EE 779: Assignment 1: Q.2

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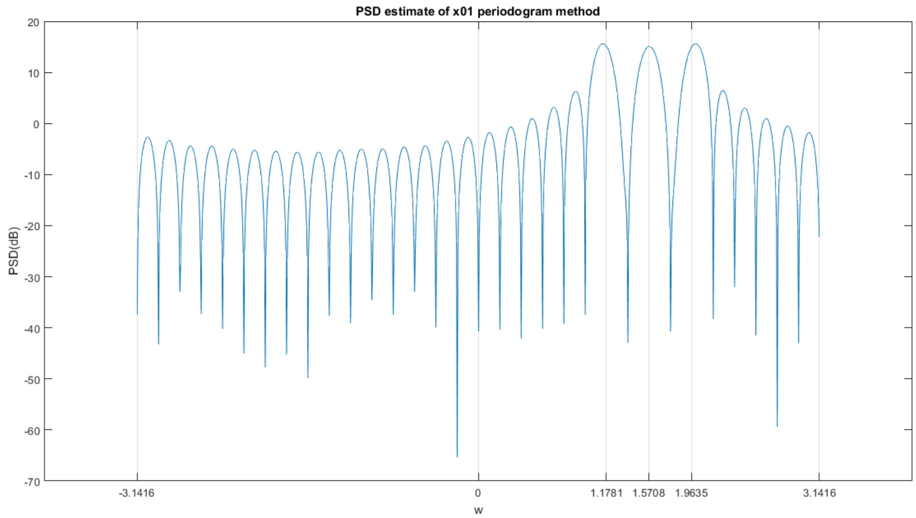
```
close all
clear all

% read input data
I01 = getdata('../assgn3_data/I01.dat')';
I10 = getdata('../assgn3_data/I10.dat')';
I40 = getdata('../assgn3_data/I40.dat')';
R01 = getdata('../assgn3_data/R01.dat')';
R10 = getdata('../assgn3_data/R10.dat')';
R40 = getdata('../assgn3_data/R40.dat')';

x01 = R01 + I01*1i;
x10 = R10 + I10*1i;
x40 = R40 + I40*1i;

x_list = [x01,x10,x40];
k_list = [0.01,0.10,0.40];
```

Q2 (a): periodogra for K = 0.01

Note that complex exponentials are used in input signal hence plots are shown for -pi to pi instead of just 0 to pi

```
fft_len = 1024;
N = length(x01);
% zero padding signal
x_padded = zeros(fft_len,1);
```

```
x_padded(1:N) = x01(1:N);
fft_xN = fftshift(fft(x_padded, fft_len));
freq = linspace(-pi,pi,fft_len);
prdgrm_psd_x01 = (abs(fft_xN).^2)/N;
fig = figure;
plot(freq,10*log10(prdgrm_psd_x01));
title('PSD estimate of x01 periodogram method');
xlabel('w');
ylabel('PSD(dB)')
set(gcf, 'Position', get(0, 'Screensize'));
set(gca,'xtick',[-pi,0,3*pi/8,pi/2,5*pi/8,pi]);
set(gca,'xgrid','on');
saveas(fig,'../results/Q2/PSD estimate of x01 periodogram
 method','jpg');
```

