Modelling the effects of mosquito repellent in a two patch system with mobility

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Introduction

We propose a two patch Ross-Macdonald model with Lagrangian mobility and the use of a repellent, based on the work of [Bichara2016] and [Rashkov2021]. Each patch is assumed to have separate healthcare capacity. We address the question of limiting the infected population sizes for given initial conditions in both patches below the respective capacities at all future times.

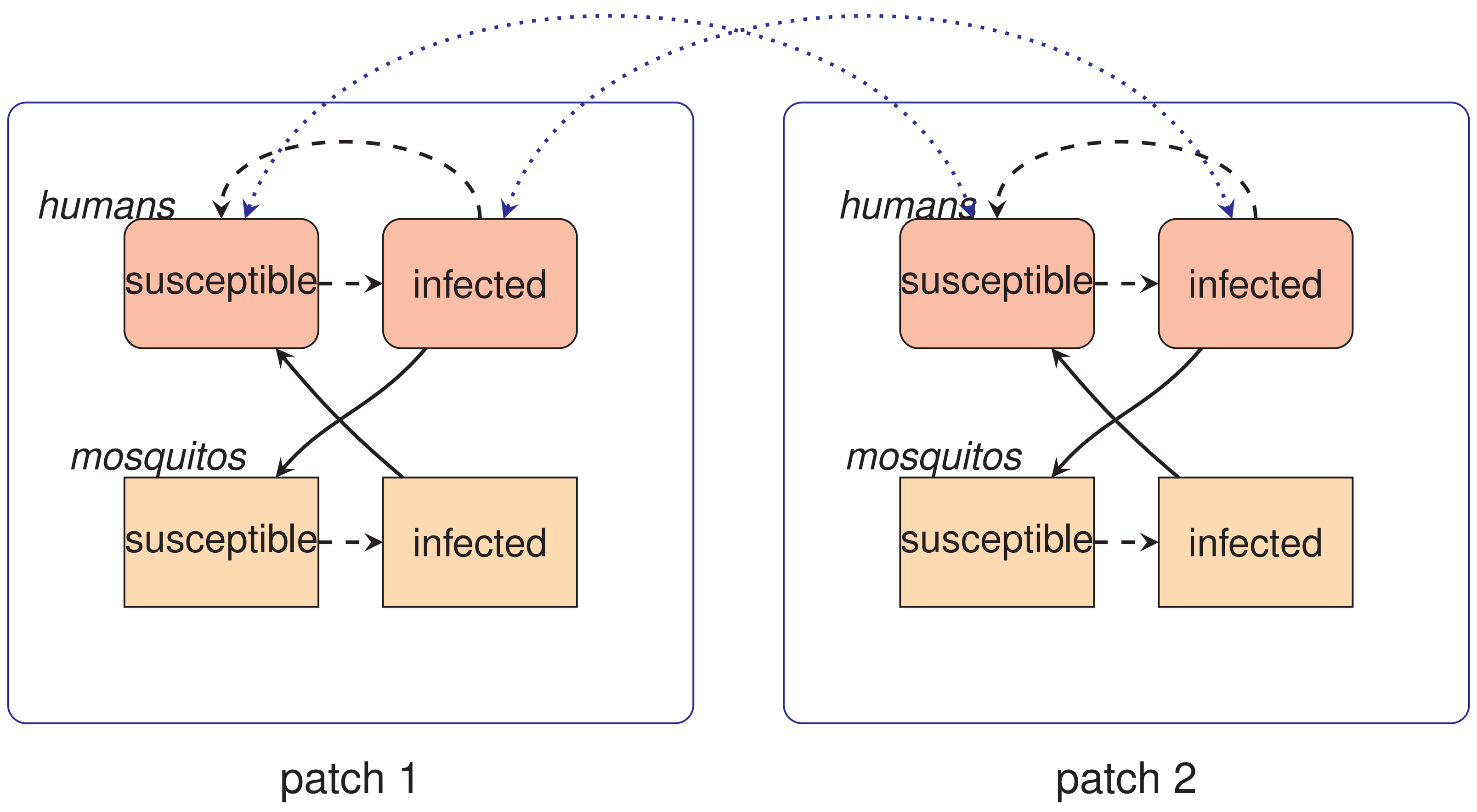

Figure 1: Model diagram

Parameter	Definition	patch 1	patch 2
β_{vh}	probability of mosquito to human transmission	0.5	
β_{hv}	probability of human to mosquito transmission	0.1	
a_i	human biting rate [day^{-1}]	0.12	0.18
M_i	female mosquito population	6×10^7	1.6×10^8
μ_i	rate of mosquito death [day^{-1}]	$\frac{1}{21}$	$\frac{1}{15}$
τ	incubation time in mosquitos [day]	10	
N_i	human population	8×10^6	2×10^7
γ_i	rate of human recovery [day^{-1}]	$\frac{1}{14}$	
p_{ij}	mobility of humans from patch i to j	varying ($p_{i1} + p_{i2} = 1$)	
κ	repellent efficiency	0.44	
$\bar{u}_i$	maximum possible repellent coverage	0.15	0.3
$\bar{I}_i$	maximum healthcare capacity	0.1	0.14

Table 1: Table of parameters

Model equations

$$\dot{X}_1(t) = \beta_{vh}(N_1 - X_1(t))\left(\frac{p_{11}e^{-\mu_1\tau}a_1(1-\kappa u_1(t))Y_1(t)}{p_{11}N_1 + p_{21}N_2} + \frac{p_{12}e^{-\mu_2\tau}a_2(1-\kappa u_1(t))Y_2(t)}{p_{12}N_1 + p_{22}N_2}\right) - \gamma_1 X_1(t)$$

