9

Mathematics

Quarter 1 – Module 8
Graph of Quadratic Function

Reminders to Learners

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in this module.
3. Read the instructions carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

About the Module

This module was designed and written with you in mind. It is here to help you master about **Graph of Quadratic Functions**. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is divided into 2 lessons, namely:

- Lesson 1 – Various Graph of Quadratic Functions
- Lesson 2 – The Effects of Changing the Values of $a, h \text{ and } k$ in the Graph of a Quadratic Function

After going through this module, you are expected to:

- graph a quadratic function using: (a) domain; (b)range; (c) intercepts; (d)axis of symmetry; (e)vertex; (f)direction of the opening of the parabola
- analyze the effects of changing the values of $a, h \text{ and } k$ in the equation $y = a(x - h)^2 + k$ of a quadratic function on its graph

What I Know (Pre-Test)

Instructions: Read each item carefully. Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. What is the vertex of the given quadratic function $y = 3(x + 6)^2 - 25$?
   A. $V\ (-6, -25)$ B. $V\ (-6, 25)$ C. $V\ (-6, -25)$ D. $V\ (6, -25)$

2. What is the value of the axis of symmetry in the function $y = (x + 5)^2 - 6$?
   A. $h = 5$ B. $h = -5$ C. $h = 6$ D. $h = -6$

3. Which of the following is the range of the function $y = (x + 6)^2 - 4$?
   A. $y \leq -4$ B. $y \geq -4$ C. $y \geq -6$ D. $y \leq -6$

4. What is the y intercept of $y = (x - 4)^2 + 2$?
   A. (0,12) B. (0,14) C. (0,16) D. (0,18)

5. If the graph of a quadratic function moves ***downward***, what is the possible value of k?
   A. positive B. zero C. negative D. undefined

6. If the graph of a quadratic function is moving to the ***right***, what would be the possible value of h?
   A. positive B. zero C. negative D. undefined

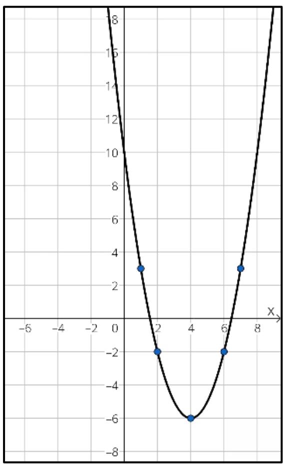
7. If the graph of a quadratic function is ***narrower*** than the parent graph $y = ax^2$, what would be the possible value of $\boldsymbol{a}$?
   A. $a = 1$ B. $a > 1$ C. $0 < a < 1$ D. $a = 0$

8. What is the minimum point of the given function $y = x^2 - 6x + 10$?
   A. $k = -1$ B. $k = -3$ C. $k = 1$ D. $k = 3$

9. What is the maximum point of the given function $y = -3(x - 6)^2 + 8$?
   A. $k = -6$ B. $k = -8$ C. $k = 6$ D. $k = 8$

10. What should be the graph of the function $y = (x - 4)^2 + 6$?

A



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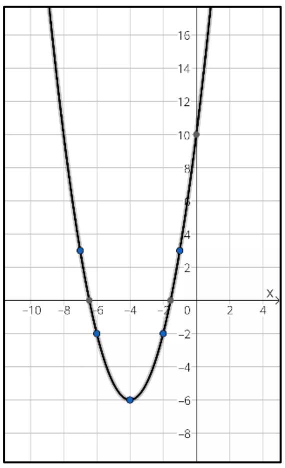

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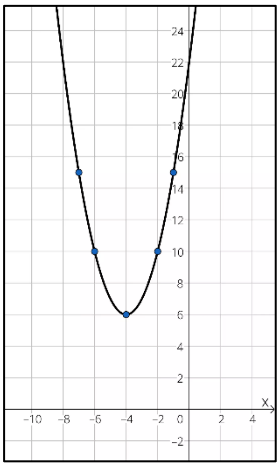

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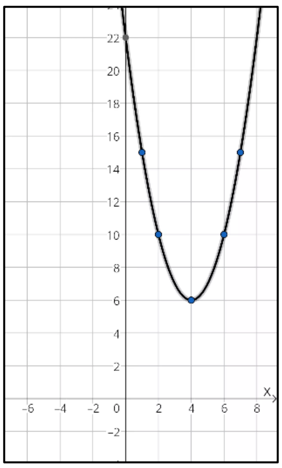


Lesson 1

Various Graph of Quadratic Functions

What I Need to Know

At the end of this lesson, you are expected to:

- graph a quadratic function using: (a) domain; (b)range; (c) intercepts; (d)axis of symmetry; (e)vertex; (f)direction of the opening of the parabola

What's In

❖ ***Flashback***

Looking back at the examples 1 and 2 we had in **Module 7 Lesson 3**, you learned how to rewrite quadratic function from standard to vertex form using HK method where $h = \frac{-b}{2a}$ and $k = f(h)$ whose value of h is plugged in to the original function represented by $y = ax^2 + bx + c$ like what is shown below.

1. $\boldsymbol{y = x^2 + 10x + 21}$		2. $\boldsymbol{y = x^2 - 4x + 6}$	
$a = 1, b = 10, c = 21$	**step1**	$a = 1, b = -4, c = 6$	**step1**
substitute the values of a and b to the formula of h		*substitute the values of a and b to the formula of h*	
$\boldsymbol{h} = -\frac{b}{2a} = -\frac{10}{2(1)} = -5$	**step2**	$\boldsymbol{h = -\frac{b}{2a} = -\frac{(-4)}{2(1)} = 2}$	**step2**
substitute the value of h to the function where $\boldsymbol{h} = \boldsymbol{-5}$		*substitute the value of h to the function where* $\boldsymbol{h} = \boldsymbol{2}$	
$k = f(h)$		$k = f(h)$	
$k = (-5)^2 + 10(-5) + 21$	**step3**	$k = (2)^2 - 4(2) + 6$	**step3**
$k = 25 - 50 + 21$		$k = 4 - 8 + 6$	
$\boldsymbol{k = -4}$		$\boldsymbol{k = 2}$	
*so, the **vertex form** of the quadratic function will be*		*so, the **vertex form** of the quadratic function will be*	
$\boldsymbol{y = (x + 5)^2 - 4}$	**step4**	$\boldsymbol{y = (x - 2)^2 + 2}$	**step4**

So, how important are the values of h *and* k to our succeeding lesson?

In what way can you use the HK method in graphing quadratic functions?

What's New

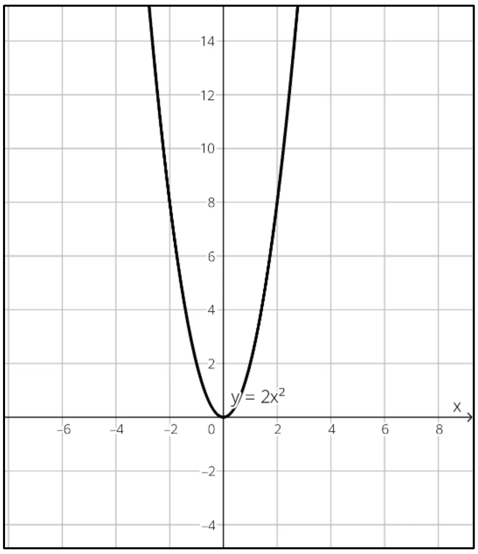
❖ ***Let's Level Up!***

In order to graph the quadratic function in standard form represented by $y = ax^2 + bx + c$, you use the HK Method for us to arrive at our vertex.

