9

Mathematics

Quarter 1 – Module 9
Finding Equation of a Quadratic Function

About the Module

This module was designed and written with you in mind. It is here to help you master about Equation of a Quadratic Functions. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module is divided into three lessons:

- Lesson 1 – Finding Equation of a Quadratic Function given a Table of Values
- Lesson 2 - Finding Equation of a Quadratic Function given a Graph
- Lesson 3 - Finding Equation of a Quadratic Function given its zeros
- Lesson 4 – Application of Quadratic Function

After going through this module, you are expected to:

1. determine the equation of a quadratic function given the following:
   (a) a table of values
   (b) graph
   (c) zeros.
2. solves problems involving quadratic functions

What I Know (Pre-Test)

Instructions: Read each item carefully. Choose the letter of the correct answer and write it on the separate sheet of paper. If answer not found in the given choices, write your correct answer.

For items 1-4, given a table of values, identify the quadratic functions.

1.

x	0	1	2	3	4
$f(x)$	5	11	19	29	41

A. $y = x^2 + 5x + 5$
B. $y = x^2 + 5x - 5$
C. $y = x^2 - 5x + 5$
D. $y = x^2 - 5x - 5$

2.

x	-5	-4	-3	-2	-1
$f(x)$	24	15	8	3	0

A. $y = x^2 + x + 1$
B. $y = x^2 + 1$
C. $y = x^2 - x + 1$
D. $y = x^2 - 1$

3.

x	-3	-2	-1	0	1
$f(x)$	18	8	2	0	2

A. $y = x^2$
B. $y = 2x^2$
C. $y = 3x^2$
D. $y = 4x^2$

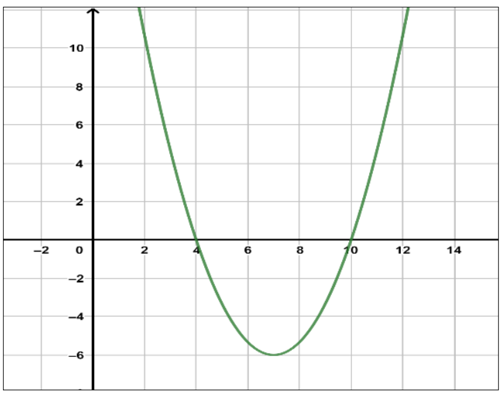
4.

x	-2	-1	0	1	2
$f(x)$	5	0	-3	-4	-3

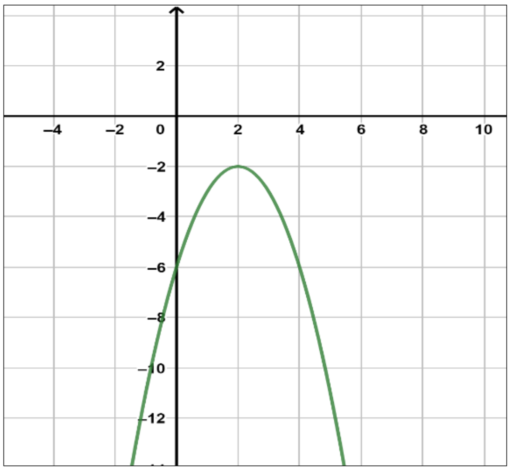
A. $y = x^2 + 2x + 3$
B. $y = x^2 + 2x - 3$
C. $y = x^2 - 2x + 3$
D. $y = x^2 - 2x - 3$

For items 5 - 8, identify the quadratic functions using the given a graph.

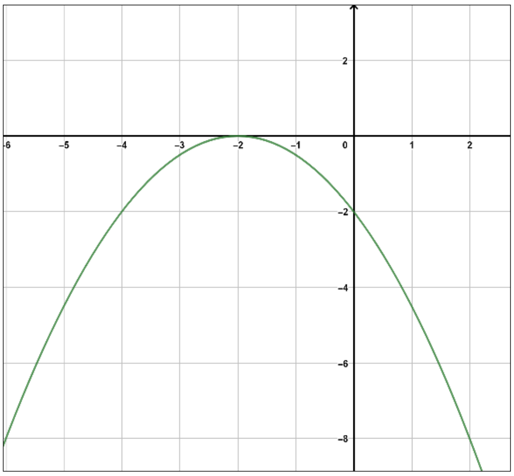
5.



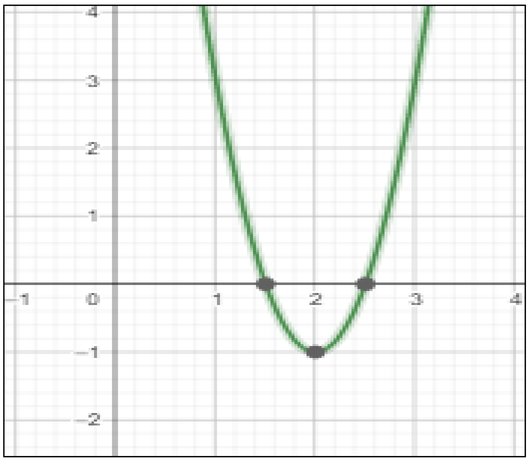
A. $y = \frac{2}{3}(x - 7)^2 - 6$
B. $y = \frac{2}{3}(x + 7)^2 + 6$
C. $y = \frac{2}{3}(x + 7)^2 - 6$
D. $y = \frac{2}{3}(x - 7)^2 + 6$

6. 

A. $y = (x-2)^2 + 2$
B. $y = -(x-2)^2 + 2$
C. $y = (x-2)^2 + 2$
D. $y = -(x-2)^2 - 2$

7. 

A. $y = \frac{1}{2}(x-2)^2$
B. $y = -\frac{1}{2}(x-2)^2$
C. $y = \frac{1}{2}(x+2)^2$
D. $y = -\frac{1}{2}(x+2)^2$

8. 

A. $y = -4(x+2)^2 + 1$
B. $y = -4(x-2)^2 - 1$
C. $y = 4(x-2)^2 + 1$
D. $y = 4(x-2)^2 - 1$

9. Which of the following is the quadratic function whose zeros are 2 and 6?

A. $y = x^2 + 8x + 12$
B. $y = x^2 + 8x - 12$
C. $y = x^2 - 8x + 12$
D. $y = x^2 - 8x - 12$

10. Which of the following choices is the quadratic function whose zeros are -3 and 8?

A. $y = x^2 + 5x + 24$
B. $y = x^2 + 5x - 24$
C. $y = x^2 - 5x + 24$
D. $y = x^2 - 5x - 24$

11. Which of the following choices is the quadratic function whose zeros are $\frac{3}{2}$ and 4?

A. $y = 2x^2 + 11x + 12$
B. $y = 2x^2 + 11x - 12$
C. $y = 2x^2 - 11x + 12$
D. $y = 2x^2 - 11x - 12$

12. During quarantine period, Richard decided to plant a tomato in their backyard. He predicted that the number of tomato plant, x, could yield $y = -20x^2 + 2800x$ tomatoes per month. How many trees should be planted to produce the maximum number of tomatoes per month?

A. 60
B. 70
C. 80
D. 90

13. Mr. Redoble is sketching the windows for a home renovation. The shape is a parabola modeled by the equation $h(w) = -w^2 + 9$, where h is the height of the window and w is the width in feet. What is the maximum height of each window?

A. 6 ft
C. 7 ft
C. 8 ft
D. 9 ft

14. John wants to make a backyard vegetable garden and build a fence around it. What are the dimensions of the rectangular garden that will enclosed it?

A. 10 m x 10 m
B. 15 m x 15 m
C. 20 m x 20 m
D. 25 m x 20 m

15. Rey and Mark recycled bottles and made them into a rocket during their vacation. The height in feet of a bottle rocket is given by $h(t) = 160t - 16t^2$ where t is the time in seconds. How long will it take for the rocket to return to the ground?

A. 5 seconds
B. 10 seconds
C. 15 seconds
D. 20 seconds

Lesson 1	**Finding Equation of a Quadratic Function Given a Table of Values**

What I Need To Know

At the end of this lesson, you are expected to:

- determine the equation of a quadratic function given a table of values.

What's In

❖ ***Let's Recall!***

Consider the quadratic function f given by $f(x) = 3x^2 + 4$. The table shows the values of $f(x)$ for some consecutive values of x and their first and second differences.

