Bilateral Filtering

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DS-GA 1013 Spring 2021

1 Motivation

Bilateral filtering is a nonlinear image smoothing technique which replaces a pixel by the weighted average of its neighbors determined by parameters on both spacial and intensity (or color) differences. This technique first appeared as an extension to Gaussian filtering which introduced the idea of local pixel averaging. Gaussian filtering uses the weighted average of the intensity of the nearby positions. The weight is defined by the Gaussian distribution in a chosen window, thereby gives the name to the technique. Notably, Gaussian filter only depends on the spatial distances among pixels, but does not account for their intensity values. This can lead to blurred edges because pixels that have large differences and discontinuities can be averaged together. To preserve edge sharpness, bilateral filtering was then developed by adding the differences in intensity values into the original Gaussian filtering calculation. Bilateral filtering not only preserves the strong smoothing ability of Gaussian filtering, but also improves edge details and other aspects of the manipulated image. Moreover, studies later on, for example [1], have taken the advantage of the straightforward and flexible form of bilateral filtering to develop further applications in computational photography including denoising, tone management, depth reconstruction and style transfer.
In this report, section 2 introduces Gaussian filtering and continues on the definition of Bilateral filtering. Section 3 shows typical applications of bilateral filtering, and compare its results to that of Gaussian filtering on selected images. Section 4 demonstrates major hyperparameter tuning effects.

2 Bilateral Filtering

Definition 2.1 (Gaussian Filtering for Gray-Level Images). *The Gaussian filter for a gray-level image is given by*

$$GC[I]_p = \sum_{q \in S} G_\sigma(||p-q||) I_q$$

where I is the intensity of a pixel, p is the center position, q is the pixel of interest, S is the set of pixel position, σ defines the neighborhood size and $G_\sigma(x)$ denotes the 2D Gaussian kernel

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}} exp(\frac{-x^2}{2\sigma^2})$$

Gaussian filter substitutes a pixel by the weighted average of its neighborhood determined by Gaussian distribution. Its strength only depends on the size parameter σ, and the weight of each neighbor decreases only with increasing distance to the central position. Bilateral filtering keeps the same idea of averaging nearby pixels; however, it requires the neighbors to have not only close spatial locations, but also similar intensity range.
Definition 2.2 (Bilateral Filtering for Gray-Level Image). *The Bilateral filter for a gray-level image is given by*

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_S}(||p-q||) G_{\sigma_r}(|I_p - I_q|) I_q$$

where the normalization factor W_p ensures pixel weights sum to 1 and has the form

$$W_p = \sum_{q \in S} G_{\sigma_S}(||p-q||) G_{\sigma_r}(|I_p - I_q|)$$

σ_S is the space weight and σ_r is the range weight.
The space and range weight parameters specify the filter threshold and strength. Generally, as σ_r increases, the range Gaussian G_{σ_r} widens and flattens, and thus becomes closer to constant over the intensity interval of the image. This makes the bilateral filter close to a Gaussian filter when σ_r is very large. As σ_S increases,

the filter smooths over larger areas. The weight decreases as the spatial distance increases and the difference in intensity increases from the window center. The following section will illustrate the effects of σ_r and σ_s with image examples.

The equation of bilateral filtering for color images has the same form with the gray-level one above. Their only difference is the scalar intensity values are substituted by 3D vector for colors.

Definition 2.3 (Bilateral Filtering for Color Image). *The Bilateral filter for a color image is given by*

$$BF[I]_p = \frac{1}{W_p} \sum_{q \in S} G_{\sigma_S}(||p - q||) G_{\sigma_r}(||C_p - C_q||) C_q$$

where C represents vector of colors on a pixel.

3 Application

To illustrate the difference between Gaussian filter and bilateral filter, we applied both filters to RGB and gray-scale of a 512×512 image. The pixel values for both RGB and gray-scale images are within [0, 1]. Figure 1 shows the result of applying a Gaussian filter with $\sigma = 5$ and a bilateral filter with $\sigma_r = 0.5, \sigma_S = 5$ to the RGB image. Figure 2 shows the result of applying the same filters to a gray-scale image. Each figure contains subplots of the original image, the filtered image and the image of the absolute difference of pixel values between original and filtered image. The difference image demonstrates what the filter changes in the original image. The effect of Gaussian filter is best described as blurring the whole image; however, the bilateral filter removes some texture on surfaces in the image, e.g. the top and side of the chocolate cake, but keeps most of the edges in the image intact, e.g. fork, chocolate puree, and strawberries. This is the reason Adobe Photoshop implements a modified version of the bilateral filter in its "surface blur" tool.

In the frequency domain, we can also see the difference between Gaussian filter and bilateral filter on a gray-scale image, as shown in figure 3. Gaussian filter retains only the peak frequencies (the bright part in the frequency frequency image) and removes other frequencies (the dark part in the frequency image). This results in a blurry image after filtering. Bilateral filter also retains the peak frequencies and certainly removes some other frequencies (there are more dark regions in the frequency image than the original one); however, bilateral filter does not remove all other frequencies, which leads to preservation of some textures of the image.

As mentioned in the previous section that the bilateral filter acts similarly to Gaussian filter when σ_r is large. Figure 4 and 5 show the result of applying a Gaussian filter with $\sigma = 5$ and a bilateral filter with $\sigma_r = 5, \sigma_S = 5$ to the RGB image. Since the pixel values are all between 0 and 1, $\sigma_r = 5$ can be considered as a large value for the range weight. We can see that the filtered images and the difference images are similar. Meanwhile, in the frequency domain, figure 6 shows that both filters act similarly while bilateral filter does not remove large scale structures as much as the Gaussian filter.