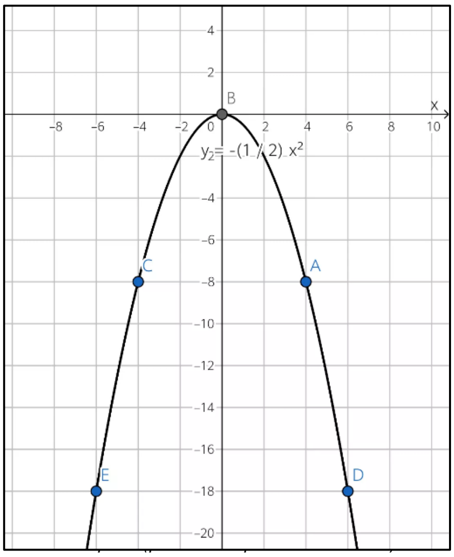
So, what does ***vertex*** represents anyway?

Vertex is the highest (maximum) or lowest (minimum) point of the graph of a quadratic function denoted by $\boldsymbol{V}\ (\boldsymbol{h}, \boldsymbol{k})$ where h is the x-coordinate and k is the y-coordinate of the vertex.

The graph of the quadratic function $y = ax^2 + bx + c$ is a **parabola** that opens **upward** when $\boldsymbol{a} > \boldsymbol{0}$ and opens **downward** when $\boldsymbol{a} < \boldsymbol{0}$ wherein $\boldsymbol{a}$ represents the coefficient of the quadratic term, ax^2.



Graph 1: Graph of $y = 2x^2$



Graph 2: Graph of $y = -\frac{1}{2}x^2$

If the parabola opens upward, the quadratic function has a **minimum value** since its vertex lies at the **bottom of the parabola**. (*see graph 1*)

If the parabola opens downward, the quadratic function has a **maximum value** since its vertex lies at the **top of the parabola.** (*see graph 2*)

While the **axis of symmetry** is an imaginary line that divides the parabola into two faces which are mirror images of each other's side, denoted by the formula $\boldsymbol{h} = -\frac{\boldsymbol{b}}{\boldsymbol{2a}}$ which is the x-coordinate of the vertex (h, k) such that $x = h$ when the given function is in standard form.

The **domain** of the graph of any quadratic function is the set of all real numbers found in the x-axis while the **range** is the set of the values of y exclusive to where the parabola faces, either upward or downward position.

The intercepts, such as x and y intercepts are points found at their respective axis where **x-intercept** is a point of intersection along the x-axis where the parabola passes through, in symbols $(x, 0)$. While **y-intercept** is a point of intersection along the y-axis where the parabola passes through, in symbols $(0, y)$.

So, these are just some of the properties that you will be using in graphing quadratic functions in our examples thereafter.

What Is It

❖ ***How should I do it?***

Example No.1 When the quadratic function is $y = ax^2$

Graph the quadratic function $\boldsymbol{y = x^2}$ and identify the following:

a. axis of symmetry
b. vertex
c. direction of the opening of the parabola
d. domain and range
e. intercepts

Solutions:

Step 1: In $y = x^2$, the following are;

a. the vertex is $V\ (0,0)$
b. the axis of symmetry is $x = 0$ since the vertex is at the origin.
c. the parabola opens upward since $a = 1$.

Step 2: Make a table of values of at least 3 points on the left and right of vertex $V(0,0)$ to solve for the values of y.

X	-3	-2	-1	**0**	1	2	3
Y	9	4	1	**0**	1	4	9

We choose 3 points to the left and right of the vertex base on the number line.

To solve for the other values of y (range), we plug-in alternately the values of x to the function $\boldsymbol{f(x) = x^2}$

$$f(-3) = (-3)^2 = 9$$
$$f(-2) = (-2)^2 = 4$$
$$f(-1) = (-1)^2 = 1$$
$$f(0) = (0)^2 = 0$$
$$f(1) = 1^2 = 1$$
$$f(2) = 2^2 = 4$$
$$f(3) = (3)^2 = 9$$

Step 3: Determine the domain and range of the function

Domain of the function is the set of real numbers such that D: $\{-\infty, +\infty\}$

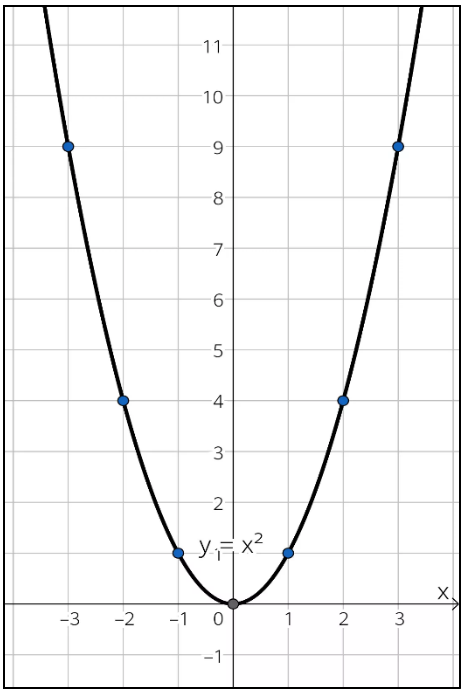
Range of the function is R: $\{y \geq 0\}$

Step 4: Determine the intercepts of the function

Since the vertex is at the origin, it contains only one intercept, that is, (0,0).

Step 5: Plot and connect the points from the table to form a parabola.

Remember: Each point must be a reflection of another point on the other side of the axis of symmetry.



Important Note!

If $a > 1$, the graph of $y = ax^2$ is narrower than $y = x^2$

If $0 < a < 1$, the graph of $y = x^2$ is wider than $y = x^2$

Example No.2 When the quadratic function is $y = ax^2 + k$

Graph the quadratic function $y = x^2 + 3$ and identify the following:

d. axis of symmetry
e. vertex
f. direction of the opening of the parabola
d. domain and range
e. intercepts

Solutions:

Step 1: In $y = x^2 + 3$, the following are;

a. the vertex is at $V\ (0,3)$
b. the axis of symmetry is $x = 0$ since the vertex falls along the y-axis.
c. the parabola opens upward since $a = 1$.

Step 2: Make a table of values of at least 3 points on the left and right of vertex $V(0,3)$ to solve for the values of y.

x	-3	-2	-1	**0**	1	2	3
y	12	7	4	**3**	4	7	12

We choose 3 points to the left and right of the vertex base on the number line.

To solve for the other values of y (range), we plug-in alternately the values of x to the function $f(x) = x^2 + 3$

$$f(-3) = (-3)^2 + 3 = 12$$
$$f(-2) = (-2)^2 + 3 = 7$$
$$f(-1) = (-1)^2 + 3 = 4$$
$$f(0) = (0)^2 + 3 = 3$$
$$f(1) = 1^2 + 3 = 4$$
$$f(2) = 2^2 + 3 = 7$$
$$f(3) = (3)^2 + 3 = 12$$

Step 3: Determine the domain and range of the function

Domain of the function is the set of real numbers such that D: $\{-\infty, +\infty\}$

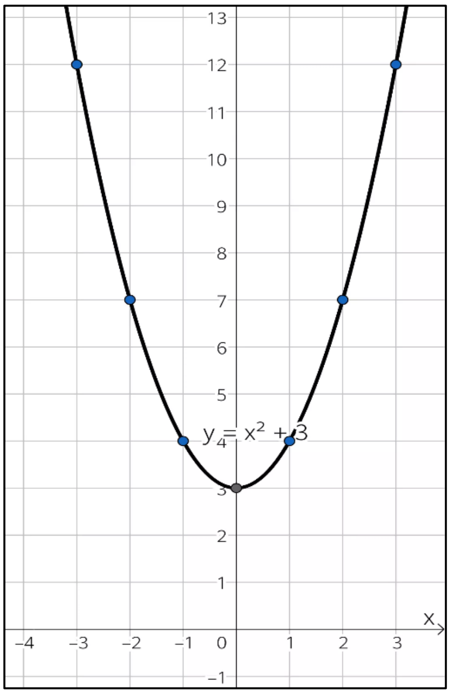
Range of the function is R: $\{y \geq 3\}$

Step 4: Determine the intercepts of the function

(0,3) is the y-intercept since it is the vertex. While there's no x-intercept since the graph is located 3 units above the x-axis.

Step 5: Plot and connect the points from the table to form a parabola.

