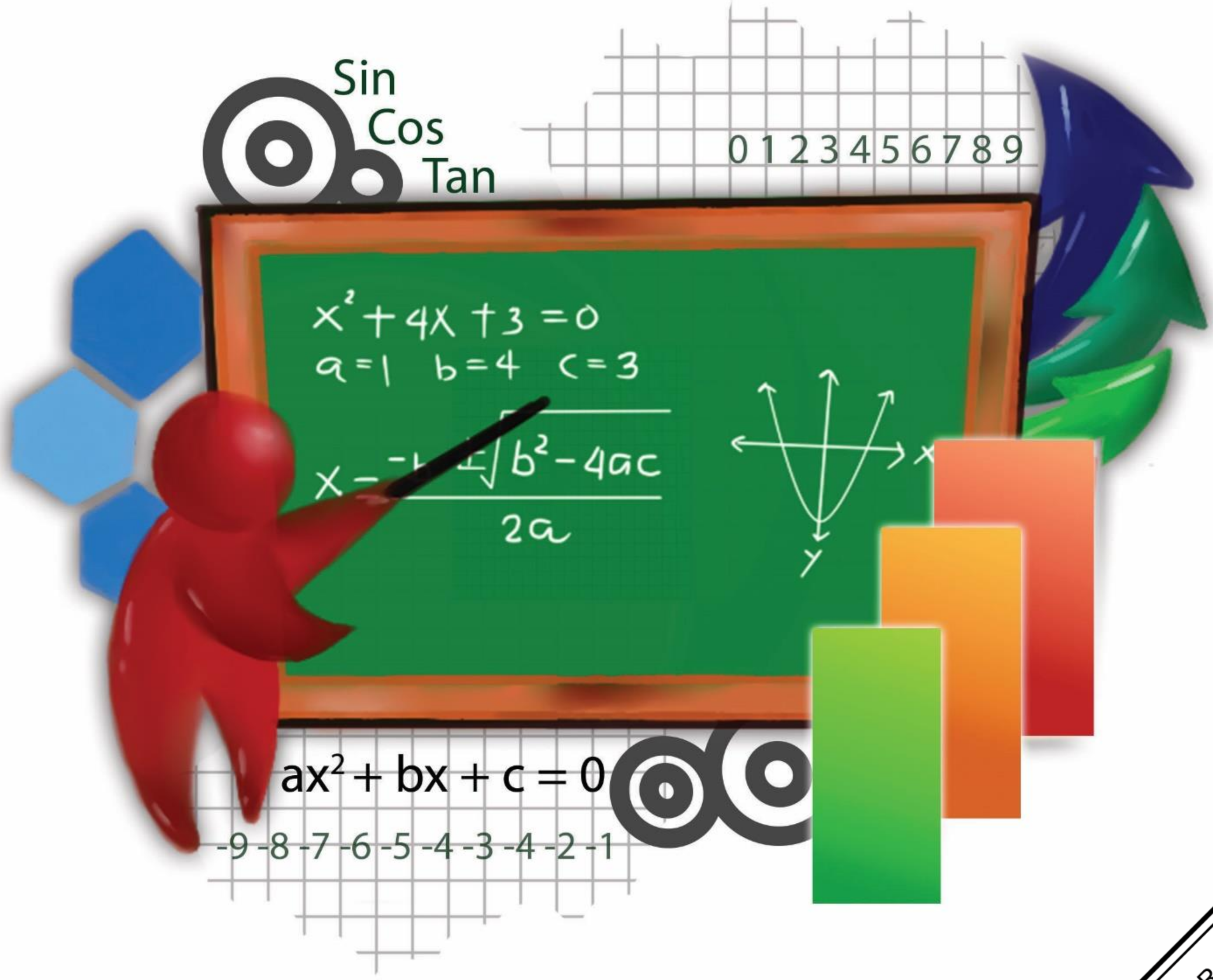
9

Mathematics

Quarter 1 – Module 1

Quadratic Equation



Mathematics – Grade 9
Quarter 1 – Module 1: Quadratic Equation

Development Team of the Module

Developer/Compiler:
IAN C. LEGASPINO – Master Teacher I, Don Sergio Osmena Sr. MNHS
LEONARDO P. CABASAN – Teacher III, Don Sergio Osmena Sr. MNHS
NISSY D. BORGA – Teacher III, Abellana National School
MIGUELINA B. JUARIZA – Teacher III, Ramon Duterte MNHS
NIÑA A. MARTINEZ – Teacher III, Mabolo NHS Night
JOHN LLOYD LISING – Teacher III, Talamban NHS
KELLIE ANN O. YNOT – Teacher I, Don Vicente Rama MNHS

Content Editors:
ROSEMARIE O. NOVABOS EdD – School Head, Mabolo NHS
NATHANAEL M. FLORES - School Head, CCNSHS

Language Editors:
MARIA FE S. MACUL – School Head, Buhisan Night HS
ELEANOR D. GALLARDO – Assistant Principal, Quiot NHS

Reviewer:
MARITES V. PATIÑO EdD – EPSvr, Mathematics

Management Team:
RHEA MAR A. ANGTUD EdD - Schools Division Superintendent
BERNADETTE A. SUSVILLA EdD, CESO VI – Assistant SDS
GRECIA F. BATALUNA – Chief EPS, CID
MARITES V. PATIÑO EdD - EPSvr, Mathematics
VANESSA L. HARAYO – EPSvr, LRMS

Printed in the Philippines by DepEd Cebu City Division, ROVII
Office Address : New Imus Road, Day-as, Cebu City
Telefax : (032) 255-1516 / (032) 253-9095
E-mail Address : cebu.city@deped.gov.ph

9

Mathematics

Quarter 1 – Module 1
Quadratic Equation

Introductory Message

Welcome to the **Mathematics 9** on **Quadratic Equation**!

