Kalpana Bahiti
310616104046
CSE-III-'A'

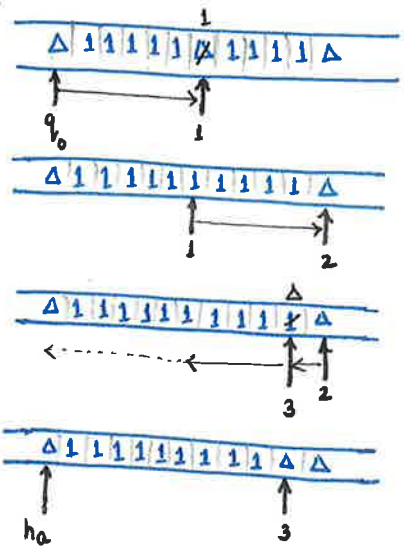
THEORY OF COMPUTATION
ASSIGNMENT-3

1) ADDITION

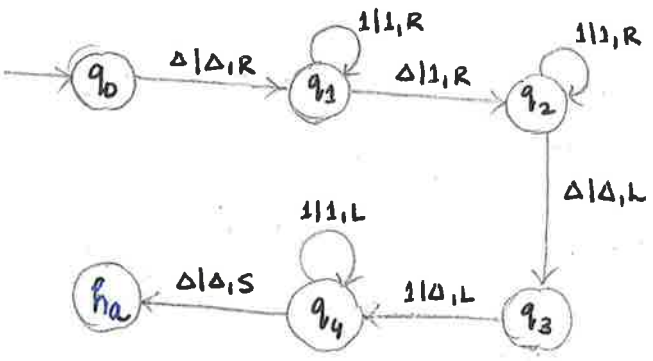
TAPE DIAGRAM :

Input : $\Delta 1^x \Delta 1^y \Delta$

Output : $\Delta 1^{x+y} \Delta$



TRANSITION DIAGRAM δ :



and hence :

$T = (\{q_0, \ldots, q_4, h_a\}, \{1\}, \{\Delta, 1\}, \delta, q_0)$

NOTE :

1) ALL NUMBERS CONSIDERED FOR THE FOLLOWING ARITHMATIC OPERATIONS ARE POSITIVE NUMBERS.
2) FOR EVERY TURING MACHINE ILLUSTRATED, A SAMPLE STRING IS RUN ON THE RIGHT HAND COLUMN HERE.

ADDITION ~

RUN FOR

w = 'Δ111Δ1111Δ'

$(q_0, \underline{\Delta}111\Delta1111\Delta) \vdash_T$

$(q_1, \Delta\underline{1}11\Delta1111\Delta) \vdash_T^*$

$(q_1, \Delta111\underline{\Delta}1111\Delta) \vdash_T$

$(q_2, \Delta1111\underline{1}111\Delta) \vdash_T^*$

$(q_2, \Delta11111111\underline{\Delta}) \vdash_T$

$(q_3, \Delta1111111\underline{1}\Delta) \vdash_T$

$(q_4, \Delta111111\underline{1}\Delta\Delta) \vdash_T^*$

$(q_4, \underline{\Delta}11111111\Delta\Delta) \vdash_T$

$(h_a, \underline{\Delta}11111111\Delta)$

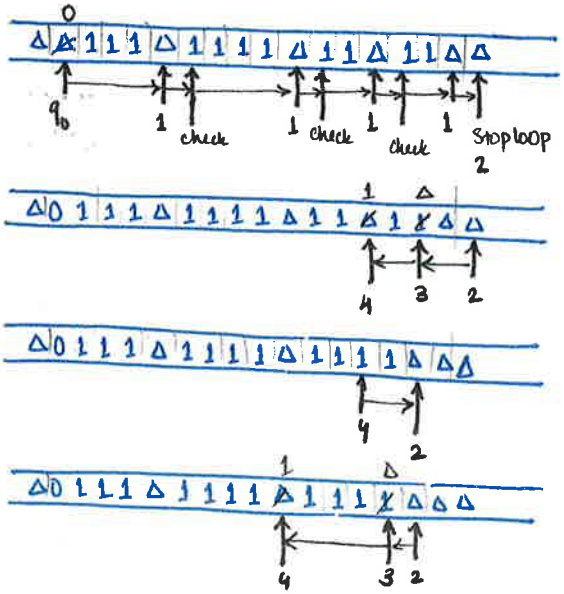
Extension (Not Compulsory):

ADDITION OF MULTIPLE NUMBERS:

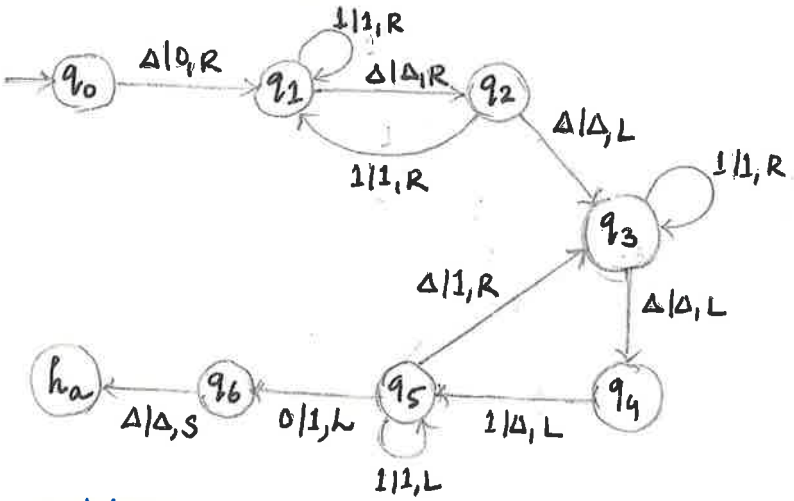
TAPE DIAGRAM:

Input: $\Delta 1^a \Delta 1^b \Delta 1^c \Delta \dots \Delta 1^n \Delta$

Output: $\Delta 1^{a+b+c+\cdots+n} \Delta$



TRANSITION DIAGRAM δ:



and hence:

$T = (\{q_0, \dots, q_6\}, \{1\}, \{0, 1\}, \delta, q_0)$

ADDITION 2 ~

RUN FOR

W = 'Δ111Δ11Δ11Δ'

$(q_0, \Delta 111 \Delta 11 \Delta 11 \Delta) \vdash_T$

$(q_1, \Delta 0 111 \Delta 11 \Delta 11 \Delta) \vdash_T^*$

$(q_1, 0111 \Delta 11 \Delta 11 \Delta) \vdash_T$

$(q_2, 0111 \Delta 11 \Delta 11 \Delta) \vdash_T$

$(q_1, 0111 \Delta 111 \Delta 11 \Delta) \vdash_T^*$

$(q_3, 0111 \Delta 111 \Delta 11 \Delta) \vdash_T$

$(q_2, 0111 \Delta 1111 \Delta 11 \Delta) \vdash_T$

$(q_1, 0111 \Delta 1111 \Delta 1 \Delta) \vdash_T^*$

$(q_2, 0111 \Delta 1111 \Delta 1 \Delta) \vdash_T$

$(q_3, 0111 \Delta 1111 \Delta \Delta) \vdash_T$

$(q_4, 0111 \Delta 1111 \Delta \Delta) \vdash_T$

$(q_5, 0111 \Delta 1111 \Delta \Delta) \vdash_T^*$

$(q_5, 0111 \Delta 1111 \Delta \Delta) \vdash_T$

$(q_3, 0111 1 1111 \Delta \Delta) \vdash_T^*$

$(q_3, 0111 1 1111 \Delta \Delta) \vdash_T$

$(q_4, 0111 1 1111 \Delta \Delta) \vdash_T$

$(q_5, 0111 1111 \Delta \Delta \Delta) \vdash_T^*$

$(q_5, 0111 1111 \Delta \Delta \Delta) \vdash_T$

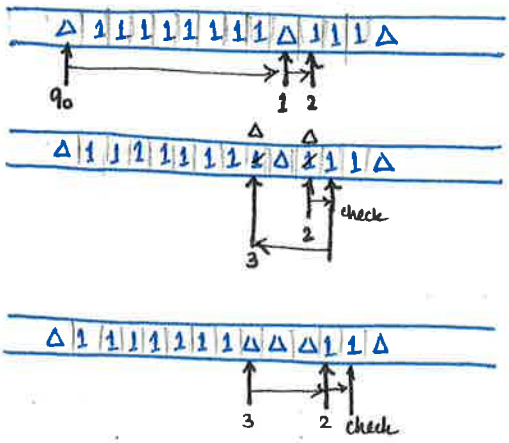
$(q_6, \Delta 111 1111 \Delta \Delta \Delta) \vdash_T$

2) SUBTRACTION

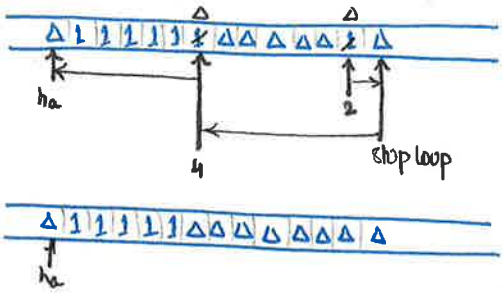
TAPE DIAGRAM:

Input: $\Delta 1^x \Delta 1^y \Delta$ where $x \geq y$

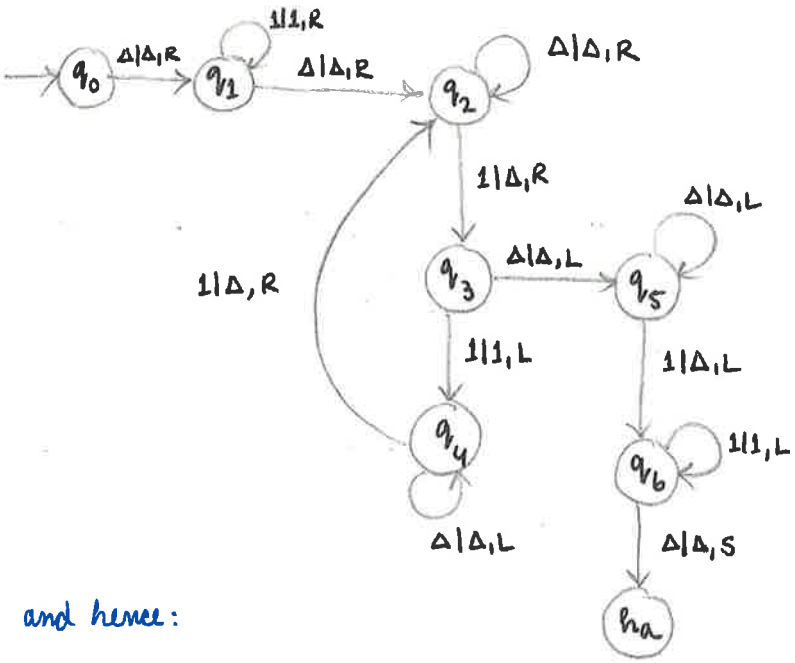
Output: $\Delta 1^{x-y} \Delta$



After few such rounds:



TRANSITION DIAGRAM δ:



and hence:

$T = (\{q_0, \ldots, q_6\}, \{1\}, \{1\}, \delta, q_0)$

SUBTRACTION ~

RUN FOR

w = 'Δ1111Δ11Δ'

$(q_0, \underline{\Delta}1111\Delta11\Delta) \vdash_T$

$(q_1, \Delta\underline{1}111\Delta11\Delta) \vdash_T^*$

$(q_1, \Delta1111\underline{\Delta}11\Delta) \vdash_T$

$(q_2, \Delta1111\Delta\underline{1}1\Delta) \vdash_T$

$(q_3, \Delta1111\Delta\Delta\underline{1}\Delta) \vdash_T$

$(q_4, \Delta1111\Delta\underline{\Delta}1\Delta) \vdash_T^*$

$(q_4, \Delta1111\underline{1}\Delta\Delta1\Delta) \vdash_T$

$(q_2, \Delta1111\Delta\underline{\Delta}\Delta1\Delta) \vdash_T^*$

$(q_2, \Delta1111\Delta\Delta\Delta\underline{1}\Delta) \vdash_T$

$(q_3, \Delta1111\Delta\Delta\Delta\Delta\underline{\Delta}) \vdash_T$

$(q_5, \Delta1111\Delta\Delta\Delta\underline{\Delta}\Delta) \vdash_T^*$

$(q_5, \Delta111\underline{1}\Delta\Delta\Delta\Delta\Delta) \vdash_T$

$(q_6, \Delta11\underline{1}\Delta\Delta\Delta\Delta\Delta) \vdash_T^*$

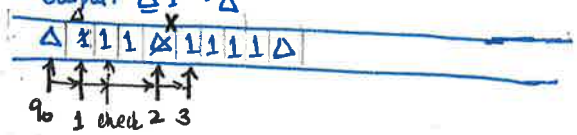
$(q_6, \underline{\Delta}11\Delta\Delta\Delta\Delta\Delta) \vdash_T$

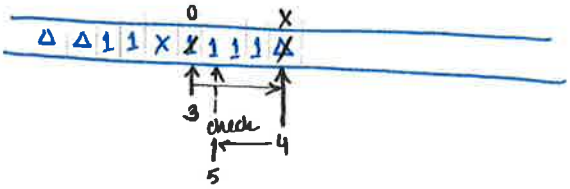
$(h_a, \underline{\Delta}11\Delta\Delta\Delta\Delta\Delta)$.

3) MULTIPLICATION

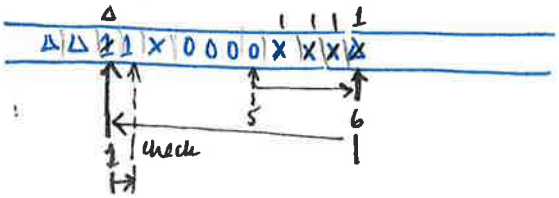
TAPE DIAGRAM:

Input: $\Delta 1^x \Delta 1^y \Delta$ ($x < y$ nonzero)
Output: $\Delta 1^{xy} \Delta$

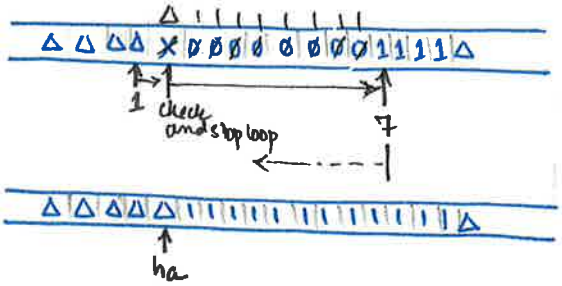




after a few rounds:



In the final round:



EXPLANATION:

1) READ A 1 on the x side, change to Δ, check next:

(a) IF next = 1:
skip all 1's and x's, if Δ comes, change Δ to x and go to the 1s of y side. (either after x or after 0's). Change 1 to 0, move right skip 1s till Δ, change Δ to X, go back to next 1. Continue till all 1s are 0's and. then immediately x's start. change all x's to 1s. Go left, Traverse all 0's, Traverse X, Traverse 1's, go to Δ and Turn right. REPEAT 1.