Remember: Each point must be a reflection of another point on the other side of the axis of symmetry.



Thing to Remember!

The graph of $y = ax^2 + k$ is the graph of $y = x^2$ moved k units upward when k is positive and k units downward when k is negative.

Example No.3 When the quadratic function is $y = a(x-h)^2$

Graph the quadratic function $\boldsymbol{y = (x-1)^2}$ and identify the following:

a. axis of symmetry
b. vertex
c. direction of the opening of the parabola
d. domain and range
e. intercepts

Solutions:

Step 1: In $y = (x-1)^2$, the following are;

a. the vertex is at V (1,0)
b. the axis of symmetry is $x = 1$ since the vertex falls along the x-axis and $h = 1$.
c. the parabola opens upward since $a = 1$.

Step 2: Make a table of values of at least 3 points on the left and right of vertex $V(1,0)$ to solve for the values of y.

x	-2	-1	0	**1**	2	3	4
y	9	4	1	**0**	1	4	9

We choose 3 points to the left and right of the vertex base on the number line.

To solve for the other values of y (range), we plug-in alternately the values of x to the function $\boldsymbol{f(x) = (x-1)^2}$

$$f(-2) = (-2-1)^2 = 9$$
$$f(-1) = (-1-1)^2 = 4$$
$$f(0) = (0-1)^2 = 1$$
$$f(1) = (1-1)^2 = 0$$
$$f(2) = (2-1)^2 = 1$$
$$f(3) = (3-1)^2 = 4$$
$$f(4) = (4-1)^2 = 9$$

Step 3: Determine the domain and range of the function

Domain of the function is the set of real numbers such that D: $\{-\infty, +\infty\}$

Range of the function is R: $\{y \geq 0\}$

Step 4: Determine the intercepts of the function

(1,0) is the x-intercept since it is the vertex at the same time, it lies on the x-axis.

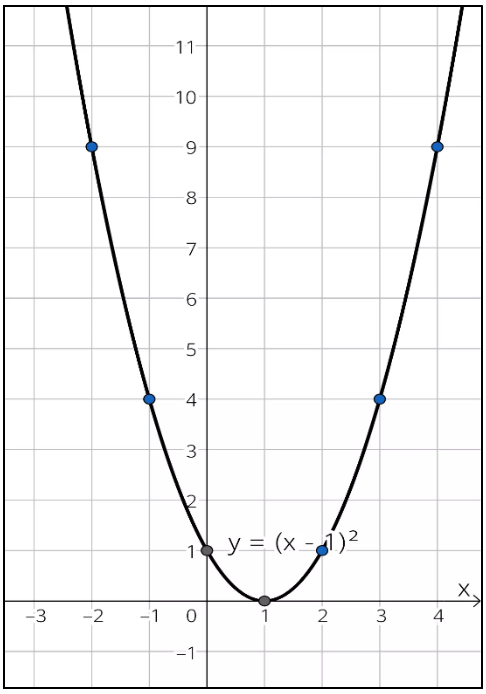
To solve for y-intercept, it is noted that $x = 0$, we substitute x with 0 in the function,

$$y = (x-1)^2$$
$$y = (0-1)^2$$
$$y = (-1)^2$$
$$y = 1$$

Therefore, y-intercept is (0,1).

Step 5: Plot and connect the points from the table to form a parabola.

Remember: Each point must be a reflection of another point on the other side of the axis of symmetry.



Thing to Remember!

The graph of $y = (x-h)^2$ is the graph of $y = x^2$ moved h units to the right when h is positive. While the graph of $y = (x+h)^2$ is the graph of $y = x^2$ moved h units to the left when h is negative.

Example No.4 When the quadratic function is $y = ax^2 + bx + c$ and $a = 1$

Graph the quadratic function $\boldsymbol{y = x^2 - 8x + 15}$ and identify the following:

d. axis of symmetry
e. vertex
f. direction of the opening of the parabola
d. domain and range
e. intercepts

Solutions:

Step 1: Identify the axis of symmetry

Given the function $\boldsymbol{y = x^2 - 8x + 15}$, where the values of $a = 1$ *and* $b = -8$

$$\boldsymbol{h} = -\frac{b}{2a} = -\frac{-8}{2(1)} = \frac{8}{2} = \mathbf{4}$$

then, the axis of symmetry passes through $\boldsymbol{x = 4}$

Step 2: Identify the vertex using HK Method

Since $\boldsymbol{h = 4,}$ then we can plug-in the value of h to the function

$k = f(h) = x^2 - 8x + 15$
$k = f(4) = 4^2 - 8(4) + 15$
$k = f(4) = 16 - 32 + 15$
$\boldsymbol{k = -1}$

then, the vertex is $\boldsymbol{V = (4, -1)}$

Step 3: Determine the opening of the parabola

Since, $a = 1$ then the parabola faces **upward** such that $a > 0$

Step 4: Determine the domain and range of the function

Domain of the function is the set of real numbers such that D: $\{-\infty, +\infty\}$

Range of the function is R: $\{y \geq -1\}$

Step 5: Make a table of values of at least 2 points on the left and right of vertex

We choose 2 points to the left and right of the vertex base on the number line

x	2	3	**4**	5	6
y	3	0	**-1**	0	3

To solve for the other values of y (range), we plug-in alternately the values of x to the function $\boldsymbol{f(x) = x^2 - 8x + 15}$

$f(2) = 2^2 - 8(2) + 15 = \mathbf{3}$
$f(3) = 3^2 - 8(3) + 15 = \mathbf{0}$
$f(4) = 4^2 - 8(4) + 15 = \mathbf{-1}$
$f(5) = 5^2 - 8(5) + 15 = \mathbf{0}$
$f(6) = 6^2 - 8(6) + 15 = \mathbf{3}$

Step 6: Determine the intercepts of the function

To determine the x and y intercepts, we need to solve using factoring.

To solve for x-intercept, we replace y=0

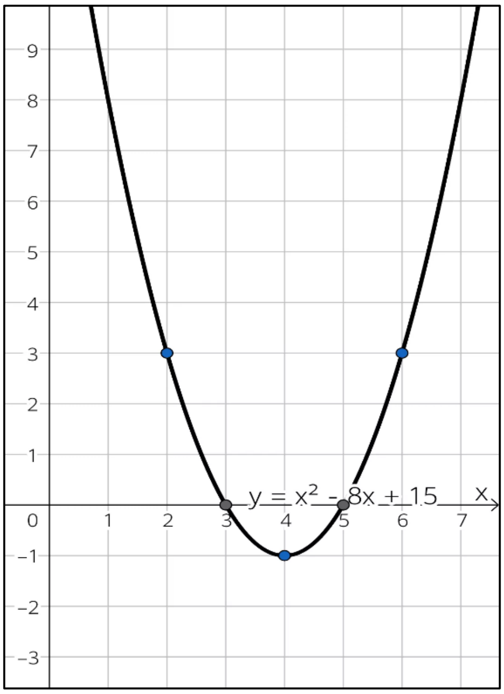
$y = x^2 - 8x + 15$	**original equation**
$0 = (x - 5)(x - 3)$	**binomial factors**
$x - 5 = 0$ *and* $x - 3 = 0$	**zero product property**
$x = 5$ *and* $x = 3$	**roots**
$(5,0)$ *and* $(3,0)$	**x-intercepts**

To solve for y-intercept, we replace x=0

$y = x^2 - 8x + 15$	**original equation**
$y = 0^2 - 8(0) + 15$	**replacing x by zero**
$y = 15$	**result**
$(0,15)$	**y-intercept**

Step 7: Plot and connect the points from the table to form a parabola.

Remember: Each point must be a reflection of another point on the other side of the axis of symmetry.



