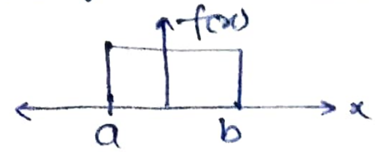
Uniform Distribution (Continuous Random Variable)



$$f(x) = \begin{cases} \frac{1}{b-a} & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases} \Bigg\} \text{ density function}$$

$E(x) = \text{Mean}(\mu) = a+b/2$, Variance $= \sigma^2 = \frac{(b-a)^2}{12}$

and Mean square value (MSV) $E(x^2)$ $= \frac{a^2+b^2+ab}{3}$

$E(x^n) = \int_{-\infty}^{\infty} x^n f(x)dx$, Ex: $E(x^3) = \int_0^1 x^3 (1) dx = \frac{1}{4}$

value of f(x) and range given in question.

$$\boxed{E(x^3) = \int_a^b x^3 \left(\frac{1}{b-a}\right) dx}$$

Exponential Distribution (Continuous Random Variable)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

Mean $= 1/\lambda$

Variance $= 1/\lambda^2$

Since $x \geq 0$ always, so its unilateral.
It has only one parameter as 'λ'.

Gaussian Distribution (Discrete Random Variable)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad ; x \geq 0$$

There are two parameters σ, & μ.

$\mu \rightarrow$ mean
$\sigma \rightarrow$ SD

→ for continuous random variable we use integration and for discrete random variable we use Σ (summation).

Poisson Distribution (Discrete Random Variable)

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} \quad ; x = 0, 1, 2 \ldots$$

λ = mean = $n \cdot p$ where; n = No. of trials, p = prob. success
x = no. of successful trials.

→ Base of x and λ should be same.

If n is large → Poisson Distribution

small → Binomial

→ Mean = Variance = μ, $SD = \sqrt{\mu}$

Mean can be said as expectation, average, weighted average, first moment about origin.

Binomial Distribution: [Discrete Random Variable]

$$P(x) = {}^nC_x \, p^x \, q^{n-x}$$

n → no. of trials, p = probability of success,
q = prob. of failure, x = no. of favourable events.

Mean (μ) = np, Variance (σ^2) = npq

* Mode = 3median − 2mean

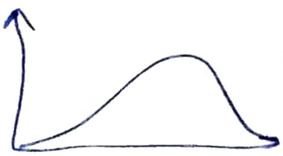
* ① for +ve skewed

Mode ≤ Median ≤ Mean

② Symmetric

Mode = Median = Mean

③ (−ve) skewed



Mean ≤ Median ≤ mode

* Coefficient of variation $= \dfrac{\sigma}{\mu}$

for continuous variable its probability density function given by,

① $f(x) \geq 0$ ② $\int_{-\infty}^{\infty} f(x)\,dx = 1$ ③ $\int_a^b f(x)\,dx$

for discrete random variable it is probability mass function.

1. $p(x) = P[X=x]$ 2. $p(x) \geq 0$ 3. $\Sigma p(x) = 1$

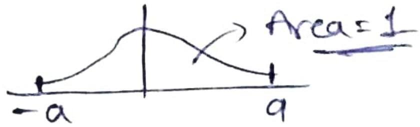
→ (Change of origin): Any constant added or subtracted than the SD remains unchanged but mean gets changed.

→ With change of scale all the 3 SD, mean and variance get changed.

→ $$I = \frac{1}{\sqrt{2\pi\sigma^2}} \int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1$$

for contineous random variable $\mu = E(X)$ then,
Standard Deviation = $E(X^2) \pm [E(X)]^2$

→ Zscore = $\frac{\text{Normalized} - \text{Mean}}{\text{SD}} = \frac{x-\mu}{\sigma}$

→ Normal distribution Curve 

→ Collectively exclusive $P(A \cup B) = P(S) = 1$

→ for standard Normal distribution $\mu = 0, \sigma^2 = 1$.

→ Second Moment $= \lambda^2 + \lambda$
first moment = mean = λ
→ Poison Distribution.