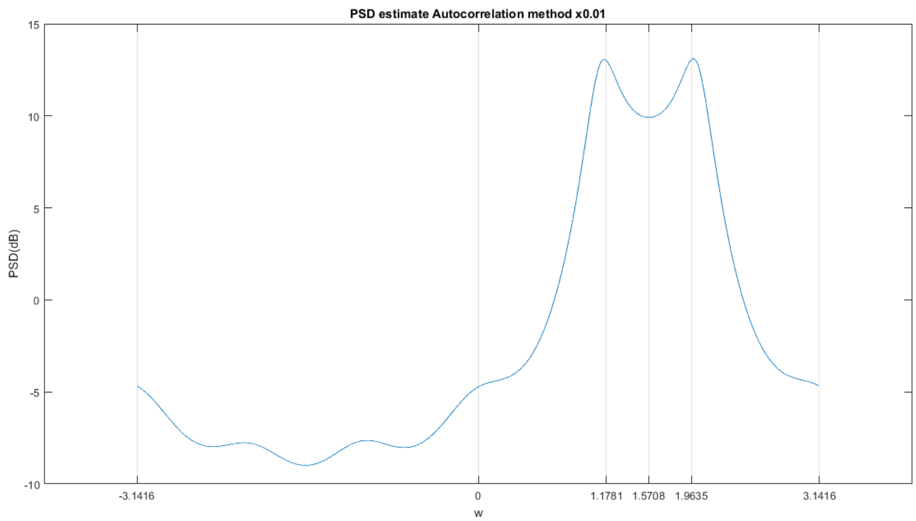


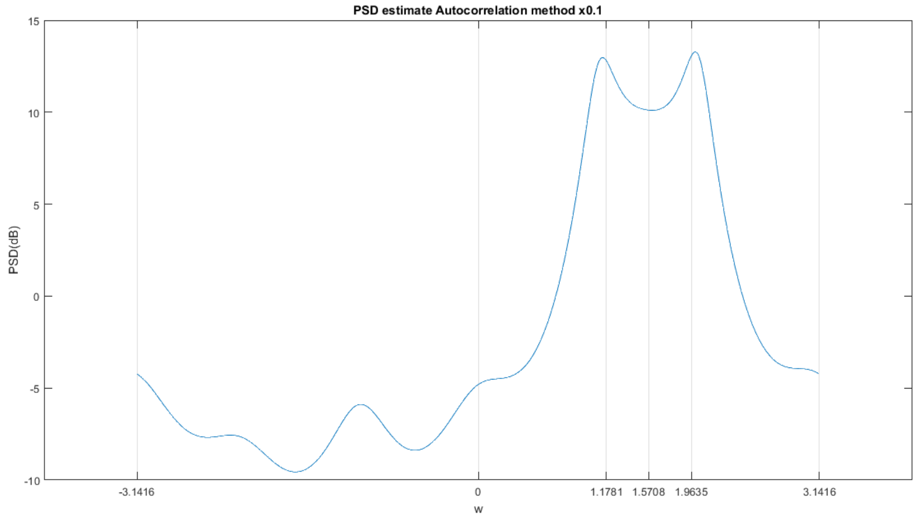
Q2 (b) Spectral Estimates for following methods