Example No. 5 When the quadratic function is $y = ax^2 + bx + c$ and $a < 1$

Graph the quadratic function $y = -x^2 + 6x - 15$ and identify the following:

a. axis of symmetry
b. vertex
c. direction of the opening of the parabola
d. domain and range
e. intercepts

Solutions:

Step 1: Identify the axis of symmetry

Given the function $y = -x^2 + 6x - 15$, where the values of $a = -1$ *and* $b = 6$

$$h = -\frac{b}{2a} = -\frac{6}{2(-1)} = -\frac{6}{-2} = 3$$

then, the axis of symmetry passes through $x = 3$

Step 2: Identify the vertex using HK Method

Since $h = 3$, then we can plug-in the value of h to the function

$$k = f(h) = -x^2 + 6x - 15$$
$$k = f(3) = -(3)^2 + 6(3) - 15$$
$$k = f(3) = -9 + 18 - 15$$
$$k = -6$$

then, the vertex is $V = (3, -6)$

Step 3: Determine the opening of the parabola

Since, $a = -1$ then the parabola faces **downward** such that $a < 0$

Step 4: Determine the domain and range of the function

Domain of the function is the set of real numbers such that D: $\{-\infty, +\infty\}$

Range of the function is R: $\{y \leq -6\}$

Step 5: Make a table of values of at least 2 points on the left and right of vertex

We choose 2 points to the left and right of the vertex base on the number line

x	1	2	**3**	4	5
y	-10	-7	**-6**	-7	-10

To solve for the other values of y (range), we plug-in alternately the values of x to the function $f(x) = -x^2 + 6x - 15$

$$f(1) = -(1)^2 + 6(1) - 15 = -10$$
$$f(2) = -(2)^2 + 6(2) - 15 = -7$$
$$f(3) = -(3)^2 + 6(3) - 15 = -6$$
$$f(4) = -(4)^2 + 6(4) - 15 = -7$$
$$f(5) = -(5)^2 + 6(5) - 15 = -10$$

Step 6: Determine the intercepts of the function

To determine the x and y intercepts, we need to solve using factoring.

To solve for x-intercept, we replace y=0

Using quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(-1)(-15)}}{2(-1)}$$
$$x = \frac{-6 \pm \sqrt{36 - 60}}{-2}$$
$$x = \frac{-6 \pm \sqrt{-24}}{-2}$$

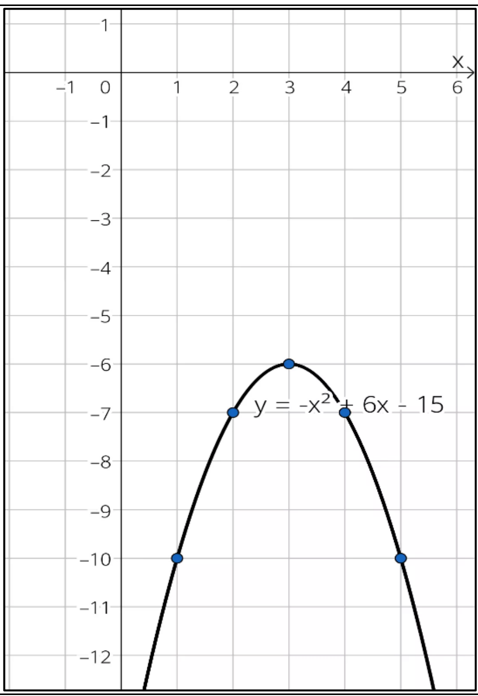
Radicand is negative

Based on the results, roots are imaginary. And the graph doesn't pass through the x-axis at any point. So, there's no x-intercept.

To solve for y-intercept, we replace x=0

$y = -x^2 + 6x - 15$	**original equation**
$y = 0^2 + 6(0) - 15$	**replacing x by zero**
$y = -15$	**result**
$(0, -15)$	**y-intercept**

Step 7: Plot and connect the points from the table to form a parabola.



Example No.6 When the quadratic function is $y = ax^2 + bx + c$ and $a > 1$

Graph the quadratic function $\boldsymbol{y = 2x^2 + 8x + 6}$ and identify the following:

a. axis of symmetry
b. vertex
c. direction of the opening of the parabola
d. domain and range
e. intercepts

Solutions:

Step 1: Identify the axis of symmetry

Given the function $\boldsymbol{y = 2x^2 + 8x + 6}$, where the values of $a = 2 \text{ and } b = 8$

$$\boldsymbol{h} = -\frac{b}{2a} = -\frac{8}{2(2)} = -\frac{8}{4} = -2$$

then, the axis of symmetry passes through $\boldsymbol{x = -2}$

Step 2: Identify the vertex using HK Method

Since $\boldsymbol{h = -2,}$ then we can plug-in the value of h to the function

$$k = f(h) = 2x^2 + 8x + 6$$
$$k = f(-2) = 2(-2)^2 + 8(-2) + 6$$
$$k = f(-2) = 2(4) - 16 + 6$$
$$\boldsymbol{k = -2}$$

then, the vertex is $\boldsymbol{V = (-2, -2)}$

Step 3: Determine the opening of the parabola

Since, $a = 2$ then the parabola faces **upward** such that $a > 0$

Step 5: Determine the domain and range of the function

Domain of the function is the set of real numbers such that D: $\{-\infty, +\infty\}$

Range of the function is R: $\{y \geq -2\}$

Step 5: Make a table of values of at least 2 points on the left and right of vertex

We choose 2 points to the left and right of the vertex base on the number line

x	-4	-3	**-2**	-1	0
y	6	0	**-2**	0	6

To solve for the other values of y (range), we plug-in alternately the values of x to the function $\boldsymbol{f(x) = 2x^2 + 8x + 6}$

$$f(-4) = 2(-4)^2 + 8(-4) + 6 = \boldsymbol{6}$$
$$f(-3) = 2(-3)^2 + 8(-3) + 6 = \boldsymbol{0}$$
$$f(-2) = 2(-2)^2 + 8(-2) + 6 = \boldsymbol{-2}$$
$$f(-1) = 2(-1)^2 + 8(-1) + 6 = \boldsymbol{0}$$
$$f(0) = 2(0)^2 + 8(0) + 6 = \boldsymbol{6}$$

Step 6: Determine the intercepts of the function

To determine the x and y intercepts, we need to solve using factoring.

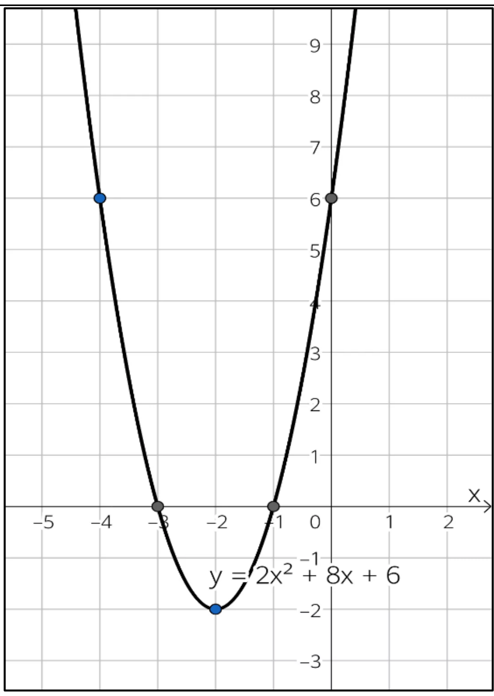
To solve for x-intercept, we replace y=0
We solve using factoring method when a>1

$y = 2x^2 + 8x + 6$	**original equation**
$2x^2 + 8x + 6 = 0$	**equating to zero**
$2x^2 + 6x + 2x + 6 = 0$	**factoring middle term**
$(2x^2 + 6x) + (2x + 6) = 0$	**grouping of binomial**
$2x(x + 3) + 2(x + 3) = 0$	**factoring binomials**
$(2x + 2)(x + 3) = 0$	**regrouping of binomial**
$2x + 2 = 0 \text{ and } x + 3 = 0$	**zero product prop.**
$x = -1 \text{ and } x = -3$	**roots**
$\boldsymbol{(-1, 0)(-3, 0)}$	**x-intercepts**

To solve for y-intercept, we replace x=0

$y = 2x^2 + 8x + 6$	**original equation**
$y = 2(0)^2 + 8(0) + 6$	**replacing x by zero**
$y = 6$	**result**
(0,6)	**y-intercept**

Step 7: Plot and connect the points from the table to form a parabola.



