ISYE 6669 – HOMEWORK 9

ANSWER 1 –

a)

Given LP –

To convert LP to standard form, we replace y with -w (where w is non- negative), z = v-u (where v and u are non-negative), convert objective to a minimization, and prepare inequalities for dual form -

Min {- x + w - v + u}

s.t.

- 10x - w - 5v + 5u >= -2 → (a in dual LP)
2w + 3v - 3u >= -3 → (b in dual LP)
- 2w - 3v + 3u >= 3 → (c in dual LP)
x - w - 2v + 2u >= -4 → (d in dual LP)

x,w,v,u >= 0

Dual LP –

Max {-2a -3b + 3c – 4d}

s.t.

-10a + d <= -1
-a + 2b - 2c – d <= 1
-5a + 3b – 3c – 2d <= -1
5a – 3b + 3c + 2d <= 1

a,b,c,d >= 0

Or in standard minimization form –

Min {2a + 3b - 3c + 4d}

s.t.

-10a + d <= -1
-a + 2b - 2c – d <= 1
-5a + 3b – 3c – 2d <= -1
5a – 3b + 3c + 2d <= 1

a,b,c,d >= 0

b)

Given LP with adjusted inequalities –

Min $\{c^T x\}$

s.t.

$-Ax >= -b$
$x >= 0$

Dual LP –

Max $\{-b^T y\}$

s.t.

$-A^T y <= c$
$y >= 0$

Or in standard minimization form –

Min $\{b^T y\}$

s.t.

$-A^T y <= c$
$y >= 0$

ANSWER 2 –

a)

The dual problem (with a and b for the two inequalities) –

Min {500a + 400b }

s.t.

2a + 3b >= 4
a + 2b >= 1
3a + b >= 3
4a + 6b >= 10

a,b >= 0

Or in standard LP format –

Min {500a + 400b }

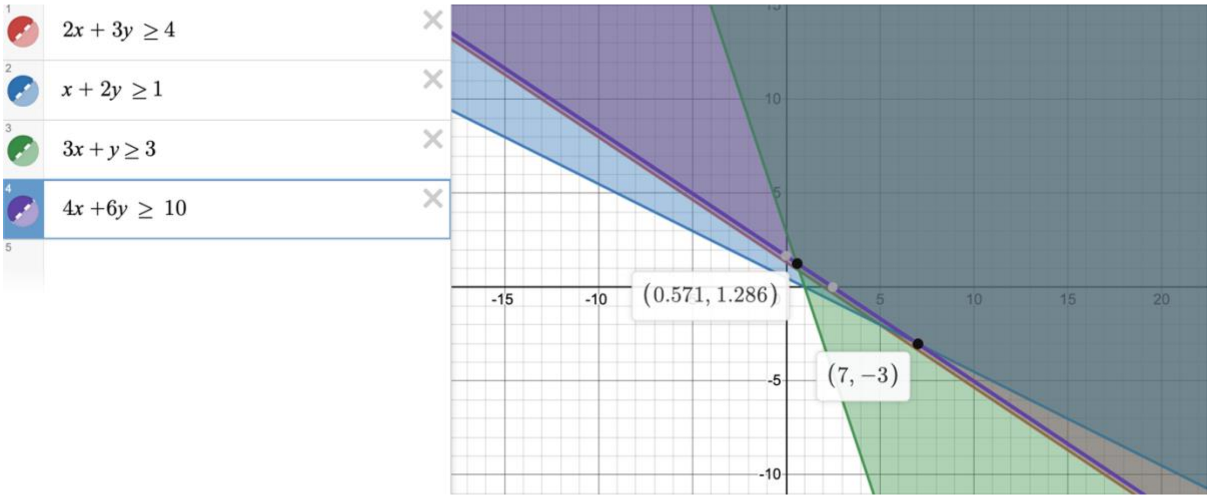
s.t.

2a + 3b >= 4
a + 2b >= 1
3a + b >= 3
4a + 6b >= 10

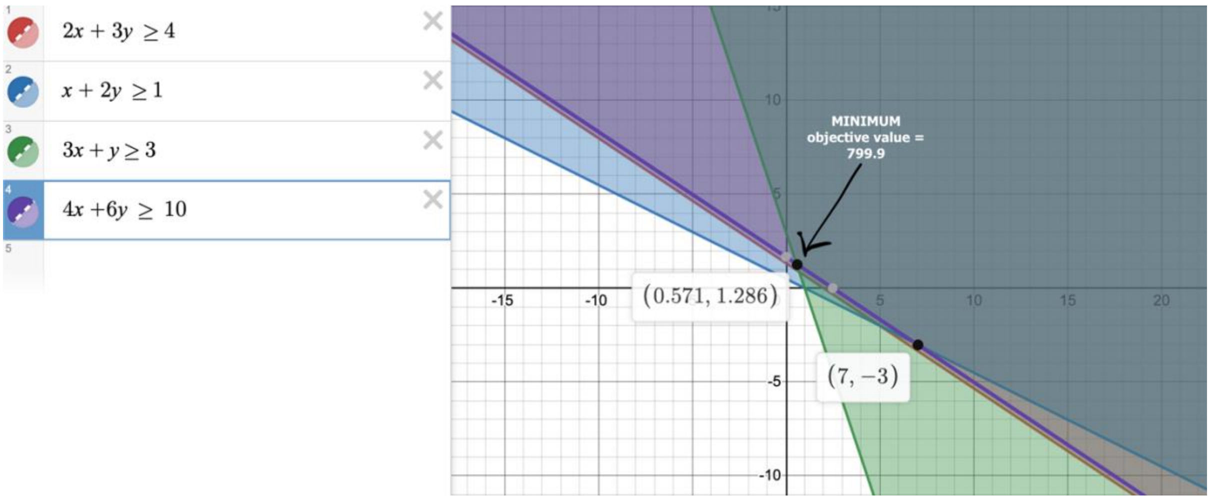
a,b >= 0

b)

The graph –



c)



d)

The optimal objective value is 800 at point 4/7 and 9/7.

e)

Both dual optimal variables are non-zero hence all primal constraints are active.

f)

The third and fourth dual constraints are active (exactly equal via the complementary slackness check), hence variables x3 and x4 are non-zero, and x1 and x2 are zero.

g)

Now we reframe our primal LP –

Max $\{3x_3 + 10x_4\}$

s.t.

$3x_3 + 4x_4 <= 500$
$x_3 + 6x_4 <= 400$
$x_3, x_4 >= 0$

The intersection of the equality version of those two lines is at point –

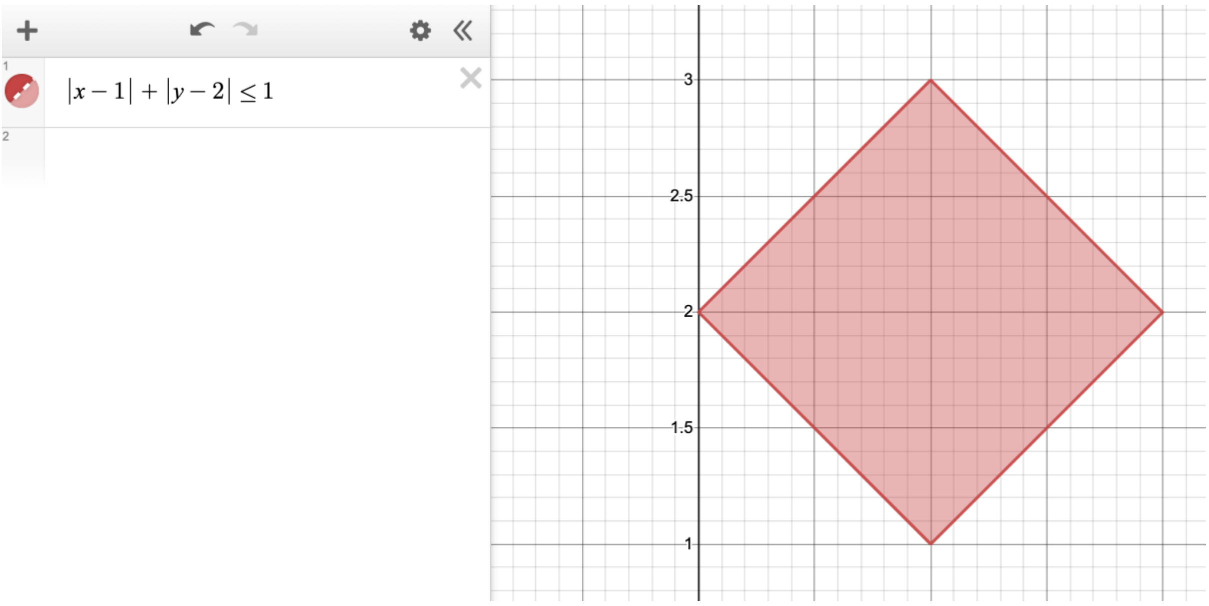
$x_3 = 100$
$x_4 = 50$

And the maximum gain is $800.
(We got around $140 more now).

ANSWER 3 –

a)

The plot –



b)

Worst case scenarios for a_1 and a_2 are the border points –

(1,3), (0,2), (1,1), (2,2)

And plugging in (x_1, x_2) = (-1, 1) in the inequality with these four values and we get the respective left hand side values of $a_1x_1 + a_2x_2$ –

2, 2, 0, 0.

The last two extremes of the hull fail the condition since they are greater than or equal to 1.

Or in other words, the points (-1, 1) DO NOT belong to the half-space for ALL values allowed by the uncertainty set.

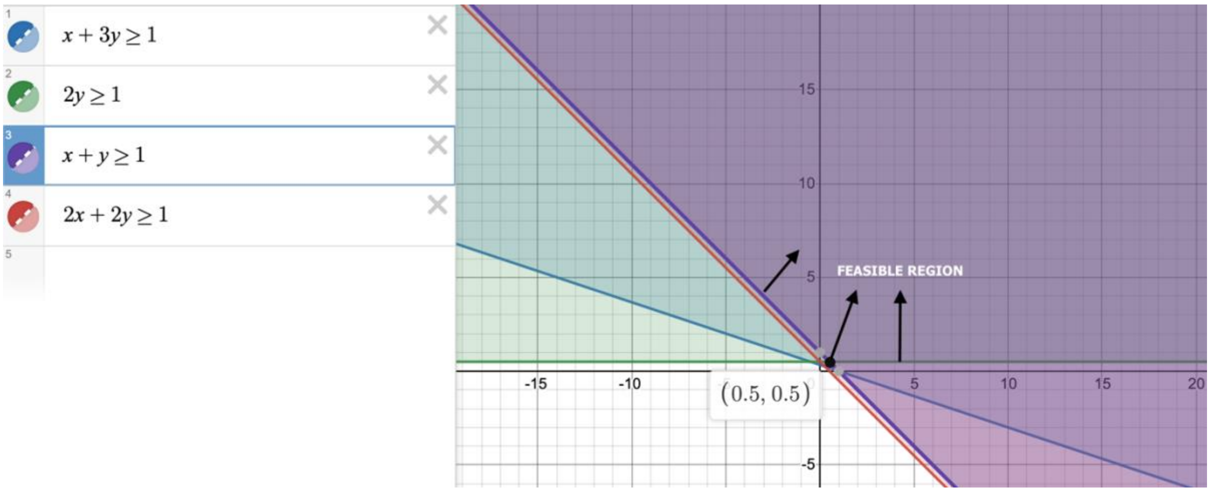
c)

New deterministic linear constraints with new variables w and y –

$a_1x_1 + a_2x_2 >= 1$
$w + y <= 1$
$-w <= a_1 – 1 <= w$
$-y <= a_2 – 2 <= y$

d)

Here is the feasible region plotted after plugging in the extreme points of the uncertainty set and generating the intersection of four half-spaces created by those extreme points –



The robust feasible region is, in fact, just the intersection of the half-spaces generated by extreme uncertainty points – (1,1) and (0,2).