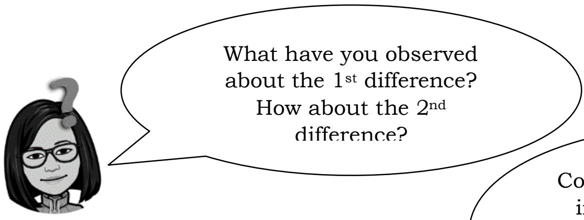
x	1	2	3	4	5	6	7
$f(x)$	7	16	31	52	79	112	151

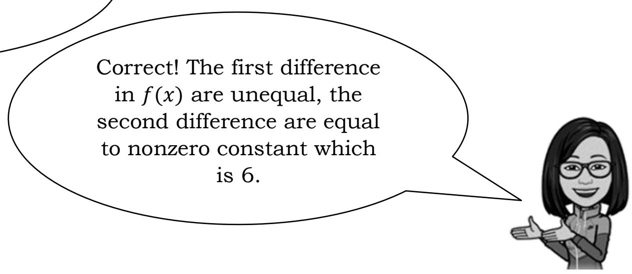
$16 - 7 = ?$ $31 - 16 = ?$ $52 - 31 = ?$ $79 - 52 = ?$ $112 - 79 = ?$ $151 - 112 = ?$

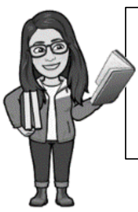
1st Difference → 9 15 21 27 33 39

$15 - 9 = ?$ $21 - 15 = ?$ $27 - 21 = ?$ $33 - 27 = ?$ $39 - 33 = ?$

2nd Difference → 6 6 6 6 6







Note: The first differences in y or $f(x)$ values are unequal while the second differences are equal are characteristics of any quadratic function.

What's New

❖ ***You Try This One!***
Instructions: Derive the quadratic function from the table of values below.

1.

x	1	2	3	4	5	6	7
$f(x)$	5	11	19	29	41	55	71

2.

x	-3	-2	-1	0	1	2	3
$f(x)$	24	16	10	6	4	4	6

What Is It

❖ ***Try to Understand!***

A table of values represents a quadratic function when the first difference of $f(x)$ or y are unequal while the second difference of $f(x)$ or y are equal. Let's take a look the activity in "*You Try This One!*".

1.

x	1	2	3	4	5	6	7
$f(x)$	5	11	19	29	41	55	71

6 8 10 12 14 16

2 2 2 2 2

To find the equation of a quadratic function, keep in mind that the standard form is $f(x) = ax^2 + bx + c$.

Note:

$\boldsymbol{f(x)}$ is the same with $\boldsymbol{y}$ which means that $f(x) = ax^2 + bx + c$ is the same with $y = ax^2 + bx + c$.

Step 1: Consider any three ordered pair from the table of values. Take $(1,5), (3,19)$, and $(4,29)$ and then substitute the values of x and y to the standard form of a quadratic equation.

$(1,5)$
$y = ax^2 + bx + c$
$(\mathbf{5}) = a(\mathbf{1})^2 + b(\mathbf{1}) + c$ *Substitute the value of $x = 1$ and $y = 5$ to form equation 1*
$5 = a + b + c \longrightarrow$ ***Eq. 1***

$(3,19)$
$y = ax^2 + bx + c$
$(\mathbf{19}) = a(\mathbf{3})^2 + b(\mathbf{3}) + c$ *Substitute the value of $x = 3$ and $y = 19$ form equation 2*
$19 = 9a + 3b + c \longrightarrow$ ***Eq. 2***

$(4,29)$
$y = ax^2 + bx + c$
$(\mathbf{29}) = a(\mathbf{4})^2 + b(\mathbf{4}) + c$ *Substitute the value of $x = 4$ and $y = 29$ form equation 1*
$29 = 16a + 4b + c \longrightarrow$ ***Eq. 3***

Step 2: Get the values of a, b and c using the three equation we have Obtained above by using either elimination by addition or subtraction.

Now, subtract eq. 1 from eq. 2 to eliminate the variable c.

Eq. 2 $19 = 9a + 3b + c$
Eq. 1 $5 = a + b + c$

$$\longrightarrow \begin{array}{r} 19 = 9a + 3b + \cancel{c} \\ -5 = -a - b - \cancel{c} \\ \hline 14 = 8a + 2b \end{array}$$

Remember the Rules of Subtraction of Integers (Always change the sign of the subtrahend and then add.)

Simplify: Divide both side by 2

$$\frac{14}{2} = \frac{8a + 2b}{2} \longrightarrow \textbf{\textit{Eq. 4}}$$
$$7 = 4a + b$$

Now do the same with eq. 3 & eq. 2 to eliminate.

Remember the Rules of Subtraction of Integers (Always change the sign of the subtrahend and then add.)

Eq. 3 $29 = 16a + 4b + c$
Eq. 2 $19 = 9a + 3b + c$

$$\longrightarrow \begin{array}{r} 29 = 16a + 4b + \cancel{c} \\ -19 = -9a - 3b - \cancel{c} \\ \hline 10 = 7a + b \end{array} \longrightarrow \textbf{\textit{Eq. 5}}$$

Also subtract eq. 5 and eq. 4 to eliminate b to solve for a.

Eq. 5 $10 = 7a + b$
Eq. 4 $7 = 4a + b$

$$\longrightarrow \begin{array}{r} 10 = 7a + \cancel{b} \\ -7 = -4a - \cancel{b} \\ \hline \end{array}$$

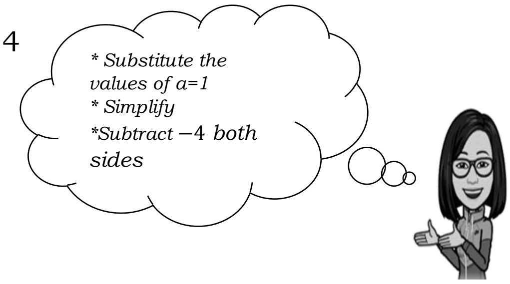
$$\frac{3}{3} = 3a$$
$$\boxed{1 = a}$$

Remember the Rules of Subtraction of Integers (Always change the sign of the subtrahend and then add.)

Simplify: Divide both side by 3

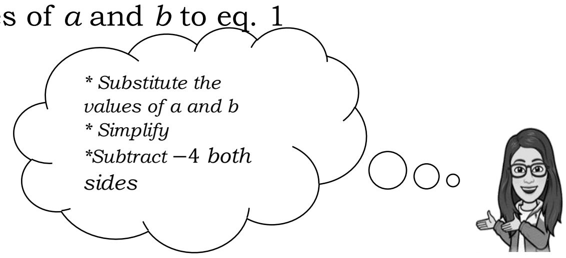
Then substitute $a = 1$ to eq. 4

$7 = 4a + b$ **eq. 4**
$7 = 4(\mathbf{1}) + b$
$7 = 4 + b$
$7 - 4 = 4 - 4 + b$
$\boxed{3 = b}$



Also, substitute the values of a and b to eq. 1

$5 = a + b + c$ **eq. 1**
$5 = (\mathbf{1}) + (\mathbf{3}) + c$
$5 = 4 + c$
$5 - 4 = 4 - 4 + b$
$\boxed{1 = b}$



Note: You can substitute values of a, b or c to any equation you have formed.

Step 3: Finally, *substitute the values of a, b, and c to the standard form of quadratic function* $f(x) = ax^2 + bx + c$.

$f(x) = ax^2 + bx + c$ *Substitute the values of a, b and c*
$f(x) = (\mathbf{1})x^2 + (\mathbf{3})x + (\mathbf{1})$ *Simplify*
$\boxed{f(x) = x^2 + 3x + 1}$

So, the quadratic function that satisfy the given table of values below is $f(x) = x^2 + 3x + 1$.

x	1	2	3	4	5	6	7
$f(x)$	5	11	19	29	41	55	71

2. Find the quadratic function that satisfy the given table of values below.

x	-3	-2	-1	0	1	2	3
$f(x)$	24	16	10	6	4	4	6

Step 1: Consider any three ordered pair from the table of values. Take $(-1, 10), (0, 6)$, and $(1, 4)$ and then substitute the values of of x and y to the standard form of a quadratic equation.