What's More

Activity 8.1: NOW IT'S YOUR TURN!

Instructions: On a separate sheet of paper or graphing paper, graph the following quadratic functions and identify the following:

a. axis of symmetry
b. vertex
c. direction of the opening of the parabola
d. domain and range
e. intercepts

1. $y = x^2 - 6x - 7$	2. $y = -2x^2 + 4x + 11$
3. $y = x^2 - 2$	4. $y = (x+3)^2$

What I Need to Remember

Study the common characteristics of each graph below that you need to remember.

Function	Axis of Symmetry	Vertex	Opening of the Direction of Parabola	Domain and Range
$y = ax^2$	$x = 0$	$(0,0)$	when $a > 1$, parabola faces upward when $a < 1$, parabola faces downward	D: $\{-\infty, +\infty\}$ R: $\{y \geq 0\}$
$y = ax^2 + k$	$x = 0$	$(0, k)$		D: $\{-\infty, +\infty\}$ R: $\{y \geq k\}$ or R: $\{y \leq k\}$
$y = a(x-h)^2$	$x = h$	(h, o)		D: $\{-\infty, +\infty\}$ R: $\{y \geq 0\}$
$y = ax^2 + bx + c$	$h = -\frac{b}{2a}$	(h, k) where $h = -\frac{b}{2a}$ and $k = f(h)$		D: $\{-\infty, +\infty\}$ R: $\{y \geq k\}$ or R: $\{y \leq k\}$

Lesson 2

The Effects of Changing the Values of a, h *and* k in the Graph of a Quadratic Function

What I Need to Know

At the end of this lesson, you are expected to:

- graph a quadratic function using: (a) domain; (b)range; (c)axis of symmetry; (d)vertex; (e)direction of the opening of the parabola
- analyze the effects of changing the values of a, h *and* k in the equation $y = a(x-h)^2 + k$ of a quadratic function on its graph

What's In

❖ ***Let's Do the Recall***

Short Quiz: Supply the values of each function as shown in the table below.

Function	Axis of Symmetry	Vertex	Opening of the Direction of Parabola	Domain and Range
1. $y = -2x^2$				
2. $y = x^2 + 4$				
3. $y = (x-2)^2$				
4. $y = x^2 + 8x + 10$				

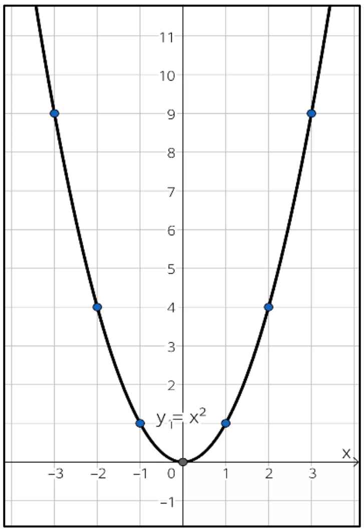
What's New

What if the given quadratic function is in **vertex form**, denoted by $\boldsymbol{y = a(x-h)^2 + k}$? How are we going to graph it?

What would be the graph of the quadratic function in vertex form as compared to that of a ***parent function,*** as shown on the right side, of a quadratic function denoted by $\boldsymbol{y = ax^2}$?

And, what is the effect on the graph if the values of a, h *and* k in the equation $y = a(x-h)^2 + k$ are changed?

Try to discover the kind of transformation it undergoes in the examples thereafter.

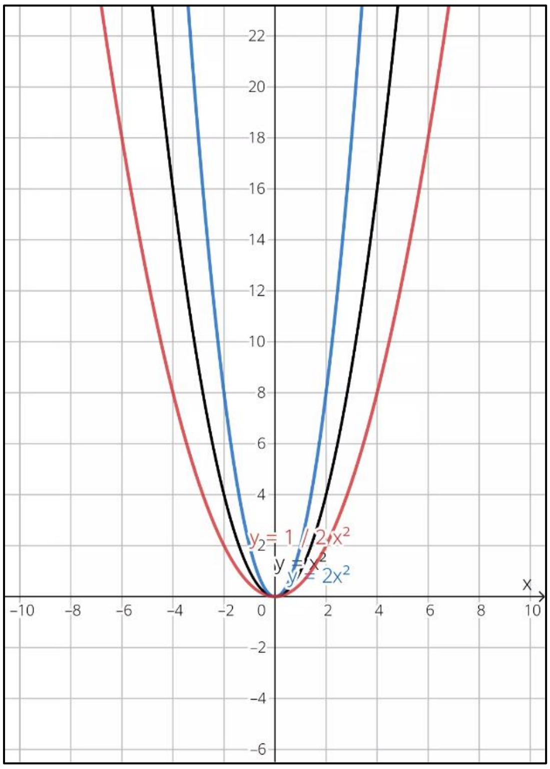


Graph 3 parent graph, $\boldsymbol{y = ax^2}$

****parent function refers to the basic graph of any quadratic function denoted by* $y = ax^2$ *where* $a = 1$

What Is It

❖ *What do we need to do first?*



Graph 4: Various Faces of a Parent Graph

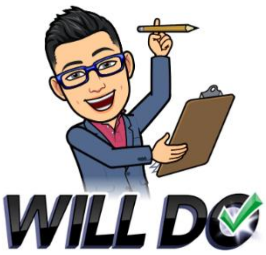
Prior to the examples on graphing quadratic functions in vertex form, visit the graph of the ***parent function***, $\boldsymbol{y = ax^2}$, which will be the basis for comparison on the transformation of the various graph (parabola) written in vertex form.

The graph on the left shows a few examples of the various faces of a ***parent graph*** whose value of $\boldsymbol{a}$ varies from one another like what is shown on the table below.

Try to take a look at the table of values for the domain and range of the 3 faces of a parent graph, as follows: $\boldsymbol{y = x^2}$, $\boldsymbol{y = 2x^2}$ **and** $\boldsymbol{y = \frac{1}{2}x^2}$.

$f(x)$	X	0	1	2	3
$y = x^2$	Y	0	1	4	9
$y = 2x^2$		0	2	8	18
$y = \frac{1}{2}x^2$		0	$\frac{1}{2}$	2	$\frac{9}{2}$

It could be noted that the inner most graph denoted by $\boldsymbol{y = 2x^2}$ is ***narrower*** than the middle graph denoted by $\boldsymbol{y = x^2}$. While the outer most graph denoted by $\boldsymbol{y = \frac{1}{2}x^2}$ is ***wider*** than the other 2 graphs.



Take note of the value of a in ax^2
If $a > 1$, the graph of $y = ax^2$ opens narrower than $y = x^2$
If $0 < a < 1$, the graph of $y = ax^2$ opens wider than $y = x^2$

This is held TRUE to ***parent function*** whose value of $\boldsymbol{a}$ is **negative.** This is made possible when the graph faces downward.

❖ *What to do next?*

Now, look at the transformation of the ***parent graph*** with the given quadratic functions in **vertex form**.

