FSR

March 6, 2021

[9]:

```python
import matplotlib.pyplot as plt
import numpy as np
from os import listdir
from sklearn import linear_model
from sklearn.linear_model import lasso_path
from warnings import filterwarnings
filterwarnings('ignore')

np.random.seed(2019)

def extract_temp(file_name,col_ind):
    data_aux = np.loadtxt(file_name, usecols=range(10))
    data = data_aux[:,col_ind]
    err_count = 0
    ind_errs = []
    for ind in range(data.shape[0]):
        if data[ind] > 100 or data[ind] < -100:
            err_count = err_count + 1
            ind_errs.append(ind)
            data[ind] = data[ind-1]
    print("File name: " + file_name)
    print("Errors: " + str(err_count) + " Indices: " + str(ind_errs))
    return data

def create_data_matrix(str_path):
    file_name_list = listdir(str_path)
    file_name_list.sort()
    col_ind = 8 # 8 = last 5 minutes, 9 = average over the whole hour
    data_matrix = []
    ind = 0
    for file_name in file_name_list:
        if file_name[0] == '.':
            continue
        else:
            print("Station " + str(ind))
            ind = ind + 1
            data_aux = extract_temp(str_path + file_name,col_ind)
            if len(data_matrix) == 0:
                data_matrix = data_aux
            else:
                data_matrix = np.vstack((data_matrix,data_aux))
    return data_matrix.T

def FSR(X,y,num_features=20):
    '''
    X: feature matrix
    y: response
    return
        coeff: coefficient betta
    '''
    S = []
    r = y
    coeff = np.zeros(shape=X.shape[1])
    iteration = 1
    position = np.linspace(0, X.shape[1]-1, num = X.shape[1])
    while iteration <= num_features:
        i_val = np.empty(shape=len(position), dtype=float)
        for i in range(len(position)):
            i_val[i] = np.linalg.norm(X[:, i].T @ r)
        i_star = np.argmax(i_val)
        S.append(int(position[i_star]))
        position = np.delete(position, i_star)
        beta = np.linalg.inv(X[:, S].T @ X[:, S]) @ X[:, S].T @ y
        r = y - X[:, S] @ beta
        for elem in range(len(S)):
            coeff[S[elem]] = beta[elem]
        iteration += 1
    return coeff
```