(b) IF next = X:
Move right, change all 0's to 1s. When you reach the 1's, Turn left, Traverse all the 1's that were recently created until you reach X, change X to Δ and STOP.

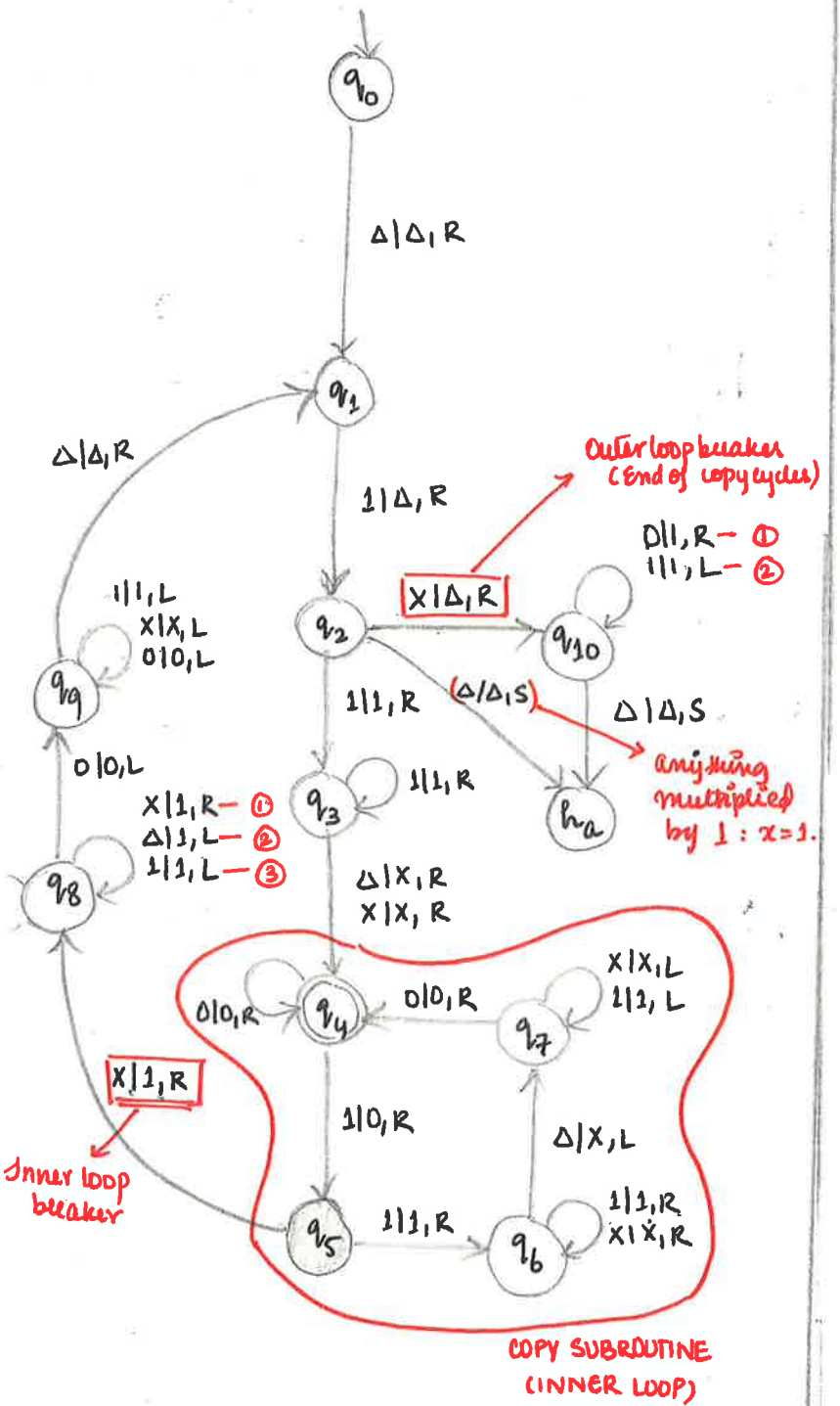
MULTIPLICATION ~

RUN FOR

w = ' Δ 1 1 Δ 1 1 1 1 Δ'

