

Mathematics

Quarter 1 - Module 5: Quadratic Inequalities



About the Module

This module was designed and written with you in mind. It is here to help you master Quadratic Inequalities. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module is divided into three lessons:

Lesson 1 – Illustrate Quadratic Inequalities

Lesson 2 – Solving Quadratic Inequalities

Lesson 3 – Word Problems Involving Quadratic Inequalities

After working through this module, you are expected to:

- illustrate quadratic inequalities by graphing
- solve quadratic inequalities; and
- solve problems involving quadratic inequalities

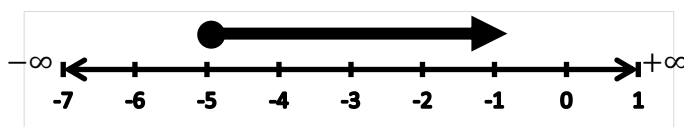


What I Know (Pre-Test)

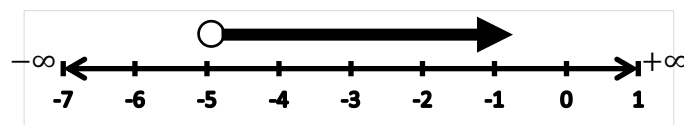
DIRECTIONS: Read and analyze the following questions carefully. Choose your answer in the given choices. Write your solutions in some of the items using a separate sheet of paper. NOTE: If there is no possible answer, solve it on your own and show your solution.

- ____ 1.) What do you call an inequality that contains a polynomial of degree 2?
A. Linear Inequality C. Quadratic Inequality
B. Linear Equation D. Quadratic Equation
- ____ 2.) What is the mathematical statement of the English statement "*a number is at most thirteen*"?
A. $x \leq 13$ B. $x \geq 13$ C. $x = 13$ D. $x > 13$
- ____ 3.) Which of the following situations below has an inequality symbol " \leq "?
i. The weight is not more than 54 kilograms.
ii. His loan is at most 500 thousand.
iii. His monthly deduction is at least ₱10, 000.00.
A. *i* and *ii* B. *i* and *iii* C. *ii*, and *iii* D. *i*, *ii*, and *iii*
- ____ 4.) Which of the following graphs represents the inequality $x \leq -5$?

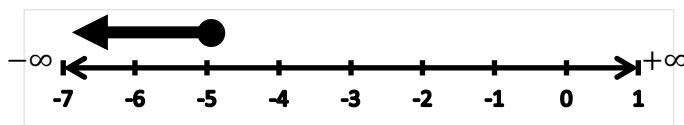
A.



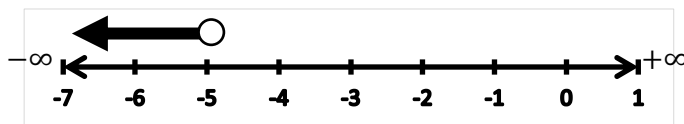
B.



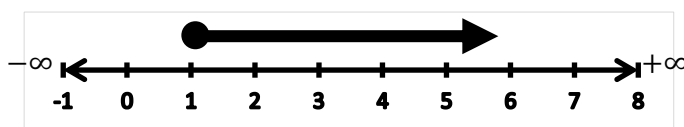
C.



D.



- ____ 5.) Which of the following inequalities represents the given graph below?



A. $x > 1$

B. $x \leq 1$

C. $x = 1$

D. $x \geq 1$

____ 6.) Which of the following is **NOT** a solution to the quadratic inequality $x^2 - 2 > 0$?

- A. 1 B. 2 C. 3 D. 4

____ 7.) Which of the following statements does **NOT** belong to the group?

- A. $x^2 + 2x > 0$ C. $x^2 < 4$
 B. $2x^2 + 3x + 4 = 0$ D. $x^2 - 3x + 5 \geq 0$

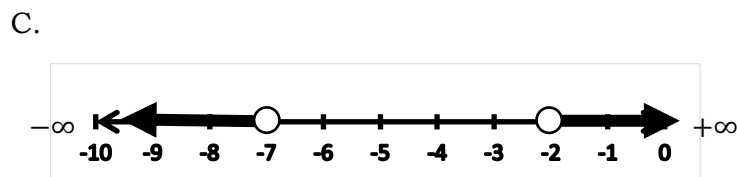
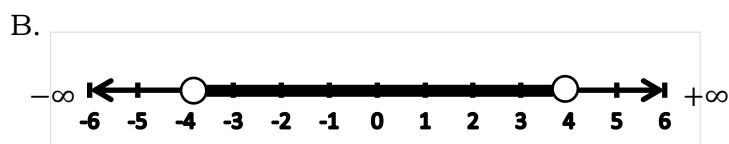
____ 8.) Which of the following statements states a quadratic inequality?

- A. $-2x + x^2 + 2 > 0$ C. $3x - 2x - 2 < 0$
 B. $2x + 5 \geq 0$ D. $3x^2 + 3x + 5 = 0$

Items 9 – 10, refer to the inequality inside the box. Then, answer the questions that follow.

$$x^2 + 9x + 14 > 0$$

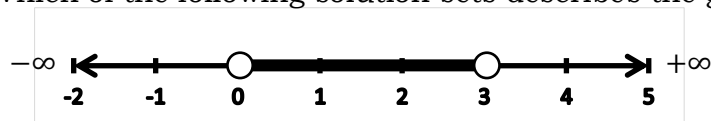
____ 9.) Which of the following is the graphs represents inequality?



____ 10.) What is the solution set of the inequality $x^2 + 9x + 14 > 0$?

- A. $(-\infty, -7)$ and $(-2, +\infty)$ C. $(-4, 4)$
 B. $(-\infty, -7]$ and $[-2, +\infty)$ D. $[-4, 4]$

____ 11.) Which of the following solution sets describes the graph below?



- A. $0 < x < 3$ B. $0 > x < 3$ C. $0 \leq x \leq 3$ D. $0 \geq x \leq 3$

____ 12.) What is the interval notation form of the solution set $-1 < x < 4$?

- A. $(-\infty, -1)$ and $(4, +\infty)$ C. $(-1, 4)$
 B. $(-\infty, -1]$ and $[4, +\infty)$ D. $[-1, 4]$

Items 13 – 16, refer to the problem inside the box. Then, answer the questions that follow.

The length of a tarpaulin is 4 more than twice the width. Its area is no less than 96 meters.

- _____ 13.) Which of the following represents the length of the tarpaulin?
A. x B. $x + 4$ C. $2x + 4$ D. $2x$
- _____ 14.) Which of the following expressions represents the width of the tarpaulin?
A. x B. $x + 4$ C. $2x + 4$ D. $2x$
- _____ 15.) Which inequality represents the situation?
A. $x(x) \geq 96$ B. $x(2x + 4) \geq 96$ C. $x(x + 4) \geq 96$ D. $x(2x) \geq 96$
- _____ 16.) What is the least length of the tarpaulin?
A. 16-meters B. 10-meters C. 6-meters D. 4-meters
- _____ 17.) The product of the two positive consecutive integers is no less than 56. Find the pairs of the integers.
A. 7 and 8 B. 7 and -8 C. -7 and 8 D. -7 and -8
- _____ 18.) The length of the swimming pool is 2-meters more than its width. If the area of the wall is no less than $48m^2$, which of the following could be its length?
A. 4-meters B. 6-meters C. 8-meters D. 10-meters
- _____ 19.) The monthly profit P (in thousand pesos) that a Sari-Sari Store earns is determined by the equation $P = 10x^2 - 280x - 1000$. What is the lowest possible value of x that will give the store a profit of at least ₱10, 000.00?
A. 5 B. 22 C. 50 D. 52
- _____ 20.) The width of the lot is 4-meters shorter than its length. How long should its width be to have an area of no more than $60m^2$?
A. 16-meters B. 10-meters C. 6-meters D. 4-meters

Lesson 1

Illustrations Of Quadratic Inequalities



What I Need To Know

At the end of this lesson, you are expected to:

1. differentiate between equations and inequalities
2. illustrate quadratic inequalities
3. graph the solution set of inequality on the number line



What's In

❖ *Flashback*

Let's Try!

Which of the following sentences below is an equation?

- 1.) $x - 2 = 0$
- 2.) $2x^2 + 2 < 3$
- 3.) $2 \geq x$
- 4.) $9 = 2x + 5$

The equation is a mathematical sentence that states that two expressions are the same.

Answer and explanation to items in "Let's Try!"

- 1.) **Equation**, because the sentence shows equality.
- 2.) **Not equation**, because the sentence does not show equality.
- 3.) **Not equation**, because the sentence does not show equality.
- 4.) **Equation**, because the sentence shows equality.

