ISYE 6669 – HOMEWORK 2

ANSWER 1 –

(a)

$x_1 + x_2 + x_3$

(b)

$x^2 + (x^4/2) + (x^3/6)$

(c)

$x^2 + x^3 + 2x^4 + x^5 + x^6$

(d)

$x_{12} + x_{13} + x_{14} + x_{22} + x_{23} + x_{24} – 3y_1 – 3y_2$

(e)

$- (x_{-2})^2 + (x_{-1})^2 + 3(x_0)^2$

(f)

$x_2y_2 + x_2y_3 + x_2y_4 + x_3y_3 + x_3y_4 + x_3y_5 + x_4y_4 + x_4y_5 + x_4y_6$

ANSWER 2 –

(a) Dimension n = 3

(b) $2x – y = (-1\ 2\ 5)^T$

(c) $x^Ty = (1x3) + (2x2) + (3x1) = 10$

(d) l2 norm = $2\sqrt{3}$

(e) l1 norm = 4

(f) l_{inf} norm = 2

(g) $x^TAy = (6\ 1\ 7) \times (3\ 2\ 1)^T = 27$

ANSWER 3 –

(a) The function is convex since the hessian –

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

- is not positive definite. Hence, the epigraph, or the set is also **NOT convex**. To picture it clearer, take the points (x1=-3, x2=-3) and (x1=3, x2=3), and upon drawing a line connecting the two, the line will cut through an area not covered by the set.

(b) The second order differential of x^2 is 2, which is positive under all cases, hence the function is **convex**.

(c) Both functions are in fact linear functions at alpha-level, which make them both concave and convex. The set is an intersection of the two sets generated by these functions, which preserves convexity. Hence, the set is **convex**.

ANSWER 4 –

(a) The function is convex (with a positive definite hessian matrix) and the set is defined by alpha-level linear functions which makes the set convex. Hence, the program is **convex**.

(b) Take the points (x1=1, x2=0) and (x1=0, x2=1). And upon drawing a line between the two, the line crosses over area not included by the set. Since the set isn't convex, the program is **NOT convex**.

(c) The variables aren't continuous here, hence the program is **NOT convex**.

ANSWER 5 –

(a)

Before we formulate it, we need to derive the absolute deviations as –

$| y_i – (a + bx_i) |$

Therefore, the program would be –

$\text{Min}\{ \Sigma \mid y_i – (a + bx_i) \mid : \{yi,xi\}^n_{i=1}\}$

It is a non-linear optimization model owing to the absolute value function. And it is also a convex function. The sum of convex functions is a convex function, hence this program is a **convex non-linear** program.

(b)

For maximum absolute deviation, the model would be –

Min{ Max(| y_i – (a + bx_i) |) : $\{yi,xi\}^n_{i=1}$}

Since the function within the max() is convex, the maximum of that function is also a convex function. Hence, this is a **convex non-linear** model.

(c)

The optimization model for the new curve to be fit would be –

Min{ Max(| y_i – (a + bx_i + cx_i^2) |) : $\{yi,xi\}^n_{i=1}$}

In this case, the function within the modulus isn’t linear and hence we cannot determine whether the function is convex or not. However, we can classify this as a **quadratic** program since the objective function has terms of power equal to two at the most.