Integration and Differentiation formulaes

$f'(x^n) = n x^{n-1}$

$f'(x \pm y) = \frac{d}{dx}(x) + \frac{d}{dx}(y)$

$f'(uv) = u f'(v) + v f'(u)$

$f'(u/v) = \frac{v f'(u) - u f'(v)}{v^2}$

$\frac{d}{dx} e^x = e^x$

$f'(a^x) = a^x \log a$

$f'(\log x) = 1/x \times (\log_e e)^{-1}$

$f'(\log_a x) = \frac{1}{x} \log_a e$

$f'(\sin x) = \cos x$, $f'(\cos x) = -\sin x$

$f'(\tan x) = \sec^2 x$, $f'(\cot x) = -\operatorname{cosec}^2 x$

$f'(\sec x) = \sec x \tan x$

$f'(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

$f'(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$, $f'(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$

$f'(\tan^{-1} x) = \frac{1}{1+x^2}$, $f'(\cot^{-1} x) = \frac{-1}{1+x^2}$

$f'(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$, $f'(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$

$I(x^n) = x^{n+1}/n+1 + c$

$I(\cos x) = \sin x + c$

$I(\sin x) = -\cos x + c$

$I(\sec^2 x) = \tan x + c$

$I(\operatorname{cosec}^2 x) = -\cot x + c$

$I(\sec x \tan x) = + \sec x$

$I\left(\frac{1}{\sqrt{1-x^2}}\right) = \sin^{-1} x$

$I(e^x) = e^x + c$, $I(1/x) = \log x$

$I(a^x) = \frac{a^x}{\log a} + c$

$I(\tan x) = \log|\sec x| + c$

$I(\cot x) = \log|\sin x| + c$

$I(\sec x) = \log|\sec x + \tan x| + c$

$I(\operatorname{cosec} x) = \log|\operatorname{cosec} x - \cot x| + c$

$I\left(\frac{1}{x^2-a^2}\right) = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

$I\left(\frac{1}{a^2-x^2}\right) = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

$I\left(\frac{1}{x^2+a^2}\right) = \frac{1}{a} \tan^{-1}\frac{x}{a} + c$

$I\left(\frac{1}{\sqrt{x^2-a^2}}\right) = \log|x + \sqrt{x^2-a^2}| + c$

$I\left(\frac{1}{\sqrt{a^2-x^2}}\right) = \sin^{-1}\frac{x}{a} + c$

$I\left(\frac{1}{\sqrt{x^2+a^2}}\right) = \log|x + \sqrt{x^2+a^2}| + c$

Limits formulas

$\lim_{x \to 0} \sin x = 0$

$\lim_{x \to 0} \cos x = 1$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$ [Max Value]

$\lim_{x \to 0} \frac{x}{\sin x} = 1$

$\lim_{x \to 0} \frac{\tan x}{x} = 1$

$\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

$\lim_{x \to 0} e^x = 1$

$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$

$\lim_{x \to 0} (1+x)^{1/x} = e$

$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$

If

$\lim_{x \to \infty} f(x) = \infty$

then

$\lim_{x \to \infty} \frac{1}{f(x)} = 0$

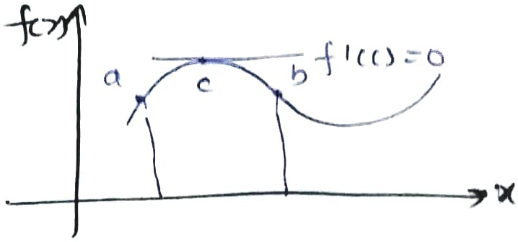
$m_1 m_2 = -1$

Mean Value theorem

Roles Theorem : Let $f(x)$ is defined is $[a,b]$ such that it satifies 3 conditions.

i) $f(x)$ is countinuous in $[a,b]$ [No holes]
ii) ——— differentiable in (a,b) [No sharp point]
iii) $f(a) = f(b)$.