Only the sentences in items 1 and 4 are considered equations based on definition. So, what do we call the sentences that do not show equality? They are called **Inequalities**. Therefore, $2x^2 + 2 < 3$ and $2 \geq x$ are inequalities.

Can you still remember what inequality is?

In your Grade 7 Mathematics, you have learned linear inequalities in one variable. So, let us have a review of what inequality is.

Inequality is a mathematical sentence which states that two expressions are unequal.

Examples: $x < 2$
 $x > -5$

So, in the given examples, we can say that the two sides of the mathematical sentence are not equal because of the relationship symbol used in the sentence. The symbols $<$, $>$, \leq , and \geq tell the inequality of a number.

Let us read the sentences:

$x < 2$	is being read as " x is less than 2"
$x + 2 > -3$	is being read as " x plus 2 is greater than negative 3"
$x \leq -10$	is being read as " x is less than or equal to negative 10"
$x \geq 5$	is being read as " x is greater than or equal to 5"
$2 < x < 4$	is being read as " x is between 2 and 4" or is " x is less than 4 but is greater than 2"



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What's New

❖ Let's Try!

What are the solutions to the equation below?

$$x^2 - 3x = -2$$

Solve the equation $x^2 - 3x = -2$ using any of the four methods in solving quadratic equations you have learned in Module 1.

The four methods in solving quadratic equations are the following: (1) *Extracting the Roots*, (2) *Factoring*, (3) *Completing the Square*, and (4) *Using the Quadratic Formula*.

Solution:

$$x^2 - 3x = -2$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x - 1 = 0$$

$$x = 1$$

$$x - 2 = 0$$

$$x = 2$$

Write the equation into its standard form

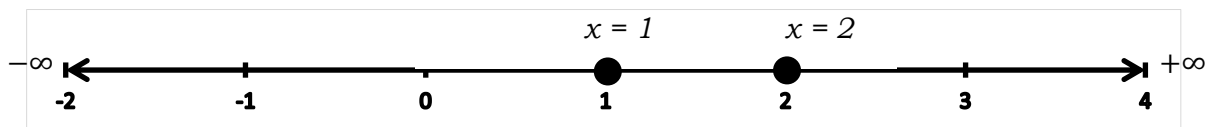
$$ax^2 + bx + c = 0$$

Solve the equation by using factoring

Note: You can use the method that is convenient for you.

Therefore, the solutions to the equation $x^2 - 3x = -2$ are 1 and 2.

Let us illustrate the equation $x^2 - 3x = -2$ using a number line. We will graph the solutions 1 and 2 on the number line.



The graph above illustrates the quadratic equation $x^2 - 3x = -2$. It means that the values of the variable x , 1 and 2, satisfy the equation and make the equation true.

What if we will use an inequality symbol in the mathematical sentence above?

Let us use the sentence $x^2 - 3x < -2$.

Will the solutions be the same?

What are the other possible solutions to inequality?



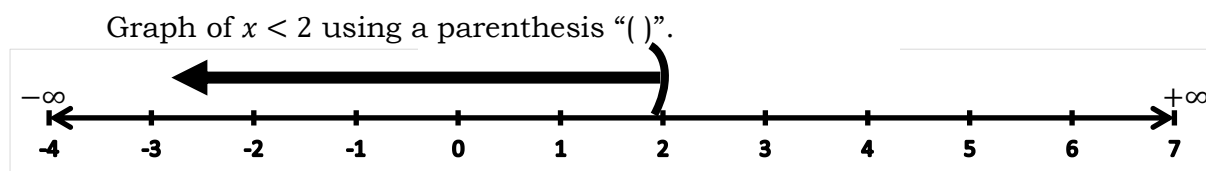
What Is It

To illustrate inequality, we are going to use a number line.

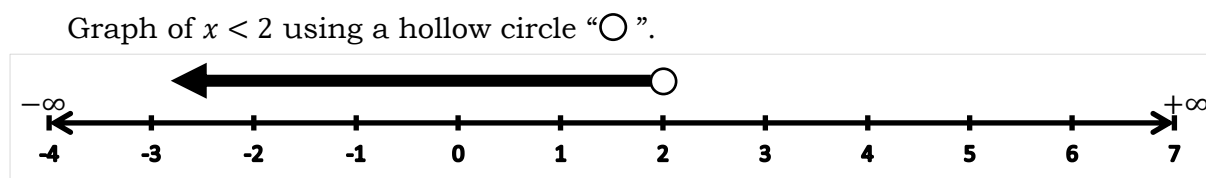
INEQUALITIES OF A NUMBER IN ONE VARIABLE

On a number line, each number corresponds to a single point. However, the inequality statement represents a part of a number line that includes an infinite number of points.

Example 1: The inequality $x < 2$ is represented by the graph below.



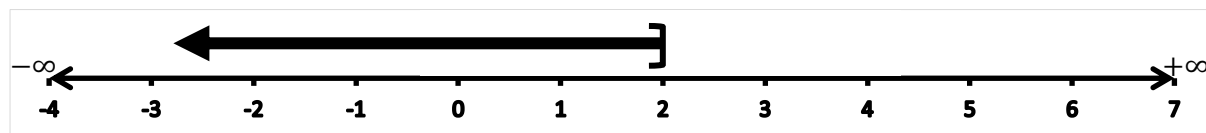
It means that all values or numbers to the left of 2 are represented by the inequality $x < 2$. Since $x = 2$ is not included as a solution set, we use parenthesis “()” as shown in the graph above.



The inequality notation $x < 2$ can also be written in the interval notation $(-\infty, 2)$. The symbol “ ∞ ” is read as infinity, and 2 is not included in the interval.

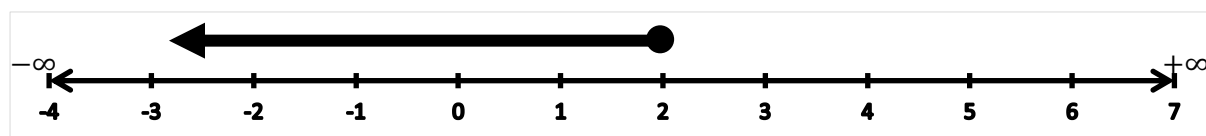
Example 2: The inequality $x \leq 2$ is represented by

Graph of $x \leq 2$ using a bracket “[]”.



It means that all numbers or values to the left of 2 are represented by the inequality $x \leq 2$. A bracket “[]” can be used in graphing the inequality as shown in the graph above, because 2 is included as a solution set.

Graph of $x \leq 2$ using a solid circle “●”



Its interval notation is $(-\infty, 2]$.

Look at the following inequalities being represented by a number line and written in interval notation.

	Graph	Interval Notation
1.) $x \geq 0$		$[0, +\infty)$ 0 is included in the interval
2.) $x \leq -1$		$(-\infty, -1]$ -1 is included in the interval
3.) $x < \frac{1}{2}$		$(-\infty, \frac{1}{2})$ $\frac{1}{2}$ is not included in the interval
4.) $-6 < x < -2$		$(-6, -2)$ -6 and -2 are not included in the interval
5.) $-6 \leq x \leq -2$		$[-6, -2]$ -6 and -2 are included in the interval

It means that the values of the x are less than -2 but greater than -6 .

It means that the values of the x are less than or equal to -2 but greater than or equal to -6 .

What is the difference between the graphs being presented above?

Can you name what kind of inequality is being illustrated in examples 1, 2, and 3? How about the illustration in examples 4 and 5? What do we call them?

These are *linear* and *quadratic* inequalities. The inequality in one variable is called **Linear Inequalities**. Its highest power of the variable is 1. The graphs in examples numbers 1, 2, and 3 illustrate linear inequality. On the other hand, **Quadratic Inequality** is an equality with degree 2. Items 4 and 5 are examples of the graphs of quadratic inequality.

Try This!

Group the following inequalities into Linear Inequality and Quadratic Inequality.

$x^2 - 3x \geq -2$

$10x + 2 > -18$

$3x \leq 1$

$x + 2 < 0$

$-4 < x^2$

$-3x - x^2 > 0$

Answer to “Try This” on page 7:

Linear Inequality	Quadratic Inequality
$3x \leq 1$	$x^2 - 3x \geq -2$
$x + 2 < 0$	$-3x - x^2 > 0$
$10x + 2 > -18$	$-4 < x^2$

Were you able to group the inequalities correctly?

In this module, you will learn about Quadratic Inequalities.

