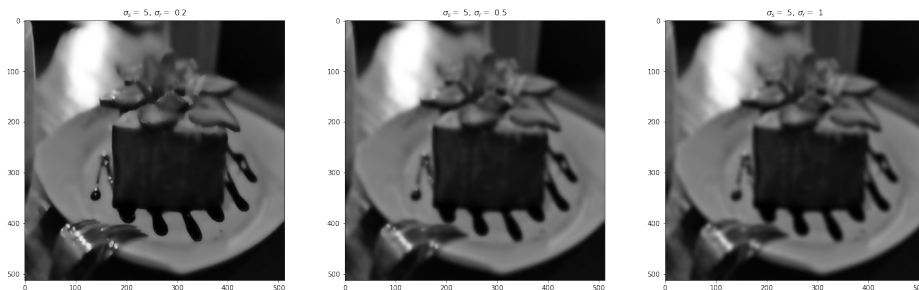


increasing σ_s ↓

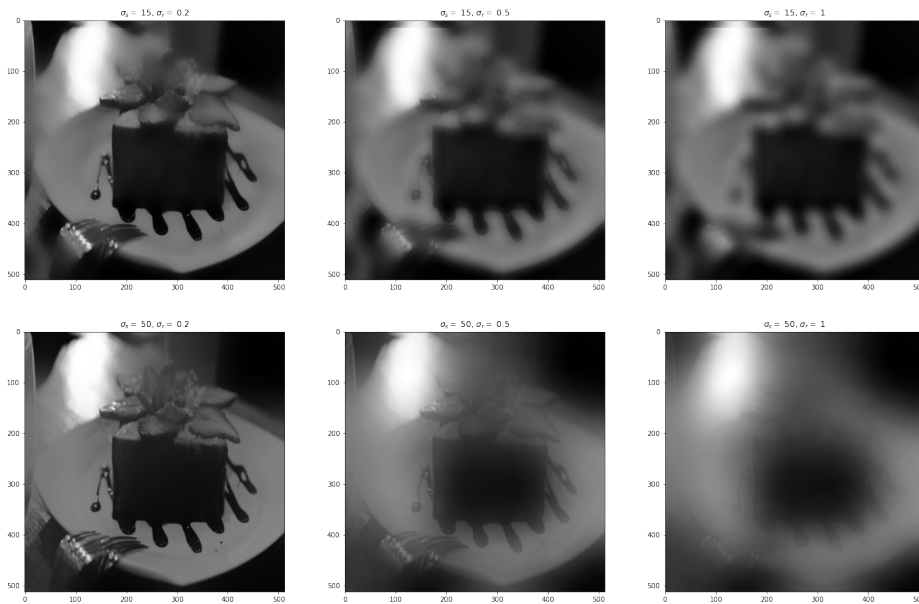


Hyperparameter Tuning

increasing σ_r →

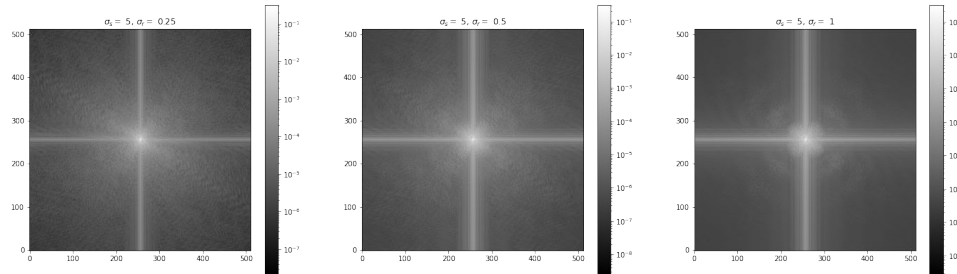


increasing σ_s ↓

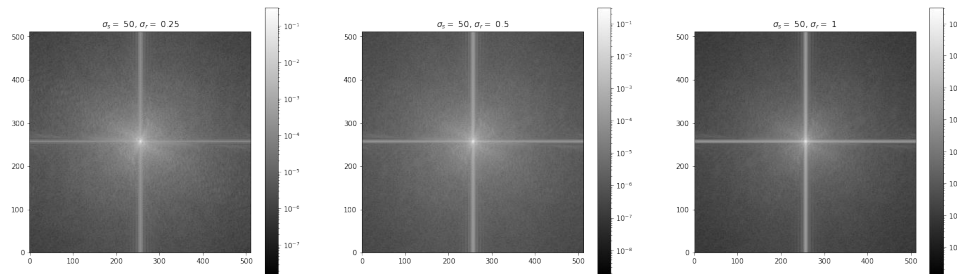
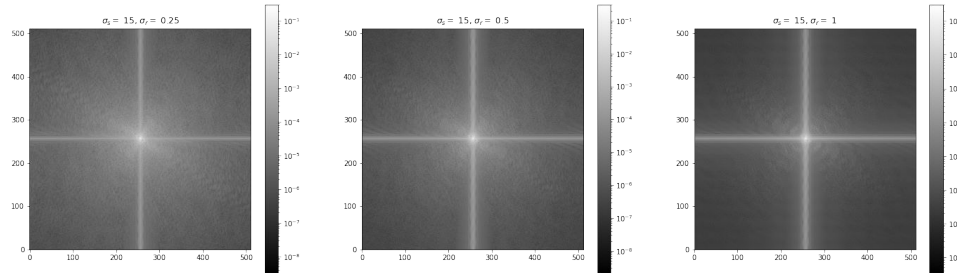


Hyperparameter Tuning

increasing σ_r →



increasing σ_s ↓





Reference

- [1] “A Gentle Introduction to Bilateral and Its Applications”. Sylvain Paris, Pierre Kornprobst, Jack Tumblin and Fredo Durand. ACM SIGGRAPH 2008.

- [2] Pierre Kornprobst, Jack Tumblin, and Frédo Durand. “Bilateral Filtering: Theory and Applications”. In: Foundations and Trends in Computer Graphics and Vision 4 (Jan. 2009), pp. 1–74