$(-1, 10)$
$y = ax^2 + bx + c$
$(\mathbf{10}) = a(-\mathbf{1})^2 + b(-\mathbf{1}) + c$
$10 = a - b + c$ $\longrightarrow$ ***Eq. 1*** *Substitute the value of* $x = -1$ *and* $y = 10$ *to form eq. 1*

$(0, 6)$
$y = ax^2 + bx + c$
$(\mathbf{6}) = a(\mathbf{0})^2 + b(\mathbf{0}) + c$
$\boxed{6 = c}$ $\longrightarrow$ ***Eq. 2*** *Substitute the value of* $x = 0$ *and* $y = 6$ *to form eq. 2*

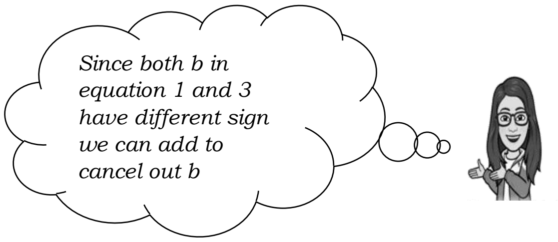
$(1, 4)$
$y = ax^2 + bx + c$
$(\mathbf{4}) = a(\mathbf{1})^2 + b(\mathbf{1}) + c$
$4 = a + b + c \longrightarrow$ ***Eq. 3*** *Substitute the value of $x = 1$ and $y = 4$ to form eq. 3*

Step 2: Get the values of a, b and c using the three equation we have obtained above by addition or subtraction.

Now, add eq. 1 from eq. 3

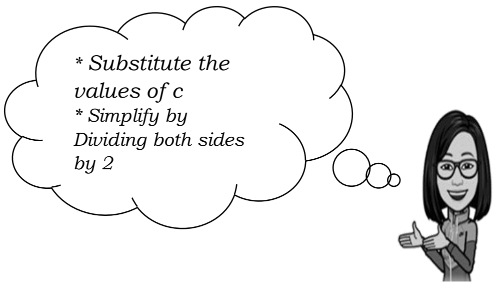
Eq. 1 $10 = a - \cancel{b} + c$
Eq. 3 $4 = a + \cancel{b} + c$

$14 = 2a + 2c \longrightarrow$ ***Eq. 4***



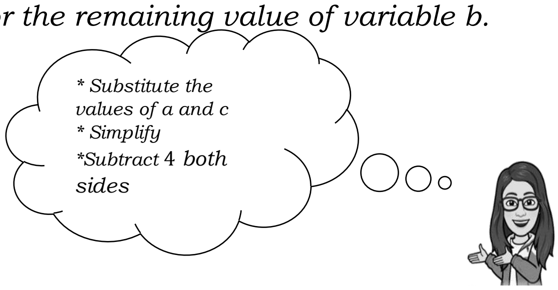
Since the value of c is already known we can now substitute the value of $c = 6$ to equation 4 to solve for a

Eq. 4 $14 = 2a + 2c$
$14 = 2a + 2(\mathbf{6})$
$14 = 2a + 12$
$14 - 12 = 2a + 12 - 12$
$\frac{2}{2} = \frac{2a}{2}$
$\boxed{1 = a}$



Since you already have the value for a and c, *substitute the $a = 1$ and $c = 6$ to equation 3 to solve for the remaining value of variable b.*

$4 = a + b + c$
$4 = (\mathbf{1}) + b + (\mathbf{6})$
$4 = 7 + b$
$4 - 7 = 7 - 7 + b$
$\boxed{-3 = b}$



Step 3: Finally, *substitute the values of a, b, and c to the standard form of quadratic function $f(x) = ax^2 + bx + c$.*

$f(x) = ax^2 + bx + c$ *Substitute the values of a, b and c*
$f(x) = (\mathbf{1})x^2 + (\mathbf{-3})x + (\mathbf{6})$ *Simplify*
$\boxed{f(x) = x^2 - 3x + 6}$

Therefore, the quadratic function that satisfy the table of values below is $f(x) = x^2 - 3x + 6$.

x	-3	-2	-1	0	1	2	3
$f(x)$	24	16	10	6	4	4	6

What's More

❖ ***Derive My Equation!***
Find the quadratic function determined by each table of values.

1.

x	-2	-1	0	1	2	3
$f(x)$	4	0	-2	-2	0	4

2.

x	-2	-1	0	1	2	3
$f(x)$	-1	3	5	5	3	-1

3.

x	-2	-1	0	1	2	3
$f(x)$	-5	-6	-3	4	15	30

What I Need To Remember

Steps in writing Equation of a Quadratic Function
Given a Table of Values

1. *Take any three ordered pairs* (x, y) *from the table of values;*
2. *Substitute the three ordered pair* (x, y) *in* $y = ax^2 + bx + c$ *to form system of linear equations in three variables;*
3. *Solve for the values of* a, b *and* c *using any method of solving system of Linear Equations;*
4. *Write the equation of the quadratic function* $f(x) = ax^2 + bx + c$.

Lesson 2 Finding Equation of a Quadratic Function Given Its Graph

What I Need To Know

At the end of this lesson, you are expected to:

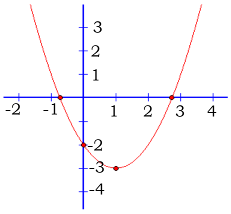
- determine the equation of a quadratic function given its graph.

What's In

❖ ***Time for Recall!***

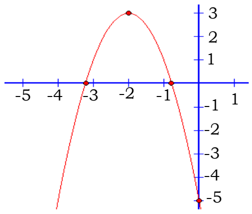
Given a graph, identify the vertex. The first number is answered for your guide.

1\.



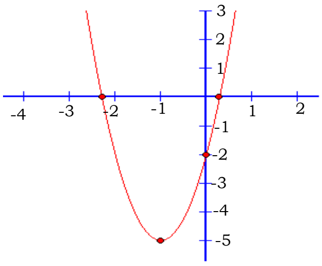
Vertex: (1, -3)

2\.



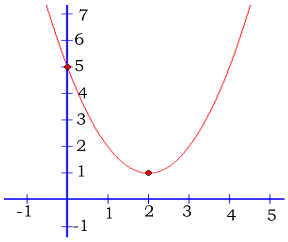
Vertex: ____________

3\.



Vertex: ____________

4\.



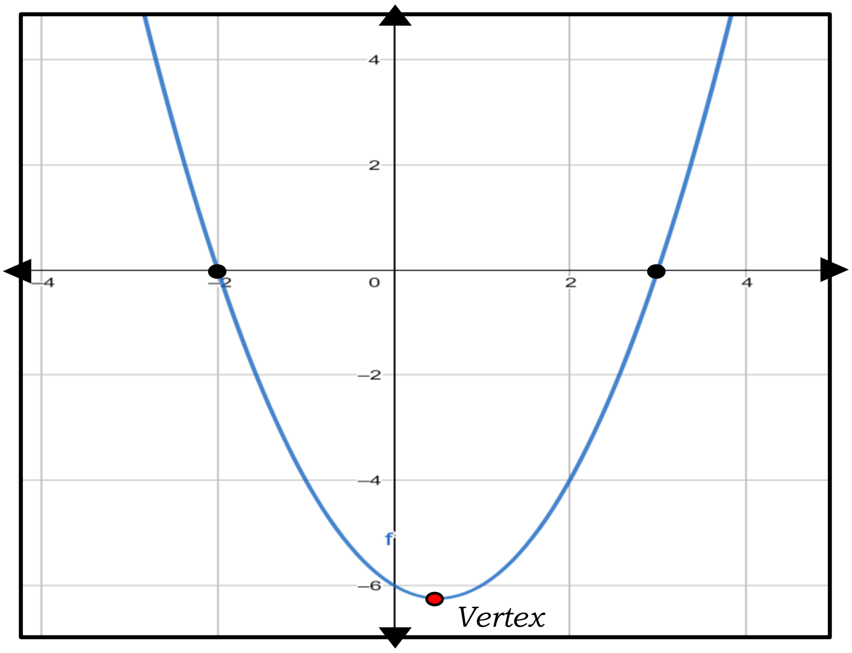
Vertex: _____________

What's New

The graph of a quadratic function is a parabola. The ***Vertex*** *of the parabola is at its lowest point when the parabola opens upward where a is less than 0 otherwise the vertex of the parabola is at its highest point if it opens downward where a is greater than 0.*

Consider the graph of the quadratic function $f(x) = x^2 - x - 6$. What can you say about the position of the vertex?