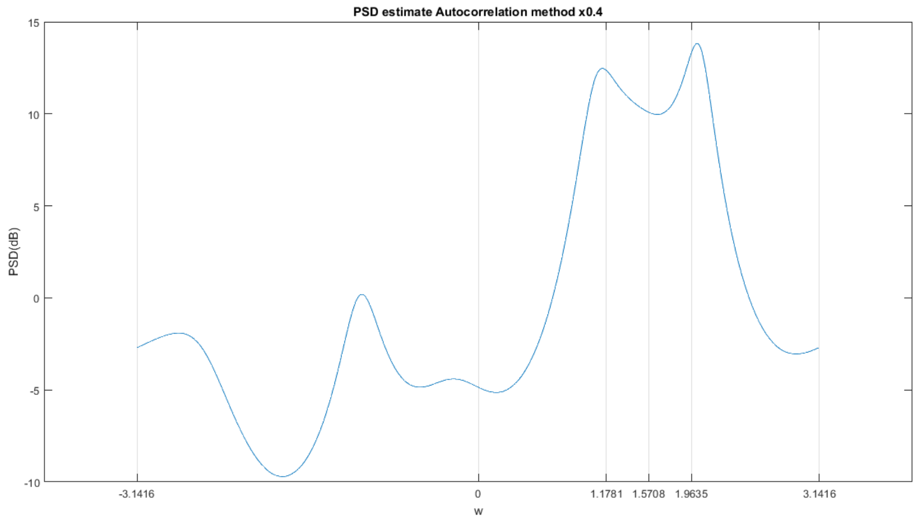
(i) : AR model autocorrelation (p = 7)

```
function [psd_AR,freq] = AR_autocorrelation_spectrum(x,p,K)
r = find_correlation(x);
[X,Rxx_autocorr_AR] = corrmtx(x,p,'autocorrelation');
%Rxx_autocorr_AR = X'*X;
temp = Rxx_autocorr_AR(2:end,2:end);
r_AR = -r(2:p+1);
a_AR = temp\r_AR;
a_AR = [1,a_AR.'].';
error_sq_AR = 0;
for j = 1:p+1
   error_sq_AR = error_sq_AR + a_AR(j)*conj(r(j));
```

```
end

error_AR = sqrt(abs(error_sq_AR));

[h,w] = freqz(error_AR,a_AR,'whole',1024);
psd_AR = abs(fftshift(h)).^2;
freq = w - pi;
end


for k = 1:length(k_list)
    x = x_list(:,k);
    [AR_autocorr_psd,freq] =
 AR_autocorrelation_spectrum(x,7,k_list(k));
    fig = figure;
    plot(freq,10*log10(AR_autocorr_psd));
    title(['PSD estimate Autocorrelation method
 x',num2str(k_list(k))]);
    xlabel('w');
    ylabel('PSD(dB)');
    set(gcf, 'Position', get(0, 'Screensize'));
    set(gca,'xtick',[-pi,0,3*pi/8,pi/2,5*pi/8,pi]);
    set(gca,'xgrid','on');
    saveas(fig,['../results/Q2/','PSD estimate Autocorrelation method
 x',num2str(k_list(k))],'jpg');
end
```



