

9

Mathematics

Quarter 1 – Module 2

- The Nature of the Roots of a Quadratic Equation**
- The Sum and Product of the Roots of Quadratic Equations**



About the Module

This module was designed and written with you in mind. It is here to help you master about The Nature, Sum and Product of the Roots of a Quadratic Equation. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

- characterizes the roots of a quadratic equation using the discriminant.
- describe the relationships between the coefficients and the roots of a quadratic equation.



What I Know (Pre-Test)

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper. If answer not found in the given choices, write your correct answer.

1. What is the nature of the roots of the quadratic equation if the value of its discriminant is zero?
 - A. The roots are not real.
 - B. The roots are irrational and not equal
 - C. The roots are rational and not equal.
 - D. The roots are real, rational, and equal.

2. What is the value of the discriminant of the quadratic equation $ax^2 + bx + c = 0$?

A. $b^2 - 4ac$	C. $b^2 + 4ac$
B. $c^2 - 4ba$	D. $c^2 - bc$

3. Which is true for the equation $x^2 - 5x + 12 = 0$?
 - A. The roots are not real.
 - B. The roots are irrational and not equal
 - C. The roots are rational and not equal.
 - D. The roots are rational and equal.

4. In a quadratic equation, what do you call the value $b^2 - 4ac$?

A. The Denominator	C. The Diameter
B. The discriminant	D. The divisor

5. Which of the following quadratic equations has no real roots?

A. $2x^2 + 4x = 3$	C. $3s^2 - 2s = -5$
B. $t^2 - 8t - 4 = 0$	D. $-2r^2 + r + 7 = 0$

6. The roots of a quadratic equations are -5 and 3 . Which of the following quadratic equations has these roots?

A. $x^2 - 8x + 15 = 0$	C. $x^2 - 2x - 15 = 0$
B. $x^2 + 8x + 15 = 0$	D. $x^2 + 2x - 15 = 0$

7. What is the sum of the roots of a quadratic equation $x^2 + 6x - 14 = 0$?

A. -3	B. -7	C. -6	D. -14
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8. What is the product of the roots of a quadratic equation $x^2 + 6x - 14 = 0$?

A. -3	B. -7	C. -6	D. -14
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9. Which of the following is the sum of the roots of $x^2 + 3x - 27 = 0$?

A. 3	B. -3	C. 27	D. -27
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10. Which of the following is the product of the roots of $x^2 + 3x - 27 = 0$?

A. 3	B. -3	C. 27	D. -27
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Lesson 1

The Nature of the Roots of a Quadratic Equation



What I Need To Know

At the end of this lesson, you are expected to:

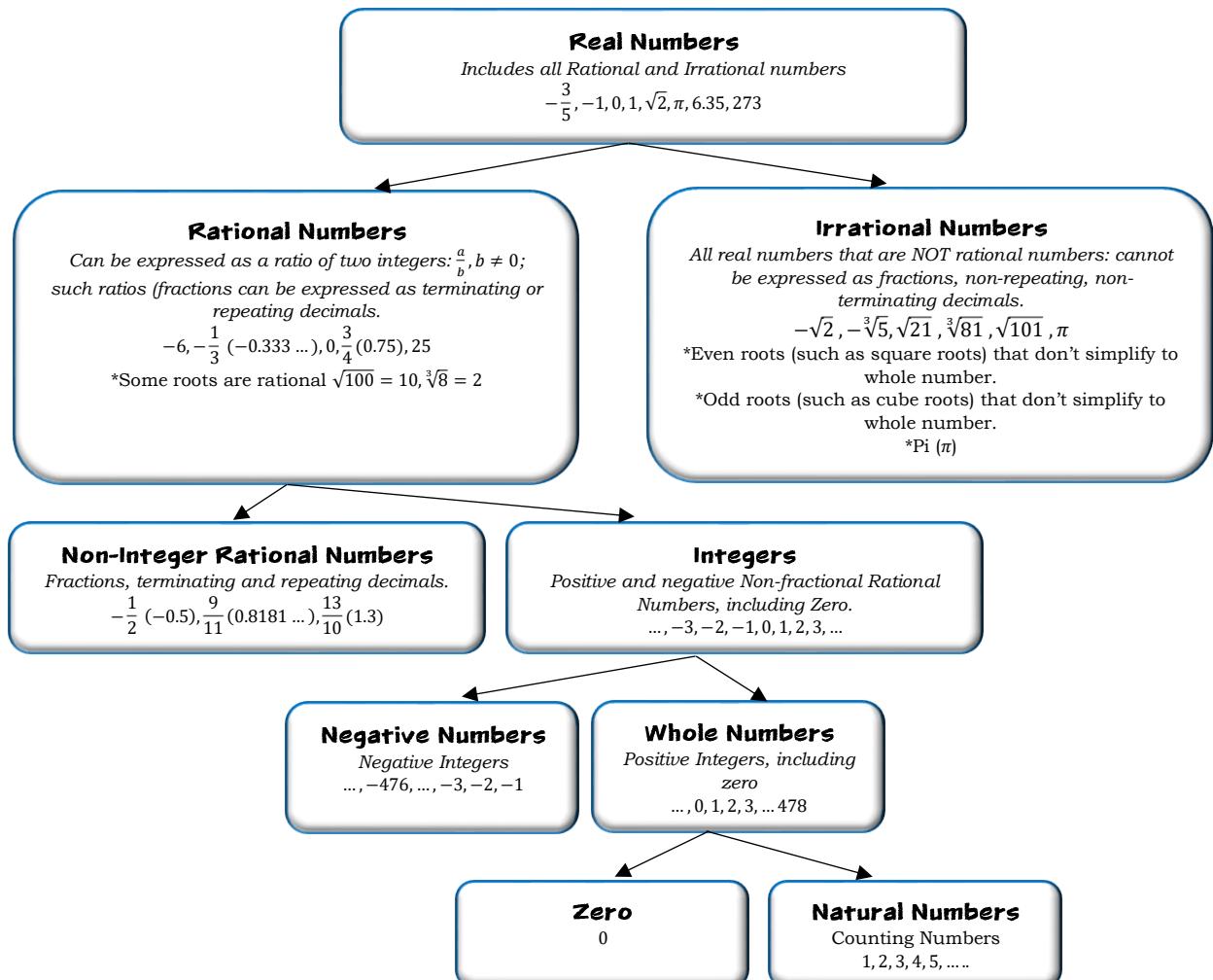
- characterize the roots of a quadratic equation using discriminant.



What's In

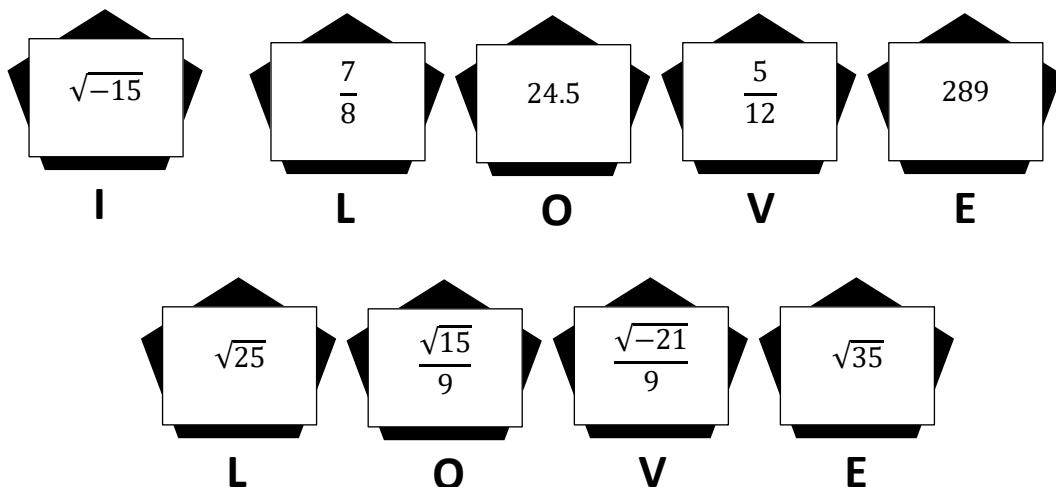
❖ Flashback

Before we begin with our lesson, lets recall on what are the real numbers by examining the diagram below and then answer the activity that follow.