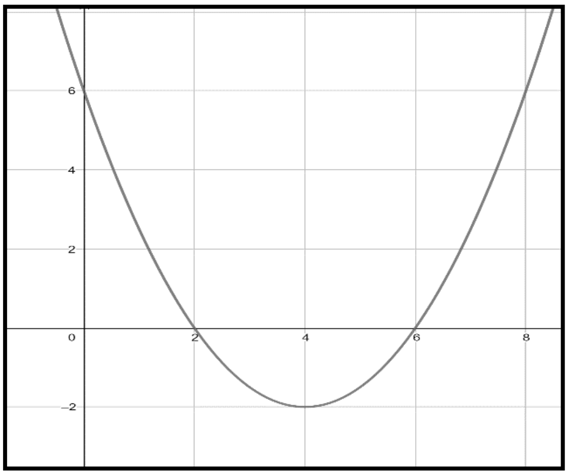
The lowest point is the vertex. We know that the standard form of the quadratic function is $y = ax^2 + bx + c$ and the vertex form is $f(x) = a(x-h)^2 + k$, where (h, k) is the vertex.



What Is It

❖ ***Follow This One!***

Study the graph of the quadratic function below. Determine the equation given in the graph by following the steps below.



1. Identify the vertex (h, k)
2. Identify the coordinates of any point on the parabola.
3. Substitute the vertex (h, k) and coordinates of any point (x, y)into the vertex form $y = a(x - h)^2 + k$.
4. Get the value of a.
5. Write the equation of the quadratic function.

❖ ***Try to Understand!***

When the vertex and any point on the parabola are clearly seen, the equation of the quadratic function can be determined by using the form of a quadratic function $y = a(x - h)^2 + k$.

Do the "***Follow This One!"*** activity above. The vertex of the graph of the quadratic function is $(4, -2)$. The graph passes through the point $(2,0)$. By replacing x and y with 2 and 0, respectively, and h and k with 4 and -2 respectively, we have

$y = a(x-h)^2 + k$ — *Substitute the vertex and the x-intercept*

$(\mathbf{0}) = a(\mathbf{2}-\mathbf{4})^2 + (-\mathbf{2})$

$0 = 4a - 2$ — *Simplify*

$\frac{\cancel{4}a}{\cancel{4}} = \frac{2}{4}$ — *Divide both sides by 4*

$\boxed{\boldsymbol{a} = \frac{\mathbf{1}}{\mathbf{2}}}$

Substitute the vertex, h=4, k=-2 and the value of a=$\frac{1}{2}$ to $y = a(x-h)^2 + k$ ***form the required quadratic function***

$y = a(x-h)^2 + k$

$\boxed{\boldsymbol{y} = \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{x}-\mathbf{4})^{\mathbf{2}} - \mathbf{2}}$

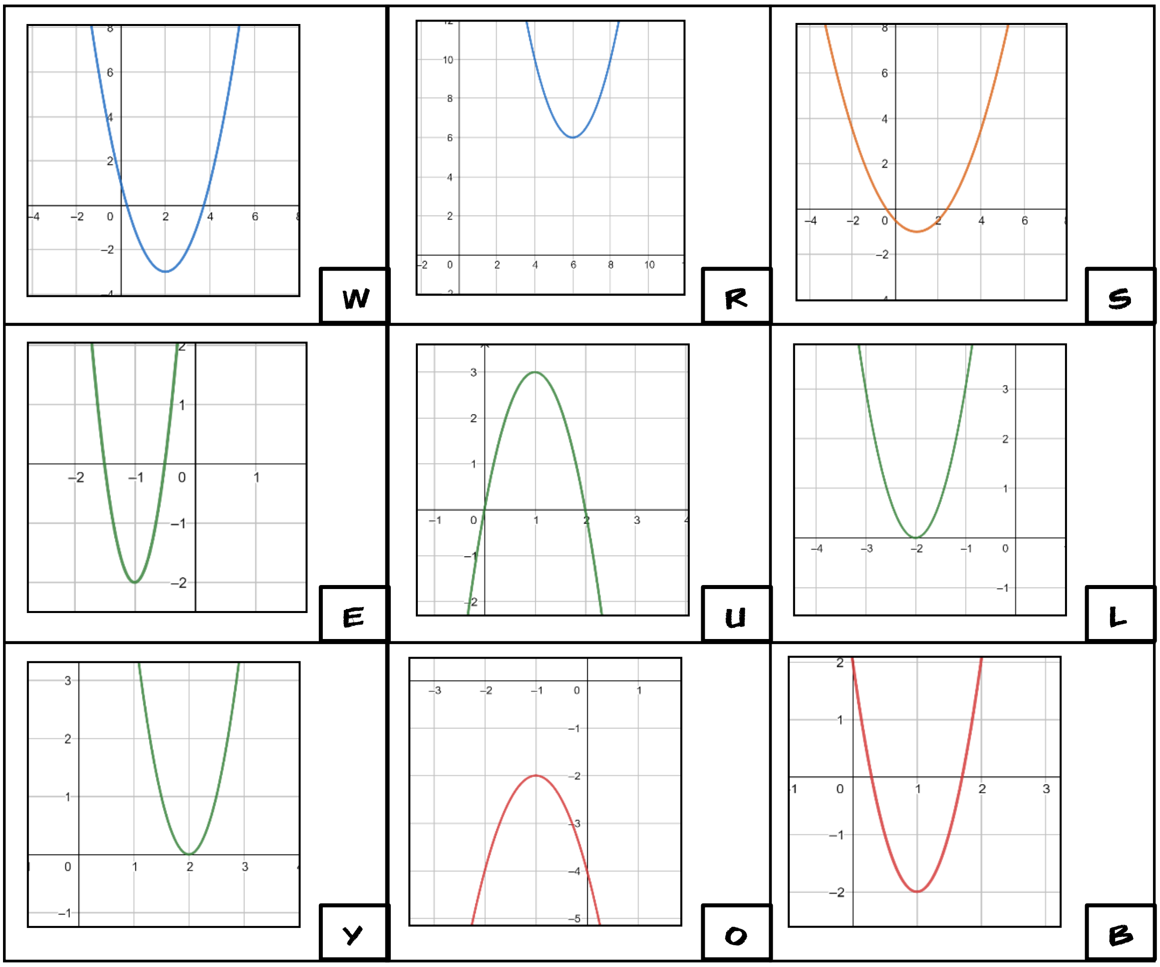
Thus, the quadratic equation is $y = \frac{1}{2}(x-4)^2 - 2$.

Note: To determine a quadratic function given its graph, you need to have the vertex any point of the graph.

What's More

❖ ***Solve the Riddle!***

Determine the equation of the following graph of a quadratic function. Then match the assigned letter of the graph into the correct equation to solve the Valentine Riddle.



"What did the boy owl say to the girl owl?"

________ $y = -2(x+1)^2$ ________ $y = (x-2)^2 - 3$ ________ $y = 3(x+2)^2$ ________ $y = 3(x+2)^2$ ________ $y = 8(x+1)^2 - 2$

________ $y = 4(x-2)^2$ ________ $y = -2(x+1)^2$ ________ $y = -3(x-1)^2 + 3$ ________ $y = (x-6)^2 + 6$ ________ $y = \frac{1}{2}(x-1)^2 - 1$

What I Need To Remember

Writing Equation of a Quadratic Function
Given its Graph, follow the steps below:

1. *Get the vertex and any point on the graph of the quadratic function;*
2. *Use the vertex form of the quadratic function,*
   a. $f(x) = a(x-h)^2 + k$;
3. *Substitute the values of* h *and* k *for the vertex and* x *and* y *for any point and solve for* a;
4. *Replace* a, h *and* k *with their values in the vertex form*
5. $f(x) = a(x-h)^2 + k$

Lesson 3 — Finding Equation of a Quadratic Function Given the Zeros

What I Need To Know

At the end of this lesson, you are expected to:

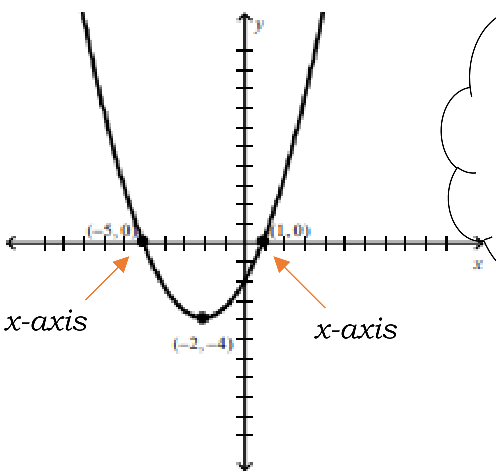
- determine the equation of a quadratic function given the zeros.

