9

Mathematics

Quarter 1 – Module 3

Solve Equations Transformable to Quadratic Equation

(including Rational Algebraic Equation)

DEPARTMENT OF EDUCATION

Introductory Message

Welcome to the **Mathematics 9** on **Solving Equations Transformable to Quadratic Equations Including Rational Algebraic Equation!**

This module was designed to provide you with opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

What I Know *(Pre-Test)*	This part includes an activity that aims to check what you already know about the lesson to take.
What I Need to Know *(Objectives)*	This will give you an idea of the skills or competencies you are expected to learn in the module.
What's In *(Review/ Springboard)*	This is a brief drill or review to help you link the current lesson with the previous one.
What's New *(Presentation of the Lesson)*	In this portion, the new lesson will be introduced to you in various ways; a story, a song, a poem, a problem opener, an activity or a situation.
What is It *(Discussion)*	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
What's More *(Application)*	This section provides activities which will help you transfer your new knowledge or skill into real life situations or concerns.
What I Need To Remember *(Generalization)*	This includes key points that you need to remember.
What I Can Do *(Enrichment Activities)*	This comprises activities for independent practice to solidify your understanding and skills of the topic.
Assessment *(Post Test)*	This is a task which aims to evaluate your level of mastery in achieving the learning competency.
Answer Key	This contains answers to the following: • What I Know • What's In • What's More

At the end of this module you will also find:

References	This is the list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in this module.
3. Read the instructions carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

About the Module

This module was designed and written with you in mind. It is here to help you master about solving equations transformable to quadratic equations including Rational Algebraic Equation. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students.

After going through this module, you are expected to:

- transform quadratic equation to its standard form;
- solve equations transformable to quadratic equation including rational algebraic expression.

What I Know (Pre-Test)

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper. If the answer is not found among the given choices, write your own answer.

A. Transform the following quadratic equations to its standard form.

1. $(\boldsymbol{x})(\boldsymbol{x}) = \mathbf{4}$

    A. $2x^2 = 4$ C. $x^2 - 4 = 0$
    B. $2x^2 - 4 = 0$ D. $x^2 + 4 = 0$

2. $\boldsymbol{x}(\boldsymbol{x} + \mathbf{3}) = \mathbf{4}$

    A. $x^2 + 7 = 0$ C. $x^2 + 3x - 4 = 0$
    B. $2x - 1 = 0$ D. $x^2 + 3x + 4 = 0$

3. $(\boldsymbol{x} + \mathbf{3})(\boldsymbol{x} - \mathbf{4}) = \mathbf{5}$

    A. $x^2 - x + 12 = 5$ C. $x^2 - x + 17 = 0$
    B. $x^2 - 7x + 12 = 5$ D. $x^2 - x - 17 = 0$

4. $(\mathbf{2}\boldsymbol{x} - \mathbf{3})(\mathbf{4}\boldsymbol{x} - \mathbf{5}) = -\mathbf{6}$

    A. $8x^2 - 22x + 9 = 0$ C. $8x^2 - 22x - 21 = 0$
    B. $8x^2 - 22x + 9 = 0$ D. $8x^2 - 22x + 21 = 0$

5. $(\boldsymbol{x} - \mathbf{6})^2 = \mathbf{7}$

    A. $x^2 - 12x + 43 = 0$ C. $x^2 - 12x + 29 = 0$
    B. $x^2 - 12x - 43 = 0$ D. $x^2 - 12 - 29 = 0$

6. $(\boldsymbol{x} - \mathbf{7})^2 + (\boldsymbol{x} + \mathbf{8})^2 = \mathbf{9}$

    A. $x^2 + x + 52 = 0$ C. $x^2 + x + 61 = 0$
    B. $x^2 - x + 52 = 0$ D. $x^2 - x + 61 = 0$

7. $\frac{\mathbf{2}\boldsymbol{x}^2}{\mathbf{5}} + \frac{\mathbf{5}\boldsymbol{x}}{\mathbf{4}} = \mathbf{3}$

    A. $8x^2 + 25x + 60 = 0$ C. $7x^2 + 9x + 12 = 0$
    B. $8x^2 + 25x - 60 = 0$ D. $7x^2 + 9x - 12 = 0$

8. $\frac{\mathbf{6}\boldsymbol{x}}{\boldsymbol{x}+\mathbf{5}} + \frac{\boldsymbol{x}-\mathbf{5}}{\mathbf{2}} = \mathbf{3}$