The brute-force implementation of bilateral filtering has the complexity of $O(|S|^2)$ where $|S|$ equals the number of pixels, and it can be slow for large images. Many faster algorithms reaches logarithmic or linear time complexity with negligible visual differences. An example technique developed by [2] approximates the bilateral by filtering downsampled copies of the image with discrete intensity kernels, and reconstructing the results by linear interpolation. Other methods includes layered approximation, seperable kernels, for example in [1], and so on. These more efficient methods were proved to have neglegible visual differences.

4 Hyperparameter Tuning

To explore the effects of σ_r and σ_S on bilateral filter, we applied bilateral filters with various configurations to the same RGB and grayscale image. The results are illustrated in figure 7 and figure 8. The result in the frequency domain for grayscale image is shown in figure 9. In general, for a fixed σ_r, increasing σ_S leads to more surface blurring in the image. In the case of RGB image, this could also result in changes in colors when pixels with similar colors are close, e.g. the green leaf among the strawberries. For a fixed σ_S, increasing σ_r leads to more global blurring in the image. This has similar effect of applying a Gaussian filter with a large σ.

References

[1] Pierre Kornprobst, Jack Tumblin, and Frédo Durand. “Bilateral Filtering: Theory and Applications”. In: *Foundations and Trends in Computer Graphics and Vision* 4 (Jan. 2009), pp. 1–74.

[2] Sylvain Paris and Frédo Durand. “A Fast Approximation of the Bilateral Filter Using a Signal Processing Approach”. In: *Computer Vision – ECCV 2006*. Ed. by Aleš Leonardis, Horst Bischof, and Axel Pinz. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006, pp. 568–580.