Examples
Graph the following functions below and identify the:

a. axis of symmetry
b. vertex
c. direction of the opening of the parabola
d. domain and range
e. table of values

1. $y = (x-2)^2 + 3$
2. $y = \frac{1}{2}(x+3)^2 - 2$
3. $y = -3(x+3)^2 + 4$
4. $y = -(x-1)^2 - 3$

> ***Take Note of This!***
> Vertex Form of Quadratic Function is denoted as $y = a(x-h)^2 + k$ where;
> a. h is positive for the expression $(x-h)^2$
> b. h is negative for the expression $(x+h)^2$
> c. sign for k follows the operation attached to it

Solutions

1. $\boldsymbol{y = (x-2)^2 + 3}$

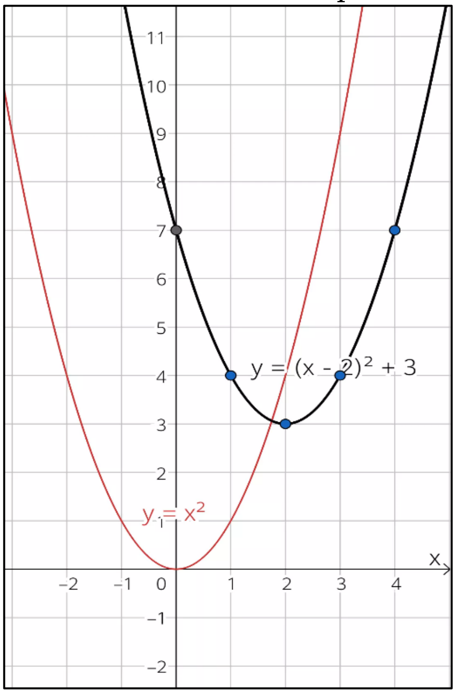
a. $h = 2$ axis of symmetry
b. $V\ (2, 3)$
c. $a = 1$ therefore, parabola faces upward since $a > 0$
d. domain is set of real numbers
range is $y \geq 3$
e. table of values

X	0	1	**2**	3	4
Y	7	4	**3**	4	7

$$\boldsymbol{f(x) = (x-2)^2 + 3}$$
$$f(0) = (0-2)^2 + 3 = 7$$
$$f(1) = (1-2)^2 + 3 = 4$$
$$f(3) = (3-2)^2 + 3 = 4$$
$$f(4) = (4-2)^2 + 3 = 7$$

Analysis of the graph:
*The graph of $y = (x-2)^2 + 3$ moves 2 units to the right of the parent graph since $h = 2$ and 3 units upward since $k = 3$ that is shown on the graph at the right side. The graph looks **the same** with the parent graph since $a = 1$.*

f. Plot and connect the points from the table to form a parabola.



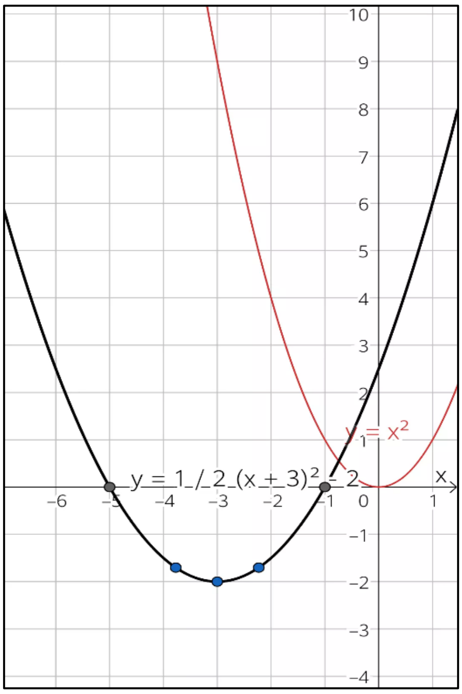
2. $\boldsymbol{y = \frac{1}{2}(x+3)^2 - 2}$

a. $h = -3$ axis of symmetry
b. $V\ (-3, -2)$
c. $a = \frac{1}{2}$ therefore, parabola faces upward since $a > 0$ and since $0 < a < 1$, then the parabola opens wider
d. domain is set of real numbers
range is $y \geq -2$
e. table of values

X	-5	-4	**-3**	-2	-1
Y	0	$-\frac{3}{2}$	**-2**	$-\frac{3}{2}$	0

$$\boldsymbol{f(x) = \frac{1}{2}(x+3)^2 - 2}$$
$$f(-5) = \frac{1}{2}(-5+3)^2 - 2 = 0$$
$$f(-4) = \frac{1}{2}(-4+3)^2 - 2 = -\frac{3}{2}$$
$$f(-2) = \frac{1}{2}(-2+3)^2 - 2 = -\frac{3}{2}$$
$$f(-1) = \frac{1}{2}(-1+3)^2 - 2 = 0$$

Analysis of the graph:
*The graph of $y = \frac{1}{2}(x+3)^2 - 2$ moves 3 units to the left of the parent graph since $h = -3$ and 2 units downward since $k = -2$ that is shown on the graph below. The graph is **wider** than the parent graph since $0 < a < 1$.*



3. $\boldsymbol{y = -3(x+3)^2 + 4}$

a. $h = -3$ axis of symmetry

b. $V\,(-3,\ 4)$

c. $a = -3$ therefore, parabola faces downward since $a < 0$ and since $|a > 1|$ then, parabola opens narrower

d. domain is set of real numbers
range is $y \leq 4$

e. table of values

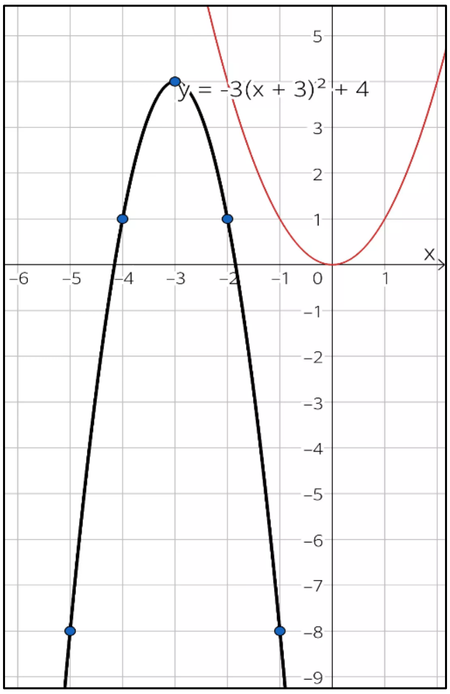
X	-5	-4	**-3**	-2	-1
Y	-8	1	**4**	1	-8

$$\boldsymbol{f(x) = -3(x+3)^2 + 4}$$
$$f(-5) = -3(-5+3)^2 + 4 = -8$$
$$f(-4) = -3(-4+3)^2 + 4 = 1$$
$$f(-2) = -3(-2+3)^2 + 4 = 1$$
$$f(-1) = -3(-1+3)^2 + 4 = -8$$

Analysis of the graph:

The graph of $y = -3(x+3)^2 + 4$ moves 3 units to the left of the parent graph since $h = -3$ and 4 units upward since $k = 4$ that is shown on the graph at the right side. The graph is ***narrower*** *than the parent graph since $a = -3$.*

f. Plot and connect the points from the table to form a parabola.



4. $\boldsymbol{y = -(x-1)^2 - 3}$

a. $h = 1$ axis of symmetry

b. $V\,(1, -3)$

c. $a = -1$ therefore, parabola faces downward since $a < 0$

d. domain is set of real numbers
range is $y \leq -3$

e. table of values

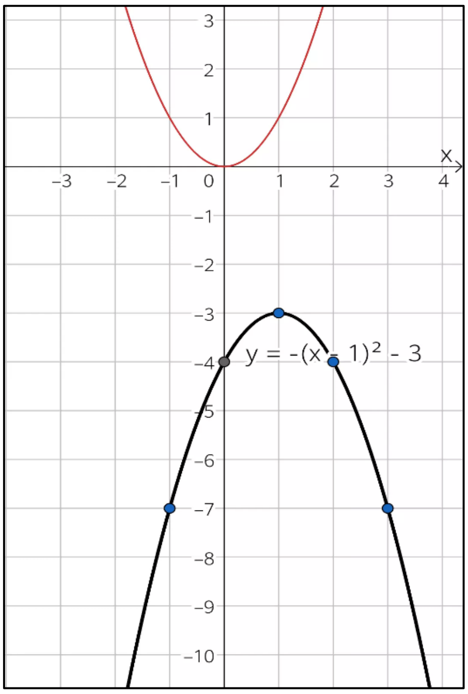
X	-1	0	**1**	2	3
Y	-7	-4	**-3**	-4	-7

$$\boldsymbol{f(x) = -(x-1)^2 - 3}$$
$$f(-1) = -(-1-1)^2 - 3 = -7$$
$$f(0) = -(0-1)^2 - 3 = -4$$
$$f(2) = -(2-1)^2 - 3 = -4$$
$$f(3) = -(3-1)^2 - 3 = -7$$

Analysis of the graph:

The graph of $y = -(x-1)^2 - 3$ moves 1 unit to the right of the parent graph since $h = 1$ and 3 units downward since $k = -3$ that is shown on the graph at the right side. The graph looks ***the same*** *with the parent graph only that it faces downward.*

f. Plot and connect the points from the table to form a parabola.



