CSE 220: Systems Fundamentals I

Stony Brook University

Number Representations

Practice Problems

Important Notes:

- These practice problems will provide you a sense of the kinds of questions you may encounter on the exams.
- The exams might include questions on material not covered by these practice problems.
- The questions on the exams might not have the same formats as these practice problems.

1. Convert the following decimal numbers to binary, octal, and hexadecimal:

   a. 1337_{10}

   Solution:
   010100111001_2, 2471_8, 539_{16}

   b. 2120_{10}

   Solution:
   100001001000_2 4110_8 848_{16}

   c. 319_{10}

   Solution:
   000100111111_2, 477_8, $13F_{16}$

   d. 7245_{10}

   Solution:
   0001110001001101_2, 16115_8, $1C4D_{16}$

2. Convert the following numbers from their base to decimal using the multiplication method.

   a. $CAFE_{16}$

   Solution:
   $C_{16} = 12_{10}$, $A_{16} = 10_{10}$, $F_{16} = 15_{10}$, $E_{16} = 14_{10}$
   $16^3 \times 12 + 16^2 \times 10 + 16^1 \times 15 + 16^0 \times 14 = 51966_{10}$

   b. $BABE_{16}$

   Solution:
   $B_{16} = 11_{10}$, $A_{16} = 10_{10}$, $B_{16} = 11_{10}$, $E_{16} = 14_{10}$
   $16^3 \times 11 + 16^2 \times 10 + 16^1 \times 11 + 16^0 \times 14 = 47806_{10}$

   c. 10011001_2

   Solution:
   $2^7 \times 1 + 2^6 \times 0 + 2^5 \times 0 + 2^4 \times 1 + 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 0 + 2^0 \times 1 = 153_{10}$

   d. 777_8

Solution:

$7_8 = 7_{10}$
$8^2 \times 7 + 8^1 \times 7 + 8^0 \times 7 = 511_{10}$

e. $C1A7_{13}$

Solution:

$C_{13} = 12_{10}$, $1_{13} = 1_{10}$, $A_{13} = 10_{10}$, $7_{13} = 7_{10}$
$13^3 \times 12 + 13^2 \times 1 + 13^1 \times 10 + 13^0 \times 7 = 26670_{10}$

f. 306_7

Solution:

$3_{13} = 3_{10}$, $0_{13} = 0_{10}$, $6_{13} = 6_{10}$
$7^2 \times 3 + 7^1 \times 0 + 7^0 \times 6 = 153_{10}$

3. Note that some of the numbers in the previous exercise are written in a base that is some power of 2 (4, 8, 16, etc.) Convert those numbers from their base into binary and then into decimal.

a. $CAFE_{16}$

Solution:

1100101011111110_2
51966_{10}

b. $BABE_{16}$

Solution:

1011101010111110_2
47806_{10}

c. 10011001_2

Solution:

10011001_2
153_{10}

d. 777_8

Solution:

111111111_2
511_{10}

4. Which of the following would be valid numbers for bases 3, 7, 8, 13, 15 and 16?

a. DEADBEAF

Solution:

16

b. DC4A

Solution:

15, 16

c. 122

Solution:
3, 7, 8, 13, 15, 16

d. 1689

Solution:
13, 15, 16

e. 1337

Solution:
8, 13, 15, 16

f. 888

Solution:
13, 15, 16

5. What is the smallest and largest numbers which can be represented with 3 digits for the corresponding base n? What is the decimal value of the largest number?

a. binary

Solution:
largest = $111_2, 7_{10}$
smallest = 0_{10}

b. base 3

Solution:
largest = $222_3, 26_{10}$
smallest = 0_{10}

c. base 5

Solution:
largest = $444_5, 124_{10}$
smallest = 0_{10}

d. octal

Solution:
largest = $777_8, 511_{10}$
smallest = 0_{10}

e. base 9

Solution:
largest = $888_9, 728_{10}$
smallest = 0_{10}

f. hexadecimal

Solution:
largest = $\text{FFF}_{16}, 4095_{10}$
smallest = 0_{10}

6. Convert the following binary numbers to base 3, 4, 6, 8 (ocatl) and 16 (hexadecimal).

   a. 1001_2

      Solution:

      100_3, 21_4, 13_6, 11_8, 9_{16}

   b. 111111_2

      Solution:

      2100_3, 333_4, 143_6, 77_8, $3F_{16}$

   c. 101010_2

      Solution:

      1120_3, 222_4, 110_6, 52_8, $2A_{16}$

   d. 101011011001110_2

      Solution:

      1010111001_3, 11123032_4, 250514_6, 53316_8, $56CE_{16}$

7. Convert 101_6 to base 9.

   Solution:

   Convert from base to decimal.
   $101_6 = 1 \times 6^2 + 0 \times 6^1 + 1 \times 6^0$
   $36 + 0 + 1 = 37_{10}$

   Convert from decimal to new base.
   $37_{10}/9 = 4$
   $37_{10} - 36_{10} = 1_{10}$ $1_{10}/9 = 1$
   $= 41_9$

8. Convert the number 423_{10} to the following bases:

   a. Base 2 (Binary)

      Solution:

      $423/2 = 211$ Remainder 1 ; Result = 1

      $211/2 = 105$ Remainder 1 ; Result = 11

      $105/2 = 52$ Remainder 1 ; Result = 111

      $52/2 = 26$ Remainder 0 ; Result = 0111

      $26/2 = 13$ Remainder 0 ; Result = 00111

      $13/2 = 6$ Remainder 1 ; Result = 100111

      $6/2 = 3$ Remainder 0 ; Result = 0100111

      $3/2 = 1$ Remainder 1 ; Result = 10100111

      $1/2 = 0$ Remainder 1; Result = 110100111

      110100111_2

b. Base 16 (Hexadecimal)

Solution:

$423 - (16^2 * 1) = 167$; Result = 1

$167 - (16^1 * 10) = 7$; Result = 1A

$7 - (16^0 * 7) = 0$; Result = 1A7

$1A7_{16}$

c. Base 5

Solution:

$423/5 = 84\frac{3}{5}$; Remainder = 3 ; Result = 3

$84/5 = 16\frac{4}{5}$; Remainder = 4 ; Result = 43

$16/5 = 3\frac{1}{5}$; Remainder = 1 ; Result = 143

$3/5 = 0\frac{3}{5}$; Remainder = 3 ; Result = 3143

3143_5

9. Given the 10-bit unsigned binary value convert to Octal, Hexadecimal, base 5 and base 6

a. 0101111001

Solution:

Octal: Group in sets of three, from right to left.

$001_2 = 1_{10}$

$111_2 = 7_{10}$

$101_2 = 5_{10}$

571_8

Hexadecimal: Group in sets of four, from right to left.

$1001_2 = 9_{10}$

$0111_2 = 7_{10}$

$0001_2 = 1_{10}$

179_{16}

Base 5: Convert to base 10 then to base 5.

$0101111001_2 = 2^0 + 2^3 + 2^4 + 2^5 + 2^6 + 2^8 = 1 + 8 + 16 + 32 + 64 + 256 = 377_{10}$

$377/5 = 75\frac{2}{5}$; Remainder 2

$75/5 = 15$; Remainder 0

$15/5 = 3$; Remainder 0

$3/5 = \frac{3}{5}$; Remainder 3

3002_5

Base 6: Convert to base 10 then to base 6.

$377/6 = 62\frac{5}{6}$; Remainder 5

$62/6 = 10\frac{2}{6}$; Remainder 2

$10/6 = 1\frac{4}{6}$; Remainder 4

$1/6 = 0\frac{1}{6}$; Remainder 1

1425_6

b. 1010101110

Solution:

Octal: Group in sets of three, from right to left.

$110_2 = 6_{10}$

$101_2 = 5_{10}$

$010_2 = 2_{10}$

$001_2 = 1_{10}$

1256_8

Hexadecimal: Group in sets of four, from right to left.