[10]:

```
str_path = "./weather/"

load_files = False
if load_files:
    str_path_2015 = str_path + "hourly/2015/"
    data_matrix = create_data_matrix(str_path_2015)
    str_path_2016 = str_path + "hourly/2016/"
    data_matrix_2016 = create_data_matrix(str_path_2016)
else:
    data_matrix = np.load(str_path +"hourly_temperature_2015.npy")
    data_matrix_2016 = np.load(str_path +"hourly_temperature_2016.npy")

file_name_list = listdir(str_path + "hourly/2015/")
file_name_list.sort()
```

```
ind_response = 78 # 53 = Manhattan, 18 = Troy has 2 correlated features
# 23 = Williams dense linear model 30 = Death Valley 16 = AL_Talladega_10_NNE good for elastic net
# 78 = ND Jamestown also good for enet
print("Response is " + str(file_name_list[ind_response]))
y_raw = data_matrix[:,ind_response]
y_2016 = data_matrix_2016[:,ind_response]
ind_X = np.hstack((np.arange(0,ind_response),np.arange(ind_response+1,data_matrix.shape[1])))
X_raw = data_matrix[:,ind_X]
X_2016 = data_matrix_2016[:,ind_X]
n_features = X_raw.shape[1]

n_train_values = 40
n_test = int(1e3)
n_val = int(1e3)
n_train_max = data_matrix.shape[0] - n_test # 5e2
n_train_min = 133
n_train_list = np.around(np.logspace(np.log10(n_train_min),np.log10(n_train_max),n_train_values))
train_error_lasso_vec = []
val_error_lasso_vec = []
test_error_lasso_vec = []
test_2016_lasso_vec = []
train_error_ridge_vec = []
val_error_ridge_vec = []
test_error_ridge_vec = []
test_2016_ridge_vec = []

train_error_fsr_vec = []
val_error_fsr_vec = []
test_error_fsr_vec = []
test_2016_fsr_vec = []

coeffs_lasso_matrix = np.zeros((n_features,len(n_train_list)))
coeffs_ridge_matrix = np.zeros((n_features,len(n_train_list)))
coeffs_fsr_matrix = np.zeros((n_features,len(n_train_list)))
lambda_lasso_vec = []
lambda_ridge_vec = []
n_features_fsr_vec = []

n_lambda = 50
lambdas_ridge_aux = np.logspace(-5, 2, n_lambda)
lambdas_lasso = np.logspace(-5, 2, n_lambda)
eps = 1e-5  # the smaller it is the longer is the path
fixed_n = n_train_list[9]
```

```
i_m = 0
for n_train in n_train_list:
    lambdas_ridge = lambdas_ridge_aux * n_train
    aux_ind = np.random.permutation(range(data_matrix.shape[0]))
    ind_test = aux_ind[:n_test]
    ind_val = aux_ind[n_test:(n_test+n_val)]
    X_test = X_raw[ind_test,:]
    y_test = y_raw[ind_test]
    X_val = X_raw[ind_val,:]
    y_val = y_raw[ind_val]
    ind_train = aux_ind[(n_test+n_val):int(n_test+n_val+n_train)]
    X_train = X_raw[ind_train,:]
    y_train = y_raw[ind_train]

    center_vec = X_train.mean(axis=0)
    X_train_centered = X_train - center_vec
    col_norms = np.linalg.norm(X_train_centered, axis=0) / np.sqrt(n_train)
    X_train_norm = np.true_divide(X_train_centered, col_norms)
    X_test_centered = X_test - center_vec
    X_test_norm = np.true_divide(X_test_centered, col_norms)
    X_val_centered = X_val - center_vec
    X_val_norm = np.true_divide(X_val_centered, col_norms)
    X_2016_centered = X_2016 - center_vec
    X_2016_norm = np.true_divide(X_2016_centered, col_norms)
    y_train_center = y_train.mean()
    y_train_centered = y_train - y_train_center
    norm_y_train = np.linalg.norm(y_train_centered) / np.sqrt(n_train)
    y_train_norm = y_train_centered / norm_y_train

    print("Computing regularization path using the lasso...")
    lambdas_lasso, coeffs_lasso, _ = lasso_path(X_train_norm, y_train_norm,␣
 ↪eps,  normalize=False,
                                          alphas = lambdas_lasso,␣
 ↪fit_intercept=False)

    min_error_val = 1e4
    i_lambda = 0
    val_error_lasso_lambdas = np.zeros(n_lambda)
    train_error_lasso_lambdas = np.zeros(n_lambda)
    for i, coeffs_est in enumerate(coeffs_lasso.T):
