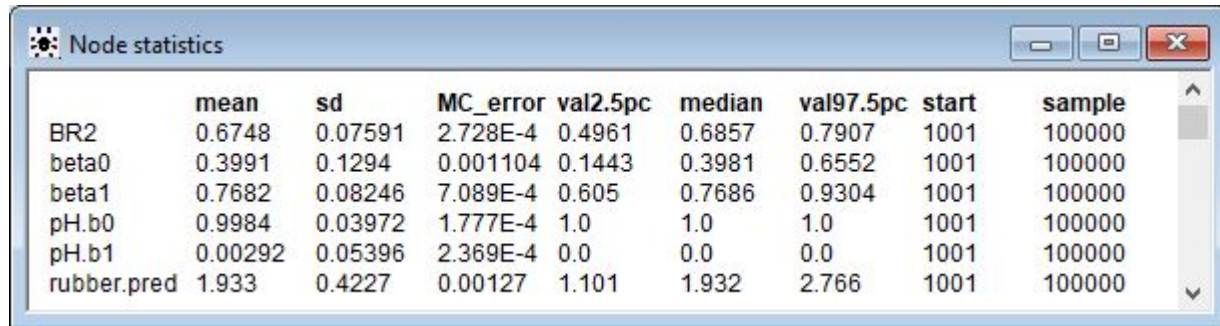


1 Paddy Soil Adhesion (6 points possible)

We run the OpenBUGS code and obtain the results shown. The code is attached.



	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
BR2	0.6748	0.07591	2.728E-4	0.4961	0.6857	0.7907	1001	100000
beta0	0.3991	0.1294	0.001104	0.1443	0.3981	0.6552	1001	100000
beta1	0.7682	0.08246	7.089E-4	0.605	0.7686	0.9304	1001	100000
pH.b0	0.9984	0.03972	1.777E-4	1.0	1.0	1.0	1001	100000
pH.b1	0.00292	0.05396	2.369E-4	0.0	0.0	0.0	1001	100000
rubber.pred	1.933	0.4227	0.00127	1.101	1.932	2.766	1001	100000

- (a) (1 point) As the mean value of β_0 and β_1 is 0.3991 and 0.7682, respectively, the fitted model is

$$y = 0.3991 + 0.7682x,$$

where the response variable y is adhesion to rubber and the variable x is adhesion to steelB.

(1 point) The Bayesian R^2 is 0.6748. In python this value is 0.7133, so please do not penalize for small differences.

- (b) (2 points) Any method students use to conclude that the model with $\beta_0 = 0$ and $\beta_1 = 1$ is not accurate is acceptable for credit. Solution 1: The credible set for β_0 is [0.1443, 0.6552] and the credible set for β_1 is [0.605, 0.9304]. Since these do not contain 0 or 1 respectively, we conclude the model with $b_0 = 0$ and $b_1 = 1$ is not accurate.

Alternate solution: The first hypothesis we monitor is $P(\beta_0 > 0)$ using the `step()` function in BUGS. From the mean of the `pH.b0` row, we see that this probability is 0.9984, so we can be confident that β_0 is not equal to 0. Similarly for $P(\beta_1 > 1)$ we get a probability of 0.00292, thus we are confident that $\beta_1 < 1$. Other hypotheses and results are perfectly valid for credit as long as the student makes the correct conclusion.

- (c) Based on the output, the predictive response has a mean of 1.933 and a 95% credible set as [1.101, 2.766]. Each of these values is worth 1 point.

Equivalent python output:

```

intercept  0.399  0.131    0.137  ...   4579.0   5402.0    1.0
beta       0.768  0.084    0.604  ...   4613.0   5034.0    1.0
sigma      0.418  0.049    0.328  ...   5978.0   5820.0    1.0
prob_H1    0.998  0.043    1.000  ...   5799.0  12000.0    1.0
prob_H2    0.004  0.060    0.000  ...   5405.0   5405.0    1.0

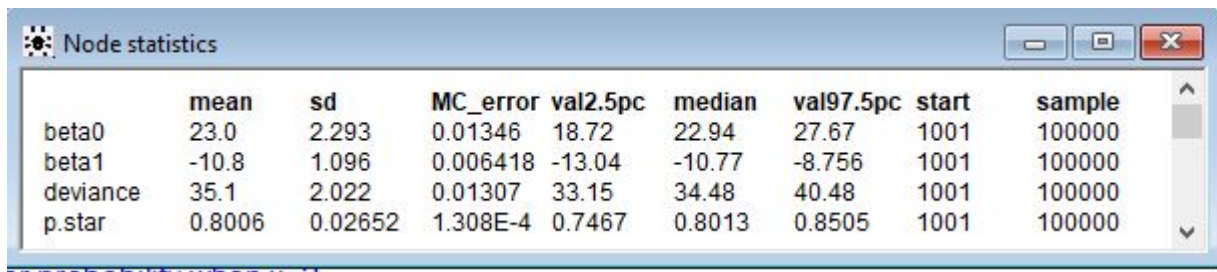
[5 rows x 9 columns]
r2          0.713324
r2_std      0.080090
dtype: float64
Prediction:

mean        1.936 | 100.00% [12000/12000 00:00<00:00]
sd          0.431
hdi_2.5%    1.115
hdi_97.5%   2.816
dtype: float64