What's More

Activity 8.2: NOW IT'S YOUR TURN!

Instructions: Graph the following functions below and identify the:

a. axis of symmetry
b. vertex
c. direction of the opening of the parabola
d. domain and range
e. table of values

1. $y = -(x-5)^2 + 3$	2. $y = -2(x+3)^2 - 9$
3. $y = (x+4)^2 - 6$	4. $y = 3(x-4)^2 + 8$

What I Need to Remember

In graphing quadratic functions in standard form $y = a(x-h)^2 + k$, we need to do the following steps:

1. identify the axis of symmetry $x = h$ and vertex $V\ (h, k)$ from the given function in vertex form $y = a(x-h)^2 + k$ where:
   a. h is positive for the expression $(x-h)^2$
   b. h is negative for the expression $(x+h)^2$
   c. sign for k follows the operation attached to it
2. identify also the opening of the parabola such that:
   a. when $a > 0$, the parabola opens upward
   b. when $a < 0$, the parabola opens downward
3. identify the domain and range of the given function;
4. make a table of values by supplying at least 2 points to the left and right of the identified vertex; and lastly;
5. plot the coordinates found at the table of values found in step 4 to graph the function.

Assessment (Post Test)

Instructions: Choose the letter of the correct answer. Write your chosen answer on a separate sheet of paper.

1. What is the vertex of the given quadratic function $y = -2(x-4)^2 - 8$?
   A. $V\ (-4, -8)$ B. $V\ (-4,8)$ C. $V\ (4, -8)$ D. $V\ (4,8)$

2. What is the value of the axis of symmetry in the function $y = \frac{2}{3}(x-8)^2 + 12$?
   A. 8 B. – 8 C. 12 D. -12

3. Which of the following is the range of the function $y = (x-5)^2 + 9$?
   A. $y \leq -5$ B. $y \geq 5$ C. $y \leq -9$ D. $y \geq 9$

4. What is the y intercept of $y = (x+6)^2 - 3$?
   A. (0,21) B. (0,33) C. (0,36) D. (0,42)

5. If the graph of a quadratic function moves ***upward***, what is the possible value of k?
   A. positive B. zero C. negative D. undefined

6. If the graph of a quadratic function is moving to the ***left***, what would be the possible value of h?
   A. positive B. zero C. negative D. undefined

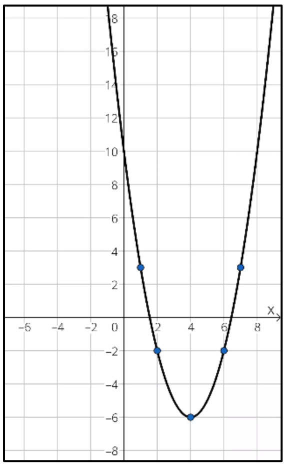
7. If the graph of a quadratic function is ***wider*** than the parent graph $y = ax^2$, what would be the possible value of ***a***?
   A. $a = 1$ B. $a > 1$ C. $0 < a < 1$ D. $a = 0$

8. What is the minimum point of the given function $y = x^2 + 4x - 12$?
   A. $k = -16$ B. $k = -12$ C. $k = 16$ D. $k = 12$

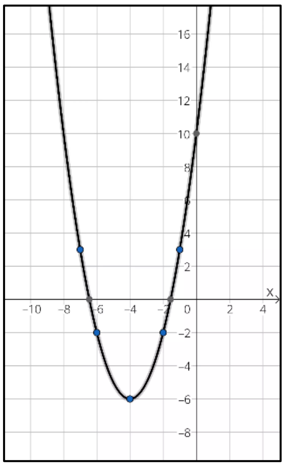
9. What is the maximum point of the given function $y = -2(x+4)^2 - 8$?
   A. $k = -6$ B. $k = -8$ C. $k = 6$ D. $k = 8$

10. What should be the graph of the function $y = (x-4)^2 + 6$?

A



B



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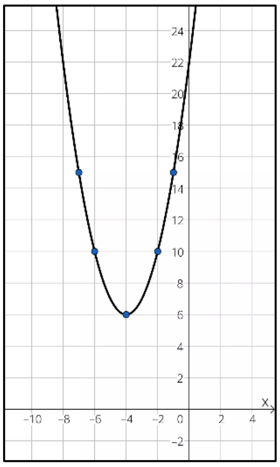

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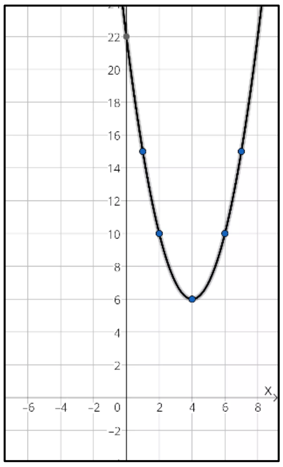


Answer Key

Remember: This portion of the module contains all the answers. Your **HONESTY** is required.