        y_val_lasso = norm_y_train * np.dot(X_val_norm, coeffs_est) +␣
 ↪y_train_center
        error_val = np.linalg.norm(y_val - y_val_lasso) / np.sqrt(len(y_val))
        # error_val_vec.append(error_val)
        if error_val < min_error_val:
            min_error_val = error_val
            lambda_best_lasso = lambdas_lasso[i]
            coeffs_lasso_best = coeffs_est
        y_train_lasso = norm_y_train * np.dot(X_train_norm, coeffs_est) +␣
↪y_train_center
        train_error_lasso_lambdas[i] =  np.linalg.norm(y_train - y_train_lasso␣
↪) / np.sqrt(len(y_train))
        val_error_lasso_lambdas[i] = error_val
    lambda_lasso_vec.append(lambda_best_lasso)
    y_train_lasso = norm_y_train * np.dot(X_train_norm, coeffs_lasso_best) +␣
↪y_train_center
    y_test_lasso = norm_y_train * np.dot(X_test_norm, coeffs_lasso_best) +␣
↪y_train_center
    y_2016_lasso = norm_y_train * np.dot(X_2016_norm, coeffs_lasso_best) +␣
↪y_train_center
    train_error_lasso = np.linalg.norm(y_train - y_train_lasso) / np.
↪sqrt(len(y_train))
    test_error_lasso = np.linalg.norm(y_test - y_test_lasso) / np.
↪sqrt(len(y_test))
    test_2016_lasso = np.linalg.norm(y_2016 - y_2016_lasso) / np.
↪sqrt(len(y_2016))
    train_error_lasso_vec.append(train_error_lasso)
    val_error_lasso_vec.append(min_error_val)
    test_error_lasso_vec.append(test_error_lasso)
    test_2016_lasso_vec.append(test_2016_lasso)
    coeffs_lasso_matrix[:,i_m] = coeffs_lasso_best


    if n_train == fixed_n:
        coeffs_lasso_fixed = np.copy(coeffs_lasso)
        plt.figure()
        plt.plot(lambdas_lasso,coeffs_lasso.T)
        plt.plot(lambdas_lasso,val_error_lasso_lambdas/10.,marker='o')
        plt.plot(lambdas_lasso,train_error_lasso_lambdas/10.,marker='x')
        plt.xscale('log')
        plt.title('Lasso: n = ' + str(n_train))

        plt.figure(figsize=(8,6))
        plt.
↪plot(lambdas_lasso,train_error_lasso_lambdas,marker='o',linestyle='none',color='purple',lab
↪error')
        plt.
↪plot(lambdas_lasso,val_error_lasso_lambdas,marker='o',linestyle='none',color='red',label='V
↪error')
        plt.xlabel(r"Regularization parameter ($\lambda$)",fontsize=14)
        plt.ylabel('Average error (deg Celsius)',fontsize=14)
        plt.gcf().subplots_adjust(bottom=0.15)
        plt.xscale('log')
        plt.legend(loc = 'lower right',fontsize=14)


    # ridge regression
    clf = linear_model.Ridge(fit_intercept=False, normalize=False,)
    min_error_val = 1e3
    lambda_best = 0
    coeffs_ridge = np.zeros((n_features,n_lambda))
    val_error_ridge_lambdas = np.zeros(n_lambda)
    for ind_a,a in enumerate(lambdas_ridge):
        # print "lambda: " + str(a)
        clf.set_params(alpha=a)
        clf.fit(X_train_norm, y_train_norm)
        coeffs_ridge[:,ind_a] = clf.coef_
        y_val_ridge = norm_y_train * np.dot(X_val_norm, clf.coef_) +␣
↪y_train_center
        error_val = np.linalg.norm(y_val - y_val_ridge) / np.sqrt(len(y_val))
        val_error_ridge_lambdas[ind_a] = error_val
        # error_val_vec.append(error_val)
        if error_val < min_error_val:
            min_error_val = error_val
            lambda_best = a
            coeffs_ridge_best = clf.coef_
    lambda_ridge_vec.append(lambda_best)
    y_train_ridge = norm_y_train * np.dot(X_train_norm, coeffs_ridge_best) +␣
↪y_train_center
    y_test_ridge = norm_y_train * np.dot(X_test_norm, coeffs_ridge_best) +␣
↪y_train_center
    y_2016_ridge = norm_y_train * np.dot(X_2016_norm, coeffs_ridge_best) +␣
↪y_train_center
    train_error_ridge = np.linalg.norm(y_train - y_train_ridge) / np.
↪sqrt(len(y_train))
    test_error_ridge = np.linalg.norm(y_test - y_test_ridge) / np.