$$\dot{X}_2(t) = \beta_{vh}(N_2 - X_2(t))\left(\frac{p_{21}e^{-\mu_1\tau}a_1(1-\kappa u_2(t))Y_1(t)}{p_{11}N_1 + p_{21}N_2} + \frac{p_{22}e^{-\mu_2\tau}a_2(1-\kappa u_2(t))Y_2(t)}{p_{12}N_1 + p_{22}N_2}\right) - \gamma_2 X_2(t)$$

$$\dot{Y}_1(t) = \beta_{hv}a_1(M_1 - Y_1(t))\frac{p_{11}(1-\kappa u_1(t))X_1(t) + p_{21}(1-\kappa u_2(t))X_2(t)}{p_{11}N_1 + p_{21}N_2} - \mu_1 Y_1(t)$$

$$\dot{Y}_2(t) = \beta_{hv}a_2(M_2 - Y_2(t))\frac{p_{12}(1-\kappa u_1(t))X_1(t) + p_{22}(1-\kappa u_2(t))X_2(t)}{p_{12}N_1 + p_{22}N_2} - \mu_2 Y_2(t)$$

$u_i(t) \in \mathcal{U}_i = \{u_i : \mathbb{R}_+ \to [0, \bar{u}_i] | u_i\text{- measurable}\}, 0 \leq \bar{u}_i < 1, i = 1, 2, \quad U = [0, \bar{u}_1] \times [0, \bar{u}_2], \quad \mathbf{u}(t) = (u_1(t), u_2(t))^T$

$\bar{\mathbf{I}} = (\bar{I}_1, \bar{I}_2)^T, \bar{I}_i \in [0, N_i], \quad \mathcal{I} = [0, \bar{I}_1] \times [0, \bar{I}_2] \times [0, M_1] \times [0, M_2]$

Equilibrium values

The equilibrium values at maximum repellent coverage can be used to observe cases in which $V(\bar{\mathbf{I}}, \bar{\mathbf{u}})$ is trivial. Fixing $u_i(t) \equiv \bar{u}_i$, $i = 1, 2$ leads to a cooperative autonomous ODE system for which either the origin is GAS or at most one endemic equilibrium $E^* = (X_1^*, X_2^*, Y_1^*, Y_2^*)^T$ exists, which is GAS. If for either patch $X_i^* > \bar{I}_i$, then $X_i(s) > \bar{I}_i$, at some time s, implying $V(\bar{\mathbf{I}}, \bar{\mathbf{u}})$ is the origin singleton.