$(q_0, \Delta 1 1 \Delta 1 1 1 1 \Delta) \vdash_T$
$(q_1, \Delta 1 1 \Delta 1 1 1 1 \Delta) \vdash_T$
$(q_2, \Delta\Delta 1 \Delta 1 1 1 1 \Delta) \vdash_T$
$(q_3, \Delta\Delta 1 \Delta 1 1 1 1 \Delta) \vdash_T$
$(q_4, \Delta\Delta 1 X 1 1 1 1 \Delta) \vdash_T$
$(q_5, \Delta\Delta 1 X 0 1 1 1 \Delta) \vdash_T$
$(q_6, \Delta\Delta 1 X 0 1 1 1 \Delta) \vdash_T^*$
$(q_6, \Delta\Delta 1 X 0 1 1 1 \Delta) \vdash_T$
$(q_7, \Delta\Delta 1 X 0 1 1 1 X \Delta) \vdash_T^*$
$(q_7, \Delta\Delta 1 X 0 1 1 1 X \Delta) \vdash_T$
$(q_4, \Delta\Delta 1 X 0 1 1 1 X \Delta) \vdash_T$
$(q_5, \Delta\Delta 1 X 0 0 1 1 X \Delta) \vdash_T$
$(q_6, \Delta\Delta 1 X 0 0 1 1 X \Delta) \vdash_T^*$
$(q_6, \Delta\Delta 1 X 0 0 1 1 X \Delta) \vdash_T$
$(q_7, \Delta\Delta 1 X 0 0 1 1 X X \Delta) \vdash_T^*$
$(q_7, \Delta\Delta 1 X 0 0 1 1 X X \Delta) \vdash_T$
$(q_4, \Delta\Delta 1 X 0 0 1 1 X X \Delta) \vdash_T$
$(q_5, \Delta\Delta 1 X 0 0 0 1 X X \Delta) \vdash_T$
$(q_6, \Delta\Delta 1 X 0 0 0 1 X X \Delta) \vdash_T^*$
$(q_6, \Delta\Delta 1 X 0 0 0 1 X X \Delta) \vdash_T$
$(q_7, \Delta\Delta 1 X 0 0 0 1 X X X \Delta) \vdash_T$
$(q_7, \Delta\Delta 1 X 0 0 0 1 X X X \Delta) \vdash_T$
$(q_4, \Delta\Delta 1 X 0 0 0 1 X X X \Delta) \vdash_T$
$(q_5, \Delta\Delta 1 X 0 0 0 0 X X X \Delta) \vdash_T$
$(q_8, \Delta\Delta 1 X 0 0 0 0 1 X X \Delta) \vdash_T^*$
$(q_8, \Delta\Delta 1 X 0 0 0 0 1 1 1 1 \Delta) \vdash_T$
$(q_9, \Delta\Delta 1 X 0 0 0 0 1 1 1 1 \Delta) \vdash_T^*$
$(q_9, \Delta\Delta 1 X 0 0 0 0 1 1 1 1 \Delta) \vdash_T$

TRANSITION DIAGRAM δ:



$(q_1, \Delta\Delta\underline{1}x0000\,1111) \vdash_T$

$(q_2, \Delta\Delta\Delta\underline{x}\,0000\,1111) \vdash_T$

$(q_{10}, \Delta\Delta\Delta\underline{0}\,0000\,1111) \vdash_T^*$

$(q_{10}, \Delta\Delta\Delta\underline{\Delta}\,1111\,1111\Delta) \vdash_T$

$(h_a, \underline{\Delta}1111\,1111\Delta)$

and hence:

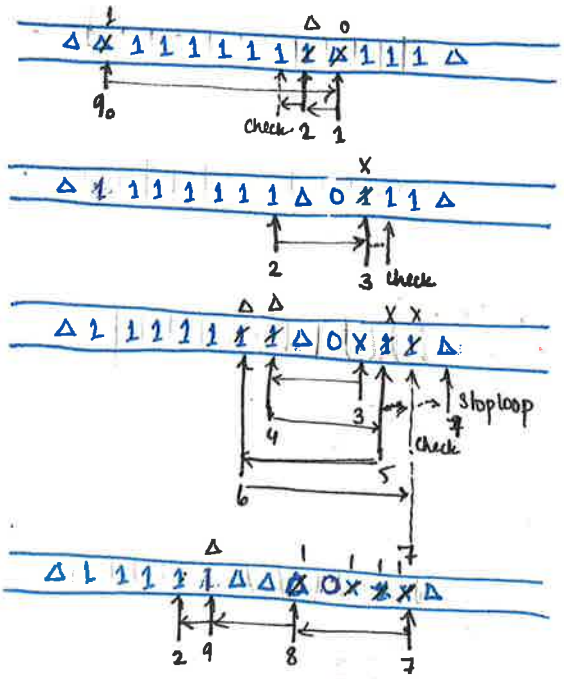
$T = (\{q_0, \ldots, q_{10}\}, \{1\}, \{0,1,X\}, \delta, q_0)$

4) DIVISION

TAPE DIAGRAM:

Input: $\Delta 1^x \Delta 1^y \Delta \quad (x, y > 0)$

Output: $\Delta 1^{x/y} \Delta 1^{x\%y} \Delta$



EXPLANATION:

1) Change first Δ to 1, go to right till Δ, change Δ to 0, go left, change last 1 to Δ.
2) Read A 1 after 0 and change to X, Check next:
   (a) IF next = 1: go left past 0 and past all 1's and Δ's till dividend part 1's and change 1 to Δ. If next on left is 1 repeat (2), else go to (3).
   (b) IF next = Δ: go left changing all X's to 1's, go past 0, change the first Δ after 0 to a 1, go past it to dividend part 1's and change 1 to Δ. If next on left is 1 repeat (2), else go to (3).
3) Go past all Δ's, 1's and 0's, change all X's to 1's and the 1's after that to Δ. Come back to 0. Change 0 to Δ, come left past all the 1's and halt at Δ.
4) STOP.