Quadratic Inequality is an inequality that contains a polynomial of degree 2, and that can be written in the following forms:

$$\begin{array}{ll}
 ax^2 + bx + c > 0 & ax^2 + bx + c \geq 0 \\
 ax^2 + bx + c < 0 & ax^2 + bx + c \leq 0 \\
 \text{where } a, b, \text{ and } c \text{ are real numbers and } a \neq 0
 \end{array}$$

So, in the activity above, the following sentences are examples of Quadratic Inequalities:

$$x^2 - 3x \geq -2 \quad -3x - x^2 > 0 \quad -4 < x^2$$

Let’s Try This!

Which of the following inequalities are quadratic?

- 1.) $2e^2 + 3 \leq 5e$
- 2.) $(x + 2)(x - 5) > 0$
- 3.) $26 < x$

Answer and explanation to “Let’s Try This”:

- 1.) Quadratic Inequality since the given is in degree 2.
- 2.) Quadratic Inequality, by using the FOIL method, it can give us the inequality $x^2 - 3x - 10 > 0$.
- 3.) Not Quadratic Inequality, because the degree of the inequality is 1.

**What’s More****NOW, IT’S YOUR TURN!****Activity 1.1: Am I A Quadratic Inequality?**

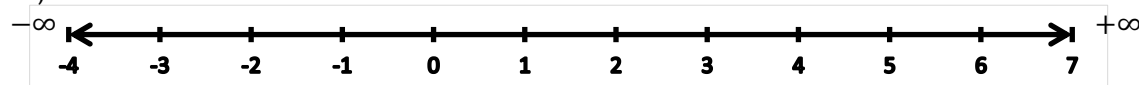
INSTRUCTIONS: Complete the table below.

Inequality	Degree	Classification (Q.I – Quadratic Inequality/ INQ – not Quadratic Inequality)
1. $6x^2 + 9x \leq 12$		
2. $(x - 3)(x + 5) > 0$		
3. $x - 5 > 0$		
4. $2w - 4 \geq 0$		
5. $(2m - 2)^2 \leq 0$		
6. $2(6k - 12) \leq 0$		
7. $5m^2 \leq -3m + 5$		
8. $3 < 3x$		

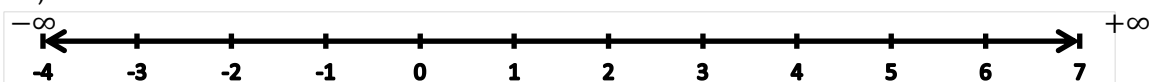
Activity 1.2: Graph me!

Graph the following inequalities on the number line and write in interval notation.

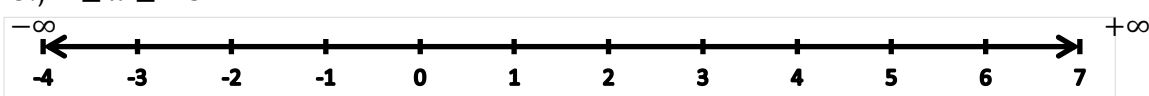
1.) $-2 < x < 4$



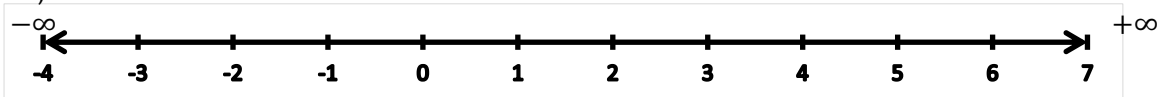
2.) $0 \leq x \leq 5$



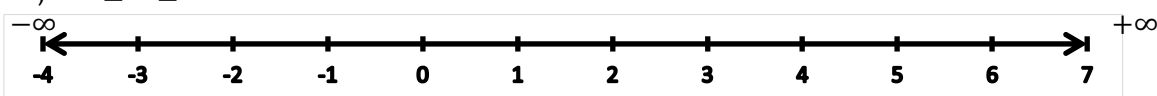
3.) $2 \geq x \geq -3$



4.) $2 > x > 0$



5.) $-1 \leq x \leq 4$



What I Need To Remember

- Quadratic Inequality can be written in the following forms:
$$\begin{array}{ll} ax^2 + bx + c > 0 & ax^2 + bx + c \geq 0 \\ ax^2 + bx + c < 0 & ax^2 + bx + c \leq 0 \end{array}$$
where a, b, and c are real numbers and $a \neq 0$
- A **solution** is a value of the variables that makes the mathematical sentence true.
- In graphing quadratic inequality, remember the following conditions:
 - If the given inequality used the symbol $<$ or $>$, a parenthesis “()” or a hollow circle “○” is used in its boundaries.
 - Use a bracket “[]” or a solid circle “●” if the inequality used is \leq or \geq .
- In writing the solution set for the quadratic inequality, remember the following conditions:
 - If the given inequality is $<$ or $>$, use a parenthesis “()”.
 - Use a bracket “[]” if the inequality is \leq or \geq .

Lesson 2

Solving Quadratic Inequalities



What I Need To Know

At the end of this lesson, you are expected to:
solve and graph quadratic inequalities



What's In

❖ Flashback

Activity 1.3: Find My Roots!

DIRECTIONS: Answer the Riddle.

RIDDLE:

What Bird can lift the Heaviest Weights?

Find the roots/solutions of the given quadratic equation by using whatever method convenient for you. Write the letter of the correct answer in the decoder at the bottom of this exercise. **Show your solution.**

- | | | |
|-------------------------|---|----------------------------------|
| 1.) $6x^2 + 12x = 0$ | R | $x = 0 \text{ \& } 10$ |
| 2.) $2x^2 - 3x - 2 = 0$ | E | $x = -2 \text{ \& } 6$ |
| 3.) $x^2 - 4x - 12 = 0$ | N | $x = 0 \text{ \& } -2$ |
| 4.) $x^2 - 4 = 0$ | A | $x = -\frac{1}{2} \text{ \& } 2$ |
| 5.) $10x - x^2 = 0$ | C | $x = -2 \text{ \& } 2$ |

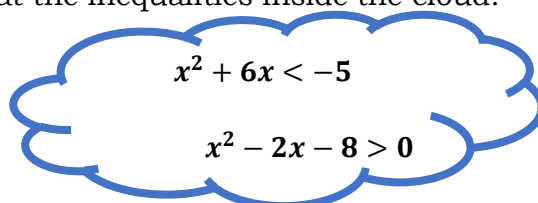
DECODER

2		4	5	2	1	3



What's New

Take a look at the inequalities inside the cloud.



How will you solve for the solutions or values of the variables of the given inequality that will make the statement true?

Can you graph the solutions of the inequalities?



What is It

In module 1, you have learned how to solve quadratic equations using the four methods, namely: *extracting the roots*, *factoring*, *completing the square*, and the *quadratic formula*. In this lesson, solving quadratic inequalities, you will also apply the methods that you have learned in solving quadratic equations.

Here are the following examples of quadratic inequalities:

$$x^2 + 6x < -5$$

$$x^2 - 2x - 8 > 0$$

$$2x^2 - x \leq 6$$

So, how are you going to solve the solutions to these quadratic inequalities? Take a look at these examples:

Example 1: Find the solution set of the inequality $x^2 + 6x < -5$.

To solve for the given inequality $x^2 + 6x < -5$, we will be looking for the values of the unknown variable that will make the expression $x^2 + 6x$ is less than -5 .

Solution:

$$x^2 + 6x < -5$$

$$x^2 + 6x = -5$$

$$x^2 + 6x + 5 = 0$$

$$(x + 1)(x + 5) = 0$$

$$x + 1 = 0 \quad | \quad x + 5 = 0$$

$$x = -1 \quad | \quad x = -5$$

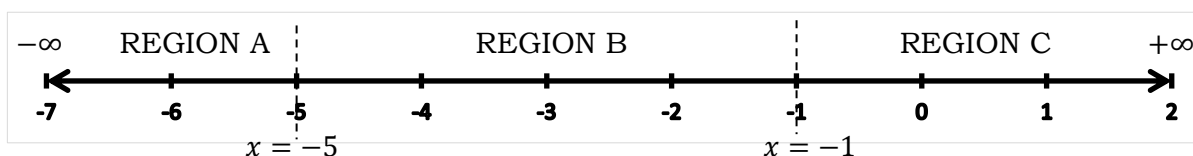
Replace the inequality sign with an equality sign

Rewrite the equation in the form of $ax^2 + bx + c = 0$

Solve the equation

So, $(x + 1)(x + 5)$ is -5 when $x = -1$ or $x = -5$.