This module was designed to provide you with opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

	***What I Know*** *(Pre-Test)*	This part includes an activity that aims to check what you already know about the lesson to take.
	***What I Need to Know*** *(Objectives)*	This will give you an idea of the skills or competencies you are expected to learn in the module.
	***What's In*** *(Review/ Springboard)*	This is a brief drill or review to help you link the current lesson with the previous one.
	***What's New*** *(Presentation of the Lesson)*	In this portion, the new lesson will be introduced to you in various ways; a story, a song, a poem, a problem opener, an activity or a situation.
	***What is It*** *(Discussion)*	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
	***What's More*** *(Application)*	This section provides activities which will help you transfer your new knowledge or skill into real life situations or concerns.
	***What I Need To Remember*** *(Generalization)*	This includes key points that you need to remember.
	***What I Can Do*** *(Enrichment Activities)*	This comprises activities for independent practice to solidify your understanding and skills of the topic.
	***Assessment*** *(Post Test)*	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
	***Answer Key***	This contains answers to the following: • What I Know • What's In • What's More

At the end of this module you will also find:

References	This is the list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in this module.
3. Read the instructions carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

About the Module

This module was designed and written with you in mind. It is here to help you master about Sets. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is divided into five lessons, namely:

- Lesson 1 – Introduction to Quadratic Equation
- Lesson 2 – Roots of the Quadratic Equation using Extracting Square Root and Quotient Rule
- Lesson 3 – Roots of the Quadratic Equation using Factoring
- Lesson 4 – Roots of the Quadratic Equation using Completing the Square
- Lesson 5 – Roots of the Quadratic Equation using Quadratic Formula

After going through this module, you are expected to:

- illustrate quadratic equation
- solve quadratic equation: by a) extracting square root; b) factoring; c) completing the square; and d) quadratic formula.

What I Know (Pre-Test)

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper.

1. What are the roots of $(y + 3)^2 = 36$?
   A. 3 and – 9 C. – 3 and 9
   B. 6 and – 3 D. – 6 and 3
2. What are the roots of the quadratic equation $x^2 - 14x + 24 = 0$?
   A. 6 and – 4 C. 12 and – 2
   B. 6 and 4 D. 12 and 2
3. What are the binomial factors of $x^2 - 20x + 100$?
   A. $(x - 5)(x + 4)$ C. $(x - 10)(x - 10)$
   B. $(x - 5)(x - 4)$ D. $(x - 10)(x + 10)$
4. What are the solutions of $b^2 + 4b = 21$?
   A. 4 and – 6 C. 7 and – 3
   B. – 4 and 6 D. – 7 and 3
5. One of the roots of $b^2 - 2b - 35 = 0$ is $b_1 = 7$, what is the other root or b_2?
   A. – 5 C. 3
   B. 5 D. – 3
6. What are the solutions of the quadratic equation $3x^2 - 108 = 0$?
   A. 12 and – 12 C. 8 and – 8
   B. 9 and – 9 D. 6 and – 6
7. What is the standard form of the quadratic equation $3x(4 - x) = 15$?
   A. $12x - 3x^2 = 12$ C. $0 = 3x^2 - 12x - 15$
   B. $3x^2 - 12x + 15 = 0$ D. $12x^2 - 3x - 15 = 0$
8. How do we write the quadratic equation whose roots are 7 and – 3?
   A. $x^2 - 4x + 21 = 0$ C. $x^2 + 4x - 21 = 0$
   B. $x^2 + 4x + 21 = 0$ D. $x^2 - 4x - 21 = 0$
9. What is the first term of this quadratic equation $4x^2 + 3x - 2 = 0$?
   A. -2 C. $3x$
   B. $4x^2$ D. x^2
10. Which of the following is a quadratic equation?
   A. $4b - 7 = 6$ C. $7x^2 + 3x \geq 9$
   B. $9x^2 + 3x - 1 < 5$ D. $y^2 + 5y - 18 = 0$
11. How many real roots does the quadratic equation $x^2 - 121 = 0$ have?
   A. 0 C. 2
   B. 1 D. 3
12. What must be added to both sides in order to complete the square in the equation $x^2 + 8x + (\,) = 9 + (\,)$?
   A. 9 C. 12
   B. 10 D. 16
13. What is the value of coefficient b in this equation $6x^2 - 8x + 10 = 0$?
   A. 10 C. 6
   B. – 8 D. 0
14. What is the leading coefficient in this equation $10x^2 - 12x + 15 = 0$?
   A. 12 C. 10
   B. – 10 D. – 12
15. What is the linear term of the given equation $3x^2 - 7x + 4 = 0$?
   A. $3x^2$ C. 4
   B. – 7x D. 0

Lesson 1	Introduction to Quadratic Equation

What I Need To Know

At the end of this lesson, you are expected to:

- illustrate and identify quadratic equations;
- write quadratic equations in standard form;
- determine the components of quadratic equation.

What's In

❖ ***Flashback***

To recall our past lesson in grade 8, first quarter, you were taught how to get the product of binomials of different terms and similar terms like those shown in the table below wherein most of the resulting product is done through FOIL method.

Product of Binomials of Different Terms	**Concept Representation**
Product of Binomials with same first term and different second terms	$(a+b)(a+c)=a^2+ac+ab+bc$ $(a+b)(a-c)=a^2-ac+ab-bc$

Special Products	**Concept Representation**
Sum and Difference of Same Terms	$(a+b)(a-b)=a^2-b^2$
Square of a Sum of Two Terms	$(a+b)^2=a^2+2ab+b^2$
Square of a Difference of Two Terms	$(a-b)^2=a^2-2ab+b^2$
Cube of a Sum of Two Terms	$(a+b)^3=a^3+3a^2b+3ab^2+b^3$
Cube of a Difference of Two Terms	$(a-b)^3=a^3-3a^2b-3ab^2+b^3$

What's New

❖ ***Let's Try This One!***

Instructions: Get the product of the following polynomials.

1. $(y+4)(y+3)$ ____________________
2. $(r+3)(r-4)$ ____________________
3. $(c-3)^2$ ____________________
4. $(m+6)(m-6)$ ____________________
5. $(z+4)^2$ ____________________
6. $(d-2)^3$ ____________________

What have you observed with each resulting product?
Did it have the same number of terms?
Which of the following products have the same number of terms?

What Is It

❖ ***Let's Brighten Up***

When we get the product of polynomials, specifically binomials, we usually arrived at getting three terms or commonly called ***trinomials***. And, based on the activity we have, most of the answers are trinomials, right? Except for numbers 2 and 6 which have a different answer among other items given.

These trinomials are special ones because of its distinct properties which is shown in numbers 1, 3, 4 and 5. And what are these properties that you have observed?

You're right! These common properties that you have observed in the resulting product or quadratic trinomials are namely as follows;

- ax^2 as its first term, quadratic term
- bx as its second term, linear term
- c as its third term, constant

And these properties are quadratic equation.

Quadratic equation is a polynomial equation in one variable expressed in the form $ax^2 + bx + c = 0$ where a, b and c are real numbers such that $a \neq 0$. Also, named as second-degree equation because its highest exponent or degree is two.