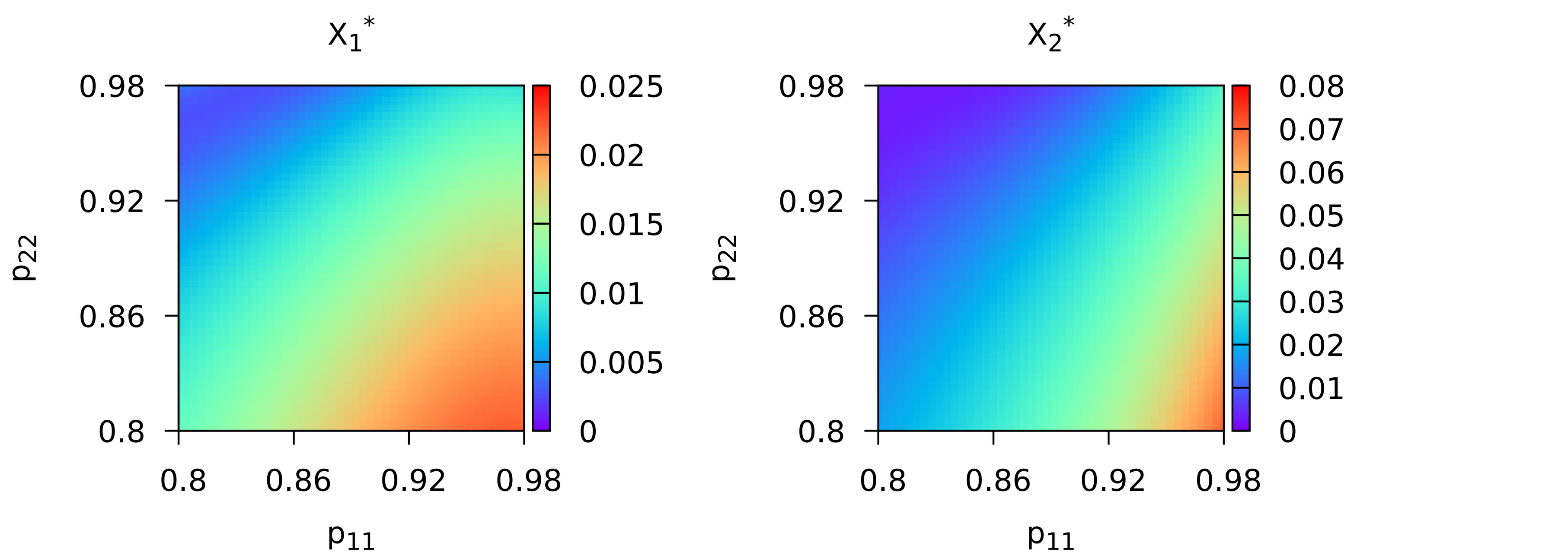

Figure 2: Effect of mobility on endemic equilibrium value at maximum repellent coverage (X_i^* as proportion)