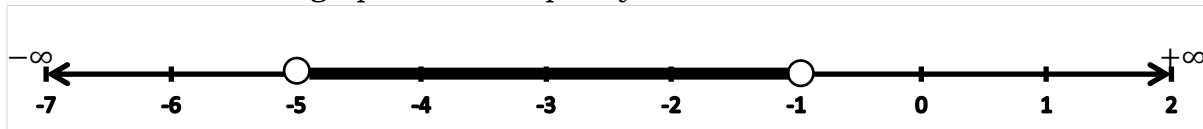
These two values of x , -1 and -5 , will be used to divide the number line below into three regions.



To test whether the inequality $x^2 + 6x < -5$ is true or false in each region, choose a number or value from each region and substitute it to the given inequality. If the resulting inequality is true, then the region containing the test point is a solution region.

Region	Test Point Value	Substitute the value of x to the inequality $x^2 + 6x < -5$	Result
A	Take any value of x in Region A Ex. $x = -6$	$x^2 + 6x < -5$ $(-6)^2 + 6(-6) < -5$ $36 - 36 < -5$ $0 < -5$	False
B	Take any value of x in Region B $x = -2$	$x^2 + 6x < -5$ $(-2)^2 + 6(-2) < -5$ $4 - 12 < -5$ $-8 < -5$	True
C	Take any value of x in Region C $x = 0$	$x^2 + 6x < -5$ $(0)^2 + 6(0) < -5$ $0 < -5$	False

In the table, the values in Region B satisfy the given inequality. And since the symbol of the inequality is $<$, it means that the values -1 and -5 are not included in the solution set. The graph of the inequality is shown below.



The solution set written in interval notation is $(-5, -1)$.

Example 2: Solve $x^2 - 2x - 8 > 0$

To solve for the given inequality

Solution:

$$x^2 - 2x - 8 = 0$$

$$(x + 2)(x - 4) = 0$$

$$x + 2 = 0$$

$$x = -2$$

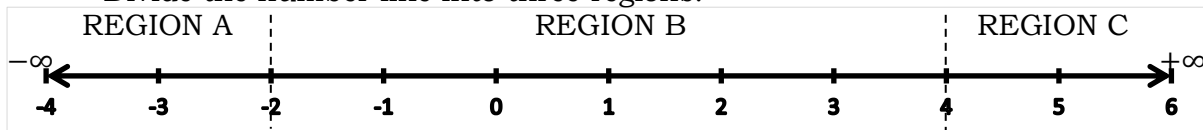
$$x - 4 = 0$$

$$x = 4$$

Replace the inequality sign with an equality sign

Solve the equation

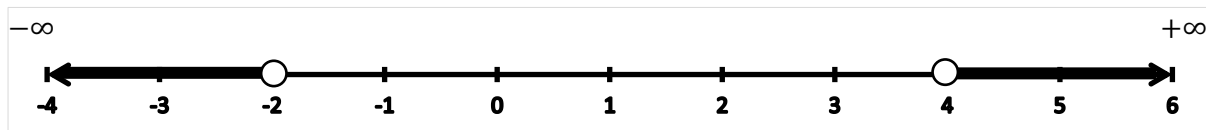
Divide the number line into three regions.



Choose the test points from each region.

Region	Test Point Value	Substitute the value of x to the inequality $x^2 - 2x - 8 > 0$	Result
A	Take any value of x in Region A Ex. $x = -4$	$x^2 - 2x - 8 > 0$ $(-4)^2 - 2(-4) - 8 > 0$ $16 + 8 - 8 > 0$ $16 > 0$	True
B	Take any value of x in Region B Ex. $x = 0$	$x^2 - 2x - 8 > 0$ $(0)^2 - 2(0) - 8 > 0$ $0 - 0 - 8 > 0$ $-8 > 0$	False
C	Take any value of x in Region C Ex. $x = 5$	$x^2 - 2x - 8 > 0$ $(5)^2 - 2(5) - 8 > 0$ $25 - 10 - 8 > 0$ $7 > 0$	True

The values in Regions A and C satisfy the inequality. The values -2 and 4 are not included in the solution set because the symbol for inequality used is $>$. The graph of the inequality is



The solution set is $(-\infty, -2)$ and $(4, +\infty)$ or in interval notation $(-\infty, -2) \cup (4, +\infty)$.

Example 3: Solve for the solution of equation $2x^2 - x \leq 6$.

Solution:

$$2x^2 - x = 6$$

$$2x^2 - x - 6 = 0$$

$$(2x + 3)(x - 2) = 0$$

$$2x + 3 = 0$$

$$2x = -3$$

$$\frac{2x}{2} = -\frac{3}{2}$$

$$x = -\frac{3}{2}$$

$$x - 2 = 0$$

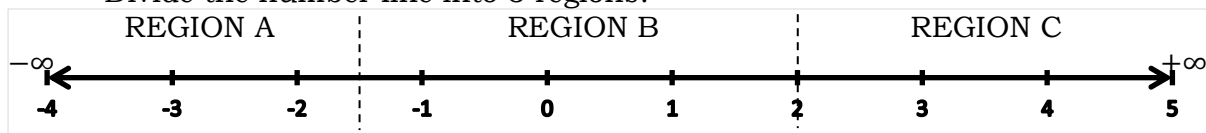
$$x = 2$$

Replace the inequality sign with an equality sign

Rewrite the equation in the form of $ax^2 + bx + c = 0$

Solve the equation

Divide the number line into 3 regions.



Test Points.

Region	Test Point Value	Substitute the value of x to the inequality $2x^2 - x \leq 6$	Result
A	Take any value of x in Region A Ex. $x = -2$	$2x^2 - x \leq 6$ $2(-2)^2 - (-2) \leq 6$ $2(4) + 2 \leq 6$ $8 + 2 \leq 6$ $10 \leq 6$	False
B	Take any value of x in Region B Ex. $x = 0$	$2x^2 - x \leq 6$ $2(0)^2 - (0) \leq 6$ $0 \leq 6$	True
C	Take any value of x in Region C Ex. $x = 3$	$2x^2 - x \leq 6$ $2(3)^2 - (3) \leq 6$ $2(9) - 3 \leq 6$ $18 - 3 \leq 6$ $15 \leq 6$	False

The values in Region B satisfy the inequality $2x^2 - x \leq 6$. Since the inequality symbol is \leq , the numbers $-\frac{3}{2}$ and 2 are included in the solution set.

The graph of the inequality.



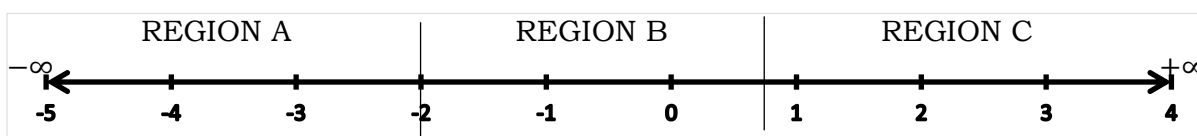
The solution set is $\left[-\frac{3}{2}, 2\right]$.

Example 4: Solve $4x^2 + 5x - 6 \geq 0$

Solution:

$$\begin{array}{lcl}
 4x^2 + 5x - 6 = 0 & \text{Replace the inequality sign with an equality sign} & \\
 (4x - 3)(x + 2) = 0 & & \\
 \left. \begin{array}{l} 4x - 3 = 0 \\ x = \frac{3}{4} \end{array} \right\} \begin{array}{l} x + 2 = 0 \\ x = -2 \end{array} & \text{Solve the equation by Factoring} &
 \end{array}$$

The regions of the number line.

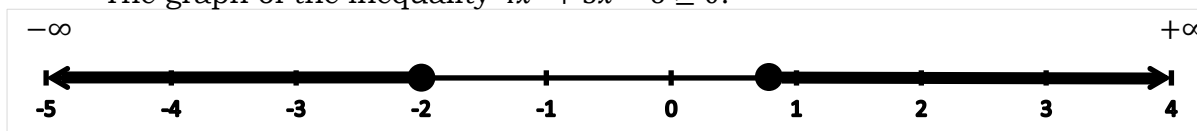


Test Points.