DIVISION ~

RUN FOR

W = 'Δ111Δ1111Δ'

$(q_0, \underline{\Delta}111\Delta1111\Delta) \vdash_T$

$(q_1, \Delta\underline{1}11\Delta1111\Delta) \vdash_T^*$

$(q_1, \Delta1\underline{1}1\Delta1111\Delta) \vdash_T$

$(q_2, \Delta11\underline{1}\Delta1111\Delta) \vdash_T$

$(q_3, \Delta111\Delta\underline{0}1111\Delta) \vdash_T$

$(q_4, \Delta11\Delta0\underline{1}111\Delta) \vdash_T$

$(q_5, \Delta11\Delta0X\underline{1}11\Delta) \vdash_T$

$(q_6, \Delta11\Delta0\underline{X}111\Delta) \vdash_T^*$

$(q_6, \Delta111\underline{\Delta}0X111\Delta) \vdash_T$

$(q_7, \Delta111\Delta0X111\Delta) \vdash_T$

$(q_8, \Delta11\underline{\Delta}\Delta0X111\Delta) \vdash_T$

$(q_3, \Delta11\underline{\Delta}\Delta0X111\Delta) \vdash_T^*$

$(q_3, \Delta11\Delta\underline{\Delta}0X111\Delta) \vdash_T$

$(q_4, \Delta11\Delta\Delta0\underline{X}111\Delta) \vdash_T^*$

$(q_4, \Delta11\Delta\Delta0X\underline{1}11\Delta) \vdash_T$

$(q_5, \Delta11\Delta\Delta0XX\underline{1}1\Delta) \vdash_T$

$(q_6, \Delta11\Delta\underline{\Delta}0XX11\Delta) \vdash_T^*$

$(q_6, \Delta11\Delta\Delta\underline{0}XX11\Delta) \vdash_T$

$(q_7, \Delta11\Delta\Delta0\underline{X}X11\Delta) \vdash_T^*$

$(q_7, \Delta11\Delta\Delta0X\underline{X}11\Delta) \vdash_T$

$(q_8, \Delta11\Delta\Delta0XX11\Delta) \vdash_T$

$(q_3, \Delta11\Delta\Delta\underline{\Delta}0XX11\Delta) \vdash_T^*$

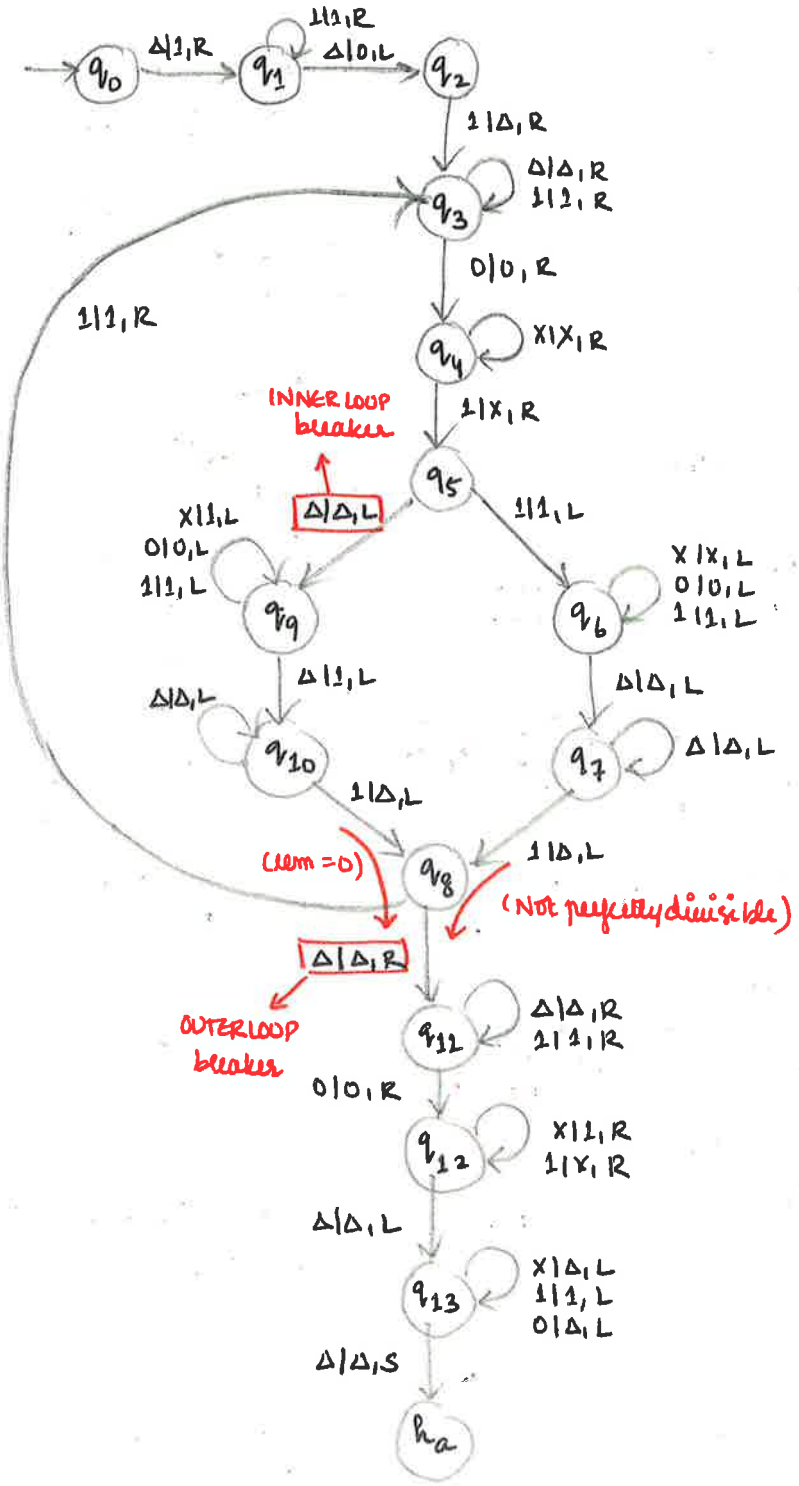
$(q_3, \Delta11\Delta\Delta\Delta\underline{0}XX11\Delta) \vdash_T$