    A. $x^2 + 6x - 45 = 0$ C. $x^2 + 6x + 55 = 0$
    B. $x^2 + 6x + 45 = 0$ D. $x^2 + 6x - 55 = 0$

B. Solve the roots of the following Quadratic Equations.

9. $(x)(x) = 9$

A. 0
B. $1 \& -1$
C. $2 \& -2$
D. $3 \& -3$

10. $x(x+4) = 5$

A. $1 \& 5$
B. $1 \& -5$
C. $-1 \& 5$
D. $-1 \& -5$

11. $(x+3)(x-4) = 8$

A. $4 \& 5$
B. $4 \& -5$
C. $-4 \& 5$
D. $-4 \& -5$

12. $(x-5)^2 = 9$

A. $2 \& 8$
B. $2 \& -8$
C. $-2 \& 8$
D. $-2 \& -8$

13. $(x-6)^2 + (x+7)^2 = 97$

A. $2 \& 3$
B. $2 \& -3$
C. $-2 \& 3$
D. $-2 \& -3$

14. $\frac{x^2}{3} + \frac{2x}{4} = 5$

A. $\frac{-3 \pm \sqrt{249}}{4}$
B. $\frac{-3 \pm \sqrt{249}}{8}$
C. $\frac{-6 \pm \sqrt{249}}{4}$
D. $\frac{-6 \pm \sqrt{249}}{8}$

15. $\frac{2x}{x+4} + \frac{x-4}{2} = 3$

A. $1 \pm \sqrt{41}$
B. $-1 \pm \sqrt{41}$
C. $2 \pm 2\sqrt{41}$
D. $-2 \pm \sqrt{41}$

Lesson 3.1	**Solve equations transformable to Quadratic Equation including Rational Algebraic Equations**

What I Need to Know

At the end of this lesson, you are expected to:

- solve equations transformable to quadratic equation in the form of:
    - $(x)(x) = c$
    - $x(x + b) = c$
    - $(x + a)(x + b) = c$; where a, b and c are real numbers.

What's In

In the previous lesson we learned that the standard form of quadratic equation is $ax^2 + bx + c = 0$, where in the degree should always be equal to 2, in which the value of a, b and c are real numbers, where $a \neq 0$.

As mention in module 2, quadratic equation in standard form is not unique. Examine the illustrative examples in the table below.

Quadratic Equations in the standard form $ax^2 + bx + c = 0$	Quadratic Equations lacking the linear coefficient or the bx	Quadratic Equations lacking the constant term or c
• $x^2 + 11x - 35 = 0$	• $x^2 + 9 = 0$	• $x^2 - 7x = 0$
• $2x^2 - 3x - 4 = 0$	• $2x^2 - 16 = 0$	• $2x^2 + 8x = 0$
• $-5x^2 - 6x + 7 = 0$	• $3x^2 + 25 = 0$	• $x^2 - 9x = 0$
• $x^2 - x - 3 = 0$	• $6x^2 + 144 = 0$	• $x^2 + 2x = 0$

What's New

A quadratic equation can also be written in a factored form. Here are examples of quadratic equation in factored form:

- $(x + 2)(x - 3) = 0$
- $(x + 1)(x + 6) = 0$
- $(x - 6)(x + 1) = 0$
- $(x - 5)(x + 2) = 0$

Here are also examples of other forms of quadratic equations:

- $(x)(x) = 36$
- $x(x - 2) = 4$
- $x(2x + 3) = 12$
- $3x(x + 8) = -2$

These forms of equations are also characterized as equations transformable to quadratic equation.

What Is It

How to solve equations transformable to quadratic equation? First, you should transform the given quadratic equation to its standard form. Then choose what method in solving the roots of a quadratic equation is appropriate and easier for you to use. Lastly, always reduce your answer to lowest term.

The table shows different examples in solving equations transformable to quadratic equation.

Solve the roots of the following Quadratic Equations:		
	Examples	**Explanation**
1	Transforming the equation: $(\boldsymbol{x})(\boldsymbol{x}) = \mathbf{9}$ $x^2 = 9$ $x^2 - 9 = 9 - 9$ $x^2 - 9 = 0$ Solving the roots: $x^2 - 9 = 0$ $x^2 = 9$ $\sqrt{x^2} = \sqrt{9}$ $\boldsymbol{x} = \boldsymbol{\pm 3}$	• Given • x times x is equal to x^2 because $(x^1)(x^1) = x^{1+1} = x^2$ • equate to zero, subtract both side by 9 (subtraction property of equality) **Think what method to be used in solving its roots:** o use extracting the roots o find the square root of both sides. ➢ $\sqrt{x^2}$ *is* x ➢ $\sqrt{9}$ is ± 3 o *the roots are* ± 3
2	Transforming the equation: $x(x+3) = 4$ $x^2 + 3x = 4$ $\boldsymbol{x^2 + 3x - 4 = 4 - 4}$ $\boldsymbol{x^2 + 3x - 4 = 0}$ Solving the roots: $\boldsymbol{x^2 + 3x - 4 = 0}$ $(x+4)(x-1) = 0$ $x + 4 = 0$ | $x - 1 = 0$ $\boldsymbol{x = -4}$ | $\boldsymbol{x = 1}$	• Given • Multiply $x(x+3)$ ➢ x times x equals x^2 ➢ x times 3 equals $3x$ • write in standard form, Subtract both side by 4 (subtraction property of equality) **Think what method to be used in solving its roots:** o use Factoring o Equate each factor to zero o *the roots are* -4 & 1