then, $\exists$ atleast one point $c \in (a,b)$ such that $f'(c) = 0$

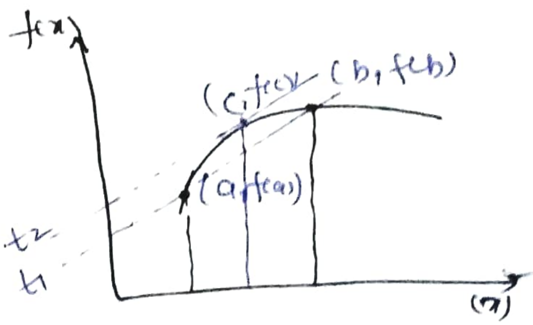


Legranges mean value : $f(x)$ is defined in $[a,b]$ such that it satisfies 2 conditions.

i) $f(x)$ is continuous in $[a,b]$
ii) $f(x)$ is differentiable in (a,b) then

there exist atleast one point $c \in (a,b)$ such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$



slope of $t_1 = \dfrac{f(b) - f(a)}{b - a}$

If t_1 and t_2 are parallel

then $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Cauchy's Mean Value Theorem :

Let $f(x)$ and $g(x)$ be defined in $[a,b]$ such that

i) $f(x)$ and $g(x)$ are continuous in $[a,b]$
ii) $f(x)$ and $g(x)$ are differentiable in (a,b)
iii) $g'(x) \neq 0 \ \forall \ x \in (a,b)$ then $\exists$ atleast one point $c \in (a,b)$, such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$g''(c) = \frac{g(b)-g(a)}{b-a}$$

Q The mean value 'c' of $f(x) = e^x[\sin x - \cos x]$ in the interval $[\frac{\pi}{4}, \frac{5\pi}{4}]$ is.

Since, $f(\frac{\pi}{4}) = f(\frac{5\pi}{4}) = 0$ so we can use rollers theorem,

$$f'(x) = 2e^x \sin x$$

So, for $c \in (\frac{\pi}{4}, \frac{5\pi}{4})$ so that $f'(c) = 0$

$$\Rightarrow 2e^c \sin c = 0$$

$$\Rightarrow \sin c = 0 \quad \text{So, } c = 0, \pm\pi, \pm 2\pi, \ldots$$

So, $c \in (\frac{\pi}{4}, \frac{5\pi}{4})$ and also c

so $\boxed{c = \pi}$ only

(2) $f(x) = x(x+3)e^{-x/2}$. find mean value 'c' in interval $[-3, 0]$

$f(-3) = f(0) = 0$ So we apply rolls theorem

After procedure,

→ Differentiable function is always continous.

$c = 3, -2$ but only $-2 \in [-3, 0]$

So, $c = -2$

(3) The mean value 'c' for $f(x) = \sqrt{x^2-4}$ in $[2, 4]$

$f(a) \neq f(b)$

So we use mean value theorem

$$f'(c) = \frac{c}{\sqrt{c^2-4}} = \frac{f(b)-f(a)}{b-a} \Rightarrow \frac{c}{\sqrt{c^2-4}} = \frac{\sqrt{12}}{2} = \sqrt{3}$$

$\Rightarrow 2c^2 = 12 \Rightarrow c = \pm\sqrt{6}$ but only $+\sqrt{6} \in [2, 4]$.

Eigen Vectors : find eigen values first then, using $|A-\lambda I|=0$

⇒ Sum of eigen values = trace

⇒ determinant of matrix from characteristic eqn.

If eqn is $a\lambda^3 + b\lambda^2 + c\lambda + d = 0$ then

$|A| = d$ (constant term)

Ex: $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ $\quad [A-\lambda I][X] = 0$

Soln: Eigen values are $\lambda_1 = 0, \lambda_2 = 3, \lambda_3 = 15$ then eigen vectors are:-

$[A-\lambda I]$ at $\lambda = 0$ is $\begin{bmatrix} 8 & -6 & 2 \\ 6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{x_1}{\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix}} = \frac{(-x_2)}{\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -6, & 7 \\ 2 & -4 \end{vmatrix}}$$ → Determinant

$$\Rightarrow \frac{x_1}{5} = \frac{-x_2}{-10} = \frac{x_3}{10} = \frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2} \Rightarrow X\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

→ Determinant of 4x4 Matrix :

$$\begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$

$$-1\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 5$$] Alternate signs (+), (-).