Activity No. 9.1

1. $\boldsymbol{y = x^2 - 6x - 7}$

   $h = 3$ *and* $k = -16$

   $V\ (3, -16)$

   $a = 1$ *so it opens upward*

x	1	2	3	4	5
y	-12	-15	-16	-15	-12

   Domain: real numbers

   Range: $y \geq -16$

   x-intercept/s: $(7,0)(-1,0)$

   y-intercepts: $(0,-7)$

2. $\boldsymbol{y = -2x^2 + 4x + 11}$

   $h = 1$ *and* $k = 13$

   $V\ (1, 13)$

   $a = -2$ *so it opens downward*

x	-1	0	1	2	3
y	5	11	13	11	5

   Domain: real numbers

   Range: $y \leq 13$

   x-intercepts: $(-1.55,0)(3.55,0)$

   y-intercepts: $(0,11)$

Activity No. 9.1

3. $\boldsymbol{y = x^2 - 2}$

   a. $V(0, -2)$

   b. $h = 0$

   c. $a = 1$ *so it opens upward*

x	-2	-1	0	1	2
y	2	-1	-2	-1	2

   Domain: real numbers

   Range: $y \geq -2$

   x-intercept/s: $(\sqrt{2},0)$

   y-intercepts: $(0,-2)$

4. $\boldsymbol{y = (x + 3)^2}$

   a. $V(-3, 0)$

   b. $h = -3$

   c. $a = 1$ *so it opens upward*

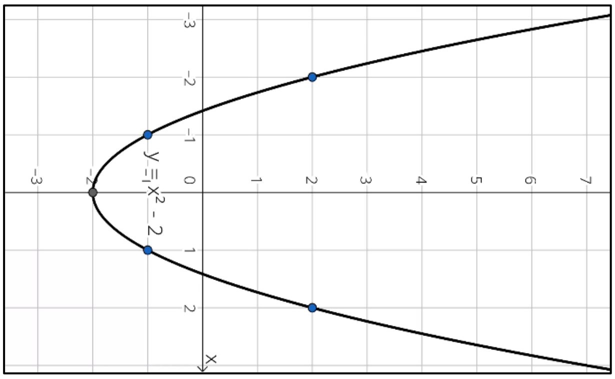
x	-5	-4	-3	-2	-1
y	4	1	0	1	4

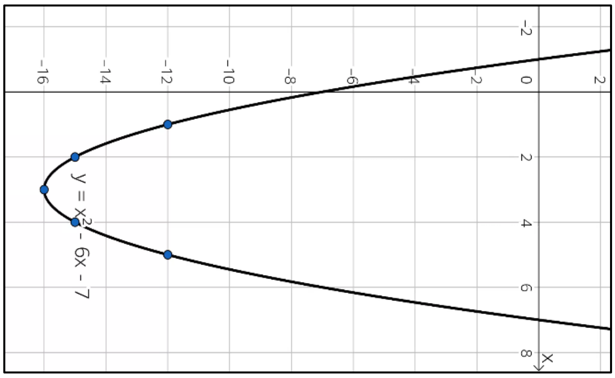
   Domain: real numbers

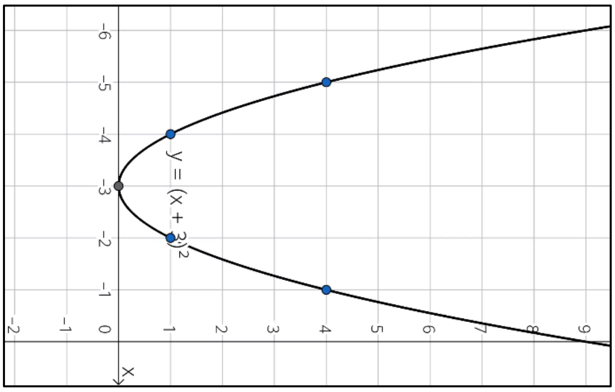
   Range: $y \geq 0$

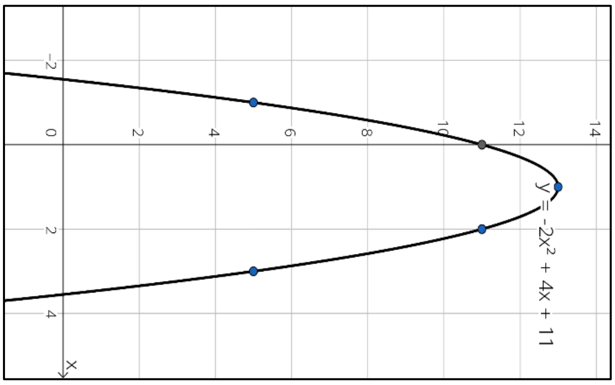
   x-intercept: $(-3,0)$

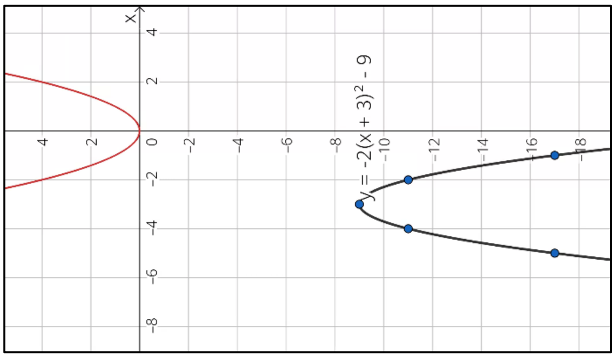
   y-intercepts: $(0,9)$

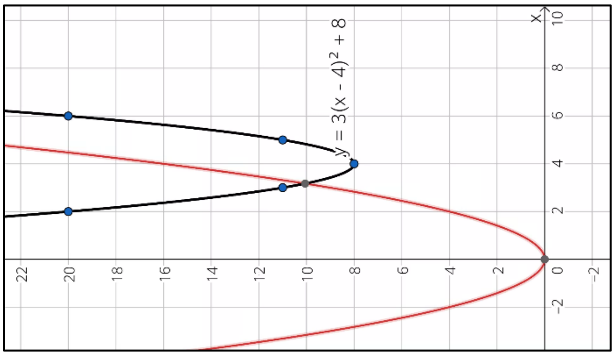


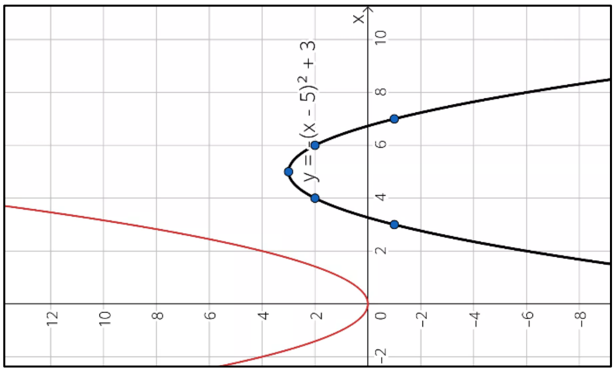


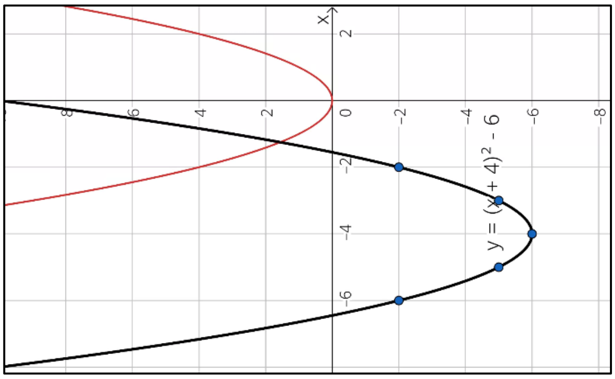












Activity No. 9.2

1. $h = 5$ *axis of symmetry*
   $V\ (5, 3)$
   $a = -1$ *so it opens downward*
   domain: real numbers
   range: $y \leq 3$

x	3	4	5	6	7
y	-1	2	3	2	-1

$f(3) = -(3-5)^2 + 3 = -1$
$f(4) = -(4-5)^2 + 3 = 2$
$f(6) = -(6-5)^2 + 3 = 2$
$f(7) = -(7-5)^2 + 3 = -1$

2. $h = -3$ *axis of symmetry*
   $V\ (-3, -9)$
   $a = -2$ *so it opens downward*
   domain: real numbers
   range: $y \leq -9$

x	-5	-4	-3	-2	-1
y	-17	-11	-9	-11	-17

$f(-5) = -2(-5+3)^2 - 9 = -17$
$f(-4) = -2(-4+3)^2 - 9 = -11$
$f(-2) = -2(-2+3)^2 - 9 = -11$
$f(-1) = -2(-1+3)^2 - 9 = -17$

Activity No. 9.2

3. $h = -4$ *axis of symmetry*
   $V\ (-4, -6)$
   $a = 1$ *so it opens upward*
   domain: real numbers
   range: $y \geq -6$

x	-6	-5	-4	-3	-2
y	-2	-5	-6	-5	-2

$f(-6) = (-6+4)^2 - 6 = -2$
$f(-5) = (-5+4)^2 - 6 = -5$
$f(-3) = (-3+4)^2 - 6 = -5$
$f(-2) = (-2+4)^2 - 6 = -2$

4. $h = 4$ *axis of symmetry*
   $V\ (4, 8)$
   $a = 3$ *so it opens upward*
   domain: real numbers
   range: $y \geq 8$

x	2	3	4	5	6
y	20	11	8	11	20

$f(2) = 3(2-4)^2 + 8 = 20$
$f(3) = 3(3-4)^2 + 8 = 11$
$f(5) = 3(5-4)^2 + 8 = 11$
$f(6) = 3(6-4)^2 + 8 = 20$

References

Text Book

Gueta, Maria Fe Rebecca D. "Quadratic Functions of the Form: $f(x) = a(x-h)^2 + k$", Math Digest: Advanced Algebra. Caneo Enterprises, 7344 A. Bonifacio Ext., San Dionisio, Parañaque City, Philippines. Copyright 2007.

Cliparts All clip arts are taken from cellphone app named Bitmoji

Graphs All graphs are taken from cellphone app named Geo Gebra

Congratulations!

You are now ready for the next module. Always remember the following:

1. Make sure every answer sheet has your
   - *Name*
   - *Grade and Section*
   - *Title of the Activity or Activity No.*
2. Follow the date of submission of answer sheets as agreed with your teacher.
3. Keep the modules with you AND return them at the end of the school year or whenever face-to-face interaction is permitted.