HW 1

1. Convert the following decimal numbers to binary, octal, and hexadecimal:

    a. 1337_{10}

        i. Binary:

        $1337 \div 2 = 668 \text{ remainder } 1$

        $668 \div 2 = 334 \text{ remainder } 0$

        $334 \div 2 = 167 \text{ remainder } 0$

        $167 \div 2 = 83 \text{ remainder } 1$

        $83 \div 2 = 41 \text{ remainder } 1$

        $41 \div 2 = 20 \text{ remainder } 1$

        $20 \div 2 = 10 \text{ remainder } 0$

        $10 \div 2 = 5 \text{ reaminder } 0$

        $5 \div 2 = 2 \text{ remainder } 1$

        $2 \div 2 = 1 \text{ remainder } 0$

        $1 \div 2 = 0 \text{ remainder } 1$

        $1337_{10} = 101\ 0011\ 1001_2$

        ii. Octal:

        $1337 \div 8 = 167 \text{ remainder } 1$

        $167 \div 8 = 20 \text{ remainder } 7$

        $20 \div 8 = 2 \text{ remainder } 4$

        $2 \div 8 = 0 \text{ remainder } 2$

        $1337_{10} = 2471_8$

        iii. Hexadecimal:

        $1337 \div 16 = 83 \text{ remainder } 9$

        $83 \div 16 = 5 \text{ remainder } 3$

$5 \div 16 = 0$ remainder 5

$1337_{10} = 539_{16}$

b. 2120_{10}

i. Binary:

$2120 \div 2 = 1060$ remainder 0

$1060 \div 2 = 530$ remainder 0

$530 \div 2 = 265$ remainder 0

$265 \div 2 = 132$ remainder 1

$132 \div 2 = 66$ remainder 0

$66 \div 2 = 33$ remainder 0

$33 \div 2 = 16$ remainder 1

$16 \div 2 = 8$ remainder 0

$8 \div 2 = 4$ remainder 0

$4 \div 2 = 2$ remainder 0

$2 \div 2 = 1$ remainder 0

$1 \div 2 = 0$ remainder 1

$2120_{10} = 1000\ 0100\ 1000_2$

ii. Octal:

$2120 \div 8 = 265$ remainder 0

$265 \div 8 = 33$ remainder 1

$33 \div 8 = 4$ remainder 1

$4 \div 8 = 0$ remainder 4

$2120_{10} = 4110_8$

iii. Hexadecimal:

$2120 \div 16 = 132$ remainder 8

$$132 \div 16 = 8 \text{ remainder } 4$$

$$8 \div 16 = 0 \text{ remainder } 8$$

$$2120_{10} = 848_{16}$$

2. Convert the following numbers from their base to decimal using the multiplication method.
   a. CAFE_{16}

      $12 * 16^3 + 10 * 16^2 + 15 * 16^1 + 14 * 16^0$

      $= 16(16(16 * 12 + 10) + 15) + 14$

      $= 51966_{10}$
   b. BABE_{16}

      $11 * 16^3 + 10 * 16^2 + 11 * 16^1 + 14 * 16^0$

      $= 16(16(16 * 11 + 10) + 11) + 14$

      $= 47806_{10}$
   c. 10011001_2

      $2^7 + 2^4 + 2^3 + 2^0$

      $2(2(2(2(2 * 2 * 2 + 1) + 1))) + 1$

      $= 153_{10}$
3. Which of the following would be valid numbers for bases 3, 7, 8, 13, 15 and 16?
   a. $\mathrm{DEADBEAF}$

      Valid for Base 16 only.
   b. $\mathrm{DC4A}$

      Valid for Base 15, 16.
   c. 122

      Valid for 3, 7, 8, 13, 15 and 16.
4. What are the smallest and largest numbers which can be represented with 3 digits for the
   corresponding base *n*? What is the decimal value of the largest number?
   a. binary