3	Transforming the equation: $$(x+5)(x-7)=-9$$ $$x^2-7x+5x-35=-9$$ $$x^2-2x-35=-9$$ $$x^2-2x-35+9=-9+9$$ $$\boldsymbol{x^2-2x-26=0}$$	• Given • Multiply $(x+5)(x-7)$ • FOIL Method ➢ x times x is equal to x^2 ➢ x times -7 equals -7x ➢ 5 times x equals $5x$ ➢ 5 times -7 equals -35 ➢ Copy -9 • Collect like terms, ➢ $-7x$ plus 5x equals $-2x$ • Write in standard form, add both sides by 9 (addition property of equality)
	Solving the roots: $$\boldsymbol{x^2-2x-26=0}$$ $$a=1, b=-2, c=-26$$ $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ $$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-26)}}{2(1)}$$ $$x=\frac{2\pm\sqrt{(4)+104}}{2}$$ $$x=\frac{2\pm\sqrt{108}}{2}$$ $$x=\frac{2\pm\sqrt{(36)(3)}}{2}$$ $$x=\frac{2\pm6\sqrt{(3)}}{2}$$ $$\boldsymbol{x=1\pm3\sqrt{(3)}}$$	**Think what method to be used in solving its roots:** ○ Use Quadratic Formula ○ Find the value of a, b, and c ○ Write the Quadratic formula ○ Substitute the value of a, b, and c. ○ Multiply the ff: ➢ $-(-2)=2$ ➢ $(-2)^2=4$ ➢ $-4(1)(-26)=104$ ➢ $2(1)=2$ ○ Add $\sqrt{4+104}=\sqrt{108}$ ○ Factor 108, make sure that when you factor in a situation like this, there should be a perfect square number in the factor. As for 108 it is (36)(3) where 36 is a perfect square number; however, if there's no perfect square number don't factor. ○ $\sqrt{(36)(3)}=\pm6\sqrt{(3)}$ ○ Reduce to lowest term ➢ 2 divided by 2 equals 1 ➢ 6 divided by 2 equals 3 ○ The roots are $x=1\pm3\sqrt{(3)}$

What's More

Activity 3.1: Transform and find the roots of the following equations. Use separate sheet of paper for your solutions.

1. $(x)(x) = 16$
2. $(x)(2x) = 50$
3. $x(x+7) = 18$
4. $x(x-9) = 22$
5. $(x-3)(x+5) = -2$

What I Need To Remember

To solve equations transformable to quadratic equation, here are the following steps to follow:

1. Transform the given quadratic equation to its standard form.
2. Think what appropriate method to be used in solving its roots.
3. Reduce your answer to lowest term.

Lesson 3.2

Solve equations transformable to Quadratic Equation including Rational Algebraic Equations

What I Need to Know

At the end of this lesson, you are expected to:

- solve equations transformable to quadratic equation involving Square of Binomials.

What's In

Can you still remember about special products? It was your lesson in grade 8 where one of its cases is the square of binomials. The square of a binomial is the sum of: the square of the first terms, twice the product of the two terms, and the square of the last term. In general forms:

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x-y)^2 = x^2 - 2xy + y^2$$

Review: Find the product of the following Square Binomials. Match Column A with Column B. Write your answer on the space provided for it.

Column A	Column B
________ 1. $(x+1)^2$	A. x^2+2x+1
________ 2. $(x-2)^2$	B. x^2+6x+9
________ 3. $(x+3)^2$	C. x^2-4x+4
________ 4. $(x-4)^2$	D. $x^2-8x+16$
________ 5. $(x+5)^2$	E. $x^2-12x+36$
	F. $x^2+10x+25$

What's New

The skills in finding the product of the square of binomials is very helpful in solving the roots of some forms of quadratic equations. Specifically, on quadratic equations written in the form with a square of binomials. Here are some examples:

- $(x+1)^2=2$
- $(x-2)^2=8$
- $(x+3)^2+(x+4)^2=10$
- $(x-5)^2+(x+6)^2=20$

What Is It

The table below shows how to solve quadratic equation involving square of binomials:

Solve the roots of the following Quadratic Equations:		
	Examples	**Explanation**
1	Transforming the equation: $(x+2)^2 = \ 9$ $x^2+4x+4=9$ $x^2+4x+4-9=9-9$ $\boldsymbol{x^2+4x-5=0}$	• Given • Find the product of $(x+2)^2$ ➢ Through special product $(x+2)^2 is\ equal\ to\ x^2+4x+4$ • Then copy $= \ 9$ • Write in standard form: ➢ subtract both sides by 9 (subtraction property of equality)
	Solving the roots: $\boldsymbol{x^2+4x-5=0}$ $(x-1)(x+5)=0$ $x-1=0$ | $x+5=0$ $\boldsymbol{x\ \ =1}$ | $\boldsymbol{x=-5}$	**Think what method to be used in solving its roots:** o In this equation you use factoring because it can be factored. o Factors of $\boldsymbol{x^2+4x-5}$ o Equate each factor to zero o Therefore, $the\ roots\ of\ the\ equation$ $x^2+4x-5=0\ are\ 1\ and -5$

2	Transforming the equation: $$(x+2)^2 + (x+3)^2 = 4$$ $$(x^2+4x+4) + (x^2+6x+9) = 4$$ $$x^2+4x+4+ x^2+6x+9 = 4$$ $$2x^2+10x+13 = 4$$ $$2x^2+10x+13-4 = 4-4$$ $$\mathbf{2x^2+10x+9=0}$$	• Given • Find the product of $(x+2)^2$ and $(x+3)^2$ • Through special product: ➢ $(x+2)^2 = x^2+4x+4$ ➢ $(x+3)^2 = x^2+6x+9$ • Remove the parenthesis • Collect like terms: ➢ $x^2+x^2 = 2x^2$ ➢ $4x+6x = 10x$ ➢ $4+9 = 13$ • Write in standard form ➢ Subtract both sides by 4, (subtraction property of equality)
	Solving the roots: $$\mathbf{2x^2+10x+9=0}$$ $$a = 2, b = 10\,, c = 9$$ $$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$ $$x = \frac{-(10) \pm \sqrt{(10)^2-4(2)(9)}}{2(2)}$$ $$x = \frac{-10 \pm \sqrt{100-72}}{4}$$ $$x = \frac{-10 \pm \sqrt{28}}{4}$$ $$x = \frac{-10 \pm \sqrt{(4)(7)}}{4}$$ $$x = \frac{-10 \pm 2\sqrt{(7)}}{4}$$	**Think what method to be used in solving its roots:** ○ Use quadratic formula ○ Find the value of a, b and c ○ Write the quadratic formula ○ Substitute the value of a, b, and c ○ Multiply the ff: ➢ $-(10) = -10$ ➢ $(10)^2 = 100$ ➢ $-4(2)(9) = -72$ ➢ $2(2) = 4$ ○ Subtract 100 – 72 = 28 ○ Factor 28, make sure that one of the factors is a perfect square number. The factor of 28 is (4)(7) where 4 is the perfect square. ○ Square root of 4 is ± 2. ○ Reduce to lowest term; ➢ -10 divided by 2 is -5 ➢ 2 divided by 2 is 1 ➢ 4 divided by 2 is 2

	$\boldsymbol{x = \frac{-5 \pm \sqrt{(7)}}{2}}$	○ Therefore, the roots of the equation $\mathbf{2x^2 + 10x + 9 = 0}$ are $\boldsymbol{x = \frac{-5 \pm \sqrt{(7)}}{2}}$
3	Transforming the equation: $(x-6)^2 + (x+4)^2 = -20$ $(x^2 - 12x + 36) + (x^2 + 8x + 16) = -20$ $x^2 - 12x + 36 + x^2 + 8x + 16 = -20$ $2x^2 - 4x + 52 = -20$ $2x^2 - 4x + 52 + 20 = -20 + 20$ $2x^2 - 4x + 72 = 0$ $\frac{2x^2 - 4x + 72}{2} = \frac{0}{2}$ $\mathbf{x^2 - 2x + 36 = 0}$ Solving the roots: $\mathbf{x^2 - 2x + 36 = 0}$ $a = 1, b = -2, c = 36$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(36)}}{2(1)}$ $x = \frac{2 \pm \sqrt{4 - 144}}{2}$ $x = \frac{2 \pm \sqrt{-140}}{2}$ $x = \frac{2 \pm \sqrt{(-1)(4)(35)}}{2}$	• Given • Find the product of $(x-6)^2$ and $(x+4)^2$ • Through special product; ➢ $(x-6)^2 = x^2 - 12x + 36$ ➢ $(x+4)^2 = x^2 + 8x + 16$ • Remove the parenthesis • Collect like terms: ➢ $x^2 + x^2 = 2x^2$ ➢ $-12x + 8x = -4x$ ➢ $36 + 16 = 52$ • Write in standard form ➢ add both sides by 20, (addition property of equality) • Reduce the equation ➢ divide both sides by 2 because the coefficient of $2x^2$ *is* 2 **Think what method to be used in solving its roots:** ○ Use quadratic equation ○ Find the value of a, b and c ○ Write the quadratic formula ○ Substitute the value of a, b, and c ○ Multiply the ff: ➢ $-(-2) = 2$ ➢ $(-2)^2 = 4$ ➢ $-4(1)(36) = -144$ ➢ $2(1) = 2$ ○ Subtract $4 - 144 = -140$ ○ Factor -140 as (-1)(4)(35) ○ $\sqrt{(-1)} = i$ ○ $\sqrt{(4)} = 2$