↪sqrt(len(y_test))
    test_2016_ridge = np.linalg.norm(y_2016 - y_2016_ridge) / np.
↪sqrt(len(y_2016))
    train_error_ridge_vec.append(train_error_ridge)
    val_error_ridge_vec.append(min_error_val)
    test_error_ridge_vec.append(test_error_ridge)
    test_2016_ridge_vec.append(test_2016_ridge)
    coeffs_ridge_matrix[:,i_m] = coeffs_ridge_best

    if n_train == fixed_n:
        coeffs_ridge_fixed = np.copy(coeffs_ridge)
        plt.figure(figsize=(8,6))
        # plt.plot(lambdas_ridge,val_error_lasso_lambdas/10.,marker='o')
        for ind in range(n_features):
            plt.plot(lambdas_ridge,coeffs_ridge[ind,:],color='skyblue')
            plt.xscale('log')
            plt.title('Ridge: n = ' + str(n_train))


    # FSR
    min_error_val = 1e3
    n_features_best_fsr = 0
    n_feature = X_train_norm.shape[1]
    coeffs_fsr = np.zeros((X_train_norm.shape[1],X_train_norm.shape[1]))
    val_error_fsr_lambdas = np.zeros(n_feature)
    train_error_fsr_lambdas = np.zeros(n_feature)
    for i,a in enumerate(range(1,n_feature)):
        coeffs_fsr = FSR(X_train_norm,y_train_norm,num_features=a)
        y_train_fsr = norm_y_train * np.dot(X_train_norm, coeffs_fsr) +␣
↪y_train_center
        y_val_fsr = norm_y_train * np.dot(X_val_norm, coeffs_fsr) +␣
↪y_train_center
        error_val = np.linalg.norm(y_val - y_val_fsr) / np.sqrt(len(y_val))
    #         error_val_vec.append(error_val)
        if error_val < min_error_val:
            min_error_val = error_val
            n_features_best_fsr = a
            coeffs_fsr_best = coeffs_est

        train_error_fsr_lambdas[i] =  np.linalg.norm(y_train - y_train_fsr ) /␣
↪np.sqrt(len(y_train))
        val_error_fsr_lambdas[i] = error_val
    n_features_fsr_vec.append(n_features_best_fsr)

    y_train_fsr = norm_y_train * np.dot(X_train_norm, coeffs_fsr_best) +␣
↪y_train_center

    y_train_fsr = norm_y_train * np.dot(X_train_norm, coeffs_fsr_best) +␣
↪y_train_center
    y_test_fsr = norm_y_train * np.dot(X_test_norm, coeffs_fsr_best) +␣
↪y_train_center
    y_2016_fsr = norm_y_train * np.dot(X_2016_norm, coeffs_fsr_best) +␣
↪y_train_center
    train_error_fsr = np.linalg.norm(y_train - y_train_fsr) / np.
↪sqrt(len(y_train))
    test_error_fsr = np.linalg.norm(y_test - y_test_fsr) / np.sqrt(len(y_test))
    test_2016_fsr = np.linalg.norm(y_2016 - y_2016_fsr) / np.sqrt(len(y_2016))
    train_error_fsr_vec.append(train_error_fsr)
    val_error_fsr_vec.append(min_error_val)
    test_error_fsr_vec.append(test_error_fsr)
    test_2016_fsr_vec.append(test_2016_fsr)

    coeffs_fsr_matrix[:,i_m] = coeffs_fsr_best

    if n_train == fixed_n:
        plt.figure(figsize=(8,6))
        plt.
↪plot(range(len(train_error_fsr_lambdas)),train_error_fsr_lambdas,marker='o',linestyle='none
↪error')
        plt.
↪plot(range(len(val_error_fsr_lambdas)),val_error_fsr_lambdas,marker='o',linestyle='none',co
↪error')
        plt.xlabel(r"Number of features",fontsize=14)
        plt.ylabel('Average error (deg Celsius)',fontsize=14)
        plt.gcf().subplots_adjust(bottom=0.15)
        plt.legend(loc = 'lower right',fontsize=14)

    i_m = i_m + 1
```

```
Response is CRNH0203-2015-ND_Medora_7_E.txt
Computing regularization path using the lasso…
Computing regularization path using the lasso…
Computing regularization path using the lasso…
Computing regularization path using the lasso…
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Computing regularization path using the lasso…
```
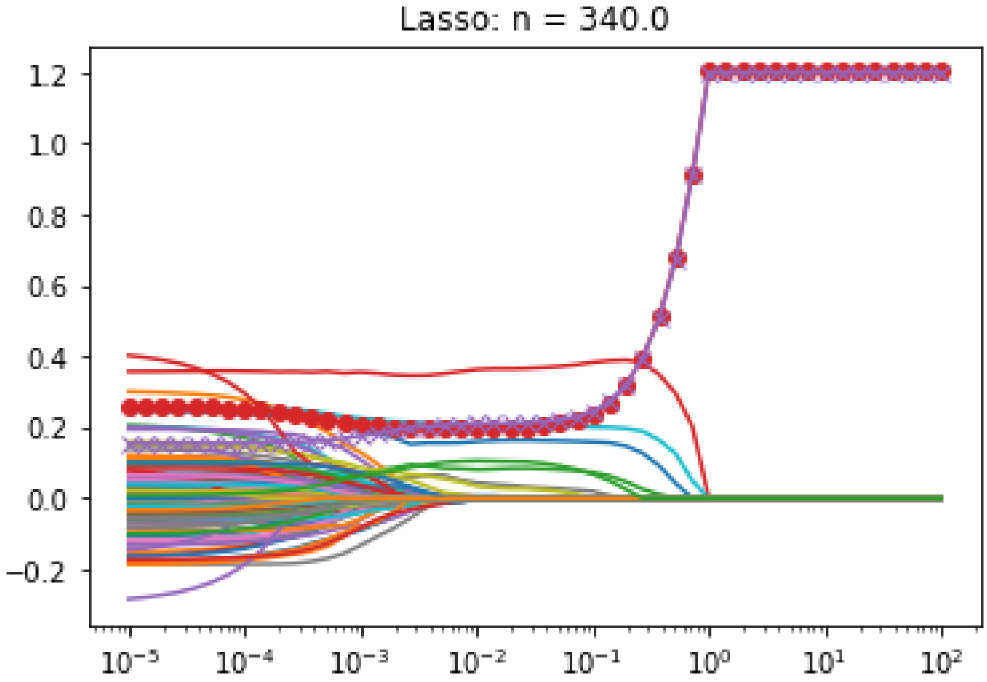
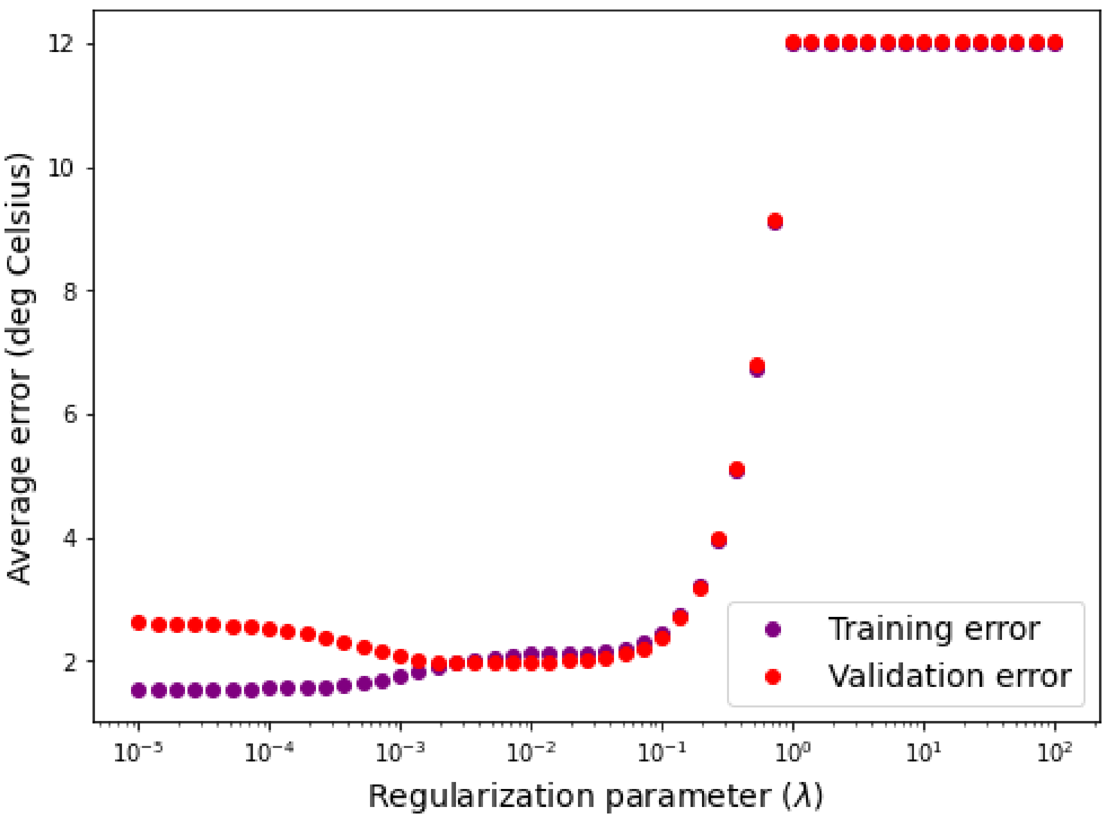
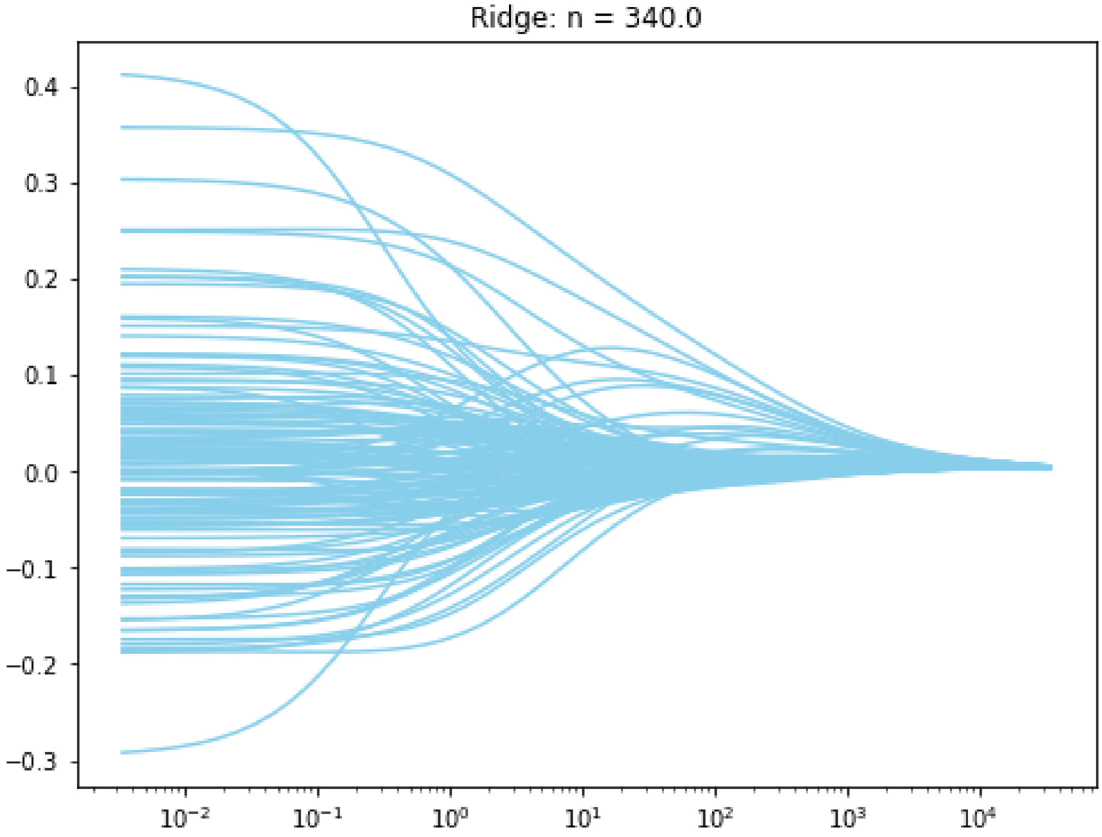
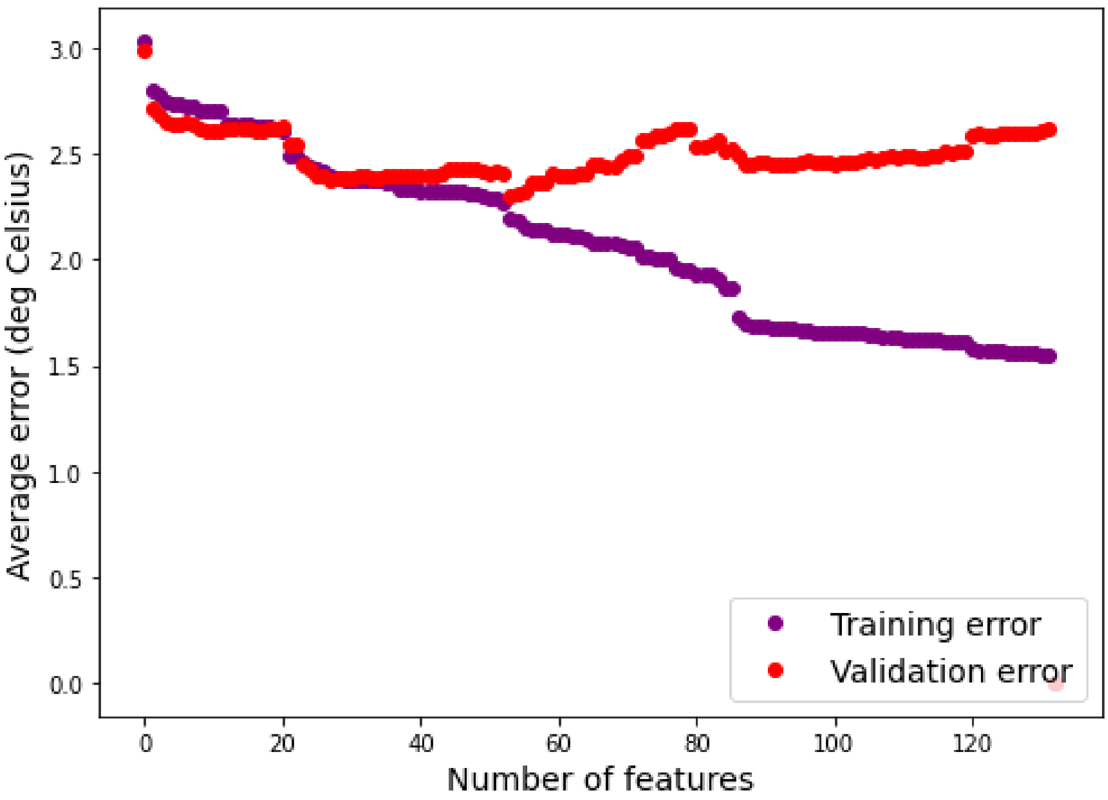