Region	Test Point Value	Substitute the value of x to the inequality $4x^2 + 5x - 6 \geq 0$	Result
A	Take any value of x in Region A Ex. $x = -3$	$4x^2 + 5x - 6 \geq 0$ $4(-3)^2 + 5(-3) - 6 \geq 0$ $4(9) - 15 - 6 \geq 0$ $36 - 21 \geq 0$ $15 \geq 0$	True
B	Take any value of x in Region B Ex. $x = 0$	$4x^2 + 5x - 6 \geq 0$ $4(0)^2 + 5(0) - 6 \geq 0$ $0 - 0 - 6 \geq 0$ $-6 \geq 0$	False
C	Take any value of x in Region C Ex. $x = 1$	$4x^2 + 5x - 6 \geq 0$ $4(1)^2 + 5(1) - 6 \geq 0$ $4(1) + 5 - 6 \geq 0$ $4 + 5 - 6 \geq 0$ $3 \geq 0$	True

The values in Regions A and C satisfy the inequality $4x^2 + 5x - 6 \geq 0$. Since the inequality symbol is \geq , the numbers -2 and $\frac{3}{4}$ are included in the solution set.

The graph of the inequality $4x^2 + 5x - 6 \geq 0$.



The solution set of the inequality is $(-\infty, -2]$ and $[\frac{3}{4}, +\infty)$.



What's More

Activity 1.4: NOW, IT'S YOUR TURN!

INSTRUCTIONS: Solve each quadratic inequality. Write the solution in interval notation. Show your solution.

1.) $x^2 + 9x - 10 < 0$

4.) $3x^2 - 5x - 2 \leq 0$

2.) $x^2 - x > 20$

5.) $x^2 - 25 \leq 0$

3.) $5x^2 + 20x \geq 0$

6.) $8x^2 - 2x > 1$



What I Need To Remember

- **The following steps can help you solve and graph for the solution of a quadratic inequality.**
 - a. Get the corresponding equation by replacing the inequality sign with an equality sign.
 - b. Solve the equation by using any of the four methods in solving quadratic equations: extracting the roots, factoring, completing the square, and using the quadratic formula.
 - c. Graph the inequality using a hollow circle “○” if the given inequality is $<$ or $>$. But if the inequality is \leq or \geq , then use the solid circle “●” in its boundaries.
 - d. Divide the number line into three regions.
 - e. Choose a representative number or value from each region.
 - f. Substitute the representative number or value into the given inequality. Whichever value satisfies the inequality is a solution and therefore it belongs to the solution set of the inequality.
 - g. Shade the region where the point belongs.

Lesson 3

Word Problems Involving Quadratic Inequalities



What I Need To Know

At the end of this lesson, you are expected to:
solve problems involving quadratic inequalities



What's In

Word problems that contain words such as “is at most”, “is at least”, “is more than”, “is no more than”, “is less than”, “is no less than” usually indicate inequality.

Try this!

Translate the statements into mathematical statements.

- The required score to pass the test is at least 38.
- The number of people is more than 60.
- The grade is no more than 90.
- The age of the applicant is at most 30 years old.

Solutions:

- The required score to pass the test is at least 38.

Translation: $\text{score} \geq 38$, in the statement, the word at least is translated with the inequality symbol “ \geq ” because the score should not be lower than 38 to pass the test.

Mathematical Statement: $s \geq 38$

- The number of people is more than 60.

Translation: $\text{number of people} > 60$, the inequality is “ $>$ ” because in the statement it says that the number of people should be more than 60 people.

Mathematical Statement: $n > 60$

- The grade is no more than 90.

Translation: $\text{grade} \leq 90$, in the statement, the word *no more than* is translated with the inequality symbol “ \leq ” because the grade should not be greater than 90 but can be exactly equal to 90.

Mathematical Statement: $g \leq 90$

- The age of the applicant is at most 30 years old.

Translation: $\text{age} \leq 30$, the inequality is “ \leq ”.

The statement says that the applicant's age should not be older than 30 years but can be equal to 30 years old.

Mathematical Statement: $a \leq 30$

❖ **Activity 1.5: Translate Me!**

INSTRUCTIONS: Read the English phrases below and translate them into Mathematical statement. To represent the unknown quantity, use the variable indicated in the statement.

<u>English Phrase</u>	<u>Mathematical Phrase</u>
1. The weight (w) is more than 2kg	_____
2. His savings (s) is less than ₱76, 000.00	_____
3. The sum of their ages (s) is no more than 24	_____
4. Her salary (s) is more than ₱25, 000.00	_____
5. The product (p) of two positive numbers is at most 100	_____
6. The time (t) Edgar ran in a marathon is at most 1 hour	_____
7. The cost(c) of a newspaper is no more than ₱25.00	_____
8. The points (p) needed to redeem a product is at least 500 points	_____
9. Her monthly sales (s) in her sari-sari store is no less than ₱90, 000.00	_____
10. The discount (d) is no more than ₱200.00	_____



What's New

❖ **Guess What It Is?**

Instructions: Read the given situation carefully. Try to make a guess on the dimensions of the floor.

The local government plans to put the asymptomatic COVID-19 patients in an isolation area. The quarantine facility has a rectangular floor which will be covered entirely with tiles. The length of the rectangular floor is 10 feet longer than its width, and the area of the floor is at least 1200ft.^2 (A)What are the possible dimensions of the floor?
(B)What are the least dimensions of the floor?



Clipart 3: Online Pictures via Microsoft 365



What Is It

You are now ready to apply the skills you have learned in solving quadratic inequalities applying it in solving word problems involving quadratic inequalities. Solving quadratic inequalities have the same procedure as solving quadratic equations.

STEPS IN SOLVING WORD PROBLEMS by POLYA

1. Analyze the problem
 - a. Read and understand the problem carefully.
 - b. Determine what is being asked in the problem. A sketch may help you.
 - c. Make a representation on the unknown number using any variable/letter.
2. Formulate the equation.
Translate the statement into mathematical expressions
3. Solve the problem and interpret
4. Check
Verify whether your answer satisfies the conditions in the problem



Clipart 4: Online Pictures via Microsoft 365

It is now time for you to solve problems involving inequalities. Try solving the problem presented in “Guess What It Is” following the steps in solving word problems.

Example 1: The local government plans to put the asymptomatic COVID-19 patients in an isolation area. The quarantine facility has a rectangular floor which will be covered entirely with tiles. The length of the rectangular floor is 10 feet longer than its width, and the area of the floor is at least 1200ft^2 .

- A. What are the possible dimensions of the floor?
- B. What are the least dimensions of the floor?

Solution:

1. Analyze the problem

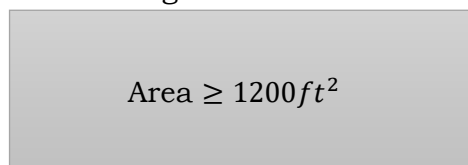
The length of the rectangular floor is 10ft . longer than its width.

Let x = be the width of the floor

$x + 10$ = be the length of the floor

$A_{\text{floor}} \geq 1200\text{ft}^2$

$$\text{length} = x + 10$$



$$\text{width} = x$$

Now, translate the conditions given in the problem into symbols to form inequality.

2. Formulate the inequality

In words: The area of the floor is at least 1200ft.^2 .

In symbols:

$$\begin{aligned}\text{Area} &\geq 1200\text{ft.}^2 \\ (\text{length})(\text{width}) &\geq 1200\text{ft.}^2\end{aligned}$$

Inequality: $(x + 10)(x) \geq 1200\text{ft.}^2$

3. Solve the inequality and state the answer.

Solving for x ,

$$(x + 10)(x) \geq 1200$$

$$(x + 10)(x) = 1200 \quad \text{Change the inequality into equality}$$

$$x^2 + 10x = 1200$$

$$x^2 + 10x - 1200 = 0$$

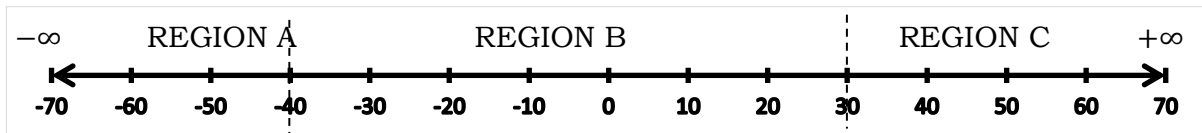
$$(x + 40)(x - 30) = 0$$

$$x = -40 \quad | \quad x = 30$$

Solve the equation

NOTE: You may use whatever method that is convenient for you.

Plot the values of x in the number line and divide it into three regions.