$1110_2 = 14_{10} = E_{16}$

$1010_2 = 10_{10} = A_{16}$

$0010_2 = 2_{10}$

$2AE_{16}$

Base 5: Convert to base 10 then to base 5.

$1010101110_2 = 2^1 + 2^2 + 2^3 + 2^5 + 2^7 + 2^9 = 512 + 128 + 32 + 8 + 4 + 2 = 686_{10}$

$686/5 = 137\frac{1}{5}$; Remainder 1

$137/5 = 27\frac{2}{5}$; Remainder 2

$27/5 = 5\frac{2}{5}$; Remainder 2

$5/5 = 1$; Remainder 0

$1/5 = 0\frac{1}{5}$; Remainder 1

10221_5

Base 6: Convert to base 10 then to base 6.

$686/6 = 114\frac{2}{6}$; Remainder 2

$114/6 = 19$; Remainder 0

$19/6 = 3\frac{1}{6}$; Remainder 1

$3/6 = 0\frac{3}{6}$; Reaminder 3

3102_6

c. 1100001111

Solution:

Octal: Group in sets of three, from right to left.

$111_2 = 7_{10}$

$001_2 = 1_{10}$

$100_2 = 4_{10}$

$001_2 = 1_{10}$

1417_8

Hexadecimal: Group in sets of four, from right to left.

$1111_2 = 15_{10} = F_{16}$

$0000_2 = 0_{10}$

$0011_2 = 3_{10}$
$30F_{16}$
Base 5: Convert to base 10 then base 5.
$1100001111_2 = 2^9 + 2^8 + 2^3 + 2^2 + 2^1 + 2^0 = 512 + 256 + 8 + 4 + 2 + 1 = 783_{10}$
$783/5 = 156\frac{3}{5}$; Remainder 3
$156/5 = 31\frac{1}{5}$; Remainder 1
$31/5 = 6\frac{1}{6}$; Remainder 1
$6/5 = 1\frac{1}{6}$; Remainder 1
$1/5 = 0\frac{1}{6}$; Remainder 1
11113_5
Base 6: Convert to base 10 then base 6.
$783/6 = 130\frac{3}{6}$; Remainder 3
$130/6 = 21\frac{4}{6}$; Remainder 4
$21/6 = 3\frac{3}{6}$; Remainder 3
$3/6 = 0\frac{3}{6}$; Remainder 3
3343_6

d. 1010110011

Solution:
Octal: Group in sets of three, from right to left.
$011_2 = 3_{10}$
$110_2 = 6_{10}$
$010_2 = 2_{10}$
$001_2 = 1_{10}$
1263_8
Hexadecimal: Group in sets of four, from right to left.
$0011_2 = 3_{10}$
$1011_2 = 11_{10} = B_{16}$
$0010_2 = 2_{10}$
$2B3_{16}$
Base 5: Convert to base 10 then to base 5.
$1010110011_2 = 2^0 + 2^1 + 2^4 + 2^5 + 2^7 + 2^9 = 512 + 128 + 32 + 16 + 2 + 1 = 691_{10}$
$691/5 = 138\frac{1}{5}$; Remainder 1
$138/5 = 27\frac{3}{5}$; Remainder 3
$27/5 = 5\frac{2}{5}$; Remainder 2
$5/5 = 1$; Remainder 0
$1/5 = 0\frac{1}{5}$; Remainder 1
10231_5
Base 6: Convert to base 10 then to base 6.
$691/6 = 115\frac{1}{6}$; Remainder 1
$115/6 = 19\frac{1}{6}$; Remainder 1
$19/6 = 3\frac{1}{6}$; Remainder 1

$3/6 = 0\frac{3}{6}$; Remainder 3

3111_6

10. Assume numbers are 7-bit. What is the range for numbers that can be stored in each representation?

   a. sign/magnitude

   Solution:

   $-63, 63$

   b. 1's complement

   Solution:

   $-63, 63$

   c. 2's complement

   Solution:

   $-64, 63$

   d. unsigned

   Solution:

   $0, 127$

11. Given the following 1's complement values, convert them from 1's complement to 2's complement.

   a. 10011001_2

   Solution:

   10011010_2

   b. 11101101_2

   Solution:

   11101110_2

   c. 01010101_2

   Solution:

   01010101_2

   d. 01001010_2

   Solution:

   01001010_2

12. Given the following 2's complement, values convert them from 2's complement to 1's complement.

   a. 10011001_2

   Solution:

   10011000_2

   b. 11101101_2

Solution:
11101100_2

c. 01010101_2

Solution:
01010101_2

d. 01001010_2

Solution:
01001010_2

13. Given the following sign/magnitude values, convert them from sign/ magnitude to 2's complement

a. 10011001_2

Solution:
11100111_2

b. 11101101_2

Solution:
10010011_2

c. 01010101_2

Solution:
01010101_2

d. 01001010_2

Solution:
01001010_2

14. Encode each of the following numbers as 16-bit sign magnitude, 1's complement, and 2's complement numbers.

a. 220_{10}

Solution:
signed magnitude: 0000000011011100
1's complement: 0000000011011100
2's complement: 0000000011011100

b. 32767_{10}

Solution:
signed magnitude: 0111111111111111
1's complement: 0111111111111111
2's complement: 0111111111111111

c. -309_{10}

Solution:
signed magnitude: 1000000100110101
1's complement: 1111111011001010
2's complement: 1111111011001011

15. In 1's complement, 2's complement, and signed magnitude number format

    a. How many representations for zero are there? ______________________

    Solution:
    1's complement: 2
    2's complement: 1
    Signed Magnitude: 2

    b. Using 6-bits, specify all representations of zero for each format.

    Solution:
    1's complement: 000000 and 111111
    2's complement: 000000
    Signed Magnitude: 000000 and 100000

16. What is the range of values for n-bit two's complement? (Give a generic form)

    Solution:
    $[-2^{n-1}, 2^{n-1} - 1]$

17. Consider the 5-bit binary number 11011_2, what is the base 10 value of this binary representation in the following encodings.

    a. Unsigned Binary

    Solution:
    $2^4 \times 1 + 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 = 16 + 8 + 0 + 2 + 1 = 27$

    27_{10}

    b. Signed Magnitude

    Solution:
    Most Significant Bit is 1, thus negative.
    $2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 = -(8 + 0 + 2 + 1) = -11$

    -11_{10}

    c. One's Complement

    Solution:
    Most Significant Bit is 1, thus negative.
    Flip bits converting into ones complement. 11011 becomes 00100

$2^4 \times 0 + 2^3 \times 0 + 2^2 \times 1 + 2^1 \times 0 + 2^0 \times 0 = -4$

-4_{10}

d. Two's Complement

Solution:

Most Significant Bit is 1, thus negative.

Flip bits converting into ones complement. 00100

Add one converting ones complement to twos complement. 00100 + 00001 = 00101

$2^4 \times 0 + 2^3 \times 0 + 2^2 \times 1 + 2^1 \times 0 + 2^0 \times 1 = -(4 + 1) = -5$

-5_{10}

18. Use 8-bits to express the indicated decimal number in the specified formats below.

a. Sign Magnitude -83_{10} __________ 83_{10} __________

Solution:

11010011_2, 01010011_2

b. One's Complement -83_{10} __________ 83_{10} __________

Solution:

10101100_2, 01010011_2

c. Two's Complement -83_{10} __________ 83_{10} __________

Solution:

10101101_2, 01010011_2

19. Consider the following addition problems for 8-bit 2's complement numbers. What is the result of the calculation? Did overflow/underflow occur?

a. 10101010_2 + 01010101_2

Solution:

11111111_2 No overflow

b. 11101010_2 + 11010101_2

Solution:

10111111_2 Carry, but no overflow

c. 11000001_2 + 01111111_2

Solution:

01000000_2 Carry, but no overflow

d. Under what general conditions will overflow occur?

Solution:

Overflow can occur when adding numbers of the same sign. Overflow can not occur when adding number of opposite sign.