12
10
8
6
4
2
Average error (deg Celsius)
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Regularization parameter (λ)
● Training error
● Validation error

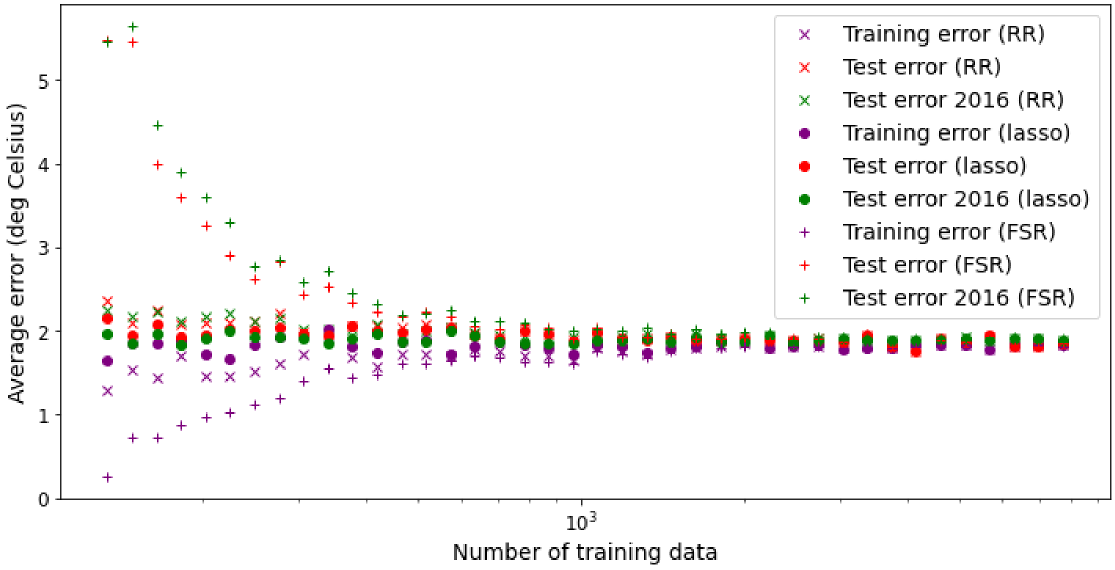

Ridge: n = 340.0
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}

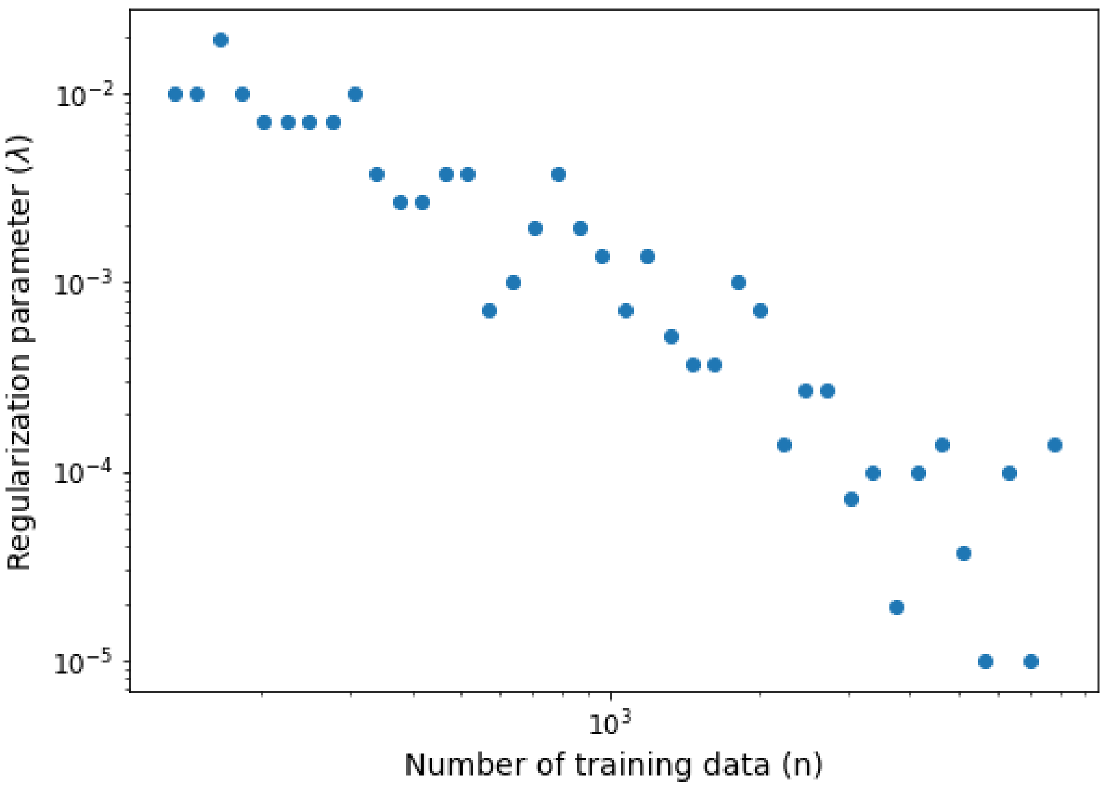


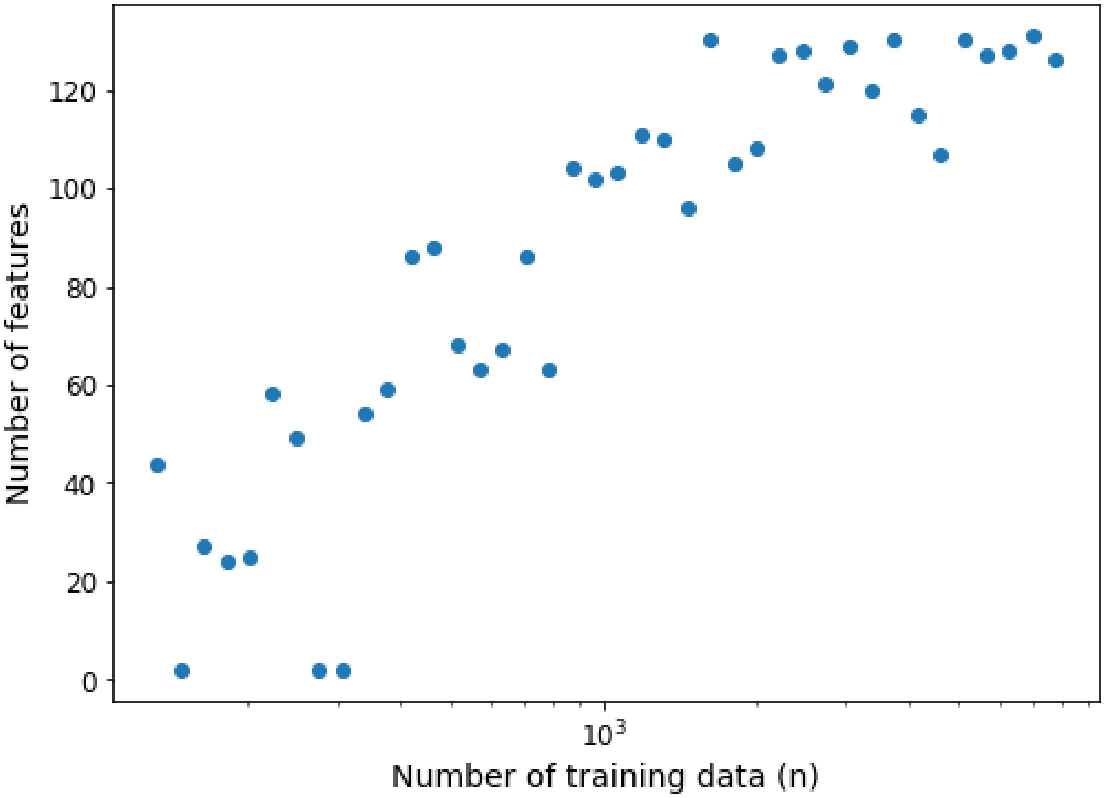
```
[11]: # xticks=(np.array([200,500,1000,2000,5000]))
      # xtick_labels = ('200','500','1000','2000','5000')
      plt.figure(figsize=(12,6))
      linstyle = 'None'
      plt.
       ↪plot(n_train_list,train_error_ridge_vec,linestyle=linstyle,marker='x',color='purple',label=
       ↪error (RR)")
      plt.
       ↪plot(n_train_list,test_error_ridge_vec,linestyle=linstyle,marker='x',color='red',label="Tes
       ↪error (RR)")
      plt.
       ↪plot(n_train_list,test_2016_ridge_vec,linestyle=linstyle,marker='x',color='green',label="Te
       ↪error 2016 (RR)")
      plt.
       ↪plot(n_train_list,train_error_lasso_vec,linestyle=linstyle,marker='o',color='purple',label=
       ↪error (lasso)")
      plt.
       ↪plot(n_train_list,test_error_lasso_vec,linestyle=linstyle,marker='o',color='red',label="Tes
       ↪error (lasso)")
```