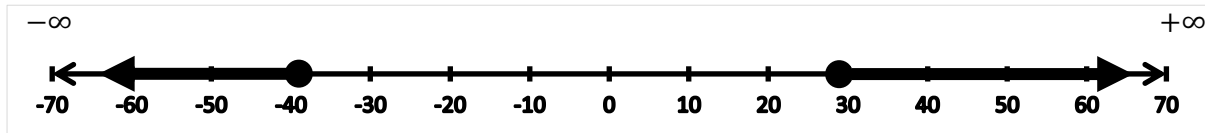


Test whether the inequality $(x + 10)(x) \geq 1200$ is true or false in each region. Choose a number or value from each region and substitute it for the given inequality. If the resulting inequality is true, then the region containing the test point is a solution region.

Region	Test Point Value	Substitute the value of x to the inequality $(x + 10)(x) \geq 1200$	Result
A	Take any value of x in Region A Ex. $x = -41$	$(x + 10)(x) \geq 1200$ $(-41 + 10)(-41) \geq 1200$ $(-31)(-41) \geq 1200$ $1271 \geq 1200$	True
B	Take any value of x in Region B Ex. $x = 0$	$(x + 10)(x) \geq 1200$ $(0 + 10)(0) \geq 1200$ $(10)(0) \geq 1200$ $0 \geq 1200$	False
C	Take any value of x in Region C Ex. $x = 31$	$(x + 10)(x) \geq 1200$ $(31 + 10)(31) \geq 1200$ $(41)(31) \geq 1200$ $1271 \geq 1200$	True

In the table, the values in Regions A and C satisfy the given inequality. Since, we are talking about measurements, we disregard the values in Region A because they consist of negative numbers, and a negative number cannot be a solution in Geometric problems. And since the symbol of the inequality is \geq it means that the

values -40 and 30 are included in the solution set. The graph of the inequality is shown below.



The solution set written in interval notation is $(-\infty, -40]$ and $[30, +\infty)$.

Hence, the width of the rectangular floor is 30 feet, since the other value $x = -40$, is unrealistic.

Solve for the length of the floor.

$$\begin{aligned}\text{length} &= x + 10 \\ &= 30 + 10 \\ &= 40\end{aligned}$$

Answer to Question A. What are the possible dimensions of the floor?

The possible dimensions of the rectangular floor are from 30 feet and 40 feet, or within the solution set $[30, +\infty)$ that will give an area of at least 1200 ft^2 .

Answer to Question B. What are the least dimensions of the floor?

The least dimensions of the floor that are 30 feet and 40 feet.

4. Checking

Check your solution using the conditions given in the problem.

Is the area of the rectangular floor at least 1200 ft^2 ?

$$\begin{aligned}A_{\text{floor}} &\geq 1200 \text{ ft}^2 \\ (\text{length})(\text{width}) &\geq 1200 \text{ ft}^2 \\ (40 \text{ ft.})(30 \text{ ft.}) &\geq 1200 \text{ ft}^2 \\ 1200 \text{ ft}^2 &\geq 1200 \text{ ft}^2\end{aligned}$$

YES

Example 2: Cebu City implemented the General Community Quarantine. Minors and senior citizens were prohibited from going outside their houses. Both Ana and Shani cannot go out. What is the greatest age of the two girls if Ana's age was 14 less the age of Shani and the product of their ages was at most 72.

Solution:

1. Analyze the problem

Ana's age was 14 less the age of Shani.

Let x = be the age of Shani

$x - 14$ = Ana's age

2. Formulate the inequality

The product of their ages was at most 72.

In words:

Ana's age	times	Shani's age	was at most 72
-----------	-------	-------------	----------------

In symbols

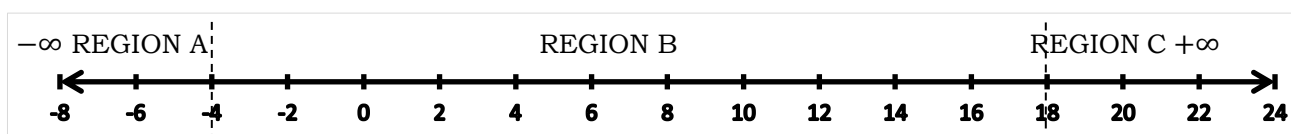
$$x - 14 \quad \cdot \quad x \quad \leq 72$$

Inequality: $(x - 14)(x) \leq 72$

3. Solve the inequality and state the answer.

Solving for x ,

$$\begin{array}{lcl} (x-14)(x) \leq 72 & \left. \begin{array}{l} \\ \\ \end{array} \right\} & \text{Change the inequality sign to equality sign} \\ (x-14)(x) = 72 & & \\ x^2 - 14x = 72 & \left. \begin{array}{l} \\ \\ \end{array} \right\} & \text{Rewrite the equation into standard form} \\ x^2 - 14x - 72 = 0 & & \text{Solve the equation} \\ (x+4)(x-18) = 0 & & \\ x = -4 \quad | \quad x = 18 & & \end{array}$$

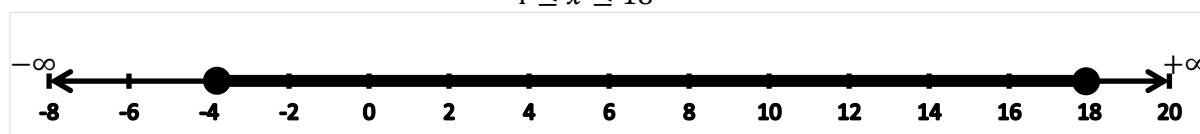


Test whether the inequality $(x-14)(x) \leq 72$ is true or false in each region

Region	Test Point Value	Substitute the value of x to the inequality $(x-14)(x) \leq 72$	Result
A	Take any value of x in Region A Ex. $x = -5$	$(x-14)(x) \leq 72$ $(-5-14)(-5) \leq 72$ $(-19)(-5) \leq 72$ $95 \leq 72$	False
B	Take any value of x in Region B Ex. $x = 0$	$(x-14)(x) \leq 72$ $(0-14)(0) \leq 72$ $(-14)(0) \leq 72$ $0 \leq 72$	True
C	Take any value of x in Region C Ex. $x = 19$	$(x-14)(x) \leq 72$ $(19-14)(19) \leq 72$ $(5)(19) \leq 72$ $95 \leq 72$	False

The range of the possible values of x is

$$-4 \leq x \leq 18$$



Since you cannot have a negative number as a solution to the problem, disregard $x = -4$ as the possible age of Shani, instead, find the greatest age of the two girls, then $x = 18$ will be used as the age of Shani.

Solve for Ana's age if Shani is 18 years old.

$$\begin{aligned} \text{Ana's age} &= x - 14 \\ &= 18 - 14 \end{aligned}$$

$$\text{Ana's age} = 4$$

Therefore, the greatest age of Shani and Ana is 18 years old and 4 years old, respectively.

4. Checking

Is the product of both girls' ages is at most 72?

$$\begin{aligned} (18-14)(18) &\leq 72 \\ (4)(18) &\leq 72 \\ 72 &\leq 72 \quad \text{YES} \end{aligned}$$



What's More

Activity 1.6: NOW, IT'S YOUR TURN!

INSTRUCTIONS: Solve the following word problems below by following the steps presented. Show your solution in a separate sheet of paper.

1. The product of the COVID-19 positive cases of the two provinces in Cebu is no less than 42. Find the figures of the positive COVID- 19 patients of the two provinces if the figures are two consecutive integers.
2. Wilma is 32 years younger than her Mother. What is Wilma's greatest age if the product of their ages is at most 900? Is Wilma allowed to go out of their house during General Community Quarantine? Yes or No? Why?
3. The length of a face mask is twice longer than its width. What are the least possible measures of the length and the width of the face mask if its area is no less than 200cm^2 ?



What I Need To Remember

- Translate word problems into mathematical symbols before finding its answer.
- Follow the following steps before solving any given word problem.
 1. Analyze the problems
 2. Formulate the equation
 3. Solve the equation and state the answer
 4. Checking
- In age problem, a negative or imaginary number cannot be a solution.
- In word problems that involve geometric figures, a negative or an imaginary number cannot be a solution to the problem.



What I Can Do

A. Am I a Solution or Not?

INSTRUCTIONS: Determine whether each of the following points is a solution to the inequality $2x^2 + x - 10 \geq 0$ or not. Write **TRUE** if the point is a solution to inequality and **FALSE** if it's not.

- 1.) -3
- 2.) 0
- 3.) 7
- 4.) 1

B. Write about Me!

INSTRUCTIONS: In what real-life situations can you apply quadratic inequalities? How helpful is this newly-acquired concept in making your decisions? Write your answer on a piece of paper.



Your reflection or output will be graded using the rubric.