20. Perform the following addition of 6-bit, 1's complement numbers: 110101 + 001110.

Solution:

```
Carry bits:  1 111
               110101
            +  001110
            ---------
             1 000011 (end around carry)
            +       1
            ---------
               000100
```

Final answer: 000100_2

21. The following binary floating-point numbers consist of a sign bit; an excess-64, radix-2 exponent; and a 16-bit fraction. Convert them to "binary"' scientific notation.

   a. 010000000001010100000001

   Solution:
   Dividing the bit representation in to the separate parts: 0 1000000 0001010100000001
   sign = 0, positive number
   exponent = 1000000 = 2^6 = 64
   stored in excess-64, therefore subtract 64-64 = 0
   fraction = 0001010100000001

   $+1.0001010100000001 \times 2^0$

   b. 001111110000001111111111

   Solution:
   Dividing the bit representation in to the separate parts: 0 0111111 0000001111111111
   sign = 0, positive number
   exponent = 0111111 = $2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$ = 63
   stored in excess-64, therefore subtract 63-64 = -1
   fraction = 0000001111111111

   $+1.0000001111111111 \times 2^{-1}$

   c. 010000111000000000000000

   Solution:
   Dividing the bit representation in to the separate parts: 0 1000011 1000000000000000
   sign = 0
   exponent = 1000011 = $2^6 + 2^1 + 2^0$ = 64+2+1 = 67
   stored in excess-64, therefore subtract 67-64 = 3
   fraction = 1000000000000000

$+1.1000000000000000 \times 2^3$

22. Show the binary representation for the following decimal floating point values in IEEE single-precision format.

    a. 7.4

    Solution:

    Convert each "part" to binary: $7 = 111_2$ $0.4 = 01100110011\ldots_2$ $111.011001100110011 * 2^0$
    normalize the number by moving the binary point
    $1.11011001100110011 * 2^2$

    positive number, therefore sign bit = 0
    exponent = 2, stored in excess-127, therefore add 2+127 = 129. Convert 129 to binary, 10000001
    fraction = 11011001100110011001100
    $01000000111011001100110011001100_2$

    b. -13/64

    Solution:

    -13/64 = -0.203125 in binary = -0.001101_2 $-0.001101_2 * 2^0$
    normalize the number by moving the binary point
    $-1.101_2 * 2^{-3}$

    positive number, therefore sign bit = 1
    exponent = -3, stored in excess-127, therefore add -3+127 = 124. Convert 124 to binary, 01111100
    fraction = 10100000000000000000000
    $10111110010100000000000000000000_2$

    c. 6.125

    Solution:

    Convert each "part" to binary: $6 = 110_2$ $0.125 = 0.001_2$ $110.001 * 2^0$ normalize the number by moving the decimal place
    $1.10001 * 2^2$

    positive number, therefore sign bit = 0
    exponent = 2, stored in excess-127, therefore add 2+127 = 129. Convert 129 to binary, 10000001
    fraction = 10001000000000000000000
    $0100000011000100000000000000000_2$

23. Assume the hexadecimal numbers are IEEE single-precision numbers. Write their values in base 2 scientific notation.

    a. B54B7F10

    Solution:

    to binary: 1011 0101 0100 1011 0111 1111 0001 0000
    Divide into fields: 1 01101010 10010110111111100010000

    sign bit = 1, therefore negative number
    exponent = 01101010, stored in excess-127, therefore 106-127 = -21
    fraction = 10010110111111100010000

    $-1.10010110111111100010000_2 \times 2^{-21}$

b. 43D00000

Solution:

to binary: 0100 0011 1101 0000 0000 0000 0000 0000
Divide into fields: 0 10000111 10100000000000000000000

sign bit = 0, therefore positive number
exponent = 10000111, stored in excess-127, therefore 135-127 = 8
fraction = 10100000000000000000000

$1.10100000000000000000000_2 \times 2^8$ (value is 416_{10})

c. C20C0000

Solution:

to binary: 1100 0010 0000 1100 0000 0000 0000 0000
Divide into fields: 1 10000100 00011000000000000000000

sign bit = 1, therefore negative number
exponent = 10000100, stored in excess-127, therefore 132-127 = 5
fraction = 00011000000000000000000