Numeric simulations

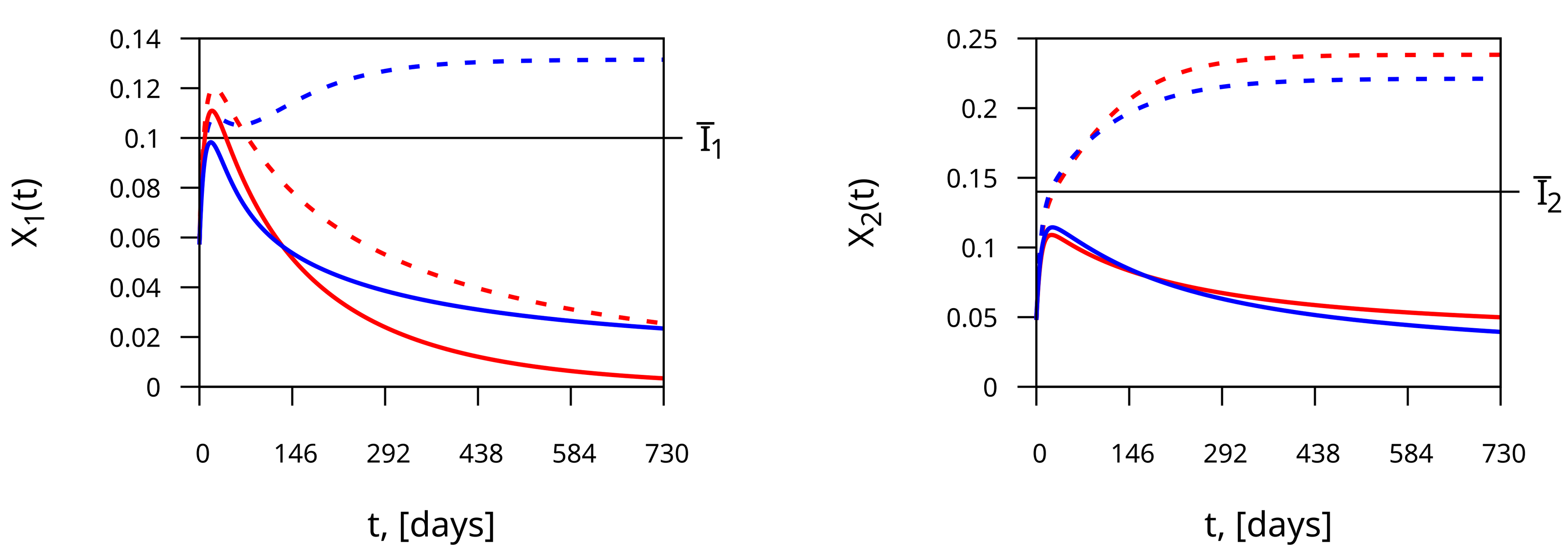

Figure 3: A plot of $X_i(t)$ as proportion with the initial condition $(0.0572, 0.048, 0.052, 0.044)^T$. Dashed: no repellent used, solid: repellent used ($u_i(t) \equiv \bar{u}_i$). Red: no mobility between the patches, blue: $p_{11} = p_{22} = 0.85$.

Viability problem

The **viability kernel** $V(\bar{\mathbf{I}}, \bar{\mathbf{u}})$ is the set of initial conditions $(X_1^0, X_2^0, Y_1^0, Y_2^0)^T$, for which a control function $\mathbf{u}(t)$ exists, such that the solutions satisfy

$$X_i(t) \leq \bar{I}_i, i = 1, 2, \quad \forall t > 0.$$

$V(\bar{\mathbf{I}}, \bar{\mathbf{u}})$ can be characterised as the sub-zero level set of a value function v [Altarovici2013].
v is the **viscosity solution** of the HJB equation

$$\min\{\lambda v(\mathbf{z}) + \mathcal{H}(\mathbf{z}, \nabla v), v(\mathbf{z}) - \Gamma(\mathbf{z})\} = 0, \quad \mathbf{z} \in \mathbb{R}^4$$

$$\mathcal{H}(\mathbf{z}, \nabla v) = \max_{\mathbf{u} \in U}\langle -\mathbf{F}(\mathbf{z}, \mathbf{u})\nabla v\rangle, \ \mathbf{F} - \text{RHS}$$

$$\Gamma(\mathbf{z}) = \begin{cases} \min_{\mathbf{z}' \in \mathcal{I}} |\mathbf{z} - \mathbf{z}'|, & \mathbf{z} \in \Omega \setminus \mathcal{I} \\ -\min_{\mathbf{z}' \in \Omega \setminus \mathcal{I}} |\mathbf{z} - \mathbf{z}'|, & \mathbf{z} \in \mathcal{I} \end{cases}$$

The HJB equation can be numerically integrated using WENO methods [Osher2003].

Mobility and the viability kernel

To measure the effects of mobility on the viability kernel $V(\bar{\mathbf{I}}, \bar{\mathbf{u}})$, the volume of its numeric approximation can be compared for various mobilities. The case of independent patches, i.e. $p_{11} = p_{22} = 1$ can be used as a reference metric.

p_{22} \ p_{11}	0.8	0.85	0.9	0.95
0.95	3.427	3.447	3.467	3.486
0.9	3.468	3.487	3.507	3.527
0.85	3.498	3.517	3.536	3.554
0.8	3.519	3.540	3.559	3.580

Table 2: Viability kernel volume (relative to no mobility)

References

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