2nd Approach: Trick

↳ No trick

Day-4

Calculus.

→ Read all the differentiation and Integration formula from the main notes.

① $\frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{x^2}{2}} = \frac{1}{2}$

→ In $\sin(x)$, put the value in radian or angle accordingly.

→ If there exist a 'y' in interval (a,b) then according immediate value theorem verify the options.

→ Various limits forms:-

① $\frac{0}{0}$ or $\frac{\infty}{\infty}$: Apply L-Hospital Rule.

② $\lim_{x \to 0} \frac{(1-\cos ax)}{x^2} = \frac{a^2}{2}$

③ for $\frac{\infty}{\infty}$ form take x as common like $\overline{x(1+5/x)}$.

④ 0^0 form take log on both sides.

⑤ 1^∞ : $\lim_{x \to 0} f(x)^{g(x)} = e^{\lim_{x \to 0} g(x)[f(x)-1]}$

⑥ $\lim_{x \to 0} \left(\frac{\sin x}{x}\right) / \left(\frac{\tan x}{x}\right) / \left(\frac{x}{\sin x}\right) = 1$

→ In Rolles theorem $f(a) = f(b)$

→ In Lagranges theorem $f(a) \neq f(b)$.

→ In Cauchy mean value theorem $\frac{f'(x)}{g'(x)} = \frac{f(b)-f(a)}{g(b)-g(a)}$.

→ Theorem: The max. value of (xAx^T) where the max. is taken over all 'x' that are eigen vector of A is the max. eigen value of A.

→ Eigen values of Upper triangular is diagonal elements.

→ If eigen value of A is $x, y, \ldots$ then for A^n it becomes $x^n, y^n, \ldots$.

The value of dot product of ~~any~~ the eigen vectors corresponding to any pair of different eigen values of any symmetric positive definite matrix is $\underline{0}$.

→ Idempotent : $A^2 = A$

→ Involuntary : $A^2 = I$

→ $(AB)^T = B^T A^T$

→ $(AB)^\theta = B^\theta A^\theta$

→ Symmetric : $A = A^T$, $A + A^T/2$

→ Skew Symm : $A^T = -A$, $A - A^T/2$

: Orthogonal → $AA^T =$

→ $A^\theta = A^{-1}$: Unitary

→ $A^{-1} = \frac{1}{|A|}(\text{Adj } A)$, $|AB| = |A||B|$, $|A^n| = |A|^n$

$|\text{adj}(A)| = |A|^{n-1}$, $|\text{adj}(\text{adj}(A))| = |A|^{(n-1)^2}$, $|A^T| = |A|$

→ $(AB)^{-1} = B^{-1}A^{-1}$; $(A^T)^{-1} = (A^{-1})^T$, $AA^T = I$

→ rank $(AB) \leq r(A)$ or $r(AB) \leq r(B)$

$r(A+B) \leq r(A) + r(B)$

If $r(A) =$ no. of vectors : linearly independent

$r(A) <$ no of vectors : ——— dependent.

→ To find eigen vector use $(A - \lambda I)X = 0$ and to verify use $AX = \lambda X$.

→ LU decomposition is important.

→ Complex eigen values occurs in pairs.

→ Determinant = product of eigen values.

→ If $|A| = 0$ it means rank$(A) < n$ (order)

→ A matrix can be diagonalized iff it has all the different eigen values or all the rows are independent, rank $= n$.

→ Remember cayley hamilton theorem.

Probability

→ Conditional Probability : $\boxed{P(F/E) = \frac{P(F \cap E)}{P(E)}}$

It means probability of (F) given that (E) have been already occured.