	$x = \dfrac{2 \pm 2i\sqrt{(35)}}{2}$	○ Reduce to lowest term ➢ 2 divided by 2 = 1 ➢ $2i$ divided by 2 = i
	$\boldsymbol{x = 1 \pm i\sqrt{(35)}}$	○ Therefore, the roots of the equation $\boldsymbol{x^2 - 2x + 36 = 0}$ are $x = 1 \pm i\sqrt{(35)}$

What's More

Activity 3.2: Transform the following quadratic equation into its standard form and find its roots. Use separate sheet of paper for your solutions.

1. $(x+5)^2 = 16$
2. $(x-6)^2 = 20$
3. $(x+4)^2 + (x+3)^2 = 11$
4. $(x+2)^2 + (x-7)^2 = 1$

What I Need To Remember

In choosing what method to be used in solving the roots of quadratic equation, the data in the table must be observed:

Method	Can be used	When to use
Extracting the Root	Sometimes	Use for $ax^2 = $ c , where the constant is a perfect square.
Factoring	Sometimes	Use when $ax^2 + bx + c = 0$ can be factored.
Completing the Square	Always	Useful in the form of $ax^2 + bx + c = 0$, where b is even.
Quadratic formula	Always	Useful in all forms of quadratic equation, especially when all other method fails.

Lesson 3.3

Solve equations transformable to Quadratic Equation including Rational Algebraic Equations

What I Need To Know

At the end of this lesson, you are expected to:

- solve Rational Algebraic Equations transformable to Quadratic Equation.

What's In

A Rational Algebraic Equation is an equation containing at least one fraction whose numerator and denominator are polynomials. These fractions may be on one or both sides of the equation.

Here are some examples of Rational Algebraic Equation.

- $\frac{x}{2}+\frac{2}{3}=4$
- $\frac{x+1}{2}+\frac{x}{3}=5$
- $\frac{x}{2}+\frac{2}{x}=3$
- $\frac{x+1}{2}=\frac{5}{x-3}$
- $\frac{x+1}{2}+\frac{2}{x-3}=5$
- $\frac{x+1}{x-2}=\frac{x+3}{2x-3}$

What's New

Do you know that some forms of Rational Algebraic Equation are transformable to quadratic equation? Yes, it can be transformed to a quadratic equation. But, how to transform it to quadratic equation? The best approach to transform rational algebraic equation is to eliminate all the denominators using the idea of LCD (least common denominator), where denominators must not equal to zero. It must also follow the definition of Quadratic Equation that the degree is 2. Example, $\frac{x}{2}+\frac{2}{x}=3$ when transformed will become $x^2-6x+4=0$, this is a quadratic equation because the degree is 2. While $\frac{x}{2}+\frac{2}{3}=4$ when transformed will become $3x-20=0$. Since, the degree of $3x-20=0$ is not 2, then it is not a quadratic equation.

After transforming Rational Algebraic Equation to standard form of quadratic equation, use the appropriate method in solving quadratic equation to find its root.

What Is It

Here is the brief discussion on how to solve Rational Algebraic Equation Transformable to Quadratic Equation.

