

1. Let  $A, B, C, D$  be four distinct points on a line, in that order. The circles with diameters  $AC$  and  $BD$  intersect at  $X$  and  $Y$ . The line  $XY$  meets  $BC$  at  $Z$ . Let  $P$  be a point on the line  $XY$  other than  $Z$ . The line  $CP$  intersects the circle with diameter  $AC$  at  $C$  and  $M$ , and the line  $BP$  intersects the circle with diameter  $BD$  at  $B$  and  $N$ . Prove that the lines  $AM, DN, XY$  are concurrent.
2. Let  $a, b, c$  be positive real numbers such that  $abc = 1$ . Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}. \quad (1)$$

3. Determine all integers  $n > 3$  for which there exist  $n$  points  $A_1, \dots, A_n$  in the plane, no three collinear, and real numbers  $r_1, \dots, r_n$  such that for  $1 \leq i < j < k \leq n$ , the area of  $\triangle A_i A_j A_k$  is  $r_i + r_j + r_k$ .
4. Find the maximum value of  $x_0$  for which there exists a sequence  $x_0, x_1, \dots, x_{1995}$  of positive reals with  $x_0 = x_{1995}$ , such that for  $i = 1, \dots, 1995$ ,

$$x_{i-1} + \frac{2}{x_{i-1}} = 2x_i + \frac{1}{x_i}. \quad (2)$$

5. Let  $ABCDEF$  be a convex hexagon with  $AB = BC = CD$  and  $DE = EF = FA$ , such that  $\angle BCD = \angle EFA = \frac{\pi}{3}$ . Suppose  $G$  and  $H$  are points in the interior of the hexagon such that  $\angle AGB = \angle DHE = \frac{2\pi}{3}$ . Prove that  $AG + GB + GH + DH + HE \geq CF$ .
6. Let  $p$  be an odd prime number. How many  $p$ -element subsets  $A$  of  $\{1, 2, \dots, 2p\}$  are there, the sum of whose elements is divisible by  $p$ ?