ISYE 6669 – HOMEWORK 7

ANSWER 1 –

a)

1. x = $\lambda_1 A + \lambda_2 B + \lambda_3 C$
2. Summation(λ_i) = 1

Let us set up a program to solve this using the python library SymPy –

```
from sympy import symbols, Eq, solve

x1, x2, x3 = symbols('x1 x2 x3')

e1 = Eq(x1*(0) + x2*(2) + x3*(1),1)
e2 = Eq(x1*(0) + x2*(0) + x3*(2),2/3)
e3 = Eq(x1 + x2 + x3,1)

soln = solve((e1,e2,e3),(x1,x2,x3))

{x1: 0.333333333333333, x2: 0.333333333333333, x3: 0.333333333333333}
```

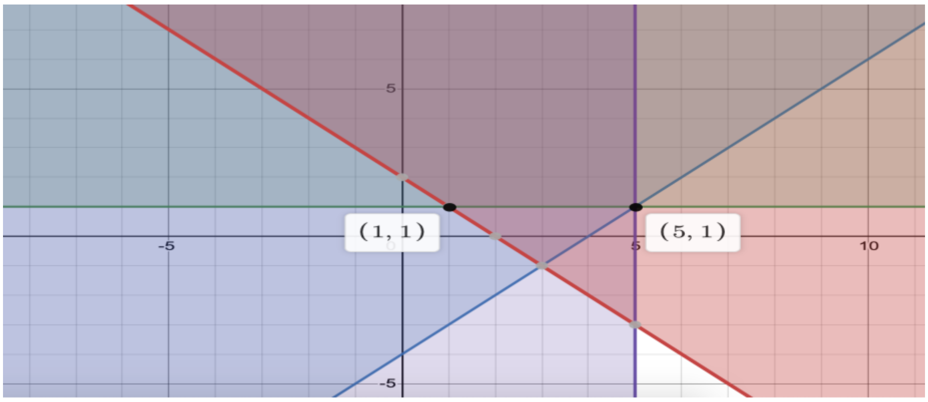
The solution for ƛ would be –
[1/3, 1/3, 1/3]

b)

Half spaces:

1. x + y >= 2
2. x – y <= 4
3. y >= 1
4. x <= 5

From the following graph –



The points –
A = (1,1)
B = (5,1)

- are the extreme points that mark the boundaries of the unbounded region of ALL four half-spaces intersecting.

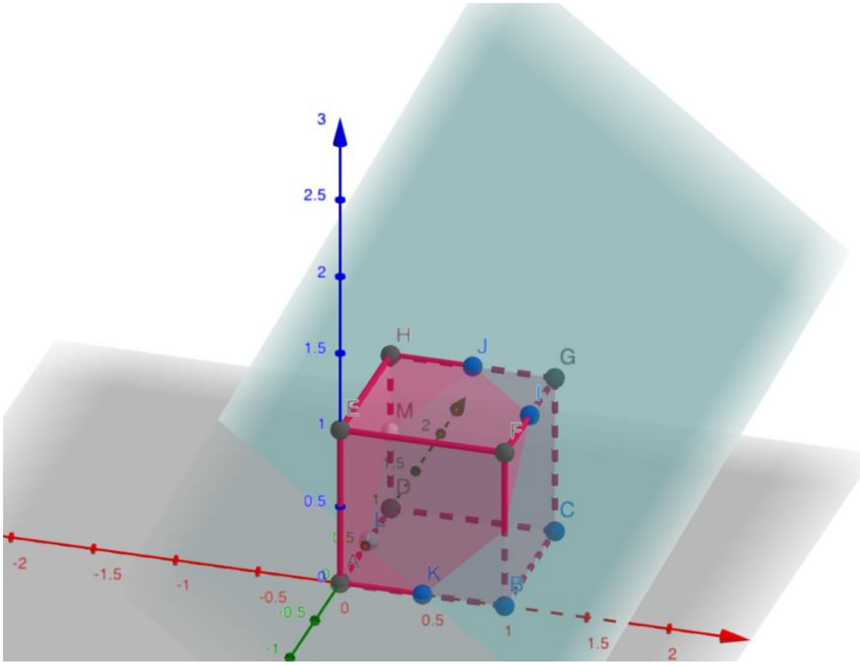
The extreme rays that define the boundaries are –

e1 = [0,1] from point A.
e2 = [-1,1] from point B.

c)

Using the following line we get 6 new extreme points added to the 4 cube vertices already present = 10 extreme points per subspace –

-0.5x - 0.5y + 0.5z = -0.25



The coordinates generated by the intersection are –

[0.5, 0, 0], [1, 0.5, 0], [0, 0, 0.5], [0, 0.5, 1], [0.5, 1, 1]

And based on which subspace we choose, we take the cube's extreme points –

-0.5x - 0.5y + 0.5z >= -0.25 : [0,0,0], [0,1,0], [0,1,1], [1,1,0]
-0.5x - 0.5y + 0.5z <= -0.25 : [1,0,0], [1,0,1], [1,0,0], [1,1,1]

ANSWER 2 –

a)

Phase 1 –

- Min { y – x }
s.t.
x + y – s_1 = 1
2x – y + s_2 = 1
X <= 0, y is free

Phase 2 –

Let x = -x_1
Let y = x_2 – x_3

Standard form –

- Min { x_2 – x_3 + x_1}

s.t.