Solve the following Rational Algebraic Equation Transformable to Quadratic Equation:	
Examples	**Explanation**
Transforming the equation:	• Given
$Ex\#1. \quad \frac{4}{x}+\frac{x}{2}=3$	• Find the LCD; the LCD is 2x
LCD = $(2)(x) = 2x \text{ divided by } 1 \text{ is } 2x$ $(2x)\left(\frac{4}{x}+\frac{x}{2}\right)=(3)(2x)$ $8+x^2=6x$	• Multiply the LCD to both sides. ➢ 2x times 4 is 8x; divided by x is 8 ➢ 2x times x is $2x^2$; divided 2 is x^2 ➢ 3 times 2x is 6x ✓ Write in standard form
$8+x^2-6x=6x-6x$ $8+x^2-6x=0$	➢ Subtract both sides by 6x
$\boldsymbol{x^2-6x+8=0}$	**Think what method to be used in solving its roots:**
Solving the roots: $\boldsymbol{x^2-6x+8=0}$	○ Use factoring ○ Factor $\boldsymbol{x^2-6x+8}$
$(x-2)(x-4)=0$	➢ $\boldsymbol{x^2}$ *is* $(x)(x)$ ➢ $\boldsymbol{8}$ *is* $(-2)(-4)$ *so that;* ➢ $\boldsymbol{-2-4=-6}$ for the middle term -6x
$(x-2)=0 \quad \vert \quad (x-4)=0$ $x-2=0 \quad \vert \quad x-4=0$ $\boldsymbol{x=4} \quad \vert \quad \boldsymbol{x=2}$	○ Equate to zero. ○ The roots are 2 and 4
Checking:	You may check your answer if it satisfies the original equation.
$\frac{4}{x}+\frac{x}{2}=3$	✓ Original equation
When $\boldsymbol{x=4}$ $\quad$ when $\boldsymbol{x=2}$	✓ Substitute the value of the roots
$\frac{4}{(4)}+\frac{(4)}{2}=3 \quad \vert \quad \frac{4}{(2)}+\frac{(2)}{2}=3$	✓ Since the value of the roots satisfies the original equation then it is correct.
$1+2=3 \quad \vert \quad 2+1=3$ $3=3$ True $\quad \vert \quad 3=3$ True	**Note:** Next examples won't be showing the checking part.

Transforming the equation:

$Ex\#2.$
$$\frac{x-4}{3}+\frac{2}{x-2}=-5$$

Find the LCD; the $LCD\ is\ (3)(x-2)$

Multiply the LCD to both sides of the equation;

$$(3)(x-2)\left(\frac{x-4}{3}\right)+(3)(x-2)\left(\frac{2}{x-2}\right)=(-5)\,(3)(x-2)$$

Cancel out similar terms of the denominator from the multiplied LCD;

$$\cancel{(3)}(x-2)\left(\frac{x-4}{\cancel{3}}\right)+(3)\cancel{(x-2)}\left(\frac{2}{\cancel{x-2}}\right)=(-5)\,(3)(x-2)$$

Multiply the remaining terms;

$$(x-2)(x-4)+(3)(2)=(-5)(3)(x-2)$$

$$x^2-6x+8+6=-15x+30$$

Collect like terms in both sides; $only\ (8+6=14)\ is\ possible.$

$$x^2-6x+14=-15x+30$$

Write in standard form, subtract both sides by $(-15x\ +30)$

$$(x^2-6x+14)-(-15x\ +30)=(-15x+30)-(-15x\ +30)$$

Remove parenthesis then collect like terms; **note:** $-(-15x\ +30)\ equals\ 15x-30$

$$x^2-6x+14+15x-30=-15x+30+15x-30$$

$$\boldsymbol{x^2+9x-16=0}$$

Think what method to be used in solving its roots:

Solving the roots:	
$\boldsymbol{x^2+9x-16=0}$	○ Use quadratic formula
$a=1, b=9, c=-16$	○ Find the value of a, b and c
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$	○ Write the quadratic formula
$x=\frac{-(9)\pm\sqrt{(9)^2-4(1)(16)}}{2(1)}$	○ Substitute the value of a, b, and c
$x=\frac{-9\pm\sqrt{81-64}}{2}$	○ Multiply the ff: ➢ $-(9)=-9$ ➢ $(9)^2=81$ ➢ $-4(1)(16)=-64$ ➢ $2(1)=2$
$\boldsymbol{x=\frac{-9\pm\sqrt{17}}{2}}$	○ Subtract $81-64=17$ ○ The roots are $x=\frac{-9\pm\sqrt{17}}{2}$

Transforming the equation:

$Ex\#3.$

$$\frac{x+2}{x-4}-\frac{x-3}{x+3}=1$$

Find the LCD; the $LCD\ is\ (x-4)(x+3)$

Multiply the LCD to both sides of the equation;

$$(x-4)(x+3)\left(\frac{x+2}{x-4}\right)-(x-4)(x+3)\left(\frac{x-3}{x+3}\right)=(x-4)(x+3)(1)$$

Cancel out similar terms of the denominator from the multiplied LCD

$$\cancel{(x-4)}(x+3)\left(\frac{x+2}{\cancel{x-4}}\right)-(x-4)\cancel{(x+3)}\left(\frac{x-3}{\cancel{x+3}}\right)=(x-4)(x+3)(1)$$