A Bilateral Filter Experiments

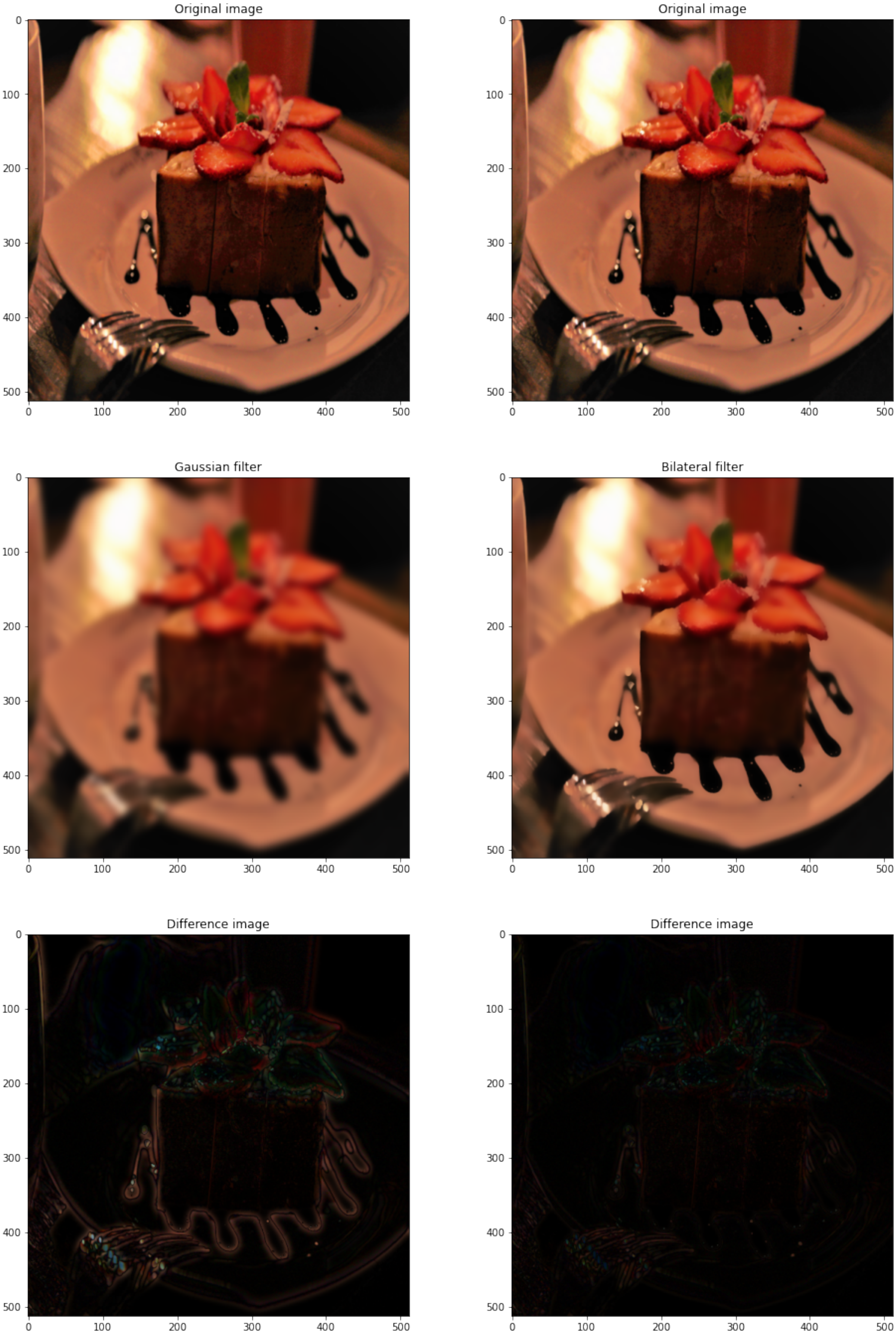

Figure 1: Bilateral vs Gaussian filter for RGB image

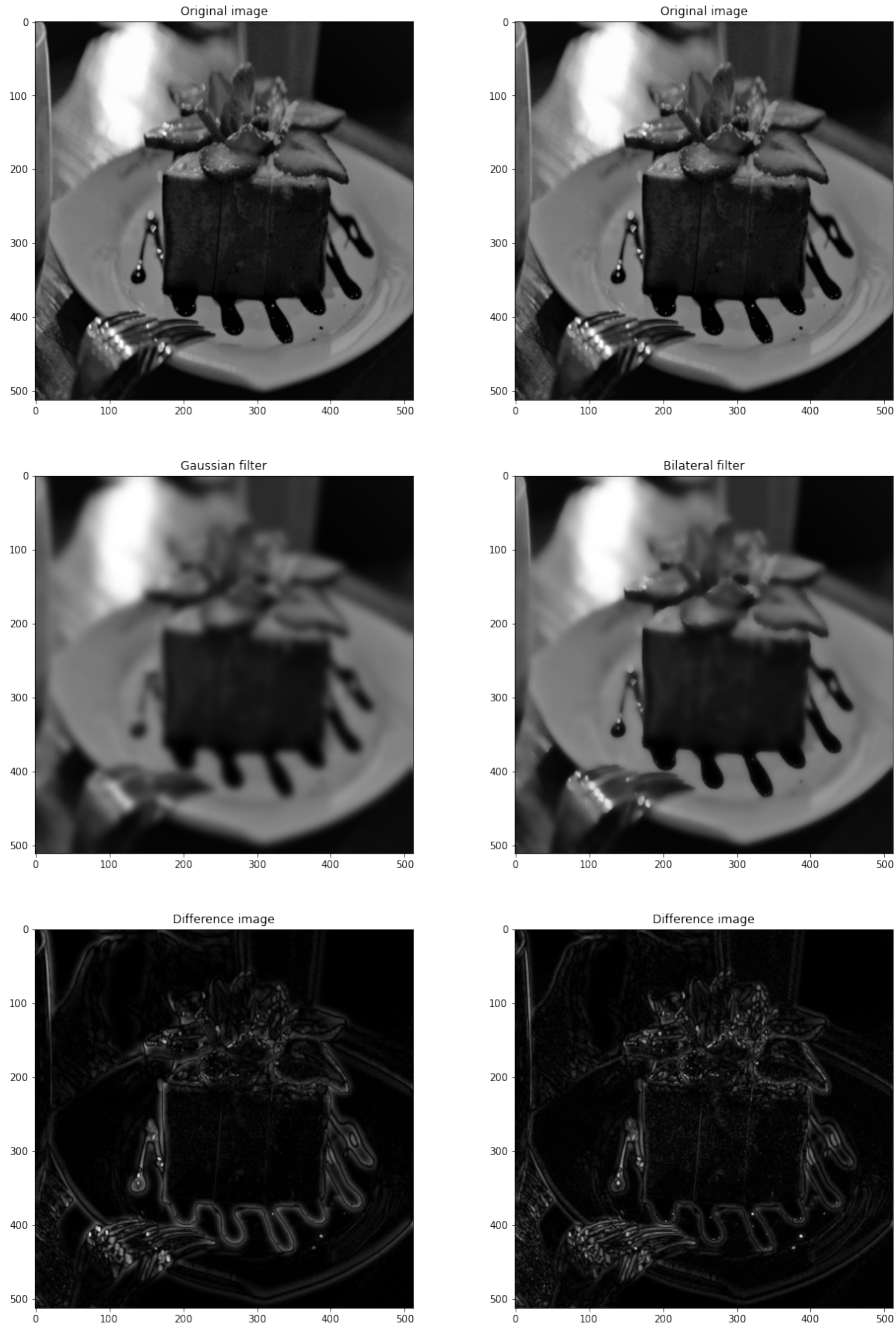