❖ **Activity 1.1: Which Are Real? Which Are Not?**

Refer to the numbers below to answer the guide questions that follow.
Write your answer in a separate paper.



Guide Questions:

1. Which of the numbers above are familiar to you? Why? Describe these numbers.
2. Which of the numbers are real? Which are not?
3. Which of the numbers are rational? Irrational? Explain your answer.
4. Which of the numbers are perfect squares? Not perfect squares?
5. How do you describe numbers that are perfect squares?



What's New

A **quadratic equation** in one variable is a mathematical sentence of degree 2 that can be written in standard form: $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$.

Any quadratic equation $ax^2 + bx + c = 0$ can be solved using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The value of the expression $b^2 - 4ac$ is called the **discriminant** of the quadratic equation.

❖ **Activity 1.2: Do Math in A, B, C**

- A. Identify the values of a , b , and c , then use the values to evaluate the expression $b^2 - 4ac$. Write your answer in a separate paper.

$$b^2 - 4ac$$

1. $x^2 - 4x + 4 = 0$ $a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$
2. $x^2 + 7x + 10 = 0$ $a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$ $\underline{\hspace{2cm}}$

3. $x^2 + 6x + 3 = 0$ $a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$

4. $x^2 + 2x + 5 = 0$ $a = \underline{\hspace{2cm}}$ $b = \underline{\hspace{2cm}}$ $c = \underline{\hspace{2cm}}$



What Is It

Consider the equation $ax^2 + bx + c = 0$

If:

$$b^2 - 4ac = 0$$

$b^2 - 4ac > 0$, perfect square

$b^2 - 4ac > 0$, not a perfect square

$b^2 - 4ac < 0$ not real

Then the roots are:

real, rational, and equal

rational numbers but are not equal

irrational number but are not equal

has no real roots

Let us analyze the different possible values of the discriminant by describing the nature of the roots of the quadratic equation found in activity 1.2.

1. $x^2 - 4x + 4 = 0$ → $a = 1$ $b = -4$ $c = 4$ *Determine the values of a, b, and c*

$$b^2 - 4ac = (-4)^2 - 4(1)(4) \quad \text{Substitute the values } a, b, \text{ and } c$$

$$= 16 - 16$$

Simplify

$$\boxed{b^2 - 4ac = 0}$$

Since the value of discriminant $b^2 - 4ac$ is 0 therefore, the roots are real numbers and are equal.

This can be checked by determining the roots of $x^2 - 4x + 4 = 0$ using any of the methods of solving quadratic equation.

If quadratic formula is used, the roots that can be obtained are the following.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$x = \frac{4 \pm 0}{2}$$

$$x = \frac{4 + 0}{2} \quad \left| \quad x = \frac{4 - 0}{2}\right.$$

$$x = \frac{4}{2}$$

$$x_1 = 2$$

$$x_2 = 2$$

One rational solution

2. $x^2 + 7x + 10 = 0$ $a = 1 \quad b = 7 \quad c = 10$ Determine the values of a , b , and c

$$b^2 - 4ac = (7)^2 - 4(1)(10)$$

$$= 49 - 40$$

$b^2 - 4ac = 9$

Since the value of discriminant $b^2 - 4ac$ is 9 and is greater than 0 therefore, the roots are rational number but are not equal.