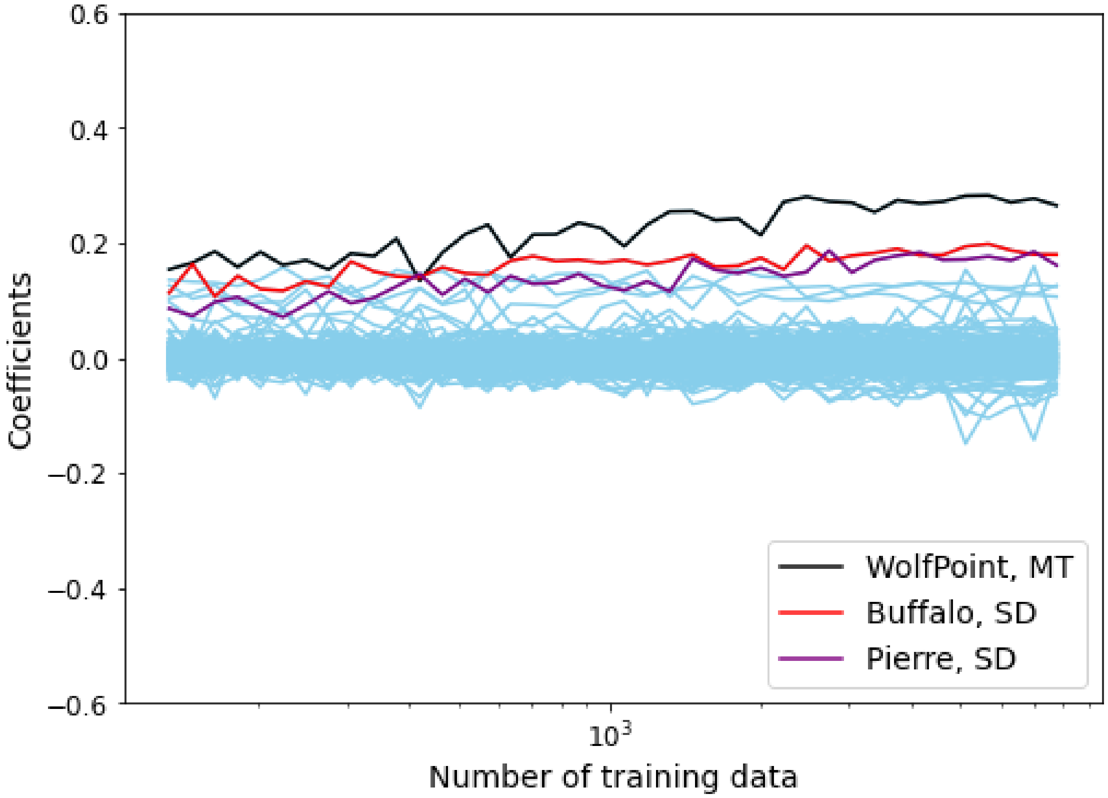
```
plt.
 ↪plot(n_train_list,test_2016_lasso_vec,linestyle=linstyle,marker='o',color='green',label="Te
 ↪error 2016 (lasso)")
plt.
 ↪plot(n_train_list,train_error_fsr_vec,linestyle=linstyle,marker='+',color='purple',label="T
 ↪error (FSR)")
plt.
 ↪plot(n_train_list,test_error_fsr_vec,linestyle=linstyle,marker='+',color='red',label="Test␣
 ↪error (FSR)")
plt.
 ↪plot(n_train_list,test_2016_fsr_vec,linestyle=linstyle,marker='+',color='green',label="Test
 ↪error 2016 (FSR)")
# plt.ylim((-0.5,7))
plt.xscale('log')
plt.xlabel('Number of training data',fontsize=14)
plt.ylabel('Average error (deg Celsius)',fontsize=14)
#plt.xticks(xticks,xtick_labels)
plt.tick_params(labelsize=12)
plt.legend(fontsize=14)
plt.gcf().subplots_adjust(bottom=0.15)

plt.figure(figsize=(8,6))
plt.plot(n_train_list,lambda_lasso_vec,marker='o',linestyle='none')
plt.xscale('log')
plt.yscale('log')
plt.ylabel(r"Regularization parameter ($\lambda$)",fontsize=14)
plt.xlabel('Number of training data (n)',fontsize=14)
# plt.xticks(xticks,xtick_labels)
plt.tick_params(labelsize=12)
# plt.gcf().subplots_adjust(left=0.2)
plt.gcf().subplots_adjust(bottom=0.15)


plt.figure(figsize=(8,6))
plt.plot(n_train_list,n_features_fsr_vec,marker='o',linestyle='none')
plt.xscale('log')
#plt.yscale('log')
plt.ylabel(r"Number of features",fontsize=14)
plt.xlabel('Number of training data (n)',fontsize=14)
# plt.xticks(xticks,xtick_labels)
plt.tick_params(labelsize=12)
# plt.gcf().subplots_adjust(left=0.2)
plt.gcf().subplots_adjust(bottom=0.15)


n_largest = 3
color_list = ['black','red','purple','green','orange']
```

