

1. Solve the differential equation:

$$y + x \frac{dy}{dx} = x - y \frac{dy}{dx}$$

2. The two adjacent sides of a parallelogram are  $2\hat{i} - 4\hat{j} - 5\hat{k}$  and  $2\hat{i} + 2\hat{j} + 3\hat{k}$ . Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, Find the area of the parallelogram.
3. Solve the equation for  $x$  :  $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$ .
4. If  $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$ , Prove that  $\frac{x^2}{a^2} - 2 \frac{xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$
5. Differentiate  $x^{\sin x} + (\sin x)^{\cos x}$  with respect to  $x$ .
6. If  $y = 2 \cos(\log x) + 3 \sin(\log x)$ , Prove that  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$ .
7. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.
8. Prove that the curve  $y^2 = 4x$  and  $x^2 = 4y$  divide the area of square bounded by  $x = 0, x = 4$  and  $y = 0$  into three equal parts.
9. Show that the binary operation  $*$  on  $A = \mathbf{R} - \{-1\}$  defined as  $a * b = a + b + ab$  for all  $a, b \in A$  is commutative and associative on  $A$ . Also find the identity element of  $*$  in  $A$  and prove that every element of  $A$  is invertible.
10. Using properties of determinants, Show that  $\triangle ABC$  is isosceles if:
 
$$\begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0.$$
11. A shopkeeper has 3 varieties of pens  $A, B$  and  $C$ . Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of  $A$  variety, 3 pens of  $B$  variety and 2 pens of  $C$  variety for ₹60. While shikha purchased 6 pens of  $A$  variety, 2 pens of  $B$  variety and 3 pens of  $C$  variety for ₹70. Using matrix method, find cost of each variety of pen.
12. There are two types of fertilisers  $A$  and  $B$ .  $A$  consists of 12% nitrogen and 5% phosphoric acid whereas  $B$  consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, Farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If  $A$  cost ₹10 per kg and  $B$  cost ₹8 per kg, Then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.
13. Find the position vector of the foot of perpendicular and the perpendicular distance from the point  $P$  with position vector  $2\hat{i} + 3\hat{j} + 4\hat{k}$  to the plane  $\vec{r} \cdot (2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$ . Also find image of  $P$  in the plane.

14. Prove that the least perimeter of an isosceles triangle in which a circle of radius  $r$  can be inscribed is  $6\sqrt{3}r$ .
15. If the sum of lengths of hypotenuse and a side of a right angled triangle is given, Show that area of triangle is maximum, when the angle between them is  $\frac{\pi}{3}$ .
16. Five bad oranges are accidentally mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.