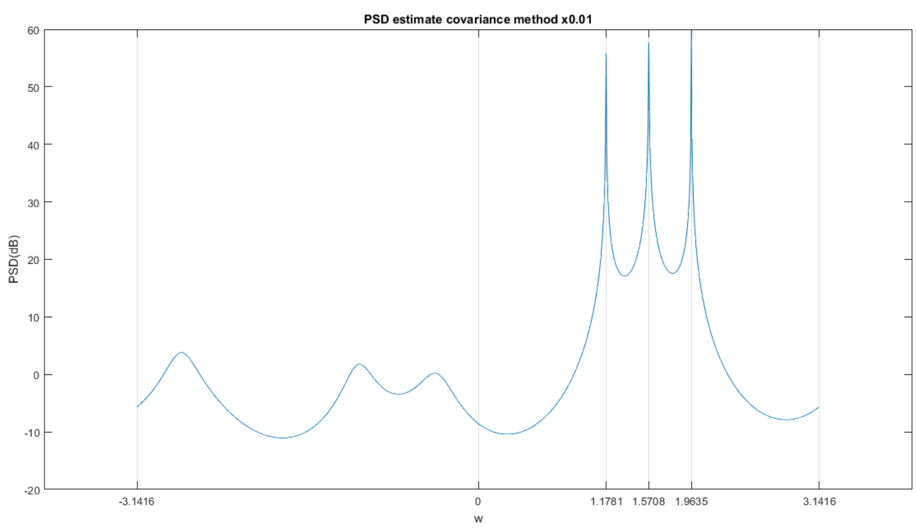


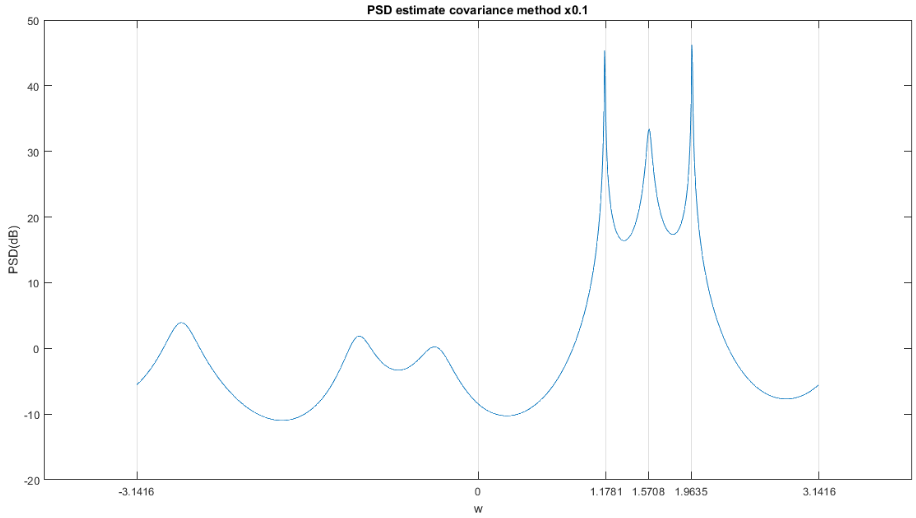


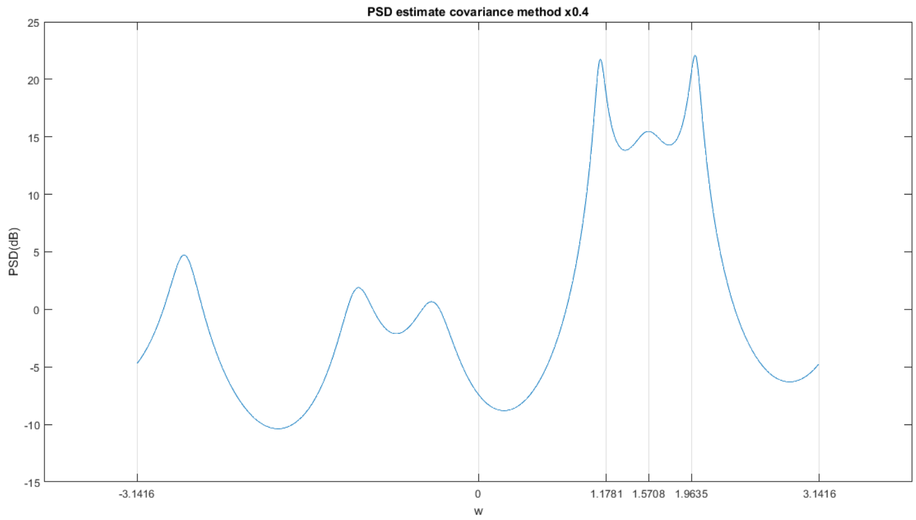
(ii) : AR model covariance (p = 7)

```
function [psd_covar,freq] = AR_covariance_spectrum(x,p,K)
N = length(x);

X = corrmtx(x,p,'covariance');
Rxx_covar = X'*X;
temp = Rxx_covar(2:length(Rxx_covar),2:length(Rxx_covar));
r_covar = -Rxx_covar(2:end,1);
a_covar = inv(temp)*r_covar;
a_covar = [1,a_covar.'].';
error_sq_covar = 0;
for j = 1:p+1
   error_sq_covar = error_sq_covar + a_covar(j)*conj(Rxx_covar(j,1));
end
```

```
error_covar = sqrt(abs(error_sq_covar));

[h,w] = freqz(1,a_covar,'whole',1024);
psd_covar = abs(fftshift(h)).^2;
freq = w - pi;
end


for k = 1:length(k_list)
    x = x_list(:,k);
    [AR_covar_psd,freq] = AR_covariance_spectrum(x,7,k_list(k));
    fig = figure;
    plot(freq,10*log10(AR_covar_psd));
    title(['PSD estimate covariance method x',num2str(k_list(k))]);
    xlabel('w');
    ylabel('PSD(dB)');
    set(gcf, 'Position', get(0, 'Screensize'));
    set(gca,'xtick',[-pi,0,3*pi/8,pi/2,5*pi/8,pi]);
    set(gca,'xgrid','on');
    saveas(fig,['../results/Q2/','PSD estimate covariance method
 x',num2str(k_list(k))],'jpg');
end
```



