ISyE6669 Homework Week 5

1. Compute the gradient $\nabla f(x)$ and Hessian $\nabla^2 f(x)$ of the Rosenbrock function

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

2. Implement the Newton's Method with line search given in Algorithm 1. Use the Newton's

Algorithm 1 Newton's Method with Line Search

Start with x^0. Set $k = 0$, $\epsilon = 10^{-4}$.
Set $d^0 \leftarrow -(\nabla^2 f(x^0))^{-1} \nabla f(x^0)$
while $\|\nabla f(x^k)\| > \epsilon$ **do**
  Choose $\bar{\alpha} > 0, \rho \in (0,1), c \in (0,1)$; Set $\alpha^k \leftarrow \bar{\alpha}$
  while $f(x^k + \alpha^k d^k) > f(x^k) + c\alpha^k \nabla f(x^k)^\top d^k$ **do**
    $\alpha^k \leftarrow \rho \alpha^k$
  end while
  $x^{k+1} \leftarrow x^k + \alpha^k d^k$
  $d^{k+1} \leftarrow -(\nabla^2 f(x^{k+1}))^{-1} \nabla f(x^{k+1})$
  $k \leftarrow k + 1$
end while

Method to minimize the Rosenbrock function in Problem 1. Set the initial stepsize $\bar{\alpha} = 1$. Select your own choice of $\rho \in (0,1), c \in (0,1)$. First run the algorithm from the initial point $x^0 = (1.2, 1.2)^\top$, and then try the more difficult starting point $x^0 = (-1.2, 1)^\top$. For each starting point, print out the step length α^k used by the algorithm as well as the point x^k for *every* step k. You should observe that Newton's Method converges very fast.

3. Figure 1 below illustrates the water network of Newvillage. The lines are water piplelines numbered from 1 through 13. The arrows on the lines are possible direction(s) of flow of water in these pipelines. The circles are water sources numbered A, B, C. The rectangles are houses D, E, F, G, H. The maximum possible capacity of the water sources are (the sources can operate at less than the maximum capacity): A: 100 Units, B: 100 Units, C: 80 Units Demands of water in the houses are: D: 50 Units, E: 60 Units, F: 40 Units, G: 30 Units, H: 70 Units Since the houses are at different elevation and the pipes are of different diameter, the cost of transporting water is different in the different pipes. These costs per unit of water are: Pipe 1: \$2, Pipe 2: \$3, Pipe 3: \$4, Pipe 4: \$2, Pipe 5: \$3, Pipe 6: \$2, Pipe 7: \$4, Pipe 8: \$1, Pipe 9: \$2, Pipe 10: \$4, Pipe 11: \$5, Pipe 12: \$1, Pipe 13: \$2. Formulate an LP to minimize the total cost of transporting water so as to meet the water demands of each house.

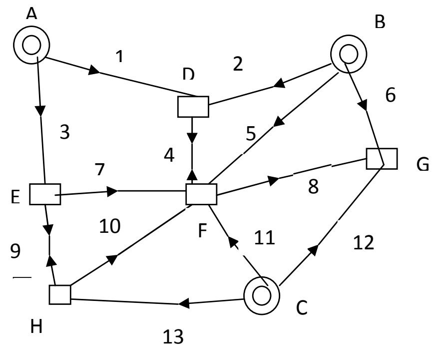

Figure 1: Water Network for Question 2

4. Consider the following electric power network shown in Figure 2. This network is taken from a real-world electric power system. Electricity generators are located at nodes 1, 3, and 5 and producing p_1, p_2, p_3 amounts of electricity, respectively. Electricity loads are located at nodes 2, 4, and 6 and are consuming d_1, d_2, d_3 amounts of electricity, respectively.

   The demand is fixed and given as $d_1 = 110,\ d_2 = 65,\ d_3 = 205$.

   Each generator i's production must be within an upper and a lower bound as $p_i^{min} \leq p_i \leq p_i^{max}$. The bounds are given as $p_1^{\min} = 20,\ p_1^{\max} = 200,\ p_2^{\min} = 20,\ p_2^{\max} = 150,\ p_3^{\min} = 10,\ p_3^{\max} = 150$.

   The flow limits over lines are given as $f_{12}^{\max} = 100, f_{23}^{\max} = 110, f_{34}^{\max} = 50, f_{45}^{\max} = 80, f_{56}^{\max} = 60, f_{61}^{\max} = 40$.

   The line parameters are given as $B_{12} = 11.6,\ B_{23} = 5.9,\ B_{34} = 13.7,\ B_{45} = 9.8,\ B_{56} = 5.6,\ B_{61} = 10.5$. The unit generation costs are given as $c_1 = 16, c_2 = 20, c_3 = 8$.

   (a) Formulate the power system scheduling problem using the model discussed in Lecture 2.

   (b) Implement and solve the model using CVXPY. Write down the optimal solution.

   (c) Find the electricity prices for demand at nodes 2, 4, 6. To do this, use the command `constraints[0].dual_value` to find the dual variable of `constraints[0]`. Hint: Recall the electricity price at node i is the dual variable for the flow conservation constraint at node i.

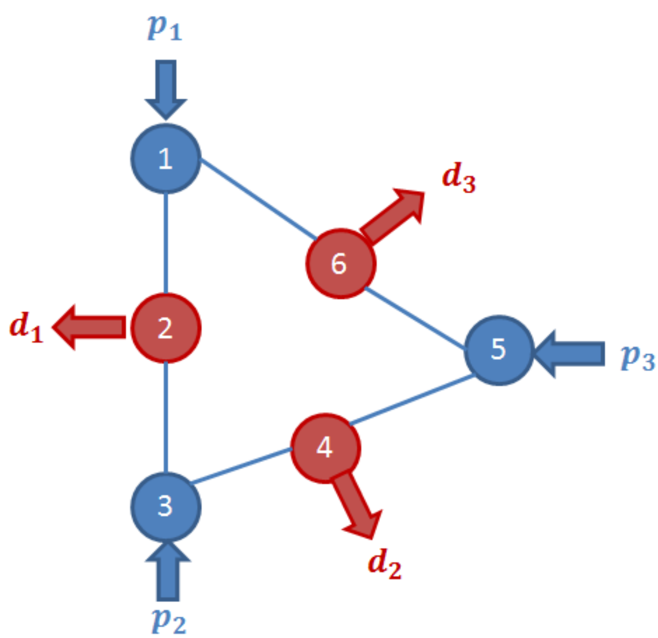

Figure 2: Electric network for Question 3.