To check, solve for the roots of $x^2 + 7x + 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{2}$$

$$x = \frac{-7 \pm 3}{2}$$

$x = \frac{-7 + 3}{2}$ $x = \frac{-4}{2}$ $x_1 = -2$	$x = \frac{-7 - 3}{2}$ $x = \frac{-10}{2}$ $x_2 = -5$
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Two rational solutions

3. $x^2 + 6x + 3 = 0$ $a = 1 \quad b = 6 \quad c = 3$ Determine the values of a , b , and c

$$b^2 - 4ac = (6)^2 - 4(1)(3)$$

$$= 36 - 12$$

$b^2 - 4ac = 24$

Since the value of discriminant $b^2 - 4ac$ is 24 and it is greater than zero but not perfect square, therefore the roots are irrational numbers and are not equal.

To check, solve for the roots of $x^2 + 6x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 12}}{2}$$

$$x = \frac{-6 \pm \sqrt{24}}{2}$$

$$\left| \begin{array}{l} x = \frac{-6 + \sqrt{24}}{2} \\ x = \frac{-6 + 2\sqrt{6}}{2} \\ x_1 = -3 + \sqrt{6} \end{array} \right| \quad \left| \begin{array}{l} x = \frac{-6 - \sqrt{24}}{2} \\ x = \frac{-6 - 2\sqrt{6}}{2} \\ x_2 = -3 - \sqrt{6} \end{array} \right.$$

Two irrational solutions

4. $x^2 + 2x + 5 = 0$  $a = 1 \quad b = 2 \quad c = 5$ Determine the values of a , b , and c

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(1)(5) && \text{Substitute the values } a, b, \text{ and } c \\ &= 4 - 20 && \text{Simplify} \end{aligned}$$

$b^2 - 4ac = -16$

Since the value of discriminant $b^2 - 4ac$ is -16 and it is less than 0, therefore the quadratic equation has no real roots.

To check, solve for the roots of $x^2 + 2x + 5 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(5)}}{2(1)} \\ &\qquad\qquad\qquad x = \frac{-2 \pm \sqrt{4 - 20}}{2} \\ &\qquad\qquad\qquad x = \frac{-2 \pm \sqrt{-16}}{2} \\ x &= \frac{-2 + \sqrt{-16}}{2} \quad \left| \quad x = \frac{-2 - \sqrt{-16}}{2} \right. \\ x_1 &= -1 + 2i \quad \left| \quad x_2 = 1 - 2i \right. \end{aligned}$$

Two complex solutions



What's More

❖ Activity 1.3: Try This!

A. Solve for the discriminant and determine the nature of the roots for each quadratic equation. Write your answer in a separate paper.

1. $x^2 + 3x + 1 = 0$

2. $3x^2 + 2 = 5x$

3. $2x^2 - 4 = 0$

4. $x^2 + 4x + 4 = 0$

5. $x^2 + 4x + 6 = 0$



What I Need To Remember

A GENERAL NOTE: THE DISCRIMINANT

For $ax^2 + bx + c = 0$, where a, b and c are real numbers, the **discriminant** is the expression under the radical sign in the quadratic formula: $b^2 - 4ac$. It tells us, the nature of the roots of a quadratic equation.

Thus,

Value of Discriminant	Nature	Number of Solutions
$b^2 - 4ac = 0$	the roots are real numbers and are equal	1 real solution
$b^2 - 4ac > 0$, perfect square	the roots are rational numbers but are not equal	2 distinct real solutions
$b^2 - 4ac > 0$, not a perfect square	the roots are irrational numbers but are not equal	2 distinct real irrational solutions
$b^2 - 4ac < 0$	has no real roots	2 complex solutions

Lesson 2

The Sum and Product of Roots of a Quadratic Equation



What I Need To Know

At the end of this lesson, you are expected to:

- describe the relationships between the coefficients and the roots of a quadratic equation.



What's In

❖ Flashback

You have observed in the previous module that the quadratic equation in standard form is not unique. For example $x^2 - 2x - 3 = 0$ can be written as the equivalent equation $-x^2 + 2x + 3 = 0$. Also $-3x^2 + 4x - 1 = 0$ can be written as the equivalent equation $3x^2 - 4x + 1 = 0$.