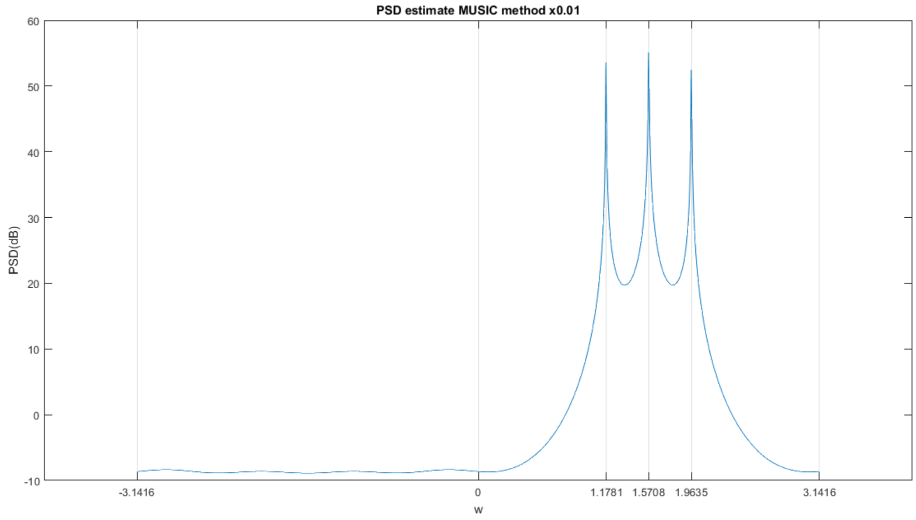


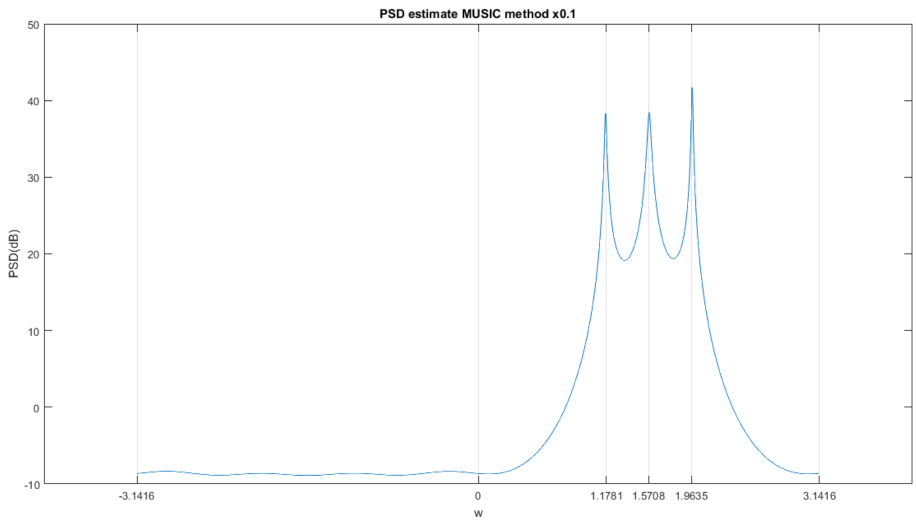


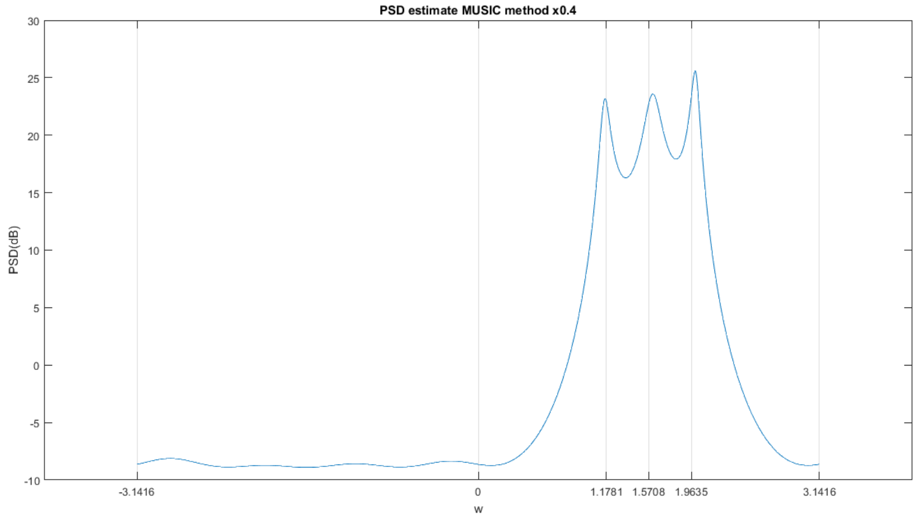
(iii): MUSIC method (8*8 covariance method matrix)

```
function [MUSIC_psd,freq] = MUSIC_spectrum(x,M,P,K)

N = length(x);
Rxx_covar = zeros(M+1,M+1);

for i = 0:M
    for j = 0:M
        sum = 0;
        for n = M+1:N
            sum = sum + x(n-j)*conj(x(n-i));
        end
```

```
        Rxx_covar(j+1,i+1) = sum;
    end
end

X = corrmtx(x,M,'covariance');
Rxx_covar = X'*X;

M = length(Rxx_covar(:,1));
[V,lambda] = eig(Rxx_covar);
spectrum_len = 1024;
freq = linspace(-pi,pi,spectrum_len);
sum_denom = zeros(1,spectrum_len);
for j = 1:length(freq)
    w = freq(j);
    sum_temp = 0;
    for i = 1:M-P
        v_i = V(:,i);
        temp = 0;
        for k = 1:M
            temp = temp + (exp(-1i*(k-1)*w))*v_i(k);
        end
        sum_temp = sum_temp + (abs(temp))^2;
    end
    sum_denom(j) = sum_temp;
end
MUSIC_psd = 1./sum_denom;
MUSIC_psd = MUSIC_psd';
end



for k = 1:length(k_list)
    x = x_list(:,k);
    [MUSIC_psd,freq] = MUSIC_spectrum(x,7,3,k_list(k));
    fig = figure;
    plot(freq,10*log10(MUSIC_psd));
    title(['PSD estimate MUSIC method x',num2str(k_list(k))]);
    xlabel('w');
    ylabel('PSD(dB)');
    set(gcf, 'Position', get(0, 'Screensize'));
    set(gca,'xtick',[-pi,0,3*pi/8,pi/2,5*pi/8,pi]);
    set(gca,'xgrid','on');
    saveas(fig,['../results/Q2/','PSD estimate MUSIC method
 x',num2str(k_list(k))],'jpg');
end
```


