$$\frac{\mathrm{d}f}{\mathrm{d}x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \tag{1}$$

This definition supposes that the change Δx becomes infinitesimal.

Instead, we can choose Δx to be a small constant, in that case we can approximate the derivative of the function by

$$\frac{\mathrm{d}f}{\mathrm{d}x} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \Delta x \ll 1 \tag{2}$$

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{\Delta x}.$$

$$f(x_{i+1}) \approx f(x_i) + f'(x_i)\Delta x.$$

1 Example

$$u' = -u.$$

Known analytical solution

$$\frac{\mathrm{d}u}{\mathrm{d}x} = -u,$$
$$\frac{1}{u}\mathrm{d}u = -\mathrm{d}x,$$

Can calculate this as

$$u_{i+1} = (1 - \Delta x)u_i, \quad \forall x = 1, 2, 3, 4, 5, \ldots.$$

We of course have to know u_0

So we have

```
x_0 = 0
x_1 = 2
N = 1001 # N should be big, so that dx becomes small

x = linspace(x_0, x_1, N) # H
dx = x[1]-x[0]

for i in range(N):
    u[i+1] = (1-dx)*u[i]

plot(x, u)
show()
```

This method is known as Euler's method