Nice — I'll trace your **heap sort** on the exact input `vector<int> arr = {9,4,3,8,10,2,5}` and **track** the important values (indices, children, selected `largest`, swaps) step-by-step. I'll show both a human-readable explanation and a compact "console-style" log you can copy.

Input

`[9, 4, 3, 8, 10, 2, 5]` (indices `0..6`, n = 7)

PHASE 1 — Build max-heap

We call `heapify` for `i = n/2 - 1` down to `0` ⇒ `i = 2, 1, 0`.

heapify(i = 2), n = 7

- `l = 2*i+1 = 5`, `r = 2*i+2 = 6`
- values: `arr[2]=3`, `arr[5]=2`, `arr[6]=5`
- compare: left(2) > root(3)? NO. right(5) > root(3)? YES → `largest = 6`
- swap `arr[2]` ↔ `arr[6]` → swap `3` and `5`
- array after swap: `[9, 4, 5, 8, 10, 2, 3]`
- recursive `heapify(6)` → `l=13, r=14` out of range → stop.

State: `[9, 4, 5, 8, 10, 2, 3]`

---

heapify(i = 1), n = 7

- `l = 3`, `r = 4`
- values: `arr[1]=4`, `arr[3]=8`, `arr[4]=10`
- left(8) > 4 → `largest = 3`; right(10) > 8 → `largest = 4`
- swap `arr[1]` ↔ `arr[4]` → swap `4` and `10`
- array: `[9, 10, 5, 8, 4, 2, 3]`
- recursive `heapify(4)` → children out of range → stop.

State: `[9, 10, 5, 8, 4, 2, 3]`

---

heapify(i = 0), n = 7

- `l = 1`, `r = 2`
- values: `arr[0]=9`, `arr[1]=10`, `arr[2]=5`

- left(10) > 9 → `largest = 1`; right(5) > 10? NO
- swap `arr[0]` ↔ `arr[1]` → swap `9` and `10`
- array: `[10, 9, 5, 8, 4, 2, 3]`
- recursive `heapify(1)`: `l=3`, `r=4`, values `arr[1]=9, arr[3]=8, arr[4]=4` → no child > 9 → stop.

Heap built: `[10, 9, 5, 8, 4, 2, 3]` (max-heap)

PHASE 2 — Sorting (extract max, reduce heap size, re-heapify)

We will swap the root `arr[0]` with `arr[i]` and `heapify` on reduced heap size `i`. I'll show each outer-loop step (`i = 6 → 1`).

---

i = 6

- swap `arr[0]` ↔ `arr[6]`: swap `10` and `3`
- array after swap (before heapify): `[3, 9, 5, 8, 4, 2, 10]` (10 is now in final place)
- call `heapify(arr, n=6, i=0)`:
  - `l=1, r=2`, vals `3,9,5` → left 9 > 3 → `largest=1`
  - swap `arr[0]↔arr[1]` → `[9,3,5,8,4,2,10]`
  - heapify(1): `l=3, r=4`, vals `3,8,4` → left 8 > 3 → `largest=3`
  - swap `arr[1]↔arr[3]` → `[9,8,5,3,4,2,10]`
  - heapify(3): children out of range → stop.

State: `[9, 8, 5, 3, 4, 2, 10]`

---

i = 5

- swap `arr[0]` ↔ `arr[5]`: swap `9` and `2`
- before heapify: `[2, 8, 5, 3, 4, 9, 10]` (9 final)
- heapify(arr, n=5, i=0):
  - `l=1, r=2`, vals `2,8,5` → left 8 > 2 → `largest=1`
  - swap → `[8,2,5,3,4,9,10]`
  - heapify(1): `l=3, r=4`, vals `2,3,4` → left 3 > 2 → `largest=3`; right 4 > 3 → `largest=4`
  - swap `arr[1]↔arr[4]` → `[8,4,5,3,2,9,10]`
  - heapify(4): children out of range → stop.