PSD estimate MUSIC method x0.01
PSD(dB)
w
-3.1416
0
1.1781
1.5708
1.9635
3.1416

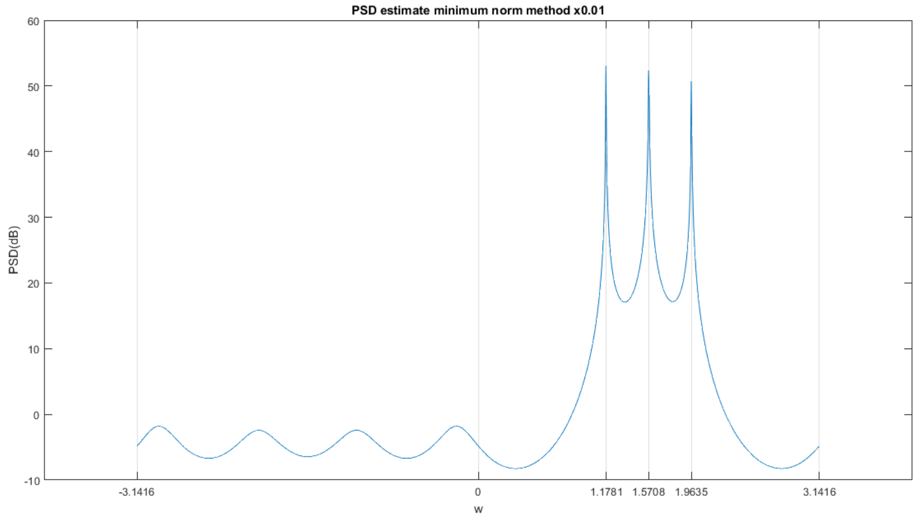

PSD estimate MUSIC method x0.1
PSD(dB)
w
-3.1416
0
1.1781
1.5708
1.9635
3.1416

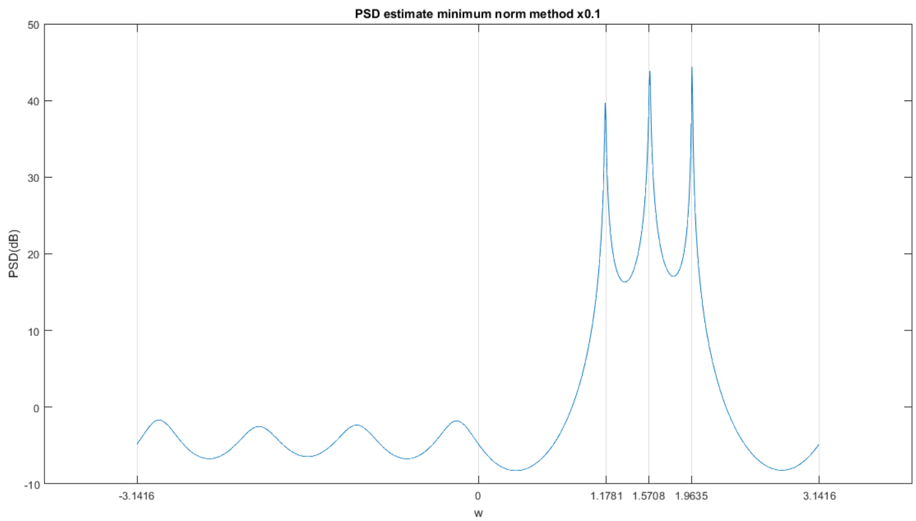


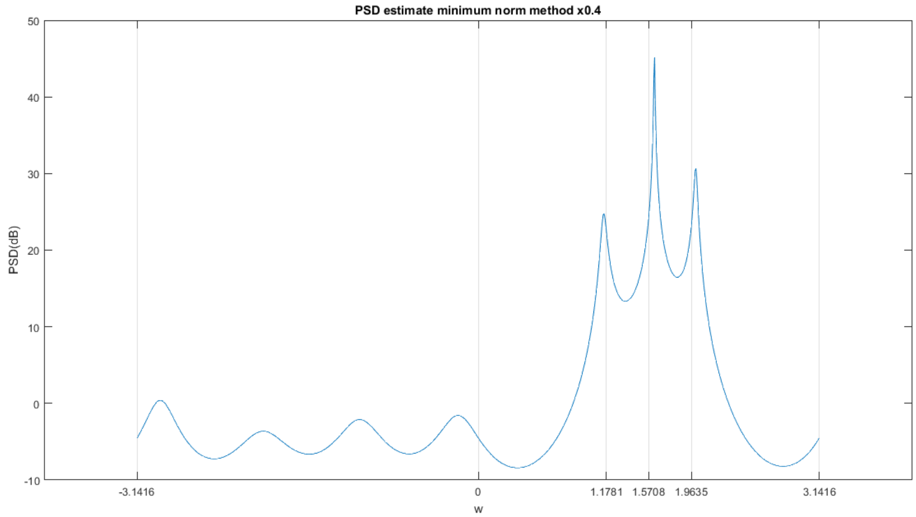
(iv) : Minimum Norm method (8*8 covariance method matrix)

```
function [minimum_norm_psd,freq] = minimum_norm_spectrum(x,M,P,K)
N = length(x);
Rxx_covar = zeros(M+1,M+1);

for i = 0:M
    for j = 0:M
        sum = 0;
        for n = M+1:N
            sum = sum + x(n-j)*conj(x(n-i));
        end
        Rxx_covar(j+1,i+1) = sum;
    end
end

X = corrmtx(x,M,'covariance');
Rxx_covar = X'*X;

M = length(Rxx_covar(:,1));
[V,lambda] = eig(Rxx_covar);
V_noise = V(:,1:M-P);
u = zeros(1,M)';
u(1) = 1;

num = (V_noise*V_noise')*u;
den = u'*(V_noise*V_noise')*u;
a = num./den;

spectrum_len = 1024;
freq = linspace(-pi,pi,spectrum_len);
```

```
sum_denom = zeros(1,spectrum_len);


for j = 1:length(freq)
    w = freq(j);