$000_2 \rightarrow 0_{10}$

$111_2 \rightarrow 7_{10}$

b. base 3

$000_3 \rightarrow 0_{10}$

$222_3 \rightarrow 2 * 3^2 + 2 * 3 + 2 = 18 + 6 + 2 = 26_{10}$

c. base 5

$000_5 \rightarrow 0_{10}$

$444_5 \rightarrow 4 * 5^2 + 4 * 5 + 4 = 100 + 20 + 4 = 124_{10}$

d. base 9

$000_9 \rightarrow 0_{10}$

$888_9 \rightarrow 8 * 9^2 + 8 * 9 + 8 = 728_{10}$

5. Convert the following binary numbers to base 3, 4, 6, 8 (octal) and 16 (hexadecimal)
   a. 1001_2
      i. Base 3

         $1001_2 \rightarrow 2^3 + 2^1 = 9_{10} \rightarrow 3^2 \rightarrow 100_3$
      ii. Base 4

         $1001_2 \rightarrow 9_{10} \rightarrow 2 * 4^1 + 1 \rightarrow 21_4$
      iii. Base 6

         $1001_2 \rightarrow 9_{10} \rightarrow 6^1 + 3 * 6^0 \rightarrow 13_6$
      iv. Base 8

         $1001_2 \rightarrow 001\ 001_2 \rightarrow 11_8$
      v. Base 16

         $1001_2 \rightarrow 9_{16}$
   b. 111111_2
      i. Base 3

         $111111_2 \rightarrow 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 \rightarrow 63_{10}$

         $63 \div 3 = 21, 0$

$21 \div 3 = 7, 0$

$7 \div 3 = 2, 1$

$2 \div 3 = 0, 2$

$63_{10} = 2100_3$

ii. Base 4

$63 \div 4 = 15, 3$

$15 \div 4 = 3, 3$

$3 \div 4 = 0, 3$

$63_{10} = 333_4$

iii. Base 6

$63 \div 6 = 10, 3$

$10 \div 6 = 1, 4$

$1 \div 6 = 0, 1$

$63_{10} = 143_6$

iv. Base 8

$111\ 111_2 \rightarrow 77_8$

v. Base 16

$0011\ 1111_2 \rightarrow 3F_{16}$

6. Convert 101_6 to base 9.

   $101_6 \rightarrow 6^2 + 1 = 37_{10}$

   $37 \div 9 = 4, 1$

   $4 \div 9 = 0, 4$

   $101_6 \rightarrow 41_9$

7. Convert the number 423_{10} to the following bases:

   a. Base 2

   $423 \div 2 = 211, 1$

   $211 \div 2 = 105, 1$

   $105 \div 2 = 52, 1$

$52 \div 2 = 26, 0$

$26 \div 2 = 13, 0$

$13 \div 2 = 6, 1$

$6 \div 2 = 3, 0$

$3 \div 2 = 1, 1$

$1 \div 2 = 0, 1$

$423_{10} = 1\ 1010\ 0111_2$

b. Base 5

$423_{10} \div 5 = 84, 3$

$84 \div 5 = 16, 4$

$16 \div 5 = 3, 1$

$3 \div 5 = 0, 3$

$423_{10} = 3143_5$

8. Assume numbers are 7-bit. What is the range for numbers that can be stored in each
representation?

   a. sign/magnitude

   $[-2^{7-1} + 1, 2^{7-1} + 1],\ \text{range: } [-63, 63]$

   b. 1's complement

   $[-2^{7-1} + 1, 2^{7-1} + 1],\ \text{range: } [-63, 63]$

   c. 2's complement

   $[-2^{7-1}, 2^{7-1} + 1],\ \text{range: } [-64, 63]$

   d. unsigned

   $2^7 - 1 = 127,\ \text{range: } [0, 127]$

9. Given the following 1's complement values, convert them from 1's complement to 2's complement.

   (Find first 1 from right hand side, drag that one down, flip everything to the left of it.)

   1. 10011001_2

$10011001_2 + 00000001_2 \rightarrow 10011010_2$

2. 11101101_2

   $11101101_2 + 00000001_2 \rightarrow 11101110_2$

10. Given the following 2's complement, values convert them from 2's complement to 1's complement.

    a. 10011001_2

       $10011001_2 - 00000001_2 \rightarrow 10011000_2$

    b. 11101101_2

       $11101101_2 - 00000001_2 \rightarrow 11101100_2$

11. Given the following sign/magnitude values, convert them from sign/ magnitude to 2's complement

    Find the first 1 from the right, drag it down and flip everything else to the left of it