State: `[8, 4, 5, 3, 2, 9, 10]`

---

i = 4

- swap `arr[0]` ↔ `arr[4]`: swap `8` and `2`
- before heapify: `[2,4,5,3,8,9,10]` (8 final)
- heapify(arr, n=4, i=0):
  - `l=1,r=2`, vals `2,4,5` → left 4 > 2 → `largest=1`; right 5 > 4 → `largest=2`
  - swap `arr[0]`↔`arr[2]` → `[5,4,2,3,8,9,10]`
  - heapify(2): children out of range → stop.

State: `[5, 4, 2, 3, 8, 9, 10]`

---

i = 3

- swap `arr[0]` ↔ `arr[3]`: swap `5` and `3`
- before heapify: `[3,4,2,5,8,9,10]` (5 final)
- heapify(arr, n=3, i=0):
  - `l=1,r=2`, vals `3,4,2` → left 4 > 3 → `largest=1`
  - swap `arr[0]`↔`arr[1]` → `[4,3,2,5,8,9,10]`
  - heapify(1): children out of range → stop.

State: `[4, 3, 2, 5, 8, 9, 10]`

---

i = 2

- swap `arr[0]` ↔ `arr[2]`: swap `4` and `2`
- before heapify: `[2,3,4,5,8,9,10]` (4 final)
- heapify(arr, n=2, i=0):
  - `l=1,r=2`, vals `2,3` → left 3 > 2 → `largest=1`
  - swap `arr[0]`↔`arr[1]` → `[3,2,4,5,8,9,10]`
  - heapify(1): children out of range → stop.

State: `[3, 2, 4, 5, 8, 9, 10]`

---

i = 1

- swap `arr[0]` ↔ `arr[1]`: swap `3` and `2`
- array becomes `[2,3,4,5,8,9,10]`
- heapify(arr, n=1, i=0) → no children → stop.

Final sorted (ascending): `[2, 3, 4, 5, 8, 9, 10]`

Initial: [9, 4, 3, 8, 10, 2, 5]

-- Build heap --

heapify(i=2): l=5(r=6), vals=(3,2,5) -> largest=6 -> swap idx2(3) with idx6(5)
-> [9, 4, 5, 8, 10, 2, 3]

heapify(i=1): l=3(r=4), vals=(4,8,10) -> largest=4 -> swap idx1(4) with idx4(10)
-> [9, 10, 5, 8, 4, 2, 3]

heapify(i=0): l=1(r=2), vals=(9,10,5) -> largest=1 -> swap idx0(9) with idx1(10)
-> [10, 9, 5, 8, 4, 2, 3]

-- After build: [10, 9, 5, 8, 4, 2, 3]

-- Sorting --

i=6: swap idx0(10) with idx6(3) -> [3,9,5,8,4,2,10]
heapify(0,n=6): -> swap idx0(3) with idx1(9) -> [9,3,5,8,4,2,10]
heapify(1): -> swap idx1(3) with idx3(8) -> [9,8,5,3,4,2,10]

i=5: swap idx0(9) with idx5(2) -> [2,8,5,3,4,9,10]
heapify(0,n=5): -> swap idx0(2) with idx1(8) -> [8,2,5,3,4,9,10]
heapify(1): -> swap idx1(2) with idx4(4) -> [8,4,5,3,2,9,10]

i=4: swap idx0(8) with idx4(2) -> [2,4,5,3,8,9,10]
heapify(0,n=4): -> swap idx0(2) with idx2(5) -> [5,4,2,3,8,9,10]

i=3: swap idx0(5) with idx3(3) -> [3,4,2,5,8,9,10]
heapify(0,n=3): -> swap idx0(3) with idx1(4) -> [4,3,2,5,8,9,10]

i=2: swap idx0(4) with idx2(2) -> [2,3,4,5,8,9,10]

heapify(0,n=2): -> swap idx0(2) with idx1(3) -> [3,2,4,5,8,9,10]

i=1: swap idx0(3) with idx1(2) -> [2,3,4,5,8,9,10]

done.