    sum_temp = 0;
    for k = 1:M
        sum_temp = sum_temp + (exp(-1i*(k-1)*w))*a(k);
    end
    sum_denom(j) = (abs(sum_temp))^2;
end

minimum_norm_psd = 1./sum_denom;


minimum_norm_psd = minimum_norm_psd';

end



for k = 1:length(k_list)
    x = x_list(:,k);
    [minimum_norm_psd,freq] = minimum_norm_spectrum(x,7,3,k_list(k));
    fig = figure;
    plot(freq,10*log10(minimum_norm_psd));
    title(['PSD estimate minimum norm method x',num2str(k_list(k))]);
    xlabel('w');
    ylabel('PSD(dB)');
    set(gcf, 'Position', get(0, 'Screensize'));
    set(gca,'xtick',[-pi,0,3*pi/8,pi/2,5*pi/8,pi]);
    set(gca,'xgrid','on');
    saveas(fig,['../results/Q2/','PSD estimate minimum norm method
 x',num2str(k_list(k))],'jpg');

end
```


PSD estimate minimum norm method x0.01
PSD(dB)
w
60
50
40
30
20
10
0
-10
-3.1416
0
1.1781
1.5708
1.9635
3.1416


PSD estimate minimum norm method x0.1
PSD(dB)
w
50
40
30
20
10
0
-10
-3.1416
0
1.1781
1.5708
1.9635
3.1416

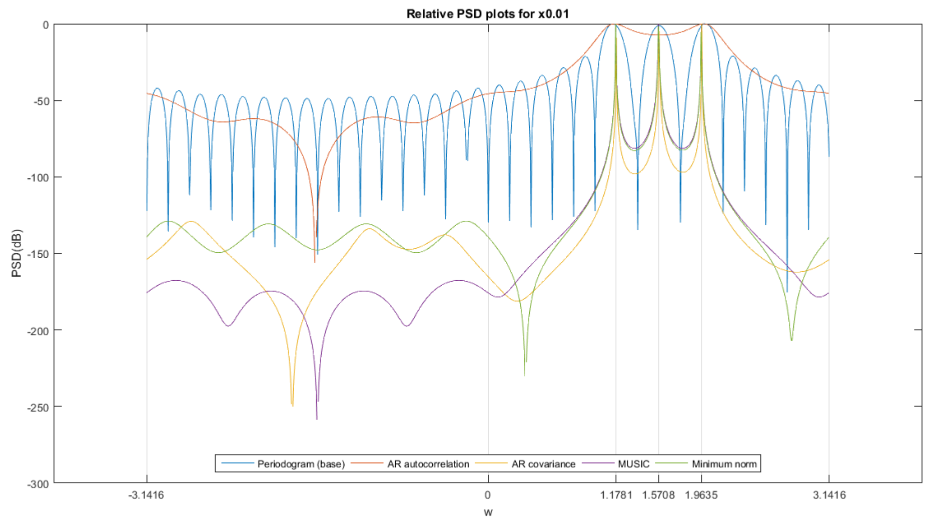