Figure 2: Bilateral vs Gaussian filter for grayscale image

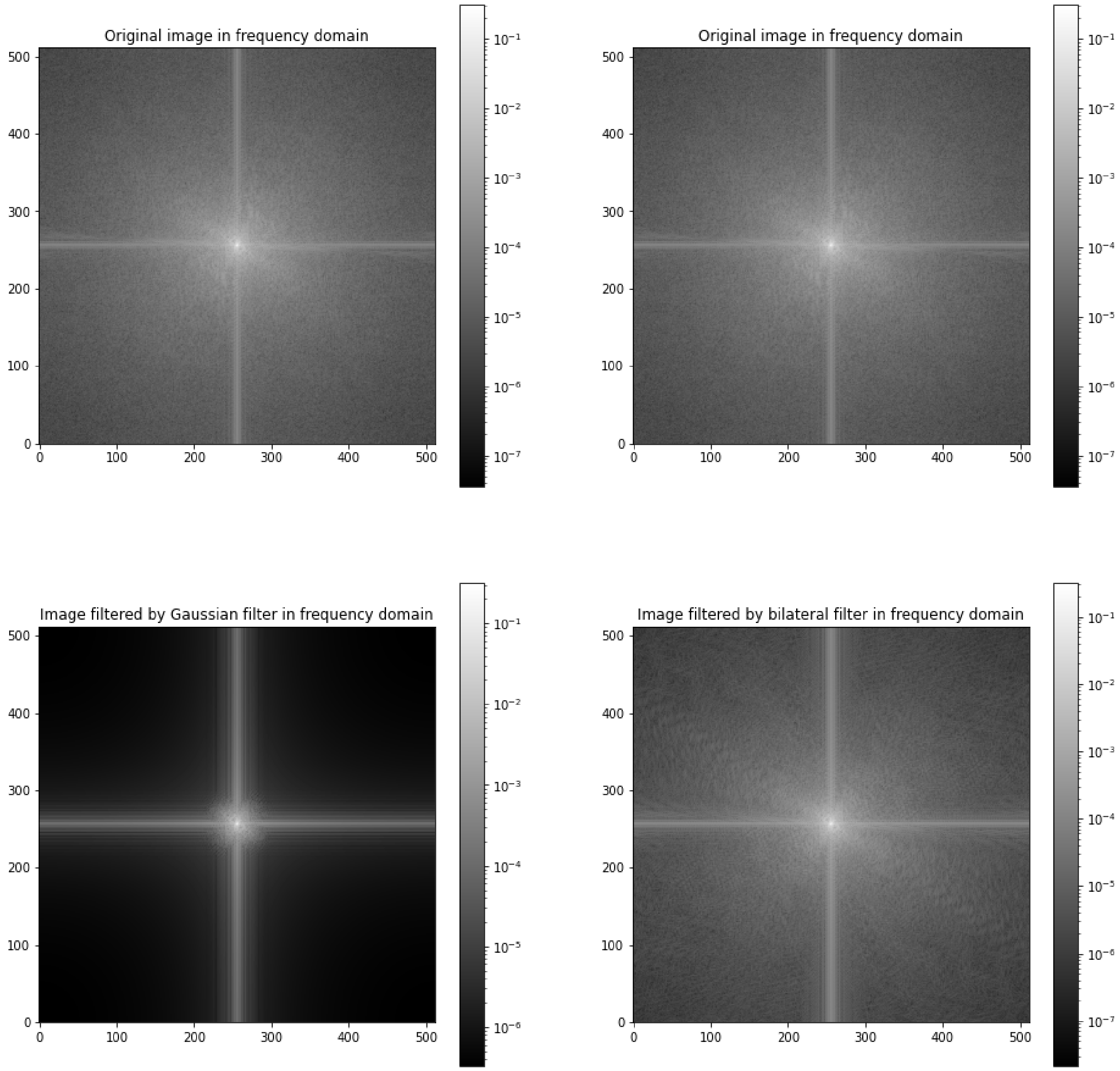

Figure 3: Bilateral vs Gaussian filter for grayscale image in the frequency domain