-x_1 + (x_2 – x_3) – s_1 = 1
-2x_1 - (x_2 – x_3) + s_2 = 1
x_1, x_2, x_3, s_1, s_2 >= 0

b)

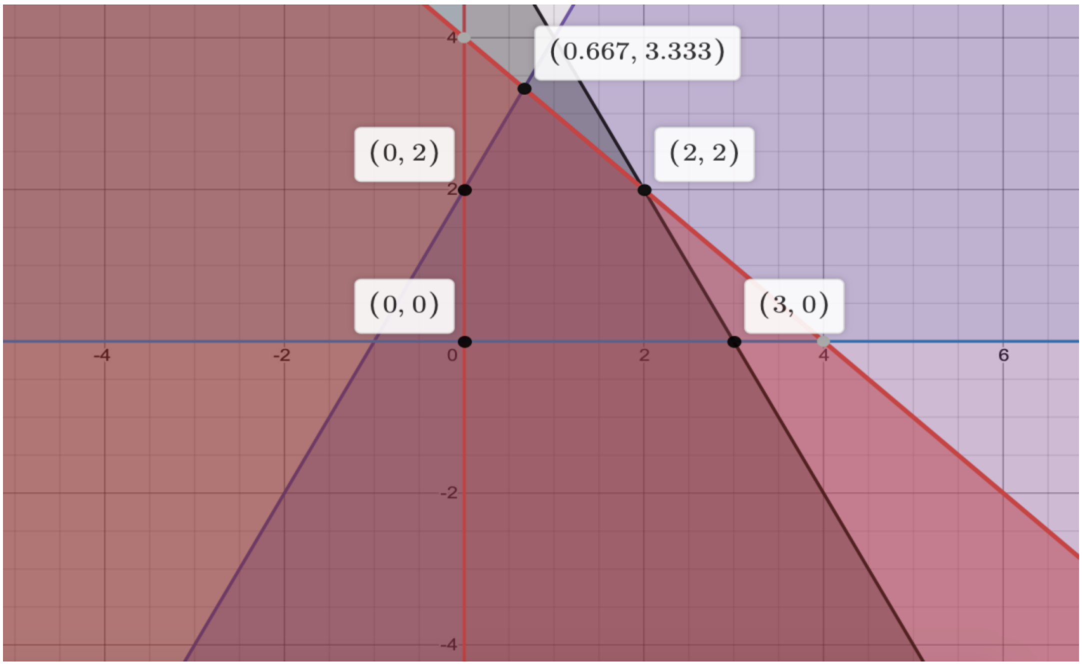
We introduce a new variables z and w and rewrite it as –

Min {z + w}
s.t.
x <= z
y <= z
x+y <= z
z <= 10
-z <= x+y <= z (from the objective function modulus)
-w <= x-2y <= w (from the objective function second modulus)

ANSWER 3 –

a)

The feasible region is graphed as the area encapsulated by the points marked –



b)

Standard form –

Min { $-x_1 - 2x_2$ }
s.t.

$x_1 + x_2 + x_3 = 4$
$-2x_1 + x_2 + x_4 = 2$
$2x_1 + x_2 + x_5 = 6$
$x_1, x_2 >= 0$

c = [-1, -2, 0, 0, 0]

A =
[[1, 1, 1, 0, 0]
[-2, 1, 0, 1, 0]
[2, 1, 0, 0, 1]]

b = [4, 2, 6]

c)

Solution 0 – x1 and x2 = 0, as demonstrated in the problems example. The cost coefficients are equal to 0 as well for x1, and x2, the active costs are all zero, while the non-basic costs are

[-1, -2]. Here are the results for the other combinations. ***Please note that the variables and cost coefficients that have not been mentioned are by default non-active constraints.***

Basic Solutions –

```
Basic Solution  1 :  Variables:  [1, 2, 3]

Basic Matrix:

[[ 1  1  1]
 [-2  1  0]
 [ 2  1  0]]


Basic Solution:
        [ 1.  4. -1.]

Active Cost Matrix:
        [-1, -2, 0]

Remaining variables and costs (inactive) assigned to 0


Basic Solution  2 :  Variables:  [1, 2, 4]

Basic Matrix:

[[ 1  1  0]
 [-2  1  1]
 [ 2  1  0]]


Basic Solution:
        [2. 2. 4.]

Active Cost Matrix:
        [-1, -2, 0]

Remaining variables and costs (inactive) assigned to 0


Basic Solution  3 :  Variables:  [1, 2, 5]

Basic Matrix:

[[ 1  1  0]
 [-2  1  0]
 [ 2  1  1]]


Basic Solution:
        [0.66666667 3.33333333 1.33333333]

Active Cost Matrix:
        [-1, -2, 0]

Remaining variables and costs (inactive) assigned to 0
```

Basic Solution 4 : Variables: [1, 3, 4]