But, how about quadratic equations that has only two terms? Can we still consider them as quadratic equation? Yes of course, if it contains the quadratic term.

❖ ***How should I do it?***

Example A:

Instructions: Identify which of the following equations are quadratic?

1. $x^2 + 5x = 24$
2. $8x^2 - 8 = 0$
3. $x - 3y^2 = 0$
4. $\sqrt{x} + 4 = 7$
5. $\frac{x}{x+2} = \frac{1}{x}$

Solutions:

1. QE, since the given equation is in degree 2 and the terms are complete.
2. QE, since it contains the quadratic term.
3. Not, because there are 2 variables involved
4. Not, because there is no quadratic term, rather it contains rational exponent
5. QE, using proportion, it could give us the equation $x^2 - x - 2 = 0$

Example B

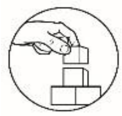
Instructions: Write the following in standard form $ax^2 + bx + c = 0$. Identify the quadratic, linear and the constant term of each equation. Then, identify the value of a, b and c.

1. $9x + 28 = 9x^2$
2. $7x^2 - 20 = 6x$
3. $x^2 = 3x$
4. $x^2 - 4(2 + x) = 13$
5. $6(1 - x) + 2 = 3x^2 - 17$

Solutions:

- *By rearranging the terms in descending powers of x, we can identify a, b and c for numbers 1 – 3.*
- *Numbers 4 and 5 will be rewritten also in standard form by simplifying the operations using PEMDAS rule*

Equation	*quadratic term*	*linear term*	*constant term*	*Values of a, b and c*
1. $0 = 9x^2 - 9x - 28$	$9x^2$	$-9x$	-28	$a = 9, b = -9, c = -28$
2. $7x^2 - 6x - 20 = 0$	$7x^2$	$-6x$	-20	$a = 7, b = -6, c = -20$
3. $x^2 - 3x = 0$	x^2	$-3x$	0	$a = 1, b = -3, c = 0$
4. $x^2 - 4(2 + \mathrm{x}) = 13$ $x^2 - 8 - 4x - 13 = 0$ $x^2 - 4x - 21 = 0$	x^2	$-4x$	-21	$a = 1, b = -4, c = -21$
5. $6(1 - x) + 2 = 3x^2 - 17$ $6 - 6x + 2 = 3x^2 - 17$ $0 = 3x^2 + 6x - 17 - 2 - 6$ $0 = 3x^2 + 6x - 25$	$3x^2$	$6x$	-25	$a = 3, b = 6, c = -25$



What's More

Activity 1.1: NOW IT'S YOUR TURN!

A. *Instructions*: Tell whether the equation is quadratic or not. Justify your answers.

1. $x^2 + 5x - 4 = 0$
2. $3x - 8 = x^2$
3. $x + 2 = \frac{5-x}{x}$
4. $4(x - 3) + 5 = 3x - 2$
5. $\frac{2x}{6} = \frac{x-2}{3x}$

B. Write the following in standard form $ax^2 + bx + c = 0$ and identify a, b and c.

1. $15 - 2x^2 = 3x$
2. $3x - 4x^2 = 8$
3. $(x + 2)^2 = 2x - 1$
4. $5 - (x + 3) = 4x^2$
5. $x^2 - 3(x + 4) = -2x$

What I Need to Remember

- Quadratic equation is a polynomial equation in one variable expressed in the form $\mathrm{ax}^2 + \mathrm{bx} + \mathrm{c} = 0$ where a, b and c are real numbers such that $\mathrm{a} \neq 0$.
- Equations can still be considered quadratic even if it has only two terms if it contains the quadratic term ax^2.
- Equations in standard form are always reversible, either the zero is on the left or right side, if the first term (quadratic term) shall be written in positive

Lesson 2	Solving Quadratic Equation by Extracting Square Root and Quotient Rule

What I Need To Know

At the end of this lesson, you are expected to:

- determine the factors of a number that is not a perfect square;
- solve quadratic equation by extracting square root.

What's In

❖ ***Let's Do the Recall***

Study the examples below on how the following equations have been transformed into its standard form and try to observe the different solutions of each item.

1. $5 - x(x-3) = 0$	2. $3x(x+2) - 15x = 0$	3. $5x(x-4) + 7x = 25 - 13x$
Solution:	***Solution:***	***Solution:***
$5 - x^2 + 3x = 0$	$3x^2 + 6x - 15x = 0$	$5x^2 - 20x + 7x = 25 - 13x$
$(-x^2 + 3x + 5 = 0)(-1)$	$\mathbf{3x^2 - 9x = 0}$	$5x^2 - 13x + 13x - 25 = 0$
$\mathbf{x^2 - 3x - 5 = 0}$	Then, the terms are:	$5x^2 - 25 = 0$
Then, the terms are:	$3x^2 = quadratic\ term$	$5x^2 = quadratic\ term$
$x^2 = quadratic\ term$	$-9x = linear\ term$	$0x = linear\ term$
$-3x = linear\ term$	$0 = constant\ term$	$-25 = constant\ term$
$-5 = constant\ term$		

What's New

Based on the samples given above, quadratic equations are sometimes incomplete but still considered quadratic if it contains the **quadratic term** $\boldsymbol{ax^2}$ paired with the **linear term, *bx*,** or **constant term, *c*,** like those seen on numbers 2 and 3, respectively.

So, how are we going to solve for the roots or solutions of these two incomplete quadratic equations? What method/s are we going to use?

What Is It

❖ ***How should I do it?***

Since there are two samples of incomplete quadratic equations above, we will solve it separately and we will use the ***square root property*** on number 3.

Why square root property and not any other method?

We can only use the ***square root property*** when the equation contains ONLY the first term or **quadratic term, ax^2,** and third term or **constant term, c,** in symbols, $ax^2 \pm c = 0$.

Square Root Property
If $x^2 = k$, and k is a non-negative integer, then $x = \pm\sqrt{k}$

Since there are only 2 terms given, each term will stay on both sides of the equation before we apply the square root property.

In case that the quadratic term, ax^2, has a coefficient that is NOT a perfect square then, eliminate first the coefficient by dividing same number to both sides before extracting the root of both sides and factor out the radicand to simplify your answer like what is shown in sample 2, in symbols,

$$ax^2 \pm c = 0$$

$$\frac{ax^2}{a} = \pm\frac{c}{a}$$

$$\sqrt{x^2} = \pm\sqrt{\frac{c}{a}}$$

$$x = \pm\sqrt{\frac{c}{a}}$$; where a and c real numbers such that $a \neq 0$

Examples using Square Root Property

Solve for the roots of each quadratic equation using square root property.