$(q_4, \Delta11\Delta\Delta\Delta0\underline{X}X11\Delta) \vdash_T^*$

$(q_4, \Delta11\Delta\Delta\Delta0X\underline{X}11\Delta) \vdash_T$

R T-0

TRANSITION DIAGRAM δ:



And hence:

$T = (\{q_0, \ldots, q_{13}\}, \{1\}, \{0,1,X\}, \delta, q_0)$

$(q_5, \Delta 1 \Delta\Delta\Delta 0XX\underline{X}1\Delta) \vdash_T$

$(q_6, \Delta 1 \Delta\Delta\Delta 0XX\underline{X}1\Delta) \vdash_T^*$

$(q_6, \Delta 1 \Delta\Delta\underline{\Delta} 0XX\underline{X}1\Delta) \vdash_T$

$(q_7, \Delta 1 \Delta\underline{\Delta}\Delta 0XX\underline{X}1\Delta) \vdash_T^*$

$(q_7, \Delta\underline{1}\Delta\Delta 0XX\underline{X}1\Delta) \vdash_T$

$(q_8, \Delta\Delta\Delta\Delta\Delta 0XX\underline{X}1\Delta) \vdash_T$

$(q_{11}, \Delta\underline{\Delta}\Delta\Delta\Delta 0XX\underline{X}1\Delta) \vdash_T^*$

$(q_{11}, \Delta\Delta\Delta\Delta\underline{\Delta}\underline{0}XX\underline{X}1\Delta) \vdash_T$

$(q_{12}, \Delta\Delta\Delta\Delta\Delta 0\underline{X}XX\underline{X}1\Delta) \vdash_T^*$

$(q_{12}, \Delta\Delta\Delta\Delta\Delta 0111\underline{X}\Delta) \vdash_T^*$

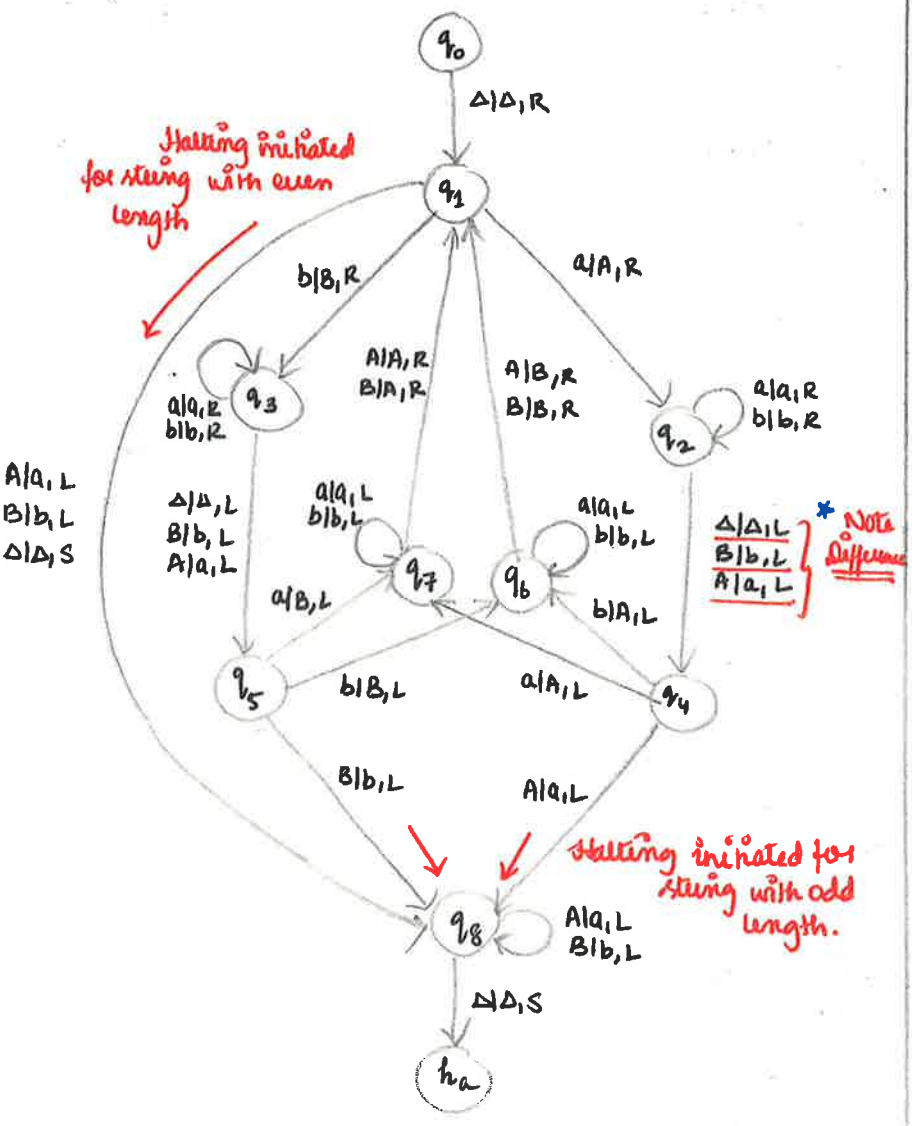
$(q_{13}, \Delta\Delta\Delta\Delta\Delta 0111\underline{X}\Delta) \vdash_T^*$