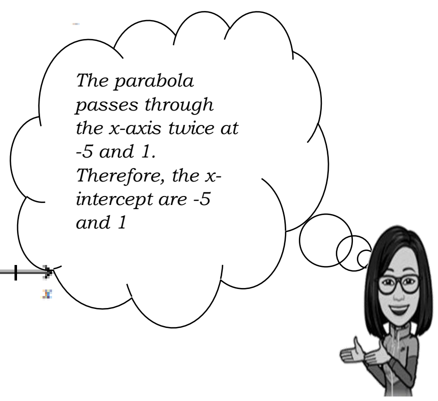
What's In

❖ ***Do the Recall!***

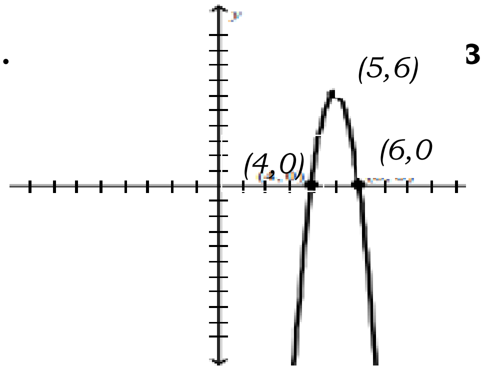
Given the graph below, find the *x-intercepts.* The first item is done as an example.

1.





2.



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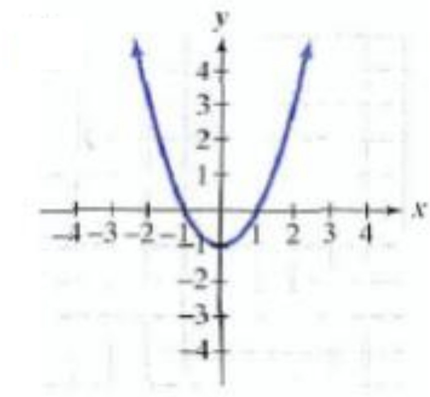


What's New

The *x-intercept* is the abscisa or the x-coordinateof the point where the parabola touches the *x-axis*. The *x-intercepts* of the quadratic function $f(x) = ax^2 + bx + c$, $a \neq 0$ are also known as the zeros of the quadratic function.

❖ ***Try This One!***

Determine the equation of a quadratic function whose zeros are the following.

1. $\{-3, 2\}$

2. $\frac{3 \pm \sqrt{2}}{3}$

What Is It

❖ ***Try to Get the Idea!***

To determine the equation of a quadratic function given the zeros is just like writing quadratic equation given its roots. Let us answer now the "Try This One!" activity above.

1. Determine the equation of a quadratic function whose zeros are $\{-3, 2\}$.

Solution:

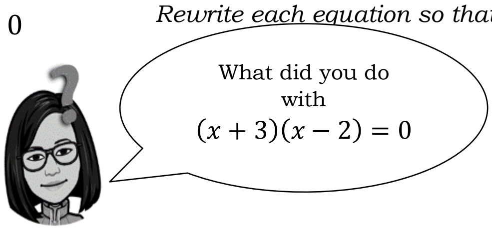
$x = -3$ or $x = 2$ — *Write the zeros as solutions of two equations.*

$x + 3 = 0$ or $x - 2 = 0$ — *Rewrite each equation so that it equals to 0.*

$(x + 3)(x - 2) = 0$

$x^2 + x - 6 = 0$

$\boxed{f(x) = x^2 + x - 6}$



Apply the Converse of Zero Product Property to write a product that equals 0. Then, multiply the binomials (FOIL). And lastly, replace 0 with $f(x)$.

2. Determine the equation of a quadratic function whose zeros are $\frac{3+\sqrt{2}}{3}$.

> *Note:*
>
> *A quadratic expression with irrational roots cannot be written as a product of linear factors with rational coefficients. In this case, we can use another method.*

Solution:

Since the zeros are $\frac{3\pm\sqrt{2}}{3}$

then,

$x = \frac{3 \pm \sqrt{2}}{3}$ *Write the zeros as solutions*

$3x = 3 \pm \sqrt{2}$

$3x - 3 = \pm\sqrt{2}$

What did you do?

First, cross multiply and then *isolate the radical in the right side of equation*

Square both sides of the equation and simplify. We get

$(3x-3)^2 = \left(\pm\sqrt{2}\right)^2$ *Square both sides*

$9x^2 - 18x + 9 = 2$ *Simplify: Square of Binomial*

$9x^2 - 18x + 7 = 0$

$f(x) = 9x^2 - 18x + 7$

What's More

❖ ***Match the Zeros!***

Each zeros of the quadratic function has a corresponding letter. Similarly, each box with the quadratic function inside has a corresponding blank below. Write the indicated letter of the zeros on the corresponding blank below the box containing the quadratic function to get the hidden message.

Y $\left\{\frac{5}{2}, -\frac{5}{2}\right\}$

V $\left\{\frac{4}{3}, -\frac{4}{3}\right\}$

G $\{-3, -3\}$

U $\{7, -3\}$

E $\{9, -4\}$

L $\{5, -4\}$

D $\left\{-\frac{3}{2}, 1\right\}$

O $\left\{\frac{2}{3}, \frac{1}{2}\right\}$

S $\left\{-\frac{4}{3}, \frac{1}{2}\right\}$

Message:

$f(x) = x^2 + 6x + 9$	$f(x) = 6x^2 - 7x + 2$	$f(x) = 2x^2 + x - 3$
________	________	________

$f(x) = x^2 - x - 20$	$f(x) = 6x^2 - 7x + 2$	$f(x) = 9x^2 - 16$	$f(x) = x^2 - 5x - 36$	$f(x) = 6x^2 + 5x - 4$
________	________	________	________	________

$f(x) = 4x^2 - 25$	$f(x) = 6x^2 - 7x + 2$	$f(x) = x^2 - 4x - 21$
________	________	________

What I Need To Remember

Steps writing Equation of a Quadratic Function Given its Graph

1. *Set the zeros as factors of the quadratic expression.*
2. *Simplify and determine the quadratic function.*

Lesson 4 Applications of Quadratic Functions

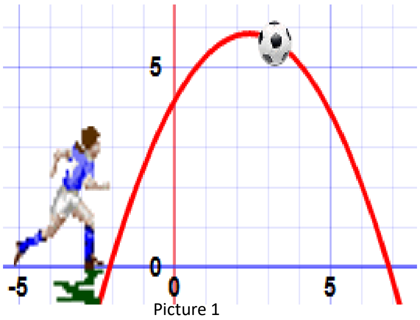
What I Need To Know

At the end of this lesson, you are expected to:

- solves problems involving quadratic functions.

What's In

❖ **Picture Analysis**



Picture 1


Picture 2

❖ ***Take Time to Think!***

1. What have you observed in the pictures.

2. How can we apply Quadratic Function in our life?

I saw a parabola. We can apply Quadratic Function in real life like the maximum height, time travelled, etc.

What's New

Steps in solving word problems by Polya

1. Analyze the problem
    a. Read and understand the problem carefully.
    b. Determine what is being asked in the problem. A sketch may help you.
    c. Make a representation for unknown quantity using a variable/letter.
2. Formulate the equation
    a. Translate the statement into mathematical expressions.
3. Solve the problem and interpret.
4. Check the solution

What Is It

❖ ***Try This One!***

Instructions: Read carefully and understand the problem.

1. What are the dimensions of the rectangular garden that can be enclosed by 80 m of fencing wire?

2. From a 96-foot building, an object is thrown straight up into the air then follows a trajectory. The height *S(t)* of the ball above the building after t seconds is given by the function $S(t) = 80t - 16t^2$.
    a. What maximum height will the object reach?
    b. How long will it take the object to reach the maximum height?
    c. Find the time at which the object is on the ground.