1. $x^2 - 64 = 0$ $\sqrt{x^2} = \sqrt{64}$ $x = \pm 8$ Therefore, the roots of the quadratic equation are 8 and – 8.	2. $5x^2 - 40 = 0$ $\frac{5x^2}{5} = \frac{40}{5}$ $\sqrt{x^2} = \sqrt{8}$ $x = \pm 2\sqrt{2}$ Therefore, the roots are $2\sqrt{2}$ and $-2\sqrt{2}$
3. $(x+2)^2 = 9$ $\sqrt{(x+2)^2} = \sqrt{9}$ $x + 2 = \pm 3$ $x = -2 \pm 3$ $x_1 = -2 + 3 = 1$ $x_2 = -2 - 3 = -5$ Therefore, the roots of the quadratic equation are 1 and – 5.	4. $4x^2 - 256 = 0$ $\sqrt{4x^2} = \sqrt{256}$ $2x = \pm 16$ $x = \pm 8$ Therefore, the roots of the quadratic equation are 8 and – 8

Let Me Explain This One
➢ For items 1 and 4, both coefficients of the first and third terms are perfect square, so it can be extracted easily. ➢ For item number 2, if the coefficient of the first term ax^2 is NOT a perfect square, you just divide both sides by the same number before extracting the root of both sides and factor out the radicand to simplify your answer. ➢ For item number 3, the base is a binomial, extract the root on both sides. To find the value of x, transfer the number to the right side of the equation, thus the sign of the number is changed from 2 to – 2. This is made possible using Subtraction Property of Equality. Then, combine it with the number with a double sign on the right side of the equation.

Another way to solve incomplete quadratic equation is ***quotient rule*** which can be used when the **quadratic term** $\boldsymbol{ax^2}$ and **linear term** $\boldsymbol{bx}$ are present ONLY, in symbols, $ax^2 \pm bx = 0$.

Quotient Rule (Law of Exponent)
If $\frac{x^m}{x^n} = x^{m-n}$ where m and n are positive integers, and $m > n$

Since there are only 2 terms given, each term will stay on both sides of the equation for us to apply the quotient rule. Then, divide both sides by the numerical coefficient and same variable as that of the quadratic term, ax^2, and simplify, in symbols,

$$ax^2 \pm bx = 0$$

$$\frac{ax^2}{ax} = \pm\frac{bx}{ax}$$

$$x = \pm\frac{b}{a}\ ;\ \text{where } a \text{ and } b \text{ are integers such that } a \neq 0$$

Examples using Quotient Rule

Solve for the roots of each quadratic equation using quotient rule.

1. $5x^2 = 125x$ $\frac{5x^2}{5x} = \frac{125x}{5x}$ $x = 25$ Therefore, the root is 25.	2. $4x^2 - 4x = 32x$ $4x^2 = 32x + 4x$ $\frac{4x^2}{4x} = \frac{36x}{4x}$ $x = 9$ Therefore, the root is 9.
3. $3x^2 + 8x = 35x$ $3x^2 = 35x - 8x$ $\frac{3x^2}{3x} = \frac{27x}{3x}$ $x = 9$ Therefore, the root is 9.	4. $2b^2 - 4b = 16b$ $2b^2 = 16b + 4b$ $\frac{2b^2}{2b} = \frac{20b}{2b}$ $b = 10$ Therefore, the root is 10.

What's More

Activity 1.2: NOW IT'S YOUR TURN!

A. Solve for the roots of the following quadratic equations using square root property. Show the solutions and encircle the final answer.

1. $x^2 = 225$
2. $4x^2 - 196 = 0$
3. $3(x + 2)^2 = 48$
4. $9x^2 = 324$
5. $(y + 3)^2 = 36$

B. Solve the following equations using quotient rule. Show your complete solution.

1. $12x^2 = 48x$
2. $3x - 12x^2 = 0$
3. $8r^2 - 15r = 25r$
4. $120x - 12x^2 = 0$
5. $210j^2 + 5j = -5j$

What I Need To Remember

- Square root property will only be used in solving quadratic equation in one variable if and only if the first and third terms of the quadratic equation are given, in symbols, $ax^2 \pm c = 0$.
- In case that the quadratic term, ax^2, has a coefficient that is NOT a perfect square then, eliminate first the coefficient by dividing same number both sides before extracting the root of both sides and factor out the radicand to simplify your answer.
- Quotient rule is used only when first and second terms are given, in symbols, $ax^2 \pm bx = 0$

Lesson 3	Solving Quadratic Equation by Factoring

Lesson 3
Solving Quadratic Equation by Factoring

What I Need To Know

At the end of this lesson, you are expected to:

- determine the binomial factors of the quadratic equation;
- solve quadratic equation by factoring.

What's In

❖ ***Let's Do the Recall***

Looking back at the examples we had in Lesson 2, items 2 and 3 were solved separately using quotient rule and square root property. How about item 1, what method are we going to use to look for its roots?

1. $5 - x(x-3) = 0$	2. $3x(x+2) - 15x = 0$	3. $5x(x-4) + 7x = 25 - 13x$
Solution:	***Solution:***	***Solution:***
$5 - x^2 + 3x = 0$	$3x^2 + 6x - 15x = 0$	$5x^2 - 20x + 7x = 25 - 13x$
$(-x^2 + 3x + 5 = 0)(-1)$	$\mathbf{3x^2 - 9x = 0}$	$5x^2 - 13x + 13x - 25 = 0$
$\mathbf{x^2 - 3x - 5 = 0}$	Then, the terms are:	$5x^2 - 25 = 0$
Then, the terms are:	$3x^2 = quadratic\ term$	$5x^2 = quadratic\ term$
$x^2 = quadratic\ term$	$-9x = linear\ term$	$0x = linear\ term$
$-3x = linear\ term$	$0 = constant\ term$	$-25 = constant\ term$
$-5 = constant\ term$		

What's New

❖ ***Fill My Window***

Let's try this!

*Here's a window filled with two numbers on the upper window. For you to fill in the lower windows, think of two numbers that when **added both**, we get the same number found on the **upper left** window and when **multiplied to each other**, we get the same number found on the **upper right** window.*

10	21
?	?

What Is It

So, what is your answer in the *Fill My Window Activity*? Did you get the correct pair of numbers? How did you do it?

As you can see, the roots of the complete quadratic equation can be solved by using a similar concept like that of a *Punnett Square* in science as to what we use in the activity above.