In the review exercises below, all answers are written in the standard form $ax^2 + bx + c = 0$ with $a > 0$ and where the *greatest common factor of all non-zero coefficient is 1*.

❖ Activity 2.1: Review Exercises

Instructions: Complete the table below by performing the following process.

1. Transform the given quadratic equation to its standard form $ax^2 + bx + c = 0$. Write your answer to column 2.
2. Identify the values of a , b , and c . Write your answer to column 3.
3. Solve for the roots of the given quadratic equation using any methods convenient to you. Write your answer to column 4.

Use separate answer sheet for this activity.

“Trace My Roots”

Quadratic Equation	Standard Form $ax^2 + bx + c = 0$	Values of a, b, c	Roots of the Quadratic Equation
1. $x^2 + 3 = 4x$			
2. $-6x = -6x^2 + 8$			
3. $x^2 - 3x = -12$			



What's New

We now discuss how the sum and product of the roots of the quadratic equation $ax^2 + bx + c = 0$ can be determined using the coefficients a , b , and c .

- ✓ Let the roots of the quadratic equation $ax^2 + bx + c = 0$, where a, b and c are real numbers and $a \neq 0$, be denoted by r_1 and r_2

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

- ✓ Notice that when the roots r_1 and r_2 if added and multiplied respectively, it will give the following relationship:

$$r_1 + r_2 = \frac{-b}{a}, \quad r_1 \cdot r_2 = \frac{c}{a}$$

- ✓ The sum and product of the roots of a quadratic equation can be found without solving for the roots.

❖ **Activity 2.2: Complete Me!**

Use the formula to solve for the sum and product of roots of the given quadratic equations below. Write your answer on a separate sheet of paper.

Quadratic Equation	Sum of Roots	Product of Roots
$x^2 + 4x + 3 = 0$		
$6x^2 - 6x - 8 = 0$		



What Is It

Sum of the Roots of Quadratic Equation

$$r_1 + r_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$r_1 + r_2 = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$

$$r_1 + r_2 = \frac{-2b}{2a}$$

$$r_1 + r_2 = \frac{-b}{a}$$

The sum of the roots of quadratic equation is $\frac{-b}{a}$.

*Add the roots of a quadratic formula

*Since both have the same denominator, combine all terms in numerator and copy the common denominator

*Simplify

Product of the Roots of Quadratic Equation

$$r_1 \cdot r_2 = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \left(\frac{-b - \sqrt{b^2 - 4ac}}{2a} \right)$$

*Multiply the roots of Quadratic Equation

$$r_1 \cdot r_2 = \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{(2a)^2}$$

*Combine similar terms and express as square

$$r_1 \cdot r_2 = \frac{b^2 - b^2 + 4ac}{4a^2}$$

*The square root sign and the square in $(\sqrt{b^2 - 4ac})^2$ cancel out each other. And simplify $(-b)^2$ and $(2a)^2$

$$r_1 \cdot r_2 = \frac{4ac}{4a^2}$$

*Simplify: Divide both numerator & denominator by 4a

$$r_1 \cdot r_2 = \frac{c}{a}$$

The product of the roots of quadratic equation is $\frac{c}{a}$.

Let us answer the Activity 2.2: Complete Me! Using the formula presented above.

