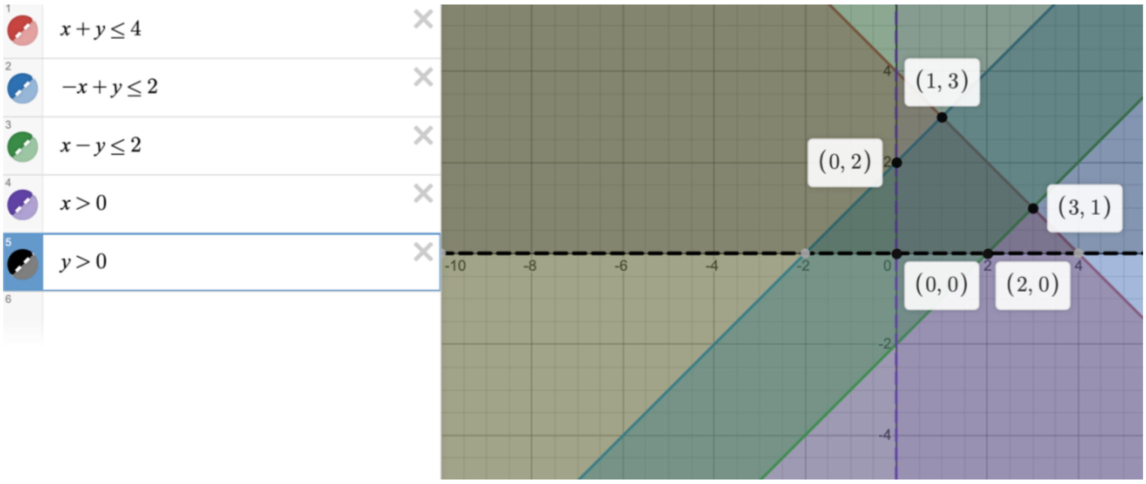
ISYE 6669 – HOMEWORK 8

ANSWER 1 –

The feasible region is enclosed by the polygon, whose vertices are marked on the graph.



ANSWER 2 –

The standard form of this linear program would be –

Min $\{-2x_1 + 4x_2\}$

Subject To –

$x_1 + x_2 + x_3 = 4$
$x_2 - x_1 + x_4 = 2$
$x_1 - x_2 + x_5 = 2$
$x_1, x_2, x_3, x_4, x_5 >= 0$

In standard vector format –

c = [-2 4 0 0 0]

A =
[
[1 1 0 0 0]
[-1 1 0 1 0]
[1 -1 0 0 1]
]

b = [4 2 2]

ANSWER 3 –

The simplex iterations over the LP –

Iteration 1 –

B: [A_1 A_2 A_5] =

```
[
[ 1  1  0]
[-1  1  0]
[ 1 -1  1]
]
```

B^{-1}:

```
[
[ 0.5 -0.5  0. ]
[ 0.5  0.5  0. ]
[ 0.   1.   1. ]
]
```

xb:
[x_1 x_2 x_5] = [1. 3. 4.] with no negative values, hence feasible.

xn:
[x_3 x_4] = [0 0]

Reduced cost c3 with A_3:
-1.0

Reduced cost c4 with A_4:
-3.0

Taking x3 as part of basis...

db:
[-0.5 -0.5 -0.] and it has negative values, hence bounded.

dn:
[1 0]

theta:
[2.00000000000000, 6.00000000000000]

min-ratio:
2.00000000000000

Hence, x1 becomes 0 and EXITS and x3 ENTERS

new xb:
[2 2 4]

xn:
[0 0]

Iteration 2 –

```
B: [A3 A2 A5] =
[
[ 1  1  0]
[ 0  1  0]
[ 0 -1  1]
]

B-1:
[
[ 1. -1.  0.]
[ 0.  1.  0.]
[ 0.  1.  1.]
]

xb:
 [x3 x2 x5] = [2 2 4] with no negative values, hence feasible.

xn:
 [x1 x4] = [0 0]

Reduced cost c1 with A1:
 4.0

Reduced cost c4 with A4:
 -8.0

Taking x4 as part of basis according to Bland's rule...

db:
 [ 1. -1. -1.] and it has negative values, hence bounded.

dn:
 [0 1]

theta:
 [2.00000000000000, 2.00000000000000, 4.00000000000000]

min-ratio:
 2.00000000000000

Hence, x3 becomes 0 and EXITS and x4 ENTERS

new xb:
 [2 4 6]

xn:
 [0 0]
```

When the reduced cost is calculated on the next round, both non-basic values are positive. Hence this is the optimal solution, and the optimal point is –

(2, 0) is the minima, with an objective value of -4.

And plotting the BFS' on the graph, we get –