→ Multiplication Theorem :

$P(A \cap B) = \underline{P(B)\,P(A/B)} = \underline{P(A)\,P(B/A)}$

or $P(E_1 \cap E_2 \cap E_3) = P(E_1) * P(E_2/E_1) * P(E_3/E_1 \cap E_2)$

→ Independent Events : $P(A/B) = P(A)$, $P(B/A) = P(B)$

or $\underline{P(A \cap B) = P(A) * P(B)}$

→ Bayes's Theorem : $P(A/R) = \frac{P(A)\;P(R/A)}{P(A)\,P(R/A) + P(B)\,P(R/B) + P(C)\,P(R/C)}$

→ Bernaulli Distribution

$P(X=1) = p$ denotes success, $P(X=0) = (1-p)$ denote failure

$\boxed{{}^nC_r\,(p)^r\,(1-p)^{n-r}}$

→ Random Variable : $S = \{HH, TH, H, T\}$

↳ $X = \{2, 1, 0\}$ → Chance of getting head

$P(X=2) : 1/4$; $P(X=1) = 1/2$; $P(X=0) : 1/4$

→ Uniform distribution : (Continuous Random Variable)

$f(x) = \begin{cases} \frac{1}{b-a} ; & a<x<b \\ 0 ; & \text{otherwise} \end{cases}$ } density f^n

$E(x) = \text{Mean}(\mu) = \frac{a+b}{2}$, Variance $(\sigma^2) = \frac{(b-a)^2}{12}$

$E(x^2)$: mean square value $= \frac{a^2+b^2+ab}{3}$

$E(x^n) = \int_{-\infty}^{\infty} x^n f(x)\,dx$ [Range of Integration $a \to b$]

Exponential Distribution : (Continuous Random Variable

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

Mean $= 1/\lambda$

Variance $= 1/\lambda^2$

Since $x \geq 0$ (always). So its unilateral with one parameter λ.

Discrete Random Variable

① Gaussian Distribution : $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$; $x \geq$

$\mu \rightarrow$ mean , $\sigma \rightarrow$ standard deviation

→ for continous random variable we use integration and for discrete we use Σ (summation).

② Poison Distribution : $\boxed{f(x) = \frac{e^{-\lambda}\lambda^x}{x!}}$; $x = 0, 1, 2, \ldots$

λ = mean = $n \cdot p$

Base of λ and x should be same.

If $n \rightarrow$ lage : poison distribution

If $n \rightarrow$ small : binomial distribution

$x \rightarrow$ sample random variable

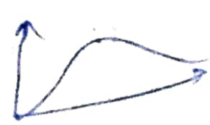
Mean = Variance = μ , SD = $\sqrt{\mu}$

③ Binomial Distribution : $P(x) = {}^nC_x \, p^x q^{n-x}$

Mean (μ) = np ; Variance (σ^2) = npq

: Mode = 3 Median − 2 Mean

: +ve skewed ↳ Mode ≤ Median ≤ Mean



: −ve skewed ↳ Mean ≤ Median ≤ Mode

- ~~Symmetric~~ → Mode = Mean = Median
- → ~~Coefficient of Variation~~ $= \sigma/\mu$

for continuous variable its probability density fn is given by:

(1) $f(x) \geq 0$ (2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ (3) $\int_{a}^{b} f(x)\,dx$

for discrete random variable it is probability mass fn: (1) $p(x) = P[X=x]$ (2) $p(x) \geq 0$ (3) $\Sigma p(x) = 1$

→ With change of origin [add or subtract constant], SD remains unchanged but mean gets changed.

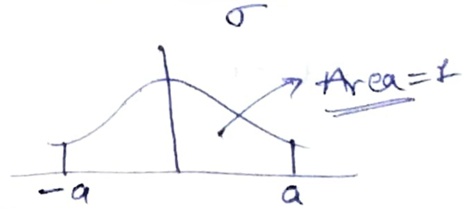
→ With change of scale [SD, mean, variance] all gets changed

→ $I = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{0}^{\infty} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\,dx = 1$

→ Imp for continuous random variable if $\mu = E(x)$ then, Standard Deviation $= E(x^2) - [E(x)]^2$

→ Z score = $\frac{\text{Normalized} - \text{Mean}}{SD} = \frac{x-\mu}{\sigma}$

→ Normal Distribution Curve



→ for Standard Normal Distribution: $\mu = 0$, $\sigma^2 = 1$

→ In Poison Distribution: first Moment = mean = λ

Second Moment = $\lambda^2 + \lambda$

→ If two fns are executing sequentially then total time taken = $\int_0^t f(x) f(t-x)\,dx$ for probability density fn.

→ $P(A \cup B) \leq P(A) + P(B)$

→ If P & Q are mutually exc. then $P(P \cap Q) = \phi$

→ $P(P \cap Q) \leq P(P)$ or $P(Q)$

→ There is no relation btw mutually exclusive & Independent events

→ Cummulative Distributed function is value upto 'x'.

→ for normal distribution or gaussian random variable Mean = Median = Mode = 0

→ Soln of Homogeneous System of Linea Equation.

: Inconsistent : (Not possible bcoz each matrix must have soln as 0)

Consistent → Unique or trivial : $|A| \neq 0$, rank = order

→ Non-trivial : Infinite : $|A| = 0$, rank < order

Soln for Non-Homogeneous :

$AX = B$ → Inconsistent ; $r[A:B] \neq r[A]$

→ Consistent : → $r[A:B] = r[A] = n$ (Unique)

→ $r[A:B] = r[A] < n$ (Infinite)

→ Standard deviation $(s) = \sqrt{\dfrac{n\sum x_i^2 - (\sum(x_i))^2}{n^2}}$

→ $E(kx) = k \cdot E(x)$, $Var[kx] = k^2 Var[x]$

$E(k-x) = k - E(x)$, $Var(k-x) = (-k)^2 Var[x]$

→ No. of distint eigen values (max) = size of matrix

→ ~~No. of~~ Sum of eigen value = trace of matrix.

→ Eigen values of Hermitian matrix are real.

→ $(x-\alpha)(x-\beta)>0$ then, $x \in (-\infty,\alpha] \cup [\beta,\infty)$

→ $(x-\alpha)(x-\beta)<0$ then, $x \in (\alpha,\beta)$ → open bracket

→ $(x-\alpha)(x-\beta)=0$ then, $\underline{x=\alpha,\beta}$.

→ If not continuous then it will not be differentiable but converse is not true.

In case of partial differential eqⁿ:

$a = \frac{\delta f}{\delta x} = 0$ to find x

$b = \frac{\delta f}{\delta y} = 0$ to find y

then,

$r = \frac{\delta^2 f}{\delta x^2}$, $t = \frac{\delta^2 f}{\delta y^2}$, $s = \frac{\delta^2 t}{\delta x \, \delta y}$

then, find $rt - s^2 = k$.

① If $k>0$, and $r<0$ then, (a,b) are points of maxima.

② If $k>0$ and $r>0$ then point of minima.

→ According to Baye's Theorem.

$$P(E_i/A) = \frac{P(E_i)\,P(A/E_i)}{\sum_{k=1} P(E_k)\,P(A/E_k)}$$

Basic Differentiation and Integration : Revise from main Notes

→ Also revise all the trigonometric Identities.

→ $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

→ If eigen values of matrix 'P' are $[a_1, a_2, \ldots]$ then if $f(P) = P^3 + P^2 + 1$ or any function given then eigen values of $f(P)$ is $[f(a_1), f(a_2), \ldots f(a_n)]$

→ The measure of skewness is dependent upon amount of dispersion.

→ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

→ Area of $\Delta ABC = \frac{1}{2}$ (product of sides) × (angle btw tha

→ for a given matrix A if eigen vector given is $\vec{V}$ and eigen value is λ then,

$$\boxed{A \cdot \vec{V} = \lambda \vec{V}}$$

→ Determinant of matrix = Product of eigen values.

→ $(A+B)^T = A^T + B^T$, $(\lambda A)^T = \lambda (A^T)$.

→ $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

→ $(A - \lambda I)X = 0$ for getting eigen vector from eigen values

$P(X \leq -1)$ and given that $\mu_x = -1$, σ_x^2 (variance) = 4

then, $z_x = \dfrac{-1-1}{\sigma_x} = -\dfrac{2}{2} = -1$ $\left(\dfrac{-1 - \mu_x}{\sigma_x}\right)$

$P(z_x \leq -1)$