Multiply the remaining terms;

$$(x+3)(x+2)-(x-4)(x-3)=(x-4)(x+3)(1)$$

$$(x^2+5x+6)-(x^2-7x+12)=(x^2-x-12)$$

Remove the parenthesis; **NOTE:** $-(x^2-7x+12)$ is $-x^2+7x-12$

$$x^2+5x+6-x^2+7x-12=x^2-x-12$$

Rewrite

$$x^2-x^2+5x+7x+6-12=x^2-x-12$$

Collect like terms in both sides;

$x^2-x^2=0$	$5x+7x=12x$	$6-12=-6$

So, it will become,

$$12x-6=x^2-x-12$$

Subtract both sides by $(12x-6)$ then collect like terms;

$$12x-6-(12x-6)=x^2-x-12-(12x-6)$$

$$12x-6-12x+6=x^2-x-12-12x+6$$

$$0=x^2-13x-6$$

Write in standard form;

$$\boldsymbol{x^2-13x-6=0}$$

Think what method to be used in solving its roots:

Solving the roots:	
$\boldsymbol{x^2+9x-16=0}$	○ Use quadratic formula
$a=1, b=9\ , c=\ -16$	○ Find the value of a, b and c
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	○ Write the quadratic formula

$x = \frac{-(9) \pm \sqrt{(9)^2 - 4(1)(16)}}{2(1)}$ $x = \frac{-9 \pm \sqrt{81 - 64}}{2}$ $\boldsymbol{x = \frac{-9 \pm \sqrt{17}}{2}}$	○ Substitute the value of a, b, and c ○ Multiply the ff: ➢ $-(9) = -9$ ➢ $(9)^2 = 81$ ➢ $-4(1)(16) = -64$ ➢ $2(1) = 2$ ○ Subtract $81 - 64 = 17$ The roots are $x = \frac{-9 \pm \sqrt{17}}{2}$
Note: There are always 2 roots in a quadratic equation, sometimes one of the roots will not satisfy against the original equation. This Root is called **Extraneous Root.**	

What's More

Activity 3.3: Transform the following Rational Algebraic Equation into a standard form of quadratic equation and solve its roots. Use separate sheet of paper for your solutions.

1. $\frac{x}{3} + \frac{12}{x} = 4$
2. $\frac{x^2}{6} + \frac{x}{2} = x + 2$
3. $\frac{x-3}{4} + \frac{2}{x-1} = 2$
4. $\frac{x+1}{x-2} - \frac{x+3}{x+2} = 1$

What I Need To Remember

- To solve Rational Algebraic Equation transformable to Quadratic Equation observed the following steps:
    1. Transform Rational Algebraic Equations to the standard form of Quadratic Equation using the LCD approach.
    2. Use appropriate method in solving its roots
    3. Reduce your answer to lowest term
    4. You may check your answer against the original equation to determine if both roots are true or one of them is an extraneous root.

- When using quadratic formula, always find the greatest perfect square number in factoring the radicand, if there's no perfect square number on the factor of the radicand, just copy the number.

Assessment (Post Test)

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper. If the answer is not found among the given choices, write your own answer.

A. Transform the following equations to quadratic equation.

1. $(\boldsymbol{x})(\boldsymbol{x}) = \mathbf{49}$

   C. $2x^2 = 49$ C. $x^2 - 49 = 0$

   D. $2x^2 - 49 = 0$ D. $x^2 + 49 = 0$

2. $\boldsymbol{x}(\boldsymbol{x} + \mathbf{6}) = \mathbf{7}$

   A. $x^2 - 1 = 0$ C. $x^2 + 6x + 7 = 0$

   B. $2x - 1 = 0$ D. $x^2 + 6x - 7 = 0$

3. $(\boldsymbol{x} - \mathbf{4})(\boldsymbol{x} + \mathbf{5}) = \mathbf{3}$

   A. $x^2 + x + 23 = 0$ C. $x^2 - x + 17 = 0$

   B. $x^2 + x - 23 = 0$ D. $x^2 - x + 17 = 0$

4. $(\mathbf{3}\boldsymbol{x} - \mathbf{2})(\mathbf{2}\boldsymbol{x} - \mathbf{1}) = -\mathbf{4}$

   A. $5x^2 - x + 6 = 0$ C. $6x^2 - x + 6 = 0$

   B. $5x^2 - 7x + 2 = 0$ D. $6x^2 - 7x + 6 = 0$

5. $(\boldsymbol{x} - \mathbf{7})^2 = \mathbf{6}$

   A. $x^2 - 14x + 43 = 0$ C. $x^2 + 14x + 55 = 0$

   B. $x^2 - 14x - 43 = 0$ D. $x^2 - 14x - 55 = 0$

6. $(\boldsymbol{x} + \mathbf{9})^2 + (\boldsymbol{x} - \mathbf{8})^2 = \mathbf{7}$