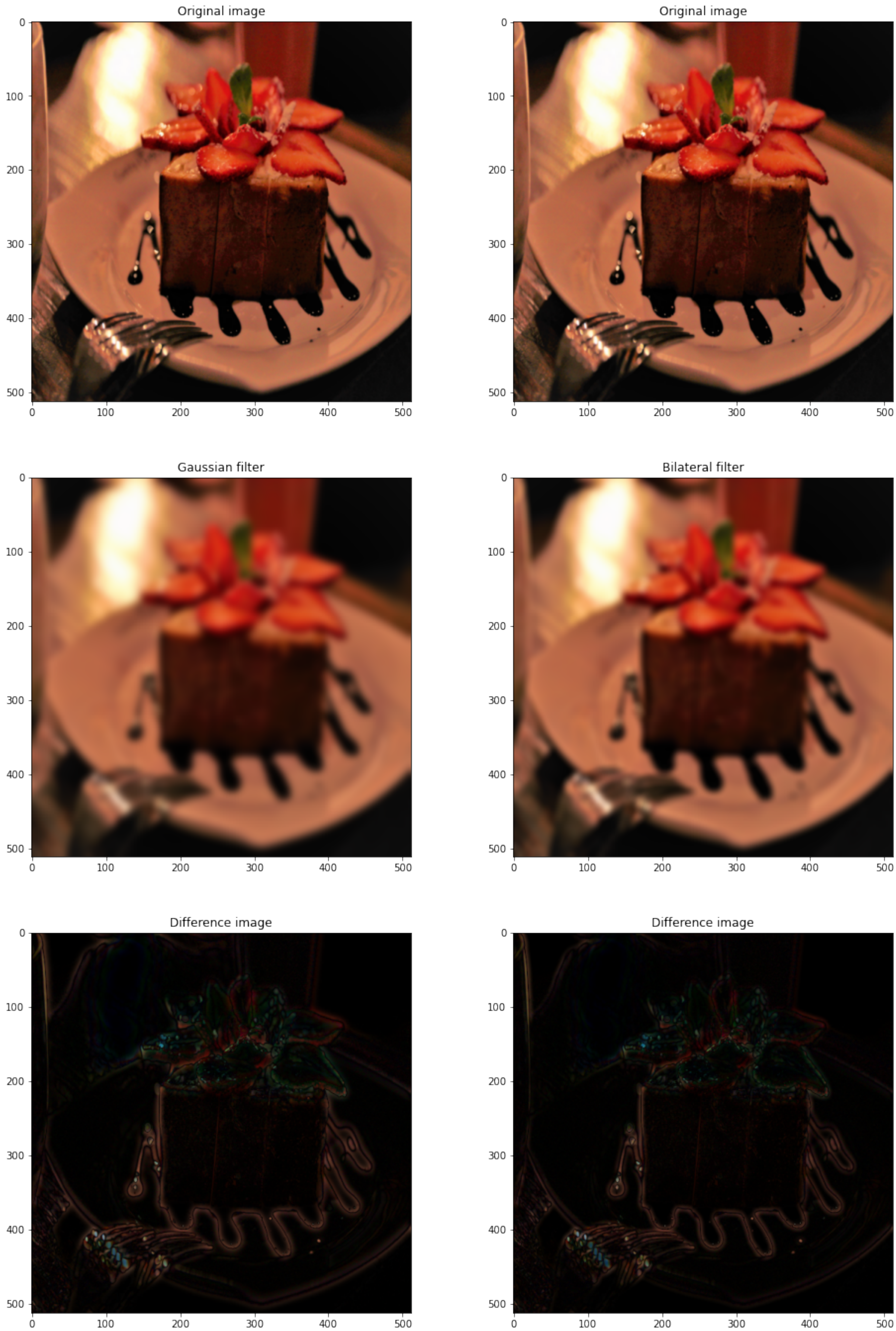

Figure 4: Bilateral vs Gaussian filter for RGB image with large σ_r

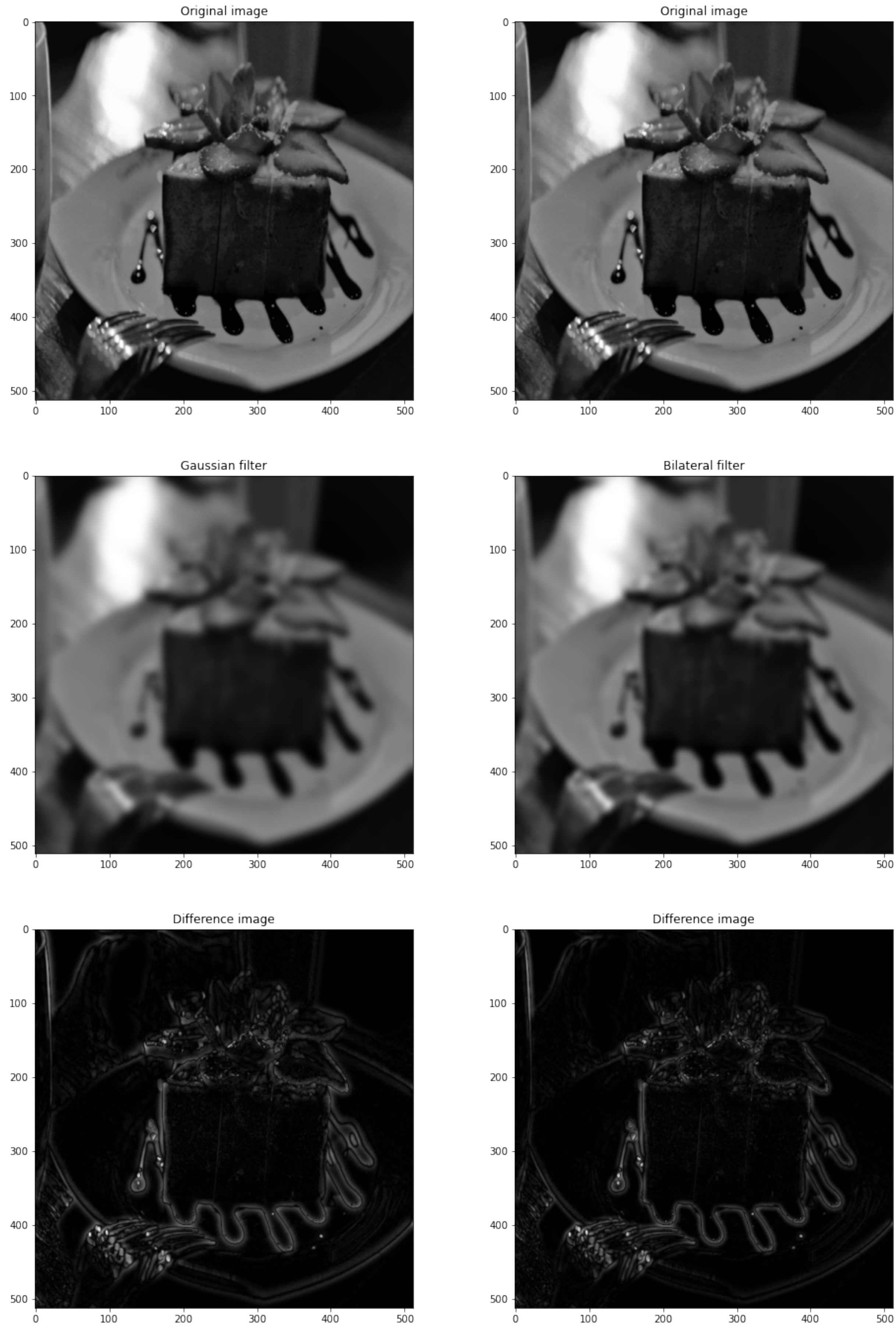

Figure 5: Bilateral vs Gaussian