Basic Matrix:

```
[[ 1  1  0]
 [-2  0  1]
 [ 2  0  0]]
```

Basic Solution:

```
        [3. 1. 8.]
```

Active Cost Matrix:

```
        [-1, 0, 0]
```

Remaining variables and costs (inactive) assigned to 0

Basic Solution 5 : Variables: [1, 3, 5]

Basic Matrix:

```
[[ 1  1  0]
 [-2  0  0]
 [ 2  0  1]]
```

Basic Solution:

```
        [-1.  5.  8.]
```

Active Cost Matrix:

```
        [-1, 0, 0]
```

Remaining variables and costs (inactive) assigned to 0

Basic Solution 6 : Variables: [1, 4, 5]

Basic Matrix:

```
[[ 1  0  0]
 [-2  1  0]
 [ 2  0  1]]
```

Basic Solution:

```
        [ 4. 10. -2.]
```

Active Cost Matrix:

```
        [-1, 0, 0]
```

Remaining variables and costs (inactive) assigned to 0

Basic Solution 7 : Variables: [2, 3, 4]

Basic Matrix:

```
[[1 1 0]
 [1 0 1]
 [1 0 0]]
```

Basic Solution:

```
        [ 6. -2. -4.]
```

Active Cost Matrix:

```
        [-2, 0, 0]
```

Remaining variables and costs (inactive) assigned to 0

Basic Solution 8 : Variables: [2, 3, 5]

Basic Matrix:

```
[[1 1 0]
 [1 0 0]
 [1 0 1]]
```

Basic Solution:

```
        [2. 2. 4.]
```

Active Cost Matrix:

```
        [-2, 0, 0]
```

Remaining variables and costs (inactive) assigned to 0

Basic Solution 9 : Variables: [2, 4, 5]

Basic Matrix:

```
[[1 0 0]
 [1 1 0]
 [1 0 1]]
```

Basic Solution:

```
        [ 4. -2.  2.]
```

Active Cost Matrix:

```
        [-2, 0, 0]
```

Remaining variables and costs (inactive) assigned to 0

Basic Solution 10 : Variables: [3, 4, 5]

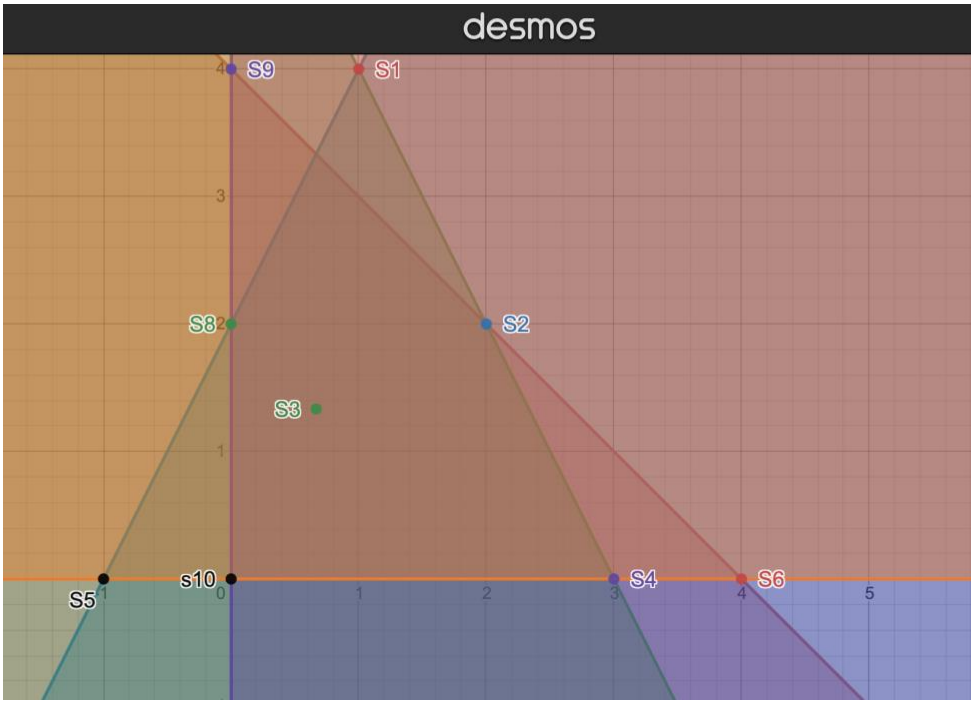
Basic Matrix:

```
[[1 0 0]
 [0 1 0]
```

```
 [0 0 1]]


Basic Solution:
        [4. 2. 6.]

Active Cost Matrix:
        [0, 0, 0]

Remaining variables and costs (inactive) assigned to 0
```

d)

The following is the plot of all feasible solutions described above –



Basic Feasible Solutions – 2, 3, 4, 8, 10

- **(2,2)**
- **(2/3,4/3)**
- **(3,0)**
- **(0,2)**
- **(0,0)**

Non-Basic Feasible Solutions – 1, 5, 6, 7, 9

Most Optimal Solution = S2 corresponding to point – (2,2)