And, we will use the *Punnett Square* concept in determining the roots through its binomial factors of a quadratic trinomial, $ax^2 + bx + c = 0$. This process of getting the binomial factors through integers using the sum and product rule, in symbols,
$x^2 - (sum)x + (pro) = 0$ is called ***factoring.***

> ***Factoring Method when $a = 1$***
>
> If $ax^2 + bx + c = 0$ where $b = sum$ and $c = pro$, we let $b = m + n$ and $c = mn$ such that m and n are integers that when added and multiplied will derive the same **sum** and **product**, then the binomial factors are $(x \pm m)(x \pm n)$. By zero product property, $(x \pm m)(x \pm n) = 0$, then $x_1 = \pm m$ and $x_2 = \pm n$.

To clearly visualize the said concept above, let us look at these examples below.

Examples for factoring quadratic equation when $a = 1$

Given the quadratic trinomials, determine the roots using factoring method.

Quadratic Equation $ax^2 + bx + c = 0$	Sum $b = m + n$	Pro $c = mn$	Integers m and n	Binomial Factors	Roots $x_1 = \pm m$ & $x_2 = \pm n$.
1. $x^2 + 10x + 16 = 0$	10	16	**2 and 8**	$(x + 2)(x + 8)$	$x_1 = -2 \quad x_2 = -8$
2. $x^2 - 10x + 24 = 0$	-10	24	**-4 and -6**	$(x - 4)(x - 6)$	$x_1 = 4 \quad x_2 = 6$
3. $x^2 - 6x - 27 = 0$	-6	-27	**-9 and 3**	$(x - 9)(x + 3)$	$x_1 = 9 \quad x_2 = -3$
4. $x^2 + 10x - 75 = 0$	10	-75	**15 and -5**	$(x + 15)(x - 5)$	$x_1 = -15 \quad x_2 = 5$

Here's how we do it!

1. Identify the sum and product by the value of b and c.
2. Think of two numbers that when added and multiplied will derive at the same sum and product.
3. Then, use these integers in writing the binomial factors following the sign (positive or negative) it contains.
4. By zero product property, determine the roots of the equation.

But how about if the numerical coefficient of the quadratic term, ax^2, denoted by a, is greater than 1? How are we going to factor it into two binomials?
So, how are we going to do it this time?

> ***Factoring Method when $a > 1$***
>
> If $ax^2 + bx + c = 0$ when factored whose $a > 1$, we do the following steps:
> 1. get the product of the coefficients of first and last term, that is $pro = ac$;
> 2. get the factors of the product, ac, in such a way that the sum, bx, could be derived;
> 3. rewrite the equation by copying the first and last term and insert the factors of the product, ac;
> 4. group the polynomial into two binomials;
> 5. factor the grouped polynomials to achieve same binomial factors;
> 6. regroup the polynomials;
> 7. using zero product property, find the roots of the QE.

To clearly visualize the said concept above, let us look at these examples below.

Examples for factoring quadratic equation when $a > 1$

Given the quadratic trinomials, determine the roots using factoring method.

1. $2x^2 - 11x + 15 = 0$

$ac = (2)(15) = 30$	step 1
$30 = \{(2,15), (3,10), (-6,-5)\}$	step 2
$2x^2 - 6x - 5x + 15 = 0$	step 3
$(2x^2 - 6x) - (5x - 15) = 0$	step 4
$2x(x-3) - 5(x-3) = 0$	step 5
$(2x-5)(x-3) = 0$	step 6
$2x - 5 = 0 \; and \; x - 3 = 0$	step 7
$2x = 5 \; and \; x = 3,$	

Therefore, the roots are $\boldsymbol{x_1 = \frac{5}{2} \; and \; x_2 = 3}$

2. $2x^2 - 27x + 36 = 0$

$ac = (2)(36) = 72$	step 1
$72 = \{(-24,-3), (4,18), (6,12)\}$	step 2
$2x^2 - 24x - 3x + 36 = 0$	step 3
$(2x^2 - 24x) - (3x - 36) = 0$	step 4
$2x(x-12) - 3(x-12) = 0$	step 5
$(2x-3)(x-12) = 0$	step 6
$2x - 3 = 0 \; and \; x - 12 = 0$	step 7
$2x = 3 \; and \; x = 12,$	

Therefore, the roots are $\boldsymbol{x_1 = \frac{3}{2} \; and \; x_2 = 12}$

What's More

Activity 1.3: NOW IT'S YOUR TURN!

A. Solve for the roots using factoring. Write your answers on your notebook

Quadratic Equation $ax^2 + bx + c = 0$	Sum $b = m + n$	Pro $c = mn$	Integers m and n	Binomial Factors	Roots $x_1 = \pm m$ & $x_2 = \pm n$.
1. $x^2 + 10x + 25 = 0$					
2. $m^2 - 8m - 20 = 0$					
3. $x^2 - 14x + 24 = 0$					
4. $a^2 + 4a - 21 = 0$					
5. $y(y-7) = 44$					

B. Solve each quadratic equation by factoring. Show your solutions and BOX the final answer on your activity notebook.

1.) $3x^2 + 12x - 15 = 0$

2.) $2x^2 + 23x - 25 = 0$

3.) $3x^2 - 5x = -12$

4.) $2(3x^2 - 1) = 11x$

What I Need to Remember

- Roots of the quadratic trinomial can be determined using a *Punnett Square* technique wherein the identified integers will take the opposite sign as final solution / roots.
- While factoring quadratic equation whose value of a is greater than 1, just follow the 6 steps.

Lesson 4 Solving Quadratic Equation by Completing the Square

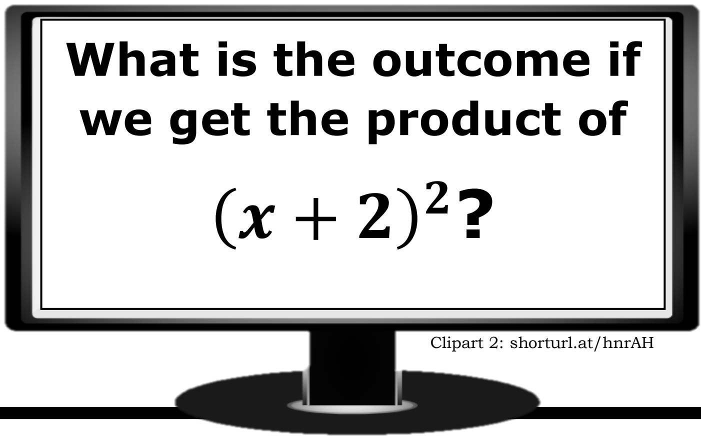
What I Need To Know

At the end of this lesson, you are expected to:

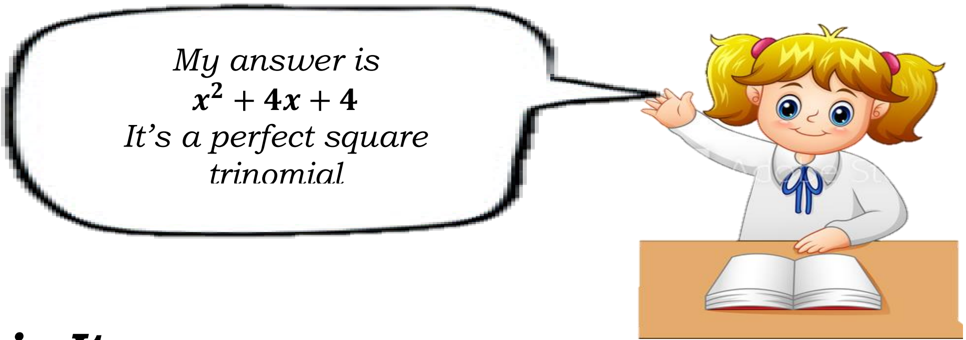
- determine the third term of the perfect square trinomial;
- solve quadratic equation by completing the square.

What's In

Clipart: shorturl.at/ghlW9



Clipart 2: shorturl.at/hnrAH



Clipart 3: shorturl.at/fqLM6

What is It

Her answer was right! But how are we going to connect that concept in our next method to use?

Here's how…

If the quadratic equation $ax^2+bx+c=0$ is **NOT factorable** into two binomials, roots can be derived by using **completing the square method**. This process involves getting the 3rd term of the perfect square trinomial by squaring the half of the coefficient of the middle term, bx, in symbols,

$$ax^2+bx+\left(\frac{b}{2}\right)^2=-c+\left(\frac{b}{2}\right)^2$$

Completing the Square Method when $a = 1$

If $ax^2 + bx + c = 0$ is not factorable whose $a = 1$, we do the following:

1. transfer the 3rd term to the other side of the equation and add 3rd term of the Perfect Square Trinomial on both sides of the equation;
2. factor the Perfect Square Trinomial on the left and add the values on the right side;
3. extract the root of both sides of the equation;
4. determine the roots by combining the numbers in the equation.

Examples for completing the square when $a = 1$

Instructions: Given the quadratic trinomials, determine the roots using completing the square method.

1. $x^2 - 3x - 28 = 0$	
$x^2 - 3x + \frac{9}{4} = 28 + \frac{9}{4}$	Step 1
$(x - \frac{3}{2})(x - \frac{3}{2}) = \frac{112 + 9}{4}$	Step 2
$\sqrt{\left(x - \frac{3}{2}\right)^2} = \sqrt{\frac{121}{4}}$	Step 3
$x - \frac{3}{2} = \pm\frac{11}{2}$	
$x = \frac{3}{2} \pm \frac{11}{2},$	Step 4
Therefore, the root are $\boldsymbol{x_1 = 7 \text{ and } x_2 = -4}$	

2. $x^2 + 8x + 12 = 0$	
$x^2 + 8x + 16 = -12 + 16$	Step 1
$(x + 4)(x + 4) = 4$	Step 2
$\sqrt{(x + 4)^2} = \sqrt{4}$	Step 3
$x + 4 = \pm 2$	
$x = -4 \pm 2$	Step 4
Therefore, the roots are $\boldsymbol{x_1 = -2 \text{ and } x_2 = -6}$	

What's More

Activity 1.4: NOW IT'S YOUR TURN!

Instructions:

A. Supply the missing term to complete the perfect square trinomial.

1. $x^2 - 16x +$ ____
2. $x^2 + 7x +$ ____
3. $x^2 - 20x +$ ____
4. $x^2 - 8x +$ ____
5. $x^2 + 3x +$ ____

B. Solve each quadratic equation using completing the square method. Show your complete solution on your activity notebook.

1. $x^2 - 8x + 12 = 0$
2. $x^2 + 16x - 17 = 0$
3. $x^2 + 6x - 7 = 0$
4. $x^2 - 8x - 9 = 0$

What I Need to Remember

- Perfect square trinomial is a trinomial derived by multiplying two identical binomials.
- To get the third term of a perfect square trinomial is squaring the half of the coefficient of the middle term, bx, in symbols,

$$\boldsymbol{ax^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2}$$

Lesson 5

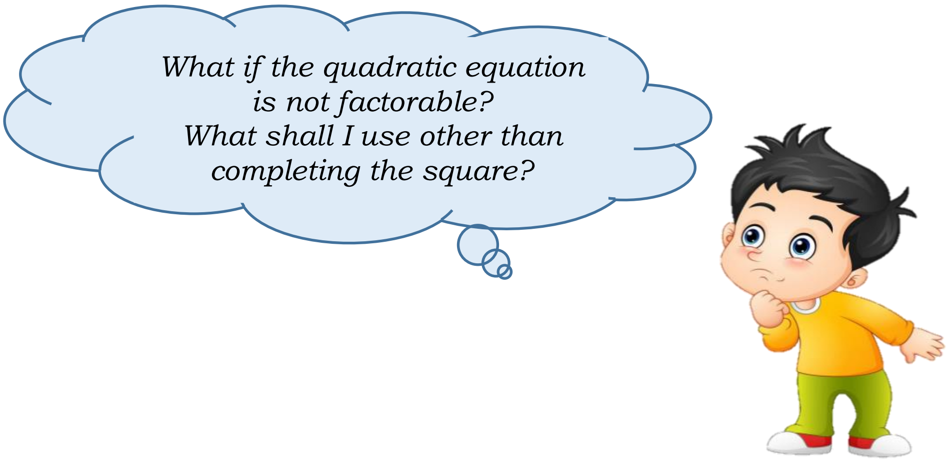
Solving Quadratic Equation by Quadratic Formula

What I Need to Know

At the end of this lesson, you are expected to:

- solve quadratic equation by quadratic formula;
- simplify the radicand to get the final roots of the quadratic equation.

What's In



Clipart 4: shorturl.at/cyBHX

What's New

Roots of the quadratic equation when not factorable can be easily solved using this method. Can you name what this technique is?