Skills	5	4	3	2
Depth of Reflection	The response demonstrates an in-depth reflection on and personalization of the theories, concepts, and strategies presented in the module.	The response demonstrates a general reflection on, personalization of the theories, concepts, and strategies presented in the module.	The response demonstrates a minimal reflection on, personalization of the theories, concepts, and strategies presented in the module.	The response demonstrates a lack of reflection on the topic, with no details provided.
Required Components	The response surpasses the required components of the selected topic.	The response includes the required components of the selected topic.	The response includes only few components of the selected topic.	The response does not include the required components of the selected topic.
Quality of Information	The response clearly relates to the main topic. It includes all the necessary supporting examples.	The response clearly relates to the main topic. It includes 1-2 necessary supporting examples.	The response clearly relates to the main topic. No examples are given.	The response has little to do or no relations to the main topic.
Conventions	The response is stated clearly, concisely, and properly. Reasons are expressed logically	The response is stated clearly, concisely, and properly. Reasons are expressed illogically.	The response is unclear and not organized. Thoughts are not expressed logically.	The response is unclear and disorganized. Thoughts make little to no sense.



Assessment (Post Test)

DIRECTIONS: Read the following questions carefully. Some items need to be solved. Solve it on a separate sheet of paper. Choose the correct answer and write it in your answer sheets. If the answer is not found in the given choices, kindly write the correct answer.

- ____ 1.) What do you call an inequality that contains a polynomial of degree 2?
A. Linear Inequality C. Quadratic Inequality
B. Linear Equation D. Quadratic Equation

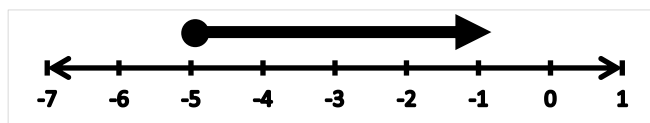
- ____ 2.) Which statement can be modeled by $x \leq 12$?
A. The price is no more than 12.
B. The number of students is less than 12.
C. The height of the pole is more than 12 feet.
D. The price of a softdrink is less than 12 pesos.

- ____ 3.) Which of the following situations below has an inequality symbol " $>$ "?
i. The number of people in the gym is more than 15 people.
ii. The grade he needs to qualify for the scholarship is no less than 90.
iii. His monthly deduction is at least ₱10, 000.00.

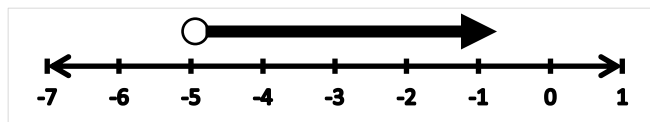
A. *i* only B. *i* and *ii* C. *ii*, and *iii* D. *i*, *ii*, and *iii*

- ____ 4.) Which of the following graphs represents the inequality $x > -5$?

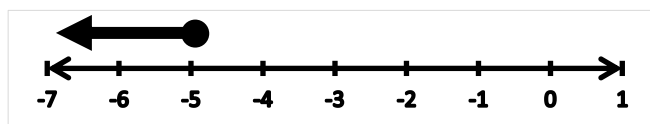
A.



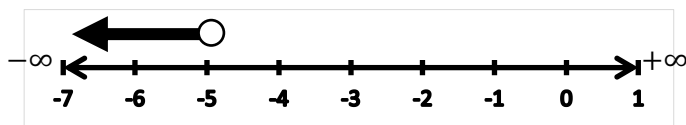
B.



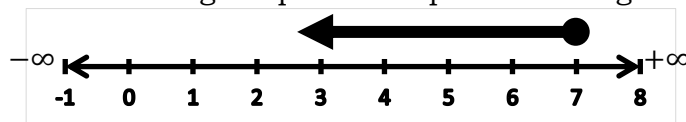
C.



D.



____ 5.) Which of the following inequalities represents the given graph below?



- A. $x < 7$ B. $x \leq 7$ C. $x = 7$ D. $x \geq 7$

____ 6.) Which of the following is **NOT** a solution to the quadratic inequality $5x^2 - 2 > -2x$?

- A. -1 B. 0 C. 1 D. 2

____ 7.) Which of the following statements is a quadratic inequality?

- A. $x^2 + 2x > x^2$ C. $4x(4 + 2) \leq 0$
 B. $2(3x + 3) = 0$ D. $2x(3x + 3) \geq 0$

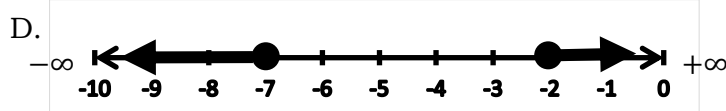
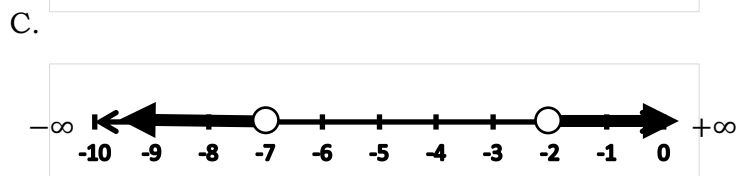
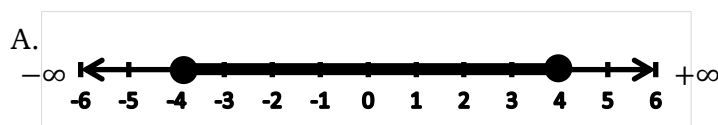
____ 8.) Which of the following statements does **NOT** state a quadratic inequality?

- A. $-2x + x^2 + 2 > 0$ C. $2x + 5 \geq 0$
 B. $3x^2 + 3x + 5 < 0$ D. $-2x^2 + 5 \geq 0$

Items 9 – 10, refer to the inequality inside the box. Then, answer the questions that follow.

$$x^2 - 16 < 0$$

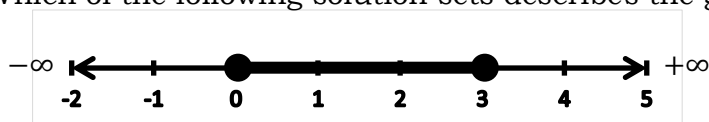
____ 9.) Which of the following graphs represents the given inequality?



____ 10.) What is the solution set of the inequality $x^2 - 16 < 0$?

- A. $(-\infty, -7)$ and $(-2, +\infty)$ C. $(-4, 4)$
 B. $(-\infty, -7]$ and $[-2, +\infty)$ D. $[-4, 4]$

____ 11.) Which of the following solution sets describes the graph below?



- A. $0 < x < 3$ B. $0 > x < 3$ C. $0 \leq x \leq 3$ D. $0 \geq x \leq 3$

- ____ 12.) What is the interval notation form of the solution set $-1 \leq x \leq 4$?
- A. $(-\infty, -1)$ and $(4, +\infty)$ C. $(-1, 4)$
 B. $(-\infty, -1]$ and $[4, +\infty)$ D. $[-1, 4]$

Items 13 – 16, refer to the problem inside the box, then answer the questions that follow.

The length of the study table is thrice its width. Its area is at least 147in^2 .

- ____ 13.) Which of the following represents the length of the table?
 A. $3x$ B. $x + 3$ C. x D. 3
- ____ 14.) Which of the following expressions represents the width of the table?
 A. $3x$ B. $x + 3$ C. x D. 3
- ____ 15.) Which inequality represents the situation?
 A. $x(x) \geq 147$ B. $x(x + 3) \geq 147$ C. $x(3x) \geq 147$ D. $x(3) \geq 147$
- ____ 16.) What is the least measurement of the length of the table?
 A. 21-inches B. 10-inches C. 7-inches D. 3 -inches
- ____ 17.) A positive integer is 2 less than the other number. If the product of the numbers is no more than 35, what are the two numbers?
 A. 5 and 7 B. - 5 and - 7 C. - 5 and 7 D. 5 and - 7
- ____ 18.) The length of the swimming pool is 2-meters more than its width. If the area of the pool is no less than 48m^2 , which of the following could be its width?
 A. 4-meters B. 6-meters C. 8-meters D. 10 -meters
- ____ 19.) The profit that a manufacturing company earns for selling a number of Face shields can be modeled by $P = 5x^2 + 110x - 600$. What is the least number of face shields that must be sold for a profit of at least ₱1, 000. 00?
 A. 10 face shields B. 20 face shields C. 32 face shields D. 50 face shields
- ____ 20.) The width of the lot is 4-meters shorter than its length. How long should its length be to have an area of no more than 60m^2 ?
 A. 16-meters B. 10-meters C. 6-meters D. 4 -meters



Answer Key (With Explanation)

Remember: This portion of the module contains all the answers. Your **HONESTY** is required.