- Find the sum and product of the quadratic equation $x^2 + 4x + 3 = 0$

$$a = 1 \quad b = 4 \quad c = 3$$

$$r_1 + r_2 = \frac{-b}{a}$$

$$= \frac{-(4)}{(1)} \quad \text{Substitute values of } a \text{ and } b$$

$$r_1 + r_2 = -4$$

$$a = 1 \quad b = 4 \quad c = 3$$

$$r_1 \cdot r_2 = \frac{c}{a}$$

$$= \frac{(3)}{(1)} \quad \text{Substitute values of } a \text{ and } c$$

$$r_1 \cdot r_2 = 3$$

- Find the sum and product of the quadratic equation $6x^2 - 6x - 8 = 0$;

$$a = 6 \quad b = -6 \quad c = -8$$

$$r_1 + r_2 = \frac{-b}{a}$$

$$= \frac{-(6)}{(6)} \quad \text{Substitute values of } a \text{ and } b$$

$$r_1 + r_2 = 1$$

$$a = 6 \quad b = -6 \quad c = -8$$

$$r_1 \cdot r_2 = \frac{c}{a}$$

$$= \frac{(-8)}{(6)} \quad \text{Substitute values of } a \text{ and } c$$

$$r_1 + r_2 = -\frac{4}{3}$$



What's More

❖ Activity 2.3: What's My Sum and Product

Find the Sum and Product of the following Quadratic Equation. Write your answer on a separate paper.

1. $8x^2 = 6x + 9$

4. $x^2 + 4x - 3 = 0$

2. $4x^2 + 8x + 3 = 0$

5. $x^2 + 4x - 21 = 0$

3. $x^2 + 4x + 3 = 0$



What I Can Do

❖ Activity 2.4: Riddle Me!

A. Find the discriminant and identify the nature of the roots of the following quadratic equations. Write your answer on the space provided. Then decode the riddle below using the choices given, write the letter in the box provided.

Equation	Discriminant	Nature
1. $6p^2 - 2p = 3$		
2. $5b^2 + b = 2$		
3. $5r + r^2 + 2 = 0$		
4. $2p^2 + 5p = -4$		
5. $2x^2 - x - 1 = 0$		
6. $x^2 + 4x + 4 = 0$		

E 76

S 17

L 9

A 0

P 41

U -7

A -76

A -41

What did the little lobsters get on its Math test?

— — — — — — —

3 1 6 2 5 4 3



Assessment (*Post Test*)

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper. If answer not found in the given choices write your correct answer.

1. What is the nature of the roots of the quadratic equation if the value of its discriminant is greater than zero and a perfect square?
 - A. The roots are not real.
 - B. The roots are irrational and not equal.
 - C. The roots are rational and not equal.
 - D. The roots are real and equal.

2. What is the value of the discriminant of the quadratic equation $ax^2 + bx + c = 0$?

A. $c^2 - bc$	C. $b^2 + 4ac$
B. $c^2 - 4ba$	D. $b^2 - 4ac$

3. Which is true for the equation $x^2 - 6x + 22 = 0$?
 - A. The roots are not real.
 - B. The roots are irrational and not equal.
 - C. The roots are rational and not equal.
 - D. The roots are rational and equal.

4. In a quadratic equation, what do you call the value $b^2 - 4ac$?

A. Denominator	C. Diameter
B. Divisor	D. Discriminant

5. Which of the following quadratic equations has no real roots?

A. $t^2 - 8t - 4 = 0$	C. $2x^2 + 4x = 3$
B. $t^2 - 8t + 4 = 0$	D. $2r^2 + r + 7 = 0$

6. The roots of a quadratic equations are -5 and 3 . Which of the following quadratic equations have these roots?

A. $x^2 - 2x + 15 = 0$	C. $x^2 + 2x + 15 = 0$
B. $x^2 - 2x - 15 = 0$	D. $x^2 + 2x - 15 = 0$

7. What is the sum of the roots of the quadratic equation $x^2 - 2x - 3 = 0$?

A. 2	C. -2
B. 3	D. -3

8. What is the product of the roots of the quadratic equation $x^2 - 2x - 3 = 0$?

A. 2	C. -2
B. 3	D. -3

9. What is the sum of the roots of $x^2 = 10x - 25$?

10. What is the product of the roots of $x^2 - 11x + 28 = 0$?



Answer Key

Remember: This portion of the module contains all the answers. Your **HONESTY** is required.