    1. 10011001_2

       $10011001_2 = -(16 + 8 + 1) = -25_{10}$

       $25_{10} = 00011001_2 \overset{flip}{\rightarrow} 11100110_2 \overset{+1}{\rightarrow} 11100111_2 = -25_{10}$

    2. 11101101_2

       $11101101_2 = -(64 + 32 + 8 + 4 + 1) = -109_{10}$

       $109_{10} = 01101101_2 \overset{flip}{\rightarrow} 10010010_2 \overset{+1}{\rightarrow} 10010011_2 = -109_{10}$

12. Encode each of the following numbers as 16-bit sign magnitude, 1's complement, and 2's complement numbers.

    a. 220_{10}

       $220 \div 2 = 110, 0$

       $110 \div 2 = 55, 0$

       $55 \div 2 = 27, 1$

       $27 \div 2 = 13, 1$

       $13 \div 2 = 6, 1$

       $6 \div 2 = 3, 0$

       $3 \div 2 = 1, 1$

       $1 \div 2 = 0, 1$

$220_{10} = 1101\ 1100_2$

- 16-bit sign magnitude

  $220_{10} = 0000\ 0000\ 1101\ 1100_2$

- 1's Complement

  $220_{10} = 0000\ 0000\ 1101\ 1100_2$

- 2's Complement

  $220_{10} = 0000\ 0000\ 1101\ 1100_2$

b. 32767_{10}

$32767 \div 2 = 16383, 1$

$16383 \div 2 = 8191, 1$

$8191 \div 2 = 4095, 1$

$4095 \div 2 = 2047, 1$

$2047 \div 2 = 1023, 1$

$1023 \div 2 = 511, 1$

$511 \div 2 = 255, 1$

$255 \div 2 = 127, 1$

$127 \div 2 = 63, 1$

$63 \div 2 = 31, 1$

$31 \div 2 = 15, 1$

$15 \div 2 = 7, 1$

$7 \div 2 = 3, 1$

$3 \div 2 = 1, 1$

$1 \div 2 = 0, 1$

- 16-bit sign magnitude

  $32767_{10} = 0111\ 1111\ 1111\ 1111_2$

- 1's Complement

  $32767_{10} = 0111\ 1111\ 1111\ 1111_2$

- 2's Complement

$32767_{10} = 0111\ 1111\ 1111\ 1111_2$

c. -309_{10}

$309 \div 2 = 154, 1$

$154 \div 2 = 77, 0$

$77 \div 2 = 38, 1$

$38 \div 2 = 19, 0$

$19 \div 2 = 9, 1$

$9 \div 2 = 4, 1$

$4 \div 2 = 2, 0$

$2 \div 2 = 1, 0$

$1 \div 2 = 0, 1$

- 16-bit sign magnitude

  $-309_{10} = 1000\ 0001\ 0011\ 0101_2$

- 1's Complement

  -309_{10}

  $309_{10} = 0000\ 0001\ 0011\ 0101_2 \xrightarrow{flip} 1111\ 1110\ 1100\ 1010_2$

- 2's Complement

  $309_{10} = 0000\ 0001\ 0011\ 0101_2 \xrightarrow{flip} 1111\ 1110\ 1100\ 1010_2 \xrightarrow{+1}$
  $1111\ 1110\ 1100\ 1011_2$

  $-309_{10} = 1111\ 1110\ 1100\ 1011_2$

13. In 1's complement, 2's complement, and signed magnitude number format

    a. How many representations for zero are there?

       i. 1's Complement, 2 representations of 0

       ii. 2'd Complement , only 1 representation of 0

       iii. signed magnitude, 2 representations of 0

    b. Using 6-bits, specify all representations of zero for each format.

i. 1's Complement, $000\ 000_2$ and $111\ 111_2$

ii. 2's Complement, $000\ 000_2$

iii. signed magnitude, $000\ 000_2$ and $100\ 000_2$

14. What is the range of values for *n*-bit two's complement? (Give a generic form)

    $[-2^{n-1}, 2^{n-1}-1]$

15. 1. Consider the 5-bit binary number 11011_2, what is the base 10 value of this binary representation in the following encodings.