3. A pharmacy sis selling about 40 facemask per week at a price of Php 100 each box. For each Php 10 decrease in price, the pharmacist found out that 5 more facemask per week were sold. Write a quadratic equation in standard form that models the revenue from facemask sales. What price produces the maximum revenue?

In solving word problem, always remember the four steps to follow:

❖ ***Try to Understand!***

Go over and answer the "Try This One!" activity.

1. What are the dimensions of the rectangular garden that can be enclosed by 80 m of fencing wire?

Solution

Let l and w be the length and width of a rectangular garden.
$P = 2l + 2w$ is the perimeter
Since $P = 80\ m$, thus,

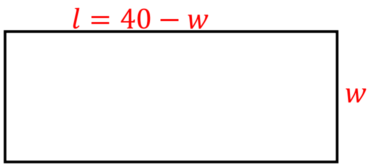
Note:

$P = 2l + 2w$ is the formula of the perimeter of a rectangle.

$$2l + 2w = 80$$

$$l + w = 40 \quad \text{Divide both side by 2}$$

$$l = 40 - w \quad \text{Expressing the length as a function of } \mathbf{w}$$



Substituting in the formula for the area A of a rectangle

$$A(w) = wl$$

$$A(w) = w(40 - w) \quad \text{Substitute the dimension}$$

$$A(w) = -w^2 + 40w \quad \text{Simplify}$$

By completing the square,

$$A(w) = -w^2 + 40w$$
$$A(w) = -(w - 20)^2 + 400$$

The vertex of the graph of the function *A(w)* is $(20, 400)$.This point indicates a maximum value of 400for *A(w)* that occurs when $w = 20$. Thus, the maximum area is $400m^2$ when the width is 20 *m*. If the width is 20 *m*, then the length is (40-20) *m* or 20 *m* also. The field with maximum area is a square.

2. From a 96-foot building, an object is thrown straight up into the air then follows a trajectory. The height *S(t)* of the ball above the building after *t* seconds is given by the function $S(t) = 80t - 16t^2$.
   a. What maximum height will the object reach?
   b. How long will it take the object to reach the maximum height?
   c. Find the time at which the object is on the ground.

Solution

a. The maximum height reached by the object is the ordinate of the vertex of the parabola of the function $S(t) = 80t - 16t^2$. By transforming this equation into the completed square form, we have,

$$S(t) = 80t - 16t^2$$

$$S(t) = -16t^2 + 80t \quad \textit{Rearrange the function}$$

$$S(t) = -16(t^2 - 5t) \quad \textit{Factored out } \boldsymbol{a}$$

$$S(t) = -16(t^2 - 5t + \frac{25}{4}) + 100 \quad \textit{Completing the square to make it perfect square trinomial}$$

$$S(t) = -16\left(t - \frac{5}{2}\right)^2 + 100 \quad \textit{Expressed the expression inside the parenthesis as square of a binomial}$$

The vertex is $\left(\frac{5}{2}, -100\right)$. Thus, the maximum height reached by the object is

100 ft. from the top of the building. This is 196 ft from the ground.

b. The time for an object to reach the maximum height is the abscissa of the vertex of the parabola or the value of h.

Since the vertex form, we get above is $S(t) = -16\left(t - \frac{5}{2}\right)^2 + 100$, the value of $h\ is \frac{5}{2}$ or 2.5, then the object is at its maximum height after 2.5 seconds.

c. To find the time it will take the object to hit the ground, let $S(t) = -96$, since the height of the building is 96 ft. The problem requires us to solve for t.

$S(t) = 80t - 16t^2$	
$-96 = 80t - 16t^2$	*Substitute $S(t) = -96$*
$16t^2 - 80t - 96 = 0$	*Put all the terms in the left side and equate to zero*
$t^2 - 5t - 6 = 0$	*Divide both side by the value of* $\boldsymbol{a}$
$(t-6)(t+1) = 0$	*Factor the trinomial*
$t = 6\ or\ t = -1$	

Thus, it will take 6 seconds before the object hits the ground.

3. **Solution**

Let x be the number of additional numbers of facemask sold.
You know that Revenue
$R(x) = (price\ per\ unit) \times (number\ of\ units\ produced\ or\ sold)$
Therefore, Revenue
$R(x) = (number\ of\ t-shirt\ sold)\ (price\ per\ shirt)$
$R(x) = (40 + 5x)(100 - 10x)$
$R(x) = -50x^2 + 100x + 4000$
If we transform the function into the form of $y = a(x-h)^2 + k$

$R(x) = -50x^2 + 100x + 4000$	
$R(x) = -50(x^2 - 2x) + 4000$	*Factored out* $\boldsymbol{a}$
$R(x) = -50(x^2 - 2x + 1) + 4000 + 50$	*Completing the square*
$R(x) = -50(x-1)^2 + 4050$	*Simplify*

The vertex is $(1, 4050)$.
Thus, the maximum revenue is Php 4, 050.
The price of the t-shirt to produce maximum revenue can be determined By

$P(x) = 100 - 10x$
$P(x) = 100 - 10(1)$ *Substitute the value of h to x*
$P(x) = 90$

Thus, Php 90 is the price of the t-shirt that produces maximum revenue

What's More

❖ ***Do It!***

Read and understand the problem carefully. Solve the problem in a separate paper.

1. The perimeter of the rectangle is 100 m, find its dimension if its area is a maximum.

2. The height (*H)* of the ball thrown into the air with a initial velocity of 9.8 m/s from a height of 2 m above the ground is given by the equation $H(t) = -4.9t^2 + 9.8t + 2$, where t is the time in seconds that the ball has been in the air.
   a. What maximum height did the object reach?
   b. How long will it take the ball to reach the maximum height?

3. Marvin has a mango plantation. If he picks the mangoes now, he will get 40 small crates and make a profit of Php 100 per crate. For every week that he delays picking, his harvest increase by 5 crates. But the selling price by Php 10 per crate. When should Marvin harvest his mangoes for him to have the maximum profit?

What I Need To Remember

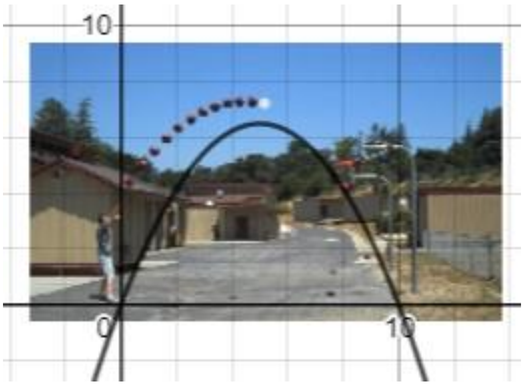
- ✓ ***Maximum value*** *of* $f(x) = ax^2 + bx + c$ *where* $a < 0$, *is the y-coordinate of the vertex*
- ✓ ***Minimum value*** *of* $f(x) = ax^2 + bx + c$ *where* $a > 0$, *is the y-coordinate of the vertex*
- ✓ ***Vertex*** *is the turning point of the parabola. It could be the lowest or the highest point of the parabola. If the quadratic function is expressed in standard form* $y = a(x - h)^2 + k$, *the vertex point is* (h, k)

What I Can Do

❖ ***Let's Apply It!***

Solve a real-life situation wherein you can apply quadratic function.

1. Roy is practicing basketball in their community during weekend. He observed that the path of the ball I formed a parabola. The function that represents the paths of the ball is $f(x) = -\frac{13}{50}x^2 + \frac{13}{5}x$. What is the maximum height of the ball?



Assessment *(Post Test)*

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper. If answer not found in the given choices write your correct answer.

For items 1-4, given a table of values, identify the quadratic equation.

