July 24^{th} '2020

LAYMAN PROOF OF FERMAT'S LAST THEOREM FOR POWERS THAT ARE MULTIPLES OF 4

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Note: The following proof does not assume that the reader is familiar with Fermat's method of Descent and only uses basic knowledge of the Pythogorean equation and the properties of primitive Pythogorean triples.

To prove: $x^{4m} + y^{4m} \neq z_{4m}$ for any x, y, z, m $\in$ N (Set of natural numbers) → (1)

Statement 1 may be reduced and its contradiction may be taken to disprove:

$(x^{2m})^2 + (y^{2m})^2 = (z^{2m})^2$, where x < y and hence, $x^{2m} < y^{2m}$, for some x, y, z, m $\in$ N which is in truth, a relation (to be proven) between three numbers (x^{2m}, y^{2m}, z^{2m}) that qualifies as a Pythogorean triple. → (2)

According to Euler's properties of primitive Pythogorean triples fulfilling; $a^2 + b^2 = c^2$, one of *a* or *b* must even and the other must be odd. In (2), let us assume for sake of simplicity that x is odd and y is even, since the converse is symmetrical and will not affect the conclusion.

Note: Any perfect square n_2, may be represented as a summation of terms of an arithmetic progression of odd numbers starting from 1 to the n_{th} element. The numbers $x^{2m}$and y^{2m} are perfect squares.

If x is odd, then x_{2m} = 1+3+…+term x_m, where x_m is odd.

And y is even, so y_{2m} = 1+3+… +term y_m, where y_m is even.

Hence, LHS of (2) becomes:

$(1+3+…+\text{term } x_m)_2 + (1+3+… +\text{term } y_m)_2$ → (3)

Since x<y, and both the terms are sum of consecutive odd numbers starting at 1, we can take the first term common.

Let comm = 1+3+…+term x_m and let x_m = O (an odd number). → (4)

Hence, comm = O_2 (through AP summation) → (5)

Then, the second term may be denoted as $(\text{comm} + \text{extra})_2$, where, extra = term (($x_m$ +2)+… +term y_m) where x_m = O (an odd number) and y_m = E (an even number). → (6)

Hence, extra = E_2 – O_2 (through AP summation). → (7)

Substituting 4 and 6 in 3, we get LHS = $comm_2$ + $(comm + extra)_2$

= $comm_2$ + $comm_2$ + 2(comm)(extra) + $extra_2$

= 2(comm)(comm+extra) + $extra_2$ → (8)

Substituting 5 and 7 in 8, we get LHS

= 2 (O_2) (O_2 + (E_2 – O_2)) + $(E_2 – O_2)_2$

= E_2 – O_2 which is the LHS of statement 2.

If statement 2 were true, then E_2 – O_2 must be a power of 4 of some natural number which means it must be a perfect square of some natural number U.

Hence, statement 2 would reduce to E_2 – O_2 = U_2 or;

E_2 = O_2 + U_2 where E is an even natural number and O is an odd natural number. According the Euler's properties of primitive Pythogorean triplets, if O is odd, U must be even. But by deduction, If E is even and O is odd, then U must be odd too, which violates the properties of primitive Pythogorean triplets. This violation is valid for all multiples of primitive Pythogorean triplets through cancellation laws.

Hence, statement 2 has been disproven through proof by contradiction, and statement 1 has been proven to hold true.

Hence;

Statement 1: x^{4m} + y^{4m} ≠ z_{4m} for any x, y, z, m ∈ N (Set of natural numbers) has been proven briefly, successfully and with minimum need of background knowledge.

There are no integer solutions for Fermat's Last Theorem for powers that are multiples of 4.

Quod Erat Demonstrandum