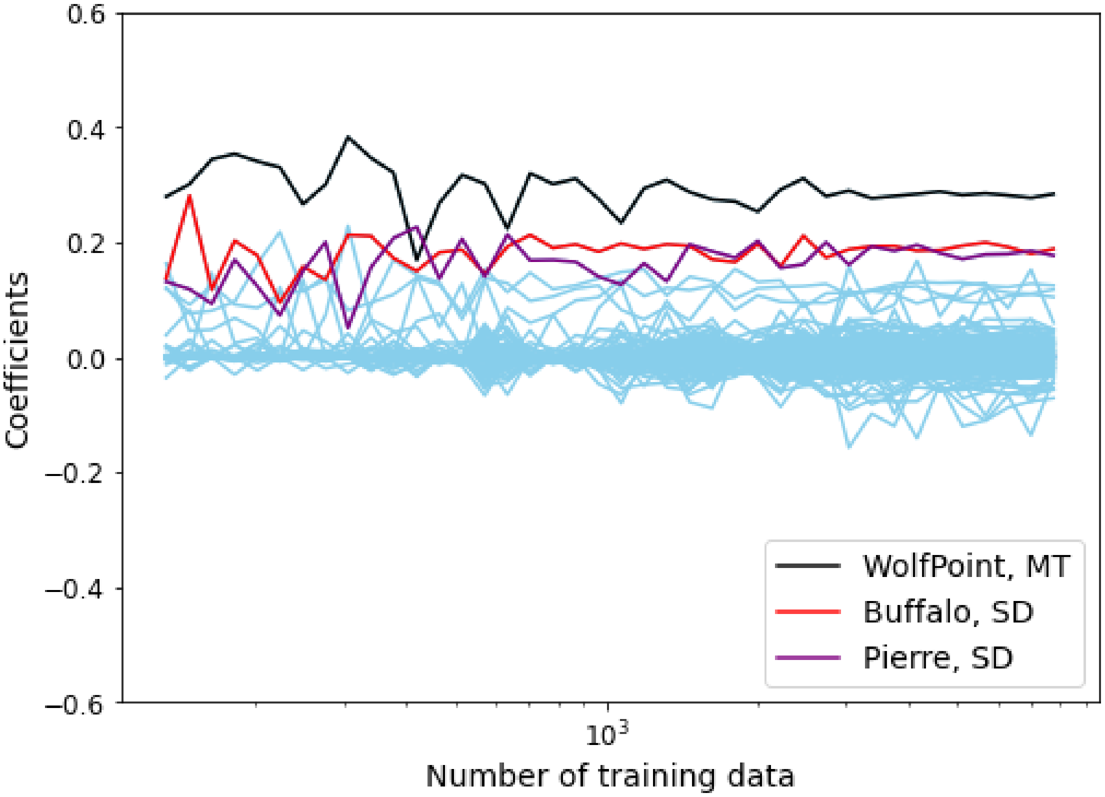
```
sorted_coeffs = np.argsort(np.abs(coeffs_ridge_matrix[:,-1]))
largest_coeffs = sorted_coeffs[-1:(-n_largest-1):-1]
plt.figure(figsize=(8,6))
for ind in range(n_features):
    plt.plot(n_train_list,coeffs_ridge_matrix[ind,:],color='skyblue')
for ind in range(len(largest_coeffs)):
    ind_name = largest_coeffs[ind]
    if ind_name>=ind_response:
        ind_name = ind_name + 1
    aux_name = file_name_list[ind_name]
    aux_name = aux_name[14:]
    aux_name = aux_name[:-7]
    table = str.maketrans(dict.fromkeys('_0123456789'))
    aux_name = aux_name.translate(table)
    aux_name = aux_name[2:] + ", " + aux_name[:2]
    plt.plot(n_train_list,coeffs_ridge_matrix[largest_coeffs[ind],:
 ↪],color=color_list[ind],label=aux_name)
plt.ylim((-0.6,0.6))
plt.xscale('log')
plt.xlabel('Number of training data',fontsize=14)
plt.ylabel('Coefficients',fontsize=14)
# plt.xticks(xticks,xtick_labels)
plt.tick_params(labelsize=12)
# plt.gcf().subplots_adjust(left=0.2)
plt.gcf().subplots_adjust(bottom=0.15)
#plt.legend(loc = 'upper right',bbox_to_anchor=(1.3, 1.))
plt.legend(loc = 'lower right',fontsize=14)
aux_name = file_name_list[ind_response]
aux_name = aux_name[14:]
aux_name = aux_name[:-7]
table = str.maketrans(dict.fromkeys('_0123456789'))
aux_name = aux_name.translate(table)
aux_name = aux_name[2:] + ", " + aux_name[:2]
# plt.title(aux_name)

sorted_coeffs = np.argsort(np.abs(coeffs_lasso_matrix[:,-1]))
largest_coeffs = sorted_coeffs[-1:(-n_largest-1):-1]
plt.figure(figsize=(8,6))
for ind in range(n_features):
    plt.plot(n_train_list,coeffs_lasso_matrix[ind,:],color='skyblue')
for ind in range(len(largest_coeffs)):
    ind_name = largest_coeffs[ind]
    if ind_name>=ind_response:
        ind_name = ind_name + 1
    aux_name = file_name_list[ind_name]
    aux_name = aux_name[14:]
    aux_name = aux_name[:-7]
    table = str.maketrans(dict.fromkeys('_0123456789'))
    aux_name = aux_name.translate(table)
    aux_name = aux_name[2:] + ", " + aux_name[:2]
    plt.plot(n_train_list,coeffs_lasso_matrix[largest_coeffs[ind],:
 ↪],color=color_list[ind],label=aux_name)
plt.ylim((-0.6,0.6))
plt.xscale('log')
plt.xlabel('Number of training data',fontsize=14)
plt.ylabel('Coefficients',fontsize=14)
# plt.xticks(xticks,xtick_labels)
plt.tick_params(labelsize=12)
# plt.gcf().subplots_adjust(left=0.2)
plt.gcf().subplots_adjust(bottom=0.15)
#plt.legend(loc = 'upper right',bbox_to_anchor=(1.3, 1.))
plt.legend(loc = 'lower right',fontsize=14)
aux_name = file_name_list[ind_response]
aux_name = aux_name[14:]
aux_name = aux_name[:-7]
table = str.maketrans(dict.fromkeys('_0123456789'))
aux_name = aux_name.translate(table)
aux_name = aux_name[2:] + ", " + aux_name[:2]
# plt.title(aux_name)

plt.figure(figsize=(8,6))
for ind in range(n_features):
    plt.plot(lambdas_ridge,coeffs_ridge_fixed[ind,:],color='skyblue')
    plt.xscale('log')
for ind in range(len(largest_coeffs)):
    ind_name = largest_coeffs[ind]
    if ind_name>=ind_response:
        ind_name = ind_name + 1
    aux_name = file_name_list[ind_name]
    aux_name = aux_name[14:]
    aux_name = aux_name[:-7]
    table = str.maketrans(dict.fromkeys('_0123456789'))
    aux_name = aux_name.translate(table)
    aux_name = aux_name[2:] + ", " + aux_name[:2]
    plt.plot(lambdas_ridge,coeffs_ridge_fixed[largest_coeffs[ind],:
 ↪],color=color_list[ind],label=aux_name)
plt.xlabel(r"Regularization parameter ($\lambda/n$)",fontsize=14)
plt.ylabel('Coefficients',fontsize=14)
plt.gcf().subplots_adjust(bottom=0.15)
plt.legend(loc = 'upper right',fontsize=14)
plt.xscale('log')

plt.figure(figsize=(8,6))
for ind in range(n_features):
    plt.plot(lambdas_lasso,coeffs_lasso_fixed[ind,:],color='skyblue')
    plt.xscale('log')
for ind in range(len(largest_coeffs)):
    ind_name = largest_coeffs[ind]
    if ind_name>=ind_response:
        ind_name = ind_name + 1
    aux_name = file_name_list[ind_name]
    aux_name = aux_name[14:]
    aux_name = aux_name[:-7]
    table = str.maketrans(dict.fromkeys('_0123456789'))
    aux_name = aux_name.translate(table)
    aux_name = aux_name[2:] + ", " + aux_name[:2]
    plt.plot(lambdas_lasso,coeffs_lasso_fixed[largest_coeffs[ind],:
 ↪],color=color_list[ind],label=aux_name)
plt.xlabel(r"Regularization parameter ($\lambda/n$)",fontsize=14)
plt.ylabel('Coefficients',fontsize=14)
plt.gcf().subplots_adjust(bottom=0.15)
plt.legend(loc = 'upper right',fontsize=14)
plt.xscale('log')
```