1.

x	-1	0	1	2	3
$f(x)$	-6	3	6	3	-6

A. $y = 3x^2 + 3$
B. $y = -3x^2 + 6x + 3$
C. $y = -3x^2 + 6x - 3$
D. $y = -3x^2 - 3$

2.

x	-1	0	1	2	3
$f(x)$	-5	1	3	1	-5

A. $y = 3x^2 - x + 1$
B. $y = -3x^2 + x + 1$
C. $y = 3x^2 + x - 1$
D. $y = -3x^2 + x - 1$

3.

x	0	1	2	3	4
$f(x)$	-4	-10	-16	-22	-28

A. $y = 2x^2 - 8x - 4$
B. $y = 2x^2 + 8x - 4$
C. $y = 2x^2 + 8x + 4$
D. $y = 2x^2 - 8x + 4$

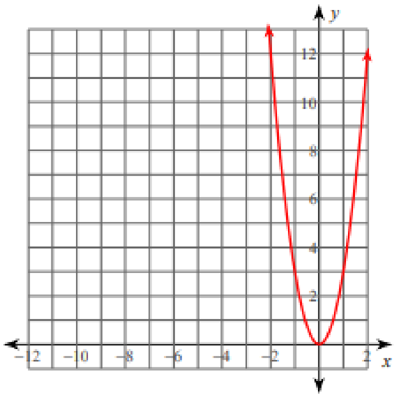
4.

x	-1	0	1	2	3
$f(x)$	8	-1	-4	-1	8

A. $y = 3x^2 - 9x - 1$
B. $y = 3x^2 - 9x + 1$
C. $y = 3x^2 + 9x - 1$
D. $y = 3x^2 - 9x + 1$

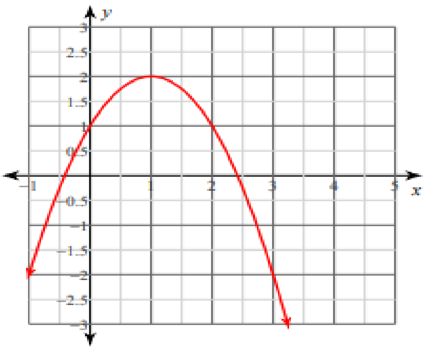
For items 5 - 8, identify the quadratic functions using the given a graph.

5.



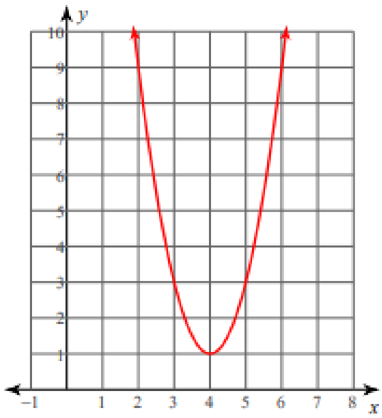
A. $y = x^2$
B. $y = 2x^2$
C. $y = 3x^2$
D. $y = 4x^2$

6.



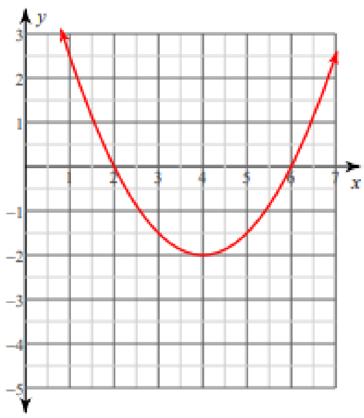
A. $y = -(x-1)^2 + 2$
B. $y = (x-1)^2 + 2$
C. $y = -(x+1)^2 + 2$
D. $y = (x-1)^2 + 2$

7.



A. $y = -2(x-4)^2 + 1$
B. $y = 2(x-4)^2 + 1$
C. $y = 2(x+4)^2 - 1$
D. $y = -2(x+4)^2 - 1$

8.



A. $y = \frac{1}{2}(x+4)^2 + 2$
B. $y = \frac{1}{2}(x-4)^2 + 2$
C. $y = \frac{1}{2}(x+4)^2 - 2$
D. $y = \frac{1}{2}(x-4)^2 - 2$

9. Which of the following choices is the equation of the quadratic function whose zeros are 8
and 6.

A. $y = x^2 + 14x + 48$
B. $y = x^2 + 14x - 48$
C. $y = x^2 - 14x + 48$
D. $y = x^2 - 14x - 48$

10. Which of the following choices is the equation of the quadratic function whose zeros are
-3 and 8.

A. $y = x^2 + 5x + 24$
B. $y = x^2 + 5x - 24$
C. $y = x^2 - 5x + 24$
D. $y = x^2 - 5x - 24$

11. Which of the following choices is the equation of the quadratic function whose zeros are
11 and 4.

A. $y = x^2 + 7x + 44$
B. $y = x^2 + 7x - 44$
C. $y = x^2 - 7x + 44$
D. $y = x^2 - 7x - 44$

12. During quarantine period, Richard decided to plant a tomato in their backyard. He predicted that the number of tomato plant, x, could yield $y = -10x^2 + 1800x$ tomatoes per month. How many trees should be planted to produce the maximum number of tomatoes per month?

A. 60
B. 70
C. 80
D. 90

13. Mr. Redoble is sketching the windows for a home renovation. The shape is a parabola modeled by the equation $h(w) = -w^2 + 6$, where h is the height of the window and w is the width in feet. What is the maximum height of each window?

A. 6 ft
C. 7 ft
C. 8 ft
D. 9 ft

14. John wants to make a backyard vegetable garden and enclosed it with a fence. What are the dimensions of the rectangular garden that can be enclosed by 60 m of fencing wire?

A. 10 m x 10 m
B. 15 m x 15 m
C. 20 m x 20 m
D. 25 m x 20 m

15. Rey and Mark recycled a bottle and make it a rocket during their vacation. The height in feet of a bottle rocket is given by $h(t) = 120t - 12t^2$ where t is the time in seconds. How long will it take for the rocket to return to the ground?

A. 5 seconds
B. 10 seconds
C. 15 seconds
D. 20 seconds

Answer Key

Remember: This portion of the module contains all the answers. Your **HONESTY** is required.

What's More (Lesson 1)

1.

$(-1,0)$
$y = ax^2 + bx + c$
$0 = a(-1)^2 + b(-1) + c$
$0 = a - b + c \quad eq.1$

$(1,-2)$
$y = ax^2 + bx + c$
$-2 = a(1)^2 + b(1) + c$
$-2 = a + b + c \quad eq.2$

$(0,-2)$
$y = ax^2 + bx + c$
$-2 = a(0)^2 + b(0) + c$
$-2 = c \quad eq.3$

Add eq. 1 & 2
$0 = a - b + c \quad eq.1$
$-2 = a + b + c \quad eq.2$
$-2 = 2a + 2c \quad eq.4$

Substitute c= -2 to eq. 4
$-2 = 2a + 2c \quad eq.4$
$-2 = 2a + 2(-2)$
$-2 = 2a - 4$
$2 = 2a$
$1 = a$

Substitute c= -2 and a=1 to eq. 2
$-2 = a + b + c \quad eq.2$
$-2 = 1 + b - 2$
$-2 = -1 + b$
$-1 = b$

Substitute a=1, b=-1and c=-2 to
$y = ax^2 + bx + c$
$y = (1)x^2 + (-1)x + (-2)$
$f(x) = x^2 - x - 2)$

2.

$(-1,3)$
$y = ax^2 + bx + c$
$3 = a(-1)^2 + b(-1) + c$
$3 = a - b + c \quad eq.1$

$(1,5)$
$y = ax^2 + bx + c$
$5 = a(1)^2 + b(1) + c$
$5 = a + b + c \quad eq.2$

$(0,5)$
$y = ax^2 + bx + c$
$5 = a(0)^2 + b(0) + c$
$5 = c \quad eq.3$

Add eq. 1 & 2
$3 = a - b + c \quad eq.1$
$5 = a + b + c \quad eq.2$
$8 = 2a + 2c \quad eq.4$

Substitute c= 5 to eq. 4
$8 = 2a + 2c \quad eq.4$
$8 = 2a + 2(5)$
$8 = 2a + 10$
$-2 = 2a$
$-1 = a$

Substitute c= 5 and a=-1 to eq. 2
$5 = a + b + c \quad eq.2$
$5 = -1 + b + 5$
$5 = 4 + b$
$1 = b$

Substitute a=-1, b=1and c=5 to
$y = ax^2 + bx + c$
$y = (-1)x^2 + (1)x + (5)$
$f(x) = -x^2 - x + 5$

1.