    a. Unsigned Binary

    $2^4+2^3+2+1=27_{10}$

    b. Signed Magnitude

    $-1*(2^3+2+1)=-11_{10}$

    c. One's Complement

    $11011_2 \overset{flip}{\rightarrow} 00100_2=4_{10}$

    $11011_2=-4_{10}$

    d. Two's Complement

    $11011_2 \overset{-1}{\rightarrow} 11010_2 \overset{flip}{\rightarrow} 00101_2=5_{10}$

    $11011_2 \rightarrow -5_{10}$

16. Consider the following additional problems for 8-bit 2's complement numbers. What is the result of the calculation? Did overflow/underflow occur?

    a. $10101010_2+01010101_2$

    $=11111111_2$

    $1111\ 1111_2 \overset{-1}{\rightarrow} 1111\ 1110_2 \overset{flip}{\rightarrow} 0000\ 0001_2=1_{10}$, since original 2's Complement MSB is 1 the answer is -1_{10}

    $10101010_2 \overset{-1}{\rightarrow} 10101001 \overset{flip}{\rightarrow} 01010110_2=86_{10}$, since 2's Complement MSB is 1. This mean $10101010_2=-86_{10}$

    $01010101_2=64+16+4+1=85_{10}$

$-86_{10} + 85_{10} = -1_{10}$

There is no overflow/underflow happens here as the result does in fact check out.

b. Under what general conditions will overflow occur?

There will be overflow whenever 2 positive number addition result in a negative number or 2 negative number addition result in a positive number.

17. The following binary floating-point numbers consist of a sign bit; an excess-64, radix-2 exponent; and a 16-bit fraction. Convert them to “binary”’ scientific notation.