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

D	U	A	R	T	Q	A	I	C

U	L	A	M	O	R	F

Answer:

What Is It

Another comfortable way of getting the roots of the quadratic equation is to use the quadratic formula, in symbols, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. To use this formula, we need to identify the values of a, b and c in the given quadratic equation and plug-in right away to the formula.

Quadratic Formula

If $ax^2 + bx + c = 0$ is not factorable, we do the following:

1. plug-in the values of a, b and c to the formula;
2. proceed with simplifying the radicand if it is: a) perfect square, b)zero, c)not perfect square and d) negative
3. determine the roots by combining the numbers in the equation.

To clearly visualize the said concept above, let us look at these examples below.

Examples for using quadratic formula

Given the quadratic trinomials, determine the roots using quadratic formula.

1. $\boldsymbol{x^2 - 5x + 4 = 0}$

Step 1

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)}$$

Step 2

$$x = \frac{5 \pm \sqrt{25 - 16}}{2}$$

$$x = \frac{5 \pm \sqrt{9}}{2}$$

Perfect Square

$$x = \frac{5 \pm 3}{2}$$

Step 3

Therefore, the roots are

$\boldsymbol{x_1 = 4 \text{ and } x_2 = 1}$

2. $\boldsymbol{x^2 - 6x + 11 = 0}$

Step 1

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(11)}}{2(1)}$$

Step 2

$$x = \frac{6 \pm \sqrt{36 - 44}}{2}$$

Step 3

$$\boldsymbol{x = \frac{6 \pm \sqrt{-8}}{2}}$$

Negative

Since there is no $\sqrt{-8}$, *the roots are said to be imaginary*

3. $\boldsymbol{3x^2 - 4x - 2 = 0}$

Step 1

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)}$$

Step 2

$$x = \frac{4 \pm \sqrt{16 + 24}}{6}$$

$$x = \frac{4 \pm \sqrt{40}}{6}$$

Not Perfect Square

$$x = \frac{4 \pm 2\sqrt{10}}{6}$$

Step 3

$$x = \frac{2 \pm 1\sqrt{10}}{3}$$

Therefore, the roots are

$$\boldsymbol{x_1 = \frac{2 + 1\sqrt{10}}{3}, x_2 = \frac{2 - 1\sqrt{10}}{3}}$$

4. $\boldsymbol{x^2 + 8x + 16 = 0}$

Step 1

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(16)}}{2(1)}$$

Step 2

$$x = \frac{-8 \pm \sqrt{64 - 64}}{2}$$

$$x = \frac{-8 \pm \sqrt{0}}{2}$$

Zero

Step 3

$$x = \frac{-8 \pm 0}{2}$$

Therefore, the roots are

$\boldsymbol{x_1 = -4 \text{ and } x_2 = -4}$

What's More

Activity 1.5: NOW IT'S YOUR TURN!

Instructions: Solve each equation by using the quadratic formula. Show each solution and encircle the final answer.

1. $x^2 + 11 \times - 12 = 0$	2. $a^2 + 7a + 13 = 0$	3. $a^2 + 10a + 25 = 0$
4. $x^2 - 8x = 3$	5. $3y^2 + y - 1 = 0$	6. $5x^2 + 8x = -1$

What I Need To Remember

If $ax^2 + bx + c = 0$ is not factorable, we use the quadratic formula by;

1. plugging-in the values of a, b and c to the formula;
2. proceed with simplifying the radicand, if it is:
   a. perfect square c. zero
   b. not perfect square d. negative
3. determine the roots by combining the numbers in the equation.

What I Can Do

❖ ***Extra Challenge***

A school project in Arts required students to make a photo frame. The teacher gave the specific dimensions that the longer side is three inches more than the shorter side. The photo frame has an area of 18 square inches. What is the exact dimension of the frame, its length and its width?

Assessment (Post Test)

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper.

1. What are the roots of $(x-4)^2 = 49$?
   A. 7 and – 4
   B. 11 and – 3
   C. – 7 and 4
   D. – 11 and 3
2. What are the roots of the quadratic equation $x^2 - 15x + 56 = 0$?
   A. – 8 and 7
   B. – 8 and – 7
   C. 8 and – 7
   D. 8 and 7
3. What are the binomial factors of $x^2 - 24x + 144$?
   A. $(x-8)(x+3)$
   B. $(x-8)(x-3)$
   C. $(x-12)(x+12)$
   D. $(x-12)(x-12)$
4. What are the solutions of $b^2 + 4b = 21$?
   A. 4 and – 6
   B. – 4 and 6
   C. 7 and – 3
   D. – 7 and 3
5. What is the other root or b_2 if one of the roots of $b^2 - 3b - 54 = 0$ is $b_1 = 9$?
   A. – 4
   B. 4
   C. 6
   D. – 6
6. What are the solutions of the quadratic equation $6x^2 - 294 = 0$?
   A. 11 and – 11
   B. 9 and – 9
   C. 8 and – 8
   D. 7 and – 7
7. What is the standard form of the quadratic equation $3x(4-x) = 15$?
   A. $12x - 3x^2 = 12$
   B. $3x^2 - 12x + 15 = 0$
   C. $0 = 3x^2 - 12x - 15$
   D. $12x^2 - 3x - 15 = 0$
8. How will you write the quadratic equation whose roots are 7 and – 3?
   A. $x^2 - 4x + 21 = 0$
   B. $x^2 + 4x + 21 = 0$
   C. $x^2 + 4x - 21 = 0$
   D. $x^2 - 4x - 21 = 0$
9. What is the middle term in this quadratic equation $4x^2 + 3x - 2 = 0$?
   A. -2
   B. $4x^2$
   C. $3x$
   D. x^2
10. Which of the following is a quadratic equation?
    A. $4b - 7 = 6$
    B. $9x^2 + 3x - 1 < 5$
    C. $7x^2 + 3x \geq 9$
    D. $y^2 + 5y - 18 = 0$
11. How many real roots does the quadratic equation $4x^2 - 324 = 0$ have?
    A. 3
    B. 2
    C. 1
    D. 0
12. What must be added to both sides in order to complete the square in the equation $x^2 + 16x + (\ ) = 9 + (\ )$?
    A. 24
    B. 32
    C. 64
    D. 128
13. What is the value of coefficient c in this equation $6x^2 - 8x + 10 = 0$?
    A. 10
    B. – 8
    C. 6
    D. 0
14. What is the coefficient of bx in this equation $10x^2 - 12x + 15 = 0$?
    A. 12
    B. – 10
    C. 10
    D. – 12
15. What is the constant term of the given equation $3x^2 - 7x + 4 = 0$?
    A. $3x^2$
    B. – 7x
    C. 4
    D. 0

Answer Key

Remember: This portion of the module contains all the answers. Your **HONESTY** is required.