Activity 1.1: Am I Quadratic Inequality?

Inequality	Degree	Classification
1. $6x^2 + 9x \leq 12$	2	Q.I
2. $(x-3)(x+5) > 0$	2 <i>By FOIL method, it will be transformed to $x^2 + 2x - 15 > 0$</i>	Q.I
3. $x - 5 > 0$	1	INQ
4. $2w - 4 \geq 0$	1	INQ
5. $(2m-2)^2 \leq 0$	2 <i>By FOIL method, it will be transformed to $4m^2 - 8m + 4 \leq 0$</i>	Q.I
6. $2(6k-12) \leq 0$	1 <i>By distributive property, it will be transformed to $12k - 24 \leq 0$</i>	INQ
7. $5m^2 \leq -3m + 5$	2	Q.I
8. $3 < 3x$	1	INQ

2	4	5	2	1	3
A	C	R	A	N	E

CRANE (Machine)

A crane is a type of machine that is used in lifting heavy things and transporting them to other places.

Solutions:

$$1.) \quad 6x^2 + 12x = 0$$

$$6x(x+2) = 0$$

$$6x = 0 \quad | \quad x+2 = 0$$

$$x = 0 \quad | \quad x = -2$$

$$2.) \quad 2x^2 - 3x - 2 = 0$$

$$(2x+1)(x-2) = 0$$

$$2x = -1 \quad | \quad x-2 = 0$$

$$x = -\frac{1}{2} \quad | \quad x = 2$$

$$3.) \quad x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x-6 = 0 \quad | \quad x+2 = 0$$

$$x = 6 \quad | \quad x = -2$$

$$4.) \quad x^2 - 4 = 0$$

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

$$5.) \quad 10x - x^2 = 0$$

$$x(10-x) = 0$$

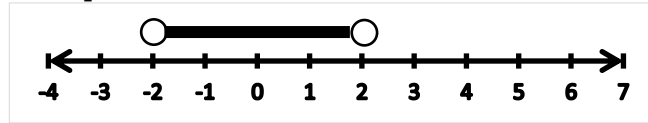
$$10-x = 0 \quad | \quad 0 = x$$

$$10 = x$$

Activity 1.2: Graph Me!

1.) $-2 < x < 4$

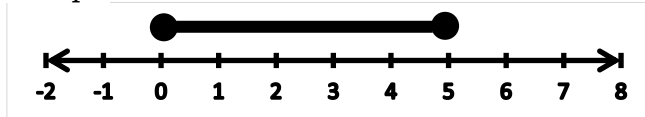
Graph:



Interval Notation: $(-2, 4)$

2.) $0 \leq x \leq 5$

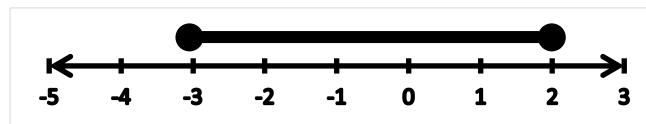
Graph



Interval Notation: $[0, 5]$

3.) $2 \geq x \geq -3$

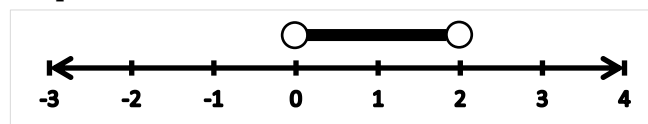
Graph



Interval Notation: $[-3, 2]$

4.) $2 > x > 0$

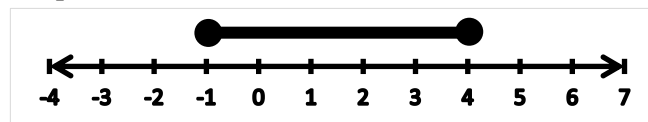
Graph



Interval Notation: $(0, 2)$

5.) $-1 \leq x \leq 4$

Graph



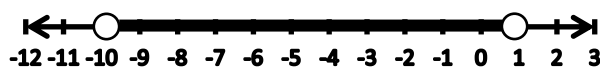
Interval Notation: $[-1, 4]$

Activity 1.4

1.) $(x - 1)(x + 10) = 0$

$x = 1$ and $x = -10$

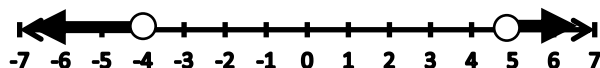
Region	Result
A	False
B	True
C	False

Solution Set: $(-10, 1)$

2.) $(x + 4)(x - 5) = 0$

$x = -4$ and $x = 5$

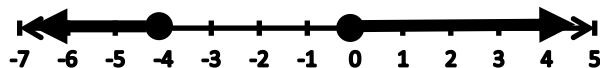
Region	Result
A	True
B	False
C	True

Solution Set: $(-\infty, -4)$ and $(5, +\infty)$

3.) $5x(x + 4) = 0$

$x = 0$ and $x = -4$

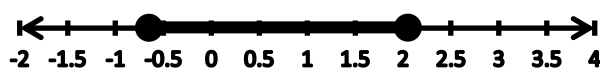
Region	Result
A	True
B	False
C	True

Solution Set: $(-\infty, -4]$ and $[0, +\infty)$

4.) $(3x + 1)(x - 2) = 0$

$x = -\frac{1}{3}$ and $x = 2$

Region	Result
A	False
B	True
C	False

Solution Set: $\left[-\frac{1}{3}, 2\right]$

5. $(x - 5)(x + 5) = 0$
 $x = 5$ and $x = -5$

Region	Result
A	False
B	True
C	False



Solution Set: $[-5, 5]$

6.) $(4x + 1)(2x - 1) = 0$
 $x = -\frac{1}{4}$ and $x = \frac{1}{2}$

Region	Result
A	True
B	False
C	True



Solution Set: $(-\infty, -\frac{1}{4})$ and $(\frac{1}{2}, \infty)$

Activity
1.5: Transform Me!

1. $w > 2$
2. $s < 76\,000$
3. $s \leq 24$
4. $s > 25\,000$
5. $p \leq 100$
6. $t \leq 1$
7. $c \leq 25$
8. $p \geq 500$
9. $s \geq 90\,000$
10. $d \leq 200$

What Can I Do

A. Am I a solution or not?

1.) -3 is a solution.
 $2x^2 + x - 10 \geq 0$
 $2(-3)^2 + (-3) - 10 \geq 0$
 $2(9) - 3 - 10 \geq 0$
 $18 - 13 \geq 0$
 $5 \geq 0$ **TRUE**

2.) 0 is not a solution.
 $2x^2 + x - 10 \geq 0$
 $2(0)^2 + (0) - 10 \geq 0$
 $0 - 10 \geq 0$
 $-10 \geq 0$ **FALSE**

3.) 7 is not a solution.
 $2x^2 + x - 10 \geq 0$
 $2(7)^2 + (7) - 10 \geq 0$
 $2(49) + 7 - 10 \geq 0$
 $98 - 3 \geq 0$
 $95 \geq 0$ **TRUE**

4.) 1 is not a solution.
 $2x^2 + x - 10 \geq 0$
 $2(1)^2 + (1) - 10 \geq 0$
 $2(1) + 1 - 10 \geq 0$
 $2 - 9 \geq 0$

What Can I Do

B. Write About Me!

Text Text text text text text
text text text text text text

What's More Activity 1.6

1.) The number of positive cases in the two provinces are 6 and 7 positive COVID – 19 cases

Sol'n: Let x = number of cases in the 1st province
 $x + 1$ = number of cases in the 2nd province
Use the inequality in solving:
 $x(x + 4) > 42$

2.) Wilma's age = 18 years old; and she is not allowed to go out in their house.
Sol'n: Let x = be the age of Wilma's mother
 $x - 32$ = Wilma's age
Use the inequality in solving:
 $x(x - 32) \leq 900$

3.) The width and length of the face mask are 10cm and 20cm, respectively
Sol'n: Let x = be the width of the face mask
 $2x$ = be the length of the face mask
Use the inequality in solving:
 $x(2x) \geq 200$

References

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Orines, Fernando B., Diaz, Zenaida E. and Mojica, Maharlika P. 2004. *Next Century Mathematics Intermediate Algebra* (Phoenix Publishing House, Inc.), 84 – 85.

Congratulations!

You are now ready for the next module. Always remember the following:

1. Make sure every answer sheet has your
 - a. *Name*
 - b. *Grade and Section*
 - c. *Title of the Activity or Activity No.*
2. Follow the date of submission as agreed with your teacher.
3. Keep the modules with you.
4. Return them at the end of the school year.