Final: [2, 3, 4, 5, 8, 9, 10]

Perfect 👍 — let's visualize **Heap Sort** step-by-step using a **binary tree layout** instead of flat arrays.

Your input:

```
[9, 4, 3, 8, 10, 2, 5]

    9

  /  \

  4    3

 /\   /\

8 10  2  5
```

Build Heap Phase

heapify(i=2)

Check node `3` with children `2` and `5`. Largest is `5`. Swap.

```
    9

  /  \

  4    5

 /\   /\

8 10  2  3
```

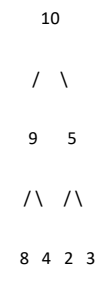
heapify(i=1)

Check node `4` with children `8` and `10`. Largest is `10`. Swap.

```
    9

  /  \

 10    5

 /\   /\

8  4  2  3
```

heapify(i=0)

Check node `9` with children `10` and `5`. Largest is `10`. Swap.

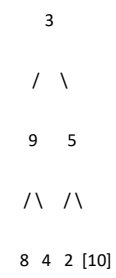


✅ **Max Heap built**

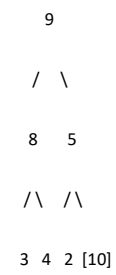
orting Phase

Step i = 6 (swap root with last element)

Swap `10` ↔ `3`

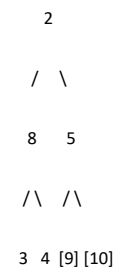


Heapify root: `3` vs `9`, `5` → largest `9`. Swap. Then fix child.

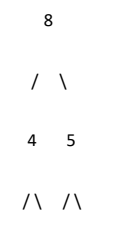


Step i = 5 (swap root with index 5)

Swap `9` ↔ `2`

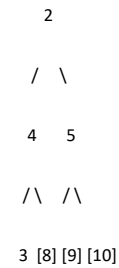


Heapify: `2` vs `8, 5` → swap with `8`. Then `2` vs `3, 4` → swap with `4`.

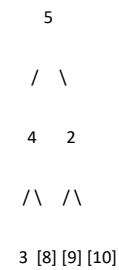


Step i = 4 (swap root with index 4)

Swap `8` ↔ `2`

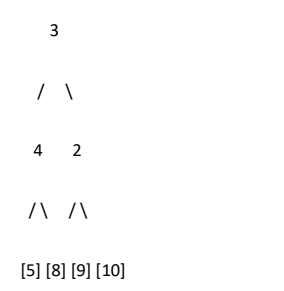


Heapify: `2` vs `4, 5` → swap with `5`.

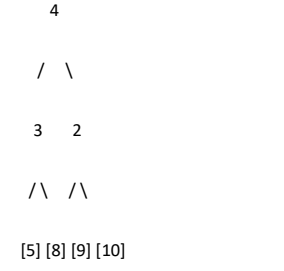


Step i = 3 (swap root with index 3)

Swap `5` ↔ `3`

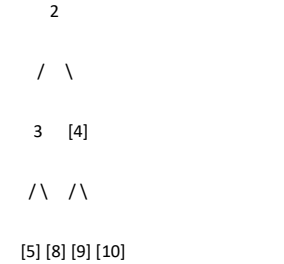


Heapify: `3` vs `4`, `2` → swap with `4`.

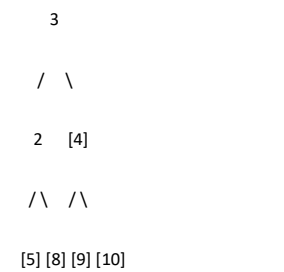


Step i = 2 (swap root with index 2)

Swap `4` ↔ `2`

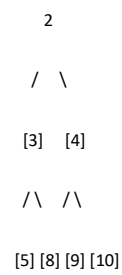


Heapify: `2` vs `3` → swap.



Step i = 1 (swap root with index 1)

Swap 3 ↔ 2



Heapify size=1 → nothing.