Let's Try This One!

1. $y^2 + 7y + 12 = 0$
2. $r^2 - r - 12 = 0$
3. $c^2 - 6c + 9 = 0$
4. $m^2 - 36 = 0$
5. $z^2 + 8z + 16 = 0$
6. $d^3 - 6d^2 + 12d - 8 = 0$

Activity 1.1A

A.
1. QE
2. QE
3. QE, the equation is $x^2 + 3x - 5 = 0$
4. Not QE, by solution the value of x=5
5. QE

Activity 1.1B

1. $0 = 2x^2 + 3x - 15$
   $a = 2, b = 4, c = -15$
2. $0 = 4x^2 - 3x + 8$
   $a = 4, b = -3, c = 8$
3. $x^2 + 2x + 5 = 0$
   $a = 1, b = 2, c = 5$
4. $0 = 4x^2 + x - 2$
   $a = 4, b = 1, c = -2$
5. $x^2 - x - 12 = 0$
   $a = 1, b = -1, c = -12$

Activity 1.2A

1. $x^2 = 225$
   $\sqrt[2]{x^2} = \sqrt[2]{225}$
   $x = \pm 15$
2. $4x^2 - 196 = 0$
   $\frac{4x^2}{4} = \frac{196}{4}$
   $\sqrt[2]{x^2} = \sqrt[2]{49}$
   $x = \pm 7$
3. $3(x+2)^2 = 48$
   $\frac{3(x+2)^2}{3} = \frac{48}{3}$
   $\sqrt[2]{(x+2)^2}$
   $= \sqrt[2]{16}$
   $x + 2 = \pm 4$
   $x = -2 \pm 4$

Activity 1.2A

4. $9x^2 = 324$
   $\frac{9x^2}{9} = \frac{324}{9}$
   $\sqrt[2]{x^2} = \sqrt[2]{36}$
   $x = \pm 6$
5. $(y+3)^2 = 36$
   $\sqrt[2]{(y+3)^2} = \sqrt[2]{36}$
   $y + 3 = \pm 6$
   $y = -3 \pm 6$
   $y_1 = -3 + 6 = 3$
   $y_2 = -3 - 6$
   $= -9$

Activity 1.2B

1. $12x^2 = 48x$
   $\frac{12x^2}{12x} = \frac{48x}{12x}$
   $x = 4$
2. $3x - 12x^2 = 0$
   $\frac{3x}{12x} = \frac{12x^2}{12x}$
   $\frac{1}{4} = x$
3. $8r^2 - 15r = 25r$
   $8r^2 = 25r + 15r$
   $\frac{8r^2}{8r} = \frac{40r}{8r}$
   $r = 5$

Activity 1.2B

4. $120x - 12x^2 = 0$
   $\frac{120x}{12x} = \frac{12x^2}{12x}$
   $10 = x$
5. $210j^2 + 5j = -5j$
   $210j^2 = -5j - 5j$
   $\frac{210j^2}{210j} = \frac{-10j}{210j}$
   $j = -\frac{1}{21}$

Activity 1.3A

Quadratic Equation	Sum	Pro	Integers	Binomial Factors	Roots
1. $x^2 + 10x + 25 = 0$	**10**	**25**	**5 and 5**	$(x+5)(x+5)$	$x_1 = -5\ x_2 = -5$
2. $m^2 - 8m - 20 = 0$	**-8**	**-20**	**-10 and 2**	$(x-10)(x+2)$	$x_1 = 10\ x_2 = -2$
3. $x^2 - 14x + 24 = 0$	**-14**	**24**	**-12 and -2**	$(x-12)(x-2)$	$x_1 = 12\ x_2 = 2$
4. $a^2 + 4a - 21 = 0$	**4**	**-21**	**7 and -3**	$(x+7)(x-3)$	$x_1 = -7\ x_2 = 3$
5. $y^2 - 7y - 44 = 0$	**-7**	**-44**	**-11 and 4**	$(x-11)(x+4)$	$x_1 = 11\ x_2 = -4$

Activity 1.3B

1. Roots are $x_1 = 1$ *and* $x_2 = -5$
2. Roots are $x_1 = -\frac{25}{2}$ *and* $x_2 = 1$
3. Roots are $x_1 = -\frac{4}{3}$ *and* $x_2 = 3$
4. Roots are $x_1 = -\frac{1}{6}$ *and* $x_2 = 2$

Activity 1.4A

1. 64
2. $\frac{49}{4}$
3. 100
4. 16
5. $\frac{9}{4}$

Activity 1.4B

1. $x_1 = 6$ and $x_2 = 2$
2. $x_1 = 1$ and $x_2 = -17$
3. $x_1 = 1$ and $x_2 = -7$
4. $x_1 = 9$ and $x_2 = -1$

Activity 1.5

1. $x_1 = 1$ and $x_2 = -12$
2. $x = \frac{-7 \pm \sqrt{-3}}{2}$
3. $x = -5$
4. $x = 4 \pm \sqrt{19}$
5. $x = \frac{-1 \pm \sqrt{13}}{6}$
6. $x = \frac{-4 \pm \sqrt{11}}{5}$

References

Text Book

Websites

"Methods to Solve a Quadratic Equation," Math Warehouse, accessed July 2, 2020, https://www.mathwarehouse.com/

"Quadratic Equations," Math is Fun, accessed June 26, 2020, https://www.mathisfun.com/algebra/quadratic-equation.html

"Quadratic Equation," Wolfram Math World, accessed July 6, 2020, https://mathworld.wolfram.com/QuadraticEquation.html

"Solving Quadratic Equations: Picking a Method," Purplemath.com, accessed June 23, 2020, https://www.purplemath.com/solvquad6.htm

Cliparts

Clipart 1: retrieved July 9, 2020, shorturl.at/ghlW9
Clipart 2: retrieved July 9, 2020, shorturl.at/hnrAH
Clipart 3: retrieved July 4, 2020, shorturl.at/fqLM6
Clipart 4: retrieved July 01, 2020, shorturl.at/cyBHX

Congratulations!

You are now ready for the next module. Always remember the following:

1. Make sure every answer sheet has your
   - *Name*
   - *Grade and Section*
   - *Title of the Activity or Activity No.*
2. Follow the date of submission of answer sheets as agreed with your teacher.
3. Keep the modules with you AND return them at the end of the school year or whenever face-to-face interaction is permitted.