$-1.00011000000000000000000_2 \times 2^5$ (value is -35_{10})

d. 42055555

Solution:

to binary: 0100 0010 0000 0101 0101 0101 0101 0101
Divide into fields: 0 10000100 00001010101010101010101

sign bit = 1, therefore negative number
exponent = 10000100, stored in excess-127, therefore 132-127 = 5
fraction = 00001010101010101010101

$1.00001010101010101010101_2 \times 2^5$ (value is $-33.333..._{10}$)

24. Convert the following values to their IEEE single precision encodings.

a. $3.22_{10} \times 2^{13}$

Solution:

3 to binary is 11_2
The fractional portion is
0.22×2 = 0.44 Rem 0
0.44×2 = 0.88 Rem 0
0.88×2 = 1.76 Rem 1
0.76×2 = 1.52 Rem 1
0.52×2 = 1.04 Rem 1
0.04×2 = 0.08 Rem 0
0.08×2 = 0.16 Rem 0

$0.16 \times 2 = 0.32$ Rem 0
$0.32 \times 2 = 0.64$ Rem 0
$0.64 \times 2 = 1.28$ Rem 1
$0.28 \times 2 = 0.56$ Rem 0
$0.56 \times 2 = 1.12$ Rem 1
$0.12 \times 2 = 0.24$ Rem 0
$0.24 \times 2 = 0.48$ Rem 0
$0.48 \times 2 = 0.96$ Rem 0
$0.96 \times 2 = 1.92$ Rem 1
$0.92 \times 2 = 1.84$ Rem 1
$0.84 \times 2 = 1.68$ Rem 1
$0.68 \times 2 = 1.36$ Rem 1
$0.36 \times 2 = 0.72$ Rem 0
$0.72 \times 2 = 1.44$ Rem 1
Repeats
$11.00111000010100011110 10_2 \times 2^{13}$
Normalize by moving decimal place 1 digit to the left $1.10011100001010001111010 \times 2^{14}$
positive number, therefore sign bit is 0
Exponent is 14 stored in excess-127, 14+127 = 141. 141 in binary is 10001101
Drop the "phantom" 1 to the left of the decimal place.

$01000110110011100001010001111010$

b. $8.123_{10} \times 2^{-14}$

Solution:

8 to binary is 1000_2
The fractional portion is
$0.123 \times 2 = 0.246$ Rem 0
$0.246 \times 2 = 0.492$ Rem 0
$0.492 \times 2 = 0.984$ Rem 0
$0.984 \times 2 = 1.968$ Rem 1
$0.968 \times 2 = 1.936$ Rem 1
$0.936 \times 2 = 1.872$ Rem 1
$0.872 \times 2 = 1.744$ Rem 1
$0.744 \times 2 = 1.488$ Rem 1
$0.488 \times 2 = 0.976$ Rem 0
$0.976 \times 2 = 1.952$ Rem 1
$0.952 \times 2 = 1.904$ Rem 1
$0.904 \times 2 = 1.808$ Rem 1
$0.808 \times 2 = 1.616$ Rem 1
$0.616 \times 2 = 1.232$ Rem 1
$0.232 \times 2 = 0.464$ Rem 0
$0.464 \times 2 = 0.928$ Rem 0
$0.928 \times 2 = 1.856$ Rem 1
$0.856 \times 2 = 1.712$ Rem 1
$0.712 \times 2 = 1.424$ Rem 1
$0.424 \times 2 = 0.848$ Rem 0

never repeats! but we only have space for 23 digits in the representation.

$1000.00011111011111001110_2 \times 2^{-14}$

Normalize by moving decimal place 3 digits to the left $1.00000011111011111001110 \times 2^{-11}$

positive number, therefore sign bit is 0

Exponent is -11 stored in excess-127, -11+127 = 116. 116 in binary is 01110100

Drop the "phantom" 1 to the left of the decimal place.

0 01110100 00000011111011111001110