   A. $x^2 - x + 74 = 0$ C. $x^2 - 17x + 69 = 0$

   B. $x^2 - x + 69 = 0$ D. $x^2 - 17x + 138 = 0$

7. $\frac{\boldsymbol{x}^2}{\mathbf{2}} + \frac{\mathbf{4}\boldsymbol{x}}{\mathbf{3}} = \mathbf{6}$

   A. $3x^2 - x + 30 = 0$ C. $3x^2 + 8x - 36 = 0$

   B. $3x^2 + 6x - 12 = 0$ D. $3x^2 + 12x + 36 = 0$

8. $\frac{\mathbf{3}\boldsymbol{x}}{\boldsymbol{x}+\mathbf{4}} + \frac{\boldsymbol{x}-\mathbf{2}}{\mathbf{3}} = \mathbf{5}$

   A. $x^2 + 4x - 52 = 0$ C. $x^2 - 26x + 52 = 0$

   B. $x^2 - 4x - 68 = 0$ D. $x^2 - 26x - 68 = 0$

B. Solve the roots of the following equations.

9. $(\boldsymbol{x})(\boldsymbol{x}) = \mathbf{144}$

A. $12 \& -12$ | C. $14 \& -14$
B. $13 \& -13$ | D. $15 \& -15$

10. $\boldsymbol{x}(\boldsymbol{x}-\mathbf{2}) = \mathbf{8}$

A. $2 \& 4$ | C. $-2 \& -4$
B. $-2 \& 4$ | D. $2 \& -4$

11. $(\boldsymbol{x}+\mathbf{4})(\boldsymbol{x}-\mathbf{6}) = -\mathbf{9}$

A. $3 \& 5$ | C. $-3 \& 5$
B. $3 \& -5$ | D. $-3 \& -5$

12. $(\boldsymbol{x}-\mathbf{8})^{\mathbf{2}} = \mathbf{11}$

A. $8 \pm \sqrt{11}$ | C. $64 \pm \sqrt{11}$
B. $-8 \pm \sqrt{11}$ | D. $-64 \pm \sqrt{11}$

13. $(\boldsymbol{x}-\mathbf{3})^{\mathbf{2}} + (\boldsymbol{x}+\mathbf{1})^{\mathbf{2}} = \mathbf{12}$

A. $1 \pm \sqrt{2}$ | C. 1 ± 2
B. $-1 \pm \sqrt{2}$ | D. -1 ± 2

14. $\dfrac{\boldsymbol{x}^{\mathbf{2}}}{\mathbf{4}} + \dfrac{\mathbf{3}\boldsymbol{x}}{\mathbf{2}} = \mathbf{6}$

A. $3 \pm \sqrt{33}$ | C. $6 \pm 2\sqrt{33}$
B. $-3 \pm \sqrt{33}$ | D. $-6 \pm \sqrt{33}$

15. $\dfrac{\boldsymbol{x}+\mathbf{3}}{\boldsymbol{x}+\mathbf{1}} + \dfrac{\boldsymbol{x}+\mathbf{4}}{\boldsymbol{x}-\mathbf{2}} = \mathbf{3}$

A. $\frac{9\pm\sqrt{65}}{2}$ | C. $\frac{9\pm\sqrt{97}}{2}$
B. $\frac{-9\pm\sqrt{65}}{2}$ | D. $\frac{-9\pm\sqrt{97}}{2}$

Answer Key

Remember: This portion of the module contains all the answers. Your **HONESTY** is required.

ACTIVITY 4.1

1. ± 4
2. ± 5
3. 2 & -9
4. -2 & 11
5. $-1 \pm \sqrt{14}$

ACTIVITY 4.2

1. 1 & 9
2. $6 \pm 2\sqrt{5}$
3. $\frac{-7 \pm \sqrt{21}}{2}$
4. -4 & 9

4.2 Review

1. A
2. C
3. B
4. D
5. F

ACTIVITY 4.3

1. ± 6
2. $\frac{-7 \pm \sqrt{21}}{2}$
3. $6 \pm \sqrt{17}$
4. $1 \pm \sqrt{13}$

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Congratulations!

You are now ready for the next module. Always remember the following:

1. Make sure every answer sheet has your
   - *Name*
   - *Grade and Section*
   - *Title of the Activity or Activity No.*
2. Follow the date of submission of answer sheets as agreed with your teacher.
3. Keep the modules with you AND return them at the end of the school year or whenever face-to-face interaction is permitted.