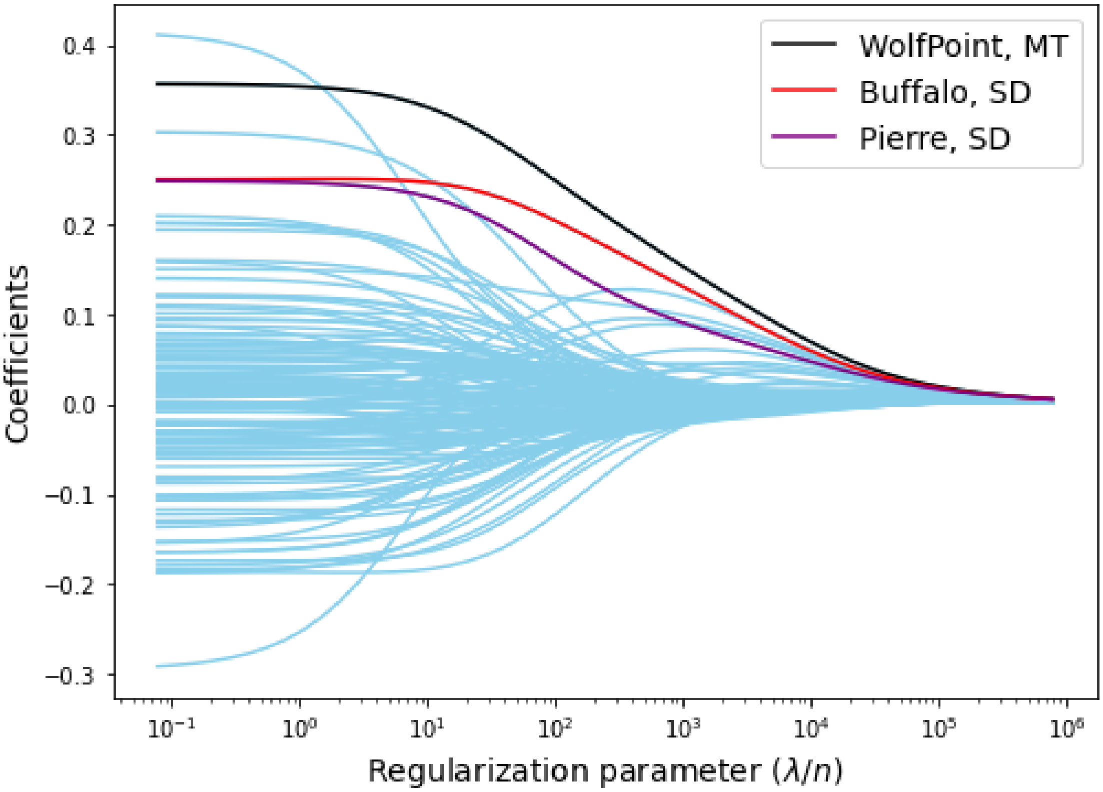


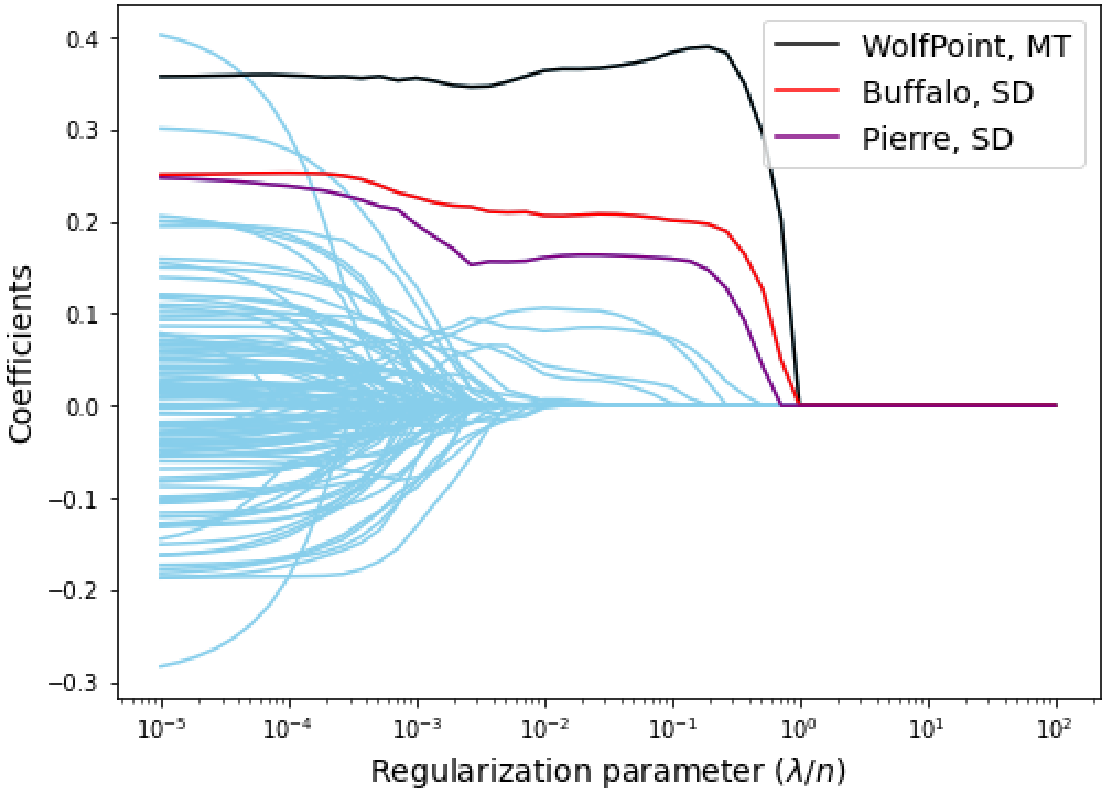

Regularization parameter (λ)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{3}
Number of training data (n)


120
100
80
60
40
20
0
Number of features
10^3
Number of training data (n)


0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Coefficients
10^3
Number of training data
WolfPoint, MT
Buffalo, SD
Pierre, SD


0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
Coefficients
10^3
Number of training data
WolfPoint, MT
Buffalo, SD
Pierre, SD


WolfPoint, MT
Buffalo, SD
Pierre, SD
Coefficients
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
Regularization parameter (λ/n)



WolfPoint, MT
Buffalo, SD
Pierre, SD
Coefficients
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Regularization parameter (λ/n)

```
[ ]:
```