5. Perfect Square number is a number that can be expressed as a square of rational number.
- Not Perfect Square: $24.5, \frac{7}{8}, \frac{5}{12}, \sqrt{25}, \frac{\sqrt{9}}{\sqrt{15}}, \sqrt{35}$
4. Perfect Square: 289
- $\frac{9}{\sqrt{15}}, \sqrt{35}$
- Rational numbers:
- 24.5, $\frac{7}{8}, \frac{5}{12}, 289, \sqrt{25}$
3. Rational numbers:
- Not Real Numbers: $\sqrt{-15}, \sqrt{\frac{9}{-21}}$
2. Real Numbers: $24.5, \frac{7}{8}, \frac{5}{12}, 289, \sqrt{25}, \frac{\sqrt{9}}{\sqrt{15}}, \sqrt{35}$
1. Varied students' responses

Activity 1.1: Which Are Real? Which Are Not?

4. $x^2 + 2x + 5 = 0$ $a = 1$ $b = 2$ $c = 5$ -16
3. $x^2 + 6x + 3 = 0$ $a = 1$ $b = 6$ $c = 3$ 24
2. $x^2 + 7x + 10 = 0$ $a = 1$ $b = 7$ $c = 10$ 9
1. $x^2 - 4x + 4 = 0$ $a = 1$ $b = -4$ $c = 4$ 0
 $b^2 - 4ac$

Activity 1.2: Do Math in A, B, C

- = -8; has no real roots
- = 16 - 24
5. $b^2 - 4ac = (4)^2 - 4(1)(6)$
= 0; real numbers and are equal
4. $b^2 - 4ac = (4)^2 - 4(1)(4)$
= 32; irrational and are not equal
3. $b^2 - 4ac = (0)^2 - 4(2)(-4)$
= 1; rational but are not equal
2. $b^2 - 4ac = (-5)^2 - 4(3)(2)$
= 5; irrational but are not equal
1. $b^2 - 4ac = (3)^2 - 4(1)(1)$
= 9 - 4

Activity 1.3: Try This!

$$r_1 + r_2 = -\frac{3}{4}$$

(6)

Substitute values

$$r_1 \cdot r_2 = \frac{a}{c}$$

$$r_1 \cdot r_2 = \frac{a}{c}$$

$$8 = -q \quad 9 = a$$

$$-9 = c \quad 9 = a$$

$$r_1 \cdot r_2 = 3$$

(1)

(3)

$$r_1 \cdot r_2 = \frac{a}{c}$$

$$a = 1 \quad b = 4 \quad c = 3$$

$$r_1 + r_2 = 1$$

(9)

Substitute values

$$r_1 + r_2 = \frac{a}{-q}$$

$$r_1 + r_2 = \frac{a}{-q}$$

$$8 = q \quad 9 = a$$

$$-9 = c \quad 9 = a$$

$$r_1 + r_2 = -4$$

(1)

(4)

$$r_1 + r_2 = \frac{a}{-q}$$

$$a = 1 \quad b = 4 \quad c = 3$$

$$1. x^2 + 4x + 3 = 0$$

$$2. 6x^2 - 6x - 8 = 0$$

Activity 2.2:

References

Text Book

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“Math Riddles”, Riddles, accessed July 9, 2020, url: <https://www.riddles.com/math-riddles?page=2>

“Real World Examples of Quadratic Equations”, Math is Fun, accessed July 15, 2020, url: <https://www.mathsisfun.com/algebra/quadratic-equation-real-world.html>

“Real Number Chart”, StudyLib, accessed July 20, 2020, url: <https://studylib.net/doc/7914704/real-number-chart>

Congratulations!

You are now ready for the next module. Always remember the following:

1. Make sure every answer sheet has your
 - *Name*
 - *Grade and Section*
 - *Title of the Activity or Activity No.*
2. Follow the date of submission of answer sheets as agreed with your teacher.
3. Keep the modules with you AND return them at the end of the school year or whenever face-to-face interaction is permitted.