filter for grayscale image with large σ_r

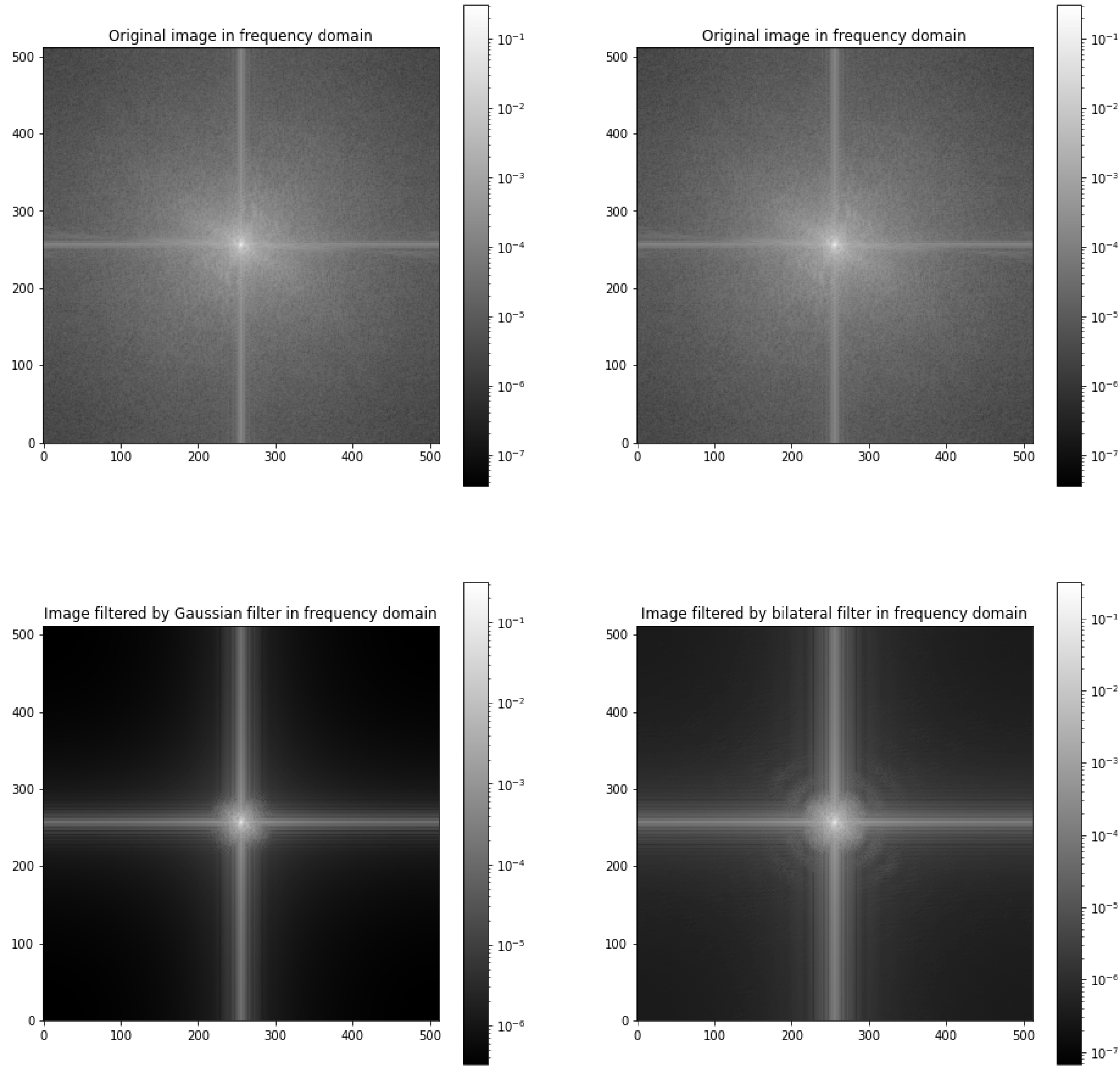

Figure 6: Bilateral vs Gaussian filter for grayscale image with large σ_r in the frequency domain