$(-1,-6)$
$y = ax^2 + bx + c$
$-6 = a(-1)^2 + b(-1) + c$
$-6 = a - b + c \quad eq.1$

$(1,4)$
$y = ax^2 + bx + c$
$4 = a(1)^2 + b(1) + c$
$4 = a + b + c \quad eq.2$

$(0,-3)$
$y = ax^2 + bx + c$
$-3 = a(0)^2 + b(0) + c$
$-3 = c \quad eq.3$

Add eq. 1 & 2
$-6 = a - b + c \quad eq.1$
$4 = a + b + c \quad eq.2$
$-2 = 2a + 2c \quad eq.4$

Substitute c= -3 to eq. 4
$-2 = 2a + 2c \quad eq.4$
$-2 = 2a + 2(-3)$
$-2 = 2a - 6$
$4 = 2a$
$2 = a$

Substitute c= -3 and a=2 to eq. 2
$4 = a + b + c \quad eq.2$
$4 = 2 + b - 3$
$4 = -1 + b$
$5 = b$

Substitute a=2, b=5and c=-3 to
$y = ax^2 + bx + c$
$y = (2)x^2 + (5)x + (-3)$
$f(x) = 2x^2 + 5x - 3$

What's In (Lesson 2)

1. $(1,-3)$ *2.* $(2,3)$ *3.* $(1,-5)$ *4.* $(2,-1)$

What's More (Lesson 2)

"OWL BE YOURS"

What's More (Lesson 3)

"GOD LOVES YOU"

Y $\left(\frac{5}{2},-\frac{5}{2}\right)$

$x-\frac{5}{2}=0 \quad or \quad x+\frac{5}{2}=0$

$\left(x-\frac{5}{2}\right)\left(x+\frac{5}{2}\right)=0$

$x^2-\frac{25}{4}=0$

$4(x^2-\frac{25}{4}=0)$

$f(x)=4x^2-25$

V $\left(\frac{4}{3},-\frac{4}{3}\right)$

$x-\frac{4}{3}=0 \quad or \quad x+\frac{4}{3}=0$

$x-\frac{5}{2}=0 \quad or \quad x+\frac{5}{2}=0$

$\left(x-\frac{4}{3}\right)\left(x+\frac{4}{3}\right)=0$

$x^2-\frac{16}{9}=0$

$9(x^2-\frac{16}{9}=0)$

$f(x)=9x^2-16$

G $(-3,-3)$

$x+3=0 \quad or \quad x+3=0$

$(x+3)(x+3)=0$

$f(x)=x^2+6x+9$

U $(7,-3)$

$x-7=0 \quad or \quad x+3=0$

$(x-7)(x+3)=0$

$f(x)=x^2-4x-21$

E $(9,-4)$

$x-9=0 \quad or \quad x+4=0$

$(x-9)(x+4)=0$

$f(x)=x^2-5x-36$

L $(5,-4)$

$x-5=0 \quad or \quad x+4=0$

$(x-5)(x+4)=0$

$f(x)=x^2-x-20$

D $\left(-\frac{3}{2},1\right)$

$x+\frac{3}{2}=0 \quad or \quad x-1=0$

$\left(x+\frac{3}{2}\right)(x-1)=0$

$2(x^2-\frac{1}{2}x-\frac{3}{2}=0)$

$f(x)=2x^2+x-3$

O $\left(\frac{2}{3},\frac{1}{2}\right)$

$x-\frac{2}{3}=0 \quad or \quad x-\frac{1}{2}=0$

$\left(x-\frac{3}{2}\right)\left(x-\frac{1}{2}\right)=0$

$6(x^2-\frac{7}{6}x-\frac{1}{3}=0)$

$f(x)=6x^2-7x+2$

S $\left(-\frac{4}{3},\frac{1}{2}\right)$

$x+\frac{4}{3}=0 \quad or \quad x-\frac{1}{2}=0$

$\left(x+\frac{4}{3}\right)\left(x-\frac{1}{2}\right)=0$

$6(x^2+\frac{5}{6}x-\frac{2}{3}=0)$

$f(x)=6x^2+5x-4$

What's In (Lesson 3)

2. 4 and 6

3. -1 and 1

$h \quad k \qquad x \quad y$

$(2,-3) \quad (1,-2)$

$y=a(x-h)^2+k$

$-2=a(1-2)^2-3$

$-2=a-3$

$-2+3=a$

$1=a$

$f(x)=(x-2)^2-3$

W

$h \quad k \qquad x \quad y$

$(6,6) \quad (4,10)$

$y=a(x-h)^2+k$

$10=a(4-6)^2+6$

$10=4a+6$

$4=4a$

$1=a$

$f(x)=(x-6)^2+6$

R

$h \quad k \qquad x \quad y$

$(1,-1) \quad (-2,4)$

$y=a(x-h)^2+k$

$4=a(-2-1)^2-1$

$4=9a-1$

$5=9a$

$\frac{5}{9}=a$

$f(x)=\frac{5}{9}(x-1)^2-1$

S

$h \quad k \qquad x \quad y$

$(-1,-2) \quad (-\frac{1}{2},0)$

$y=a(x-h)^2+k$

$0=a\left(-\frac{1}{2}+1\right)^2-2$

$0=\frac{1}{4}a-2$

$4(2)=4(\frac{1}{4}a)$

$8=a$

$f(x)=8(x+1)^2-2$

E

$h \quad k \qquad x \quad y$

$(1,3) \quad (0,0)$

$y=a(x-h)^2+k$

$0=a(0-1)^2+3$

$0=a+3$

$0-3=a$

$-3=a$

$f(x)=-3(x-1)^2+3$

U

$h \quad k \qquad x \quad y$

$(-2,0) \quad (-1,3)$

$y=a(x-h)^2+k$

$3=a(-1+2)^2+0$

$3=a$

$f(x)=3(x+2)^2$

L

$h \quad k \qquad x \quad y$

$(2,0) \quad (\frac{3}{2},1)$

$y=a(x-h)^2+k$

$1=a\left(\frac{3}{2}-2\right)^2+0$

$4(1)=4(\frac{1}{4}a)$

$4=a$

$f(x)=4(x-2)^2$

Y

$h \quad k \qquad x \quad y$

$(-1,-2) \quad (0,-4)$

$y=a(x-h)^2+k$

$-4=a(0+1)^2-2$

$-4=a-2$

$-4+2=a$

$-2=a$

$f(x)=-2(x+1)^2-2$

O

$h \quad k \qquad x \quad y$

$(1,-2) \quad (0,2)$

$y=a(x-h)^2+k$

$2=a(0-1)^2-2$

$2=a-2$

$2+2=a$

$4=a$

$f(x)=4x^2-2$

B

What's More (Lesson 1)

❖ ***Do It!***

1. $P = 2l + 2w$

$2l + 2w = 100$

$l + w = 50$

$l = 50 - w$

$A(w) = wl$

$A(w) = w(50 - w)$

$A(w) = -w^2 + 50w$

$A(w) = -1(w^2 - 50w)$

$A(w) = -1(w^2 - 50w + 625) + 625$

$A(w) = -(w - 25)^2 + 625$

Thus, the dimension is 25 m x 25 m

2. $H(t) = -4.9t^2 + 9.8t + 2$

$H(t) = -4.9(t^2 - 2t) + 2$

$H(t) = -4.9(t^2 - 2t + 1) + 2 + 4.9$

$H(t) = -4.9(t - 1)^2 + 6.9$

a.) maximum height is 6.9 m.

b. The ball is at its maximum height after 1 second

3. $R(x) = (40 + 5x)(1oo - 10x)$

$R(x) = -50x^2 + 100x + 4000$

$R(x) = -50(x^2 + 2x + 1) + 4000 + 50$

$R(x) = -50(x + 1)^2 + 4050$

Thus, Marvin should wait for 1 week to harvest his mangoes for him to have the maximum profit.

References

Text Book

Merden L. Bryant, et.al, Mathematics Learner's Material 9 (Meralco Avenue, Pasig City: Vibal Group, Inc.,), pp. 254 – 258.

Websites

"Radical Expressions", accessed September 30, 2020, url:
"Quadratic Model-Basketball", accessed October 8, 2020, url: https://www.desmos.com/calculator/djikpphgde

Pictures

From bitmoji app
https://www.mdc.edu/main/images/Radical%20Expressions_tcm6-60836.PDF
Picture 1, url: mathisfun.com
Picture 1, url: dbis.org

Congratulations!

You are now ready for the next module. Always remember the following:

1. Make sure every answer sheet has your
   - *Name*
   - *Grade and Section*
   - *Title of the Activity or Activity No.*
2. Follow the date of submission of answer sheets as agreed with your teacher.
3. Keep the modules with you and return them at the end of the school year or whenever face-to-face interaction is permitted.