$0\ 0000000\ 0000000000000000$

1. 010000000001010100000001

   $0\ 1000000\ 0001010100000001$

   $s = 0$

   $e = 64 - 64 = 0$

   $1.0001010100000001 * 2^0$

2. 001111110000001111111111

   $0\ 0111111\ 0000001111111111$

   $s = 0$

   $e = (32 + 16 + 8 + 4 + 2 + 1) - 64 = 63 - 64 = -1$

   $1.0000001111111111 * 2^{-1}$

18. Show the binary representation for the following decimal floating point values in IEEE single-precision format.

Single precision is 1 bit sign, 8 bit exponent, bias 127, fraction 23 bits

1. 7.4

   $7_{10} = 2^2 + 2^1 + 2^0 = 111_2$

   0.4_{10}

   $0.4 * 2 = 0.8 < 1, 0$

   $0.8 * 2 = 1.6 > 1, 1$

   $0.6 * 2 = 1.2 > 1, 1$

   $0.2 * 2 = 0.4 < 1, 0$

   … Cycle

   $0.4_{10} = 0.\overline{0110}_2$

   $7.4_{10} = 111.\overline{0110}_2$

   $7.4_{10} = 1.11\overline{0110}_2 * 2^2$

   $e = 2_{10} + 127_{10}$

   $e = 129_{10} = 2^7 + 2^0$

   $e = 1000\ 0001_2$

   $s = 0$

   $f = 11011001100110011001100_2$

   $7.4_{10} = 0\ 10000001\ 11011001100110011001100_2$

2. $-\frac{13}{64}$

   $-\frac{13}{64} = -0.203125$

   $0.203125_{10} \div 2^{-1} = 0.40625$

   $0.203125_{10} \div 2^{-2} = 0.8125$

   $0.203125_{10} \div 2^{-3} = 1.625$

   $\frac{13}{64} = 1.625 * 2^{-3}$

   $0.625 * 2 = 1.25 > 1, 1$

$0.25 * 2 = 0.5 < 1, 0$

$0.5 * 2 = 1$

$e = -3_{10} + 127_{10} = 124_{10} = 0111\ 1100_2$

$s = 1$

$f = 10100000000000000000000_2$

$-\frac{13}{64} = 1\ 01111100\ 10100000000000000000000_2$

3. 6.125

   $6_{10} = 2^2 + 2^1 = 110_2$

   $0.125 * 2 = 0.25 < 1, 0$

   $0.25 * 2 = 0.5 < 1, 0$

   $0.5 * 2 = 1$

   $0.125_{10} = 0.001_2$

   $6.125_{10} = 110.001_2$

   $110.001_2 = 1.10001_2 * 2^2$

   $e = 2 + 127 = 129_{10} = 1000\ 0001_2$

   $s = 0$

   $f = 10001000000000000000000_2$

   $6.125_{10} = 0\ 10000001\ 10001000000000000000000_2$

19. Convert the following values to their IEEE single precision encodings.

    a. $3.22_{10} \times 2^{13}$

       $3_{10} = 2^1 + 2^0 = 11_2$

$0.22 * 2 = 0.44 < 1, 0$

$0.44 * 2 = 0.88 < 1, 0$

$0.88 * 2 = 1.76 > 1, 1$

$0.76 * 2 = 1.52 > 1, 1$

$0.52 * 2 = 1.04 > 1, 1$

$0.04 * 2 = 0.08 < 1, 0$

$0.08 * 2 = 0.16 < 1, 0$

$0.16 * 2 = 0.32 < 1, 0$

$0.32 * 2 = 0.64 < 1, 0$

$0.64 * 2 = 1.28 > 1, 1$

$0.28 * 2 = 0.56 < 1, 0$

$0.56 * 2 = 1.12 > 1, 1$

$0.12 * 2 = 0.24 < 1, 0$

$0.24 * 2 = 0.48 < 1, 0$

$0.48 * 2 = 0.96 < 1, 0$

$0.96 * 2 = 1.92 > 1, 1$

$0.92 * 2 = 1.84 > 1, 1$

$0.84 * 2 = 1.68 > 1, 1$

$0.68 * 2 = 1.36 > 1, 1$

$0.36 * 2 = 0.72 < 1, 0$

$0.72 * 2 = 1.44 > 1, 1$

$0.44 * 2 = 0.88 < 1, 0$

… Cycle

$0.22_{10} = 0.0\overline{01110000101000111101}_2$

$3.22_{10} = 11.\overline{001110000101000111101}_2 = 1.10\overline{01110000101000111101}_2 * 2^1$

$3.22_{10} \times 2^{13} = 1.10\overline{01110000101000111101}_2 * 2^1 * 2^{13} =$
$1.10\overline{01110000101000111101}_2 * 2^{14}$

$s = 0$

$e = 14 + 127 = 141_{10} = 2^7 + 2^3 + 2^2 + 2^0 = 1000\ 1101_2$

$f = 10011100001010001111010_2$

$3.22_{10} \times 2^{13} = 0\ 10001101\ 10011100001010001111010_2$

b. 8.123×2^{-14}

$8_{10} = 1000_2$

$0.123 * 2 = 0.246 < 1, 0$

$0.246 * 2 = 0.492 < 1, 0$

$0.492 * 2 = 0.984 < 1, 0$

$0.984 * 2 = 1.968 > 1, 1$

$0.968 * 2 = 1.936 > 1, 1$

$0.936 * 2 = 1.872 > 1, 1$

$0.872 * 2 = 1.744 > 1, 1$

$0.744 * 2 = 1.488 > 1, 1$

$0.488 * 2 = 0.976 < 1, 0$

$0.976 * 2 = 1.952 > 1, 1$

$0.952 * 2 = 1.904 > 1, 1$

$0.904 * 2 = 1.808 > 1, 1$

$0.808 * 2 = 1.616 > 1, 1$

$0.616 * 2 = 1.232 > 1, 1$

$0.232 * 2 = 0.464 < 1, 0$

$0.464 * 2 = 0.928 < 0, 0$

$0.928 * 2 = 1.856 > 1, 1$

$0.856 * 2 = 1.712 > 1, 1$

$0.712 * 2 = 1.424 > 1, 1$

$0.424 * 2 = 0.848 < 1, 0$

$0.848 * 2 = 1.696 > 1, 1$

… (21st digit precision) continue as this is not going to terminate any time soon.

$0.123 = 000111110111110011101_2$ (21 precision)

$8.125_{10} = 1000.000111110111110111101_2$

$1.00000011111011111011110_2 * 2^3$ (round to 23 bits)

$8.123_{10} * 2^{-14} = 1.00000011111011111011110_2 * 2^3 * 2^{-14} =$
$1.00000011111011111011110_2 * 2^{-11}$

$s = 0$

$e = 127 - 11 = 116_{10} = 01110100_2$

$f = 00000011111011111001110_2$

$8.123_{10} * 2^{-14} = 0\ 01110100\ 00000011111011111001110_2$