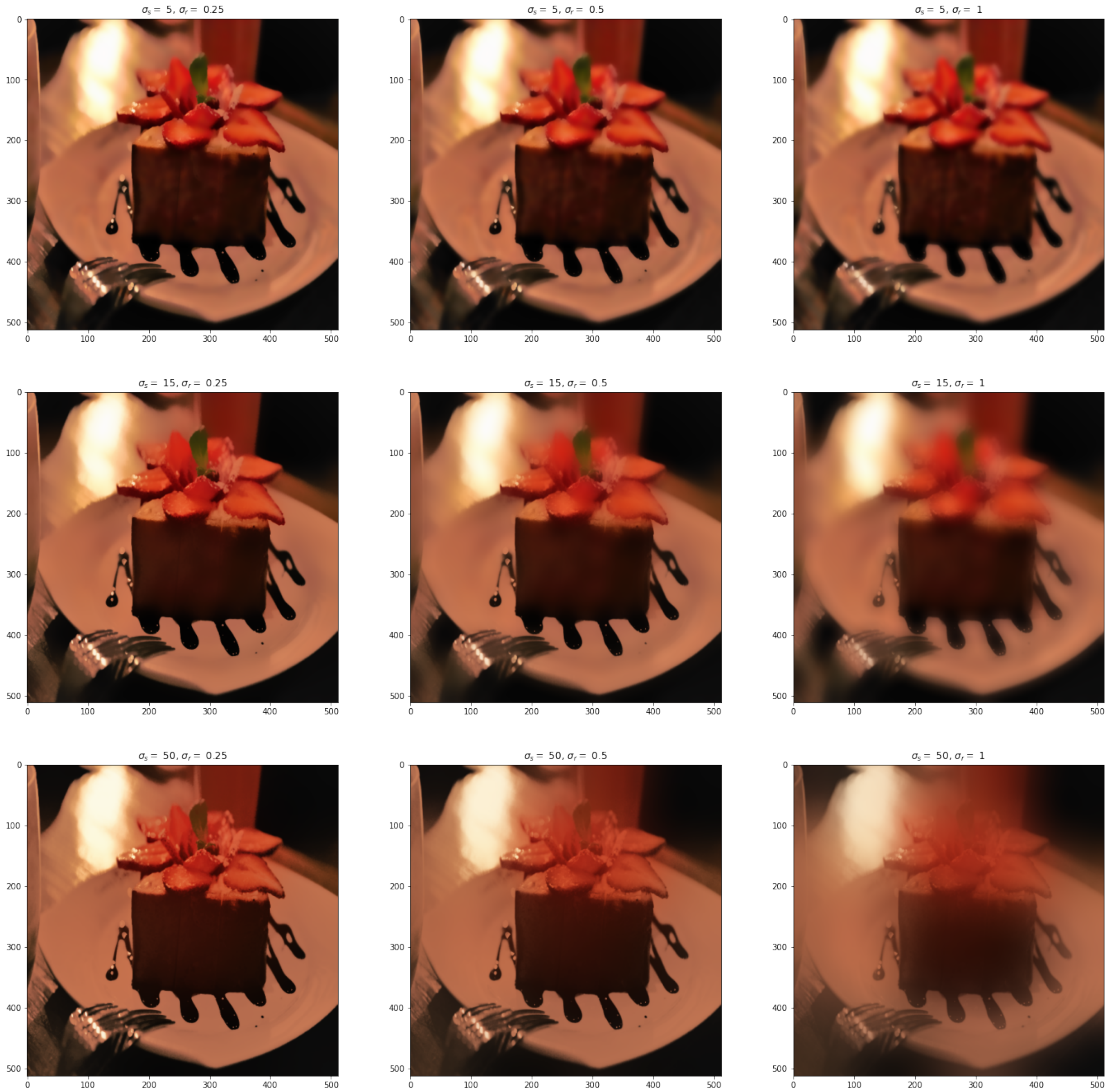

Figure 7: Space and color parameters in bilateral filter for RGB image

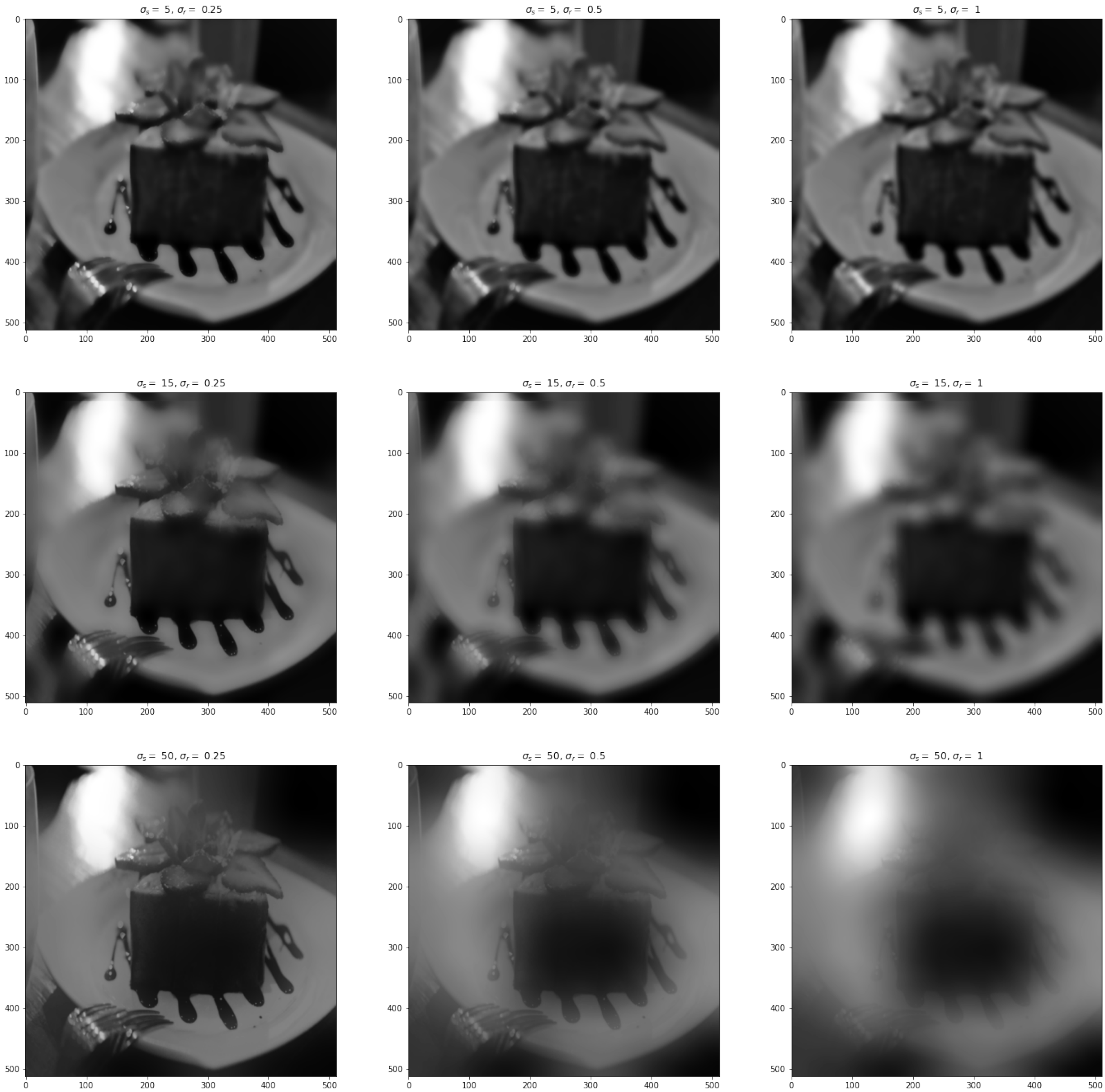

Figure 8: Space and color parameters in bilateral filter for grayscale image

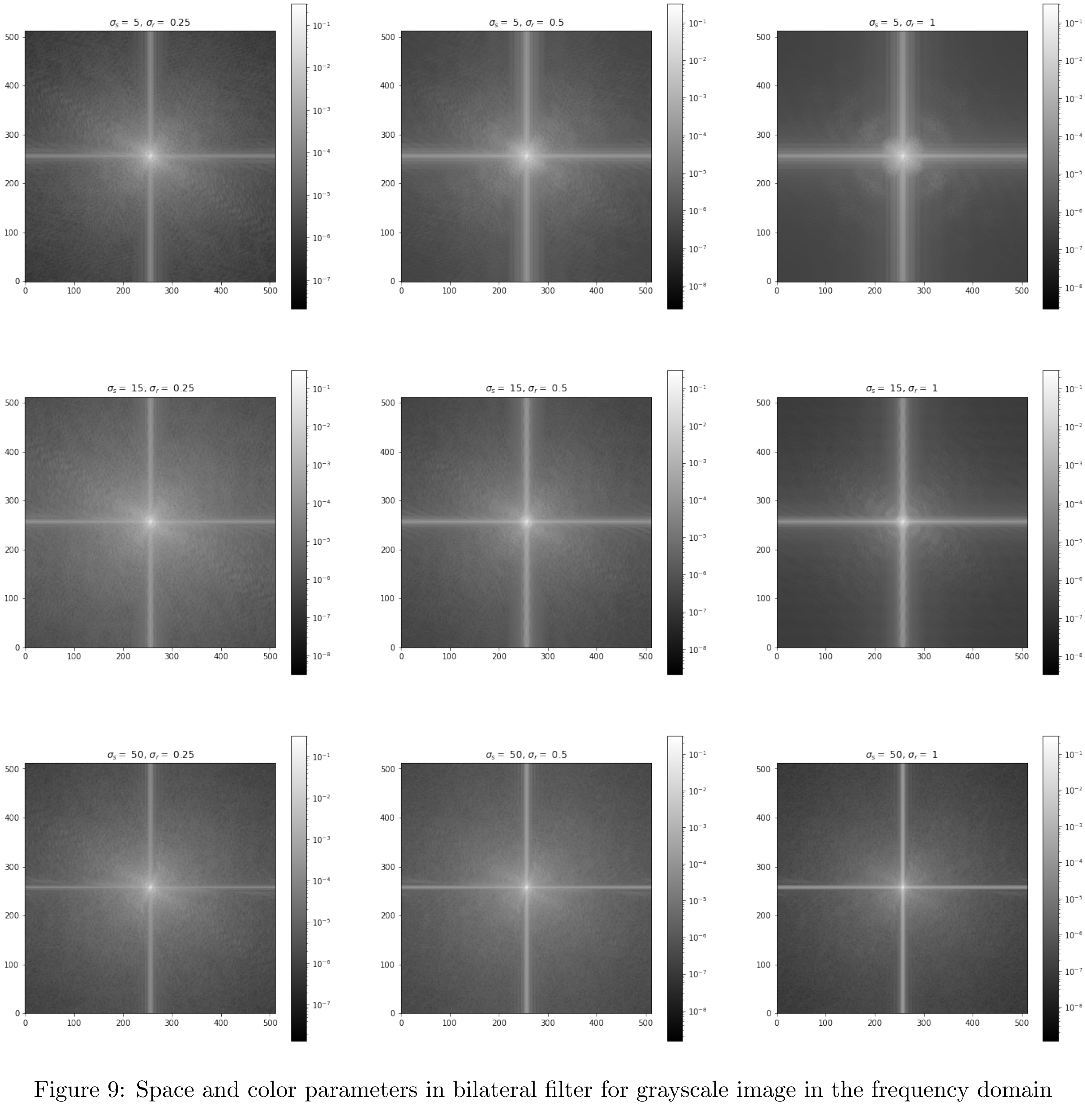

Figure 9: Space and color parameters in bilateral filter for grayscale image in the frequency domain