```

2 Third-degree Burns (7 points possible)

By running the OpenBUGS code, we obtain the results shown as follows:



	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
beta0	23.0	2.293	0.01346	18.72	22.94	27.67	1001	100000
beta1	-10.8	1.096	0.006418	-13.04	-10.77	-8.756	1001	100000
deviance	35.1	2.022	0.01307	33.15	34.48	40.48	1001	100000
p.star	0.8006	0.02652	1.308E-4	0.7467	0.8013	0.8505	1001	100000

- (a) (2 points) Based on the estimated parameters, we obtain the logistic regression model as

$$y[i] \sim \text{Bin}(n_i, p_i), \text{ where } \text{logit}(p_i) = 23.0 - 10.8x_i, i = 1, \dots, 9,$$

where y is the number of survivals, p is the probability of survival rate and n is total number patients.

(2 points) We obtain the following values for the deviance based on whether we model with a Bernoulli or Binomial likelihood:

Binomial + BUGS: 35.1

Binomial + pymc: 39.12

Bernoulli + BUGS: 337.2

Bernoulli + pymc: 339.43

- (b) (3 points) When $\log(\text{area}+1)$ equals 2, the posterior probability of survival is 0.8006.
Equivalent python output for Bernoulli likelihood:

```

      mean      sd  hdi_3%  hdi_97%  ...  mcse_sd  ess_bulk  ess_tail  r_hat
beta0  22.934  2.301  18.497  27.286  ...   0.038   1835.0   1746.0    1.0
beta1 -10.768  1.099 -12.914  -8.722  ...   0.018   1834.0   1712.0    1.0

[2 rows x 9 columns]
Computed from 12000 posterior samples and 435 observations log-likelihood matrix.

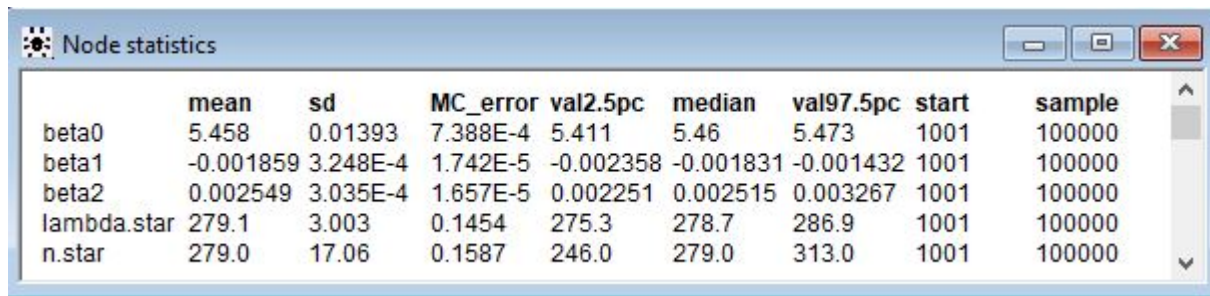
      Estimate      SE
deviance_waic   339.43   24.50
p_waic           2.14    -

Predicted probability when log(area + 1) = 2:
0.801

```

3 SO_2 , NO_2 , and Hospital Admissions (7 points possible)

By running the OpenBUGS code, we obtain the results shown as follows:



	mean	sd	MC_error	val2.5pc	median	val97.5pc	start	sample
beta0	5.458	0.01393	7.388E-4	5.411	5.46	5.473	1001	100000
beta1	-0.001859	3.248E-4	1.742E-5	-0.002358	-0.001831	-0.001432	1001	100000
beta2	0.002549	3.035E-4	1.657E-5	0.002251	0.002515	0.003267	1001	100000
lambda.star	279.1	3.003	0.1454	275.3	278.7	286.9	1001	100000
n.star	279.0	17.06	0.1587	246.0	279.0	313.0	1001	100000

- (a) (2 points) We obtain the following model with the estimated parameters.

$$y_i \sim \text{Poi}(\lambda_i), \text{ where } \lambda_i = \exp\{5.458 - 0.001859x_{i1} + 0.002549x_{i2}\}, i = 1, \dots, n.$$

(2 points for correct interpretation) Based on the estimated coefficients, we see that the level of SO_2 has a negative coefficient (-0.001859) and the level of NO_2 has a positive coefficient (0.002549). This means that the level of SO_2 has a negative impact on the hospital admission, while the level of NO_2 has a positive impact on the hospital admission.

- (b) (3 points) Based on the results, the expected number of hospital admissions is 279.0 and the 95% credible set is $[246.0, 313.0]$.

```

      mean      sd    hdi_2.5%    hdi_97.5% ...    mcse_sd    ess_bulk    ess_tail    r_hat
beta0  5.461  0.007      5.448      5.476 ...      0.0      5152.0      5132.0      1.0
beta1 -0.002  0.000     -0.002     -0.001 ...      0.0      6359.0      5976.0      1.0
beta2  0.002  0.000      0.002      0.003 ...      0.0      4963.0      4933.0      1.0

[3 rows x 9 columns]
Sampling: [likelihood]
|████████████████████| 100.00% [12000/12000 00:01<00:00]mean          278.457495
sd              16.787308
hdi_2.5%        244.686301
hdi_97.5%       309.894521
dtype: float64

```