$(q_{13}, \Delta\Delta\Delta\Delta\underline{\Delta}\Delta 111\Delta\Delta) \vdash_T$

$(h_a, \underline{\Delta}\Delta 111\Delta)$

5) STRING REVERSAL (BETTER SOLUTION)

TRANSITION DIAGRAM δ:



And hence:

$$T = (\{q_0,\ldots,q_8,h\}, \{a,b\}, \{a,b,A,B\}, \delta, q_0)$$

NOTE 1: In case of odd length strings, at state q_4/q_5, the finite state control will encounter a capital letter while moving left, meaning the middle is reached. whereas in even length string (including $|w| = 0$), the finite state control encounters a capital letter moving to the right from q_3, hence the loop breaking conditions are as stated above in the diagram.

NOTE 2: The difference* in B|b,L and A|a,L is that when the capital letters are changed to small then and there and then the control moves to left, the control needn't go all the way to right end to toggle everything back. It can start from wherever it's current position is hence, movement is minimized.

STRING REVERSAL ~

1) ODD LENGTH RUN FOR $w = \text{'}\Delta abab b\Delta\text{'}$

$(q_0, \Delta ab abba\Delta) \vdash_T$
$(q_1, \Delta a babb\Delta) \vdash_T$
$(q_2, \Delta A babb\Delta) \vdash_T$
$(q_2, \Delta A babb\Delta) \vdash_T$
$(q_4, \Delta A babb\Delta) \vdash_T$
$(q_6, \Delta A babA\Delta) \vdash_T$
$(q_6, \Delta A babA\Delta) \vdash_T$
$(q_1, \Delta B abA\Delta) \vdash_T$
$(q_2, \Delta B BabA\Delta) \vdash_T$
$(q_2, \Delta BBabA\Delta) \vdash_T$
$(q_4, \Delta BB abA\Delta) \vdash_T$
$(q_6, \Delta BB aBA\Delta) \vdash_T$
$(q_6, \Delta BBaBA\Delta) \vdash_T$
$(q_1, \Delta BB aBA\Delta) \vdash_T$
$(q_2, \Delta BBA BA\Delta) \vdash_T$
$(q_4, \Delta BBABA\Delta) \vdash_T$
$(q_8, \Delta BB ABA\Delta) \vdash_T$
$(q_8, \Delta bb ABA\Delta) \vdash_T$
$(h_a, \Delta bbABA\Delta)$

2) EVEN LENGTH RUN FOR $w = \text{'}\Delta babb b\Delta\text{'}$

$(q_0, \Delta babb b\Delta) \vdash_T$
$(q_1, \Delta babb b\Delta) \vdash_T$
$(q_3, \Delta Babb b\Delta) \vdash_T$
$(q_3, \Delta Babb b\Delta) \vdash_T$
$(q_5, \Delta B abb b\Delta) \vdash_T$
$(q_6, \Delta BabbB\Delta) \vdash_T$
$(q_6, \Delta BabbB\Delta) \vdash_T$
$(q_1, \Delta BabbB\Delta) \vdash_T$
$(q_2, \Delta BAbbB\Delta) \vdash_T$
$(q_2, \Delta BAbbB\Delta) \vdash_T$
$(q_4, \Delta BAbbA\Delta) \vdash_T$
$(q_6, \Delta BAAbA\Delta) \vdash_T$
$(q_1, \Delta BBAbA\Delta) \vdash_T$
$(q_8, \Delta BBabA\Delta) \vdash_T$
$(q_8, \Delta bbab\Delta) \vdash_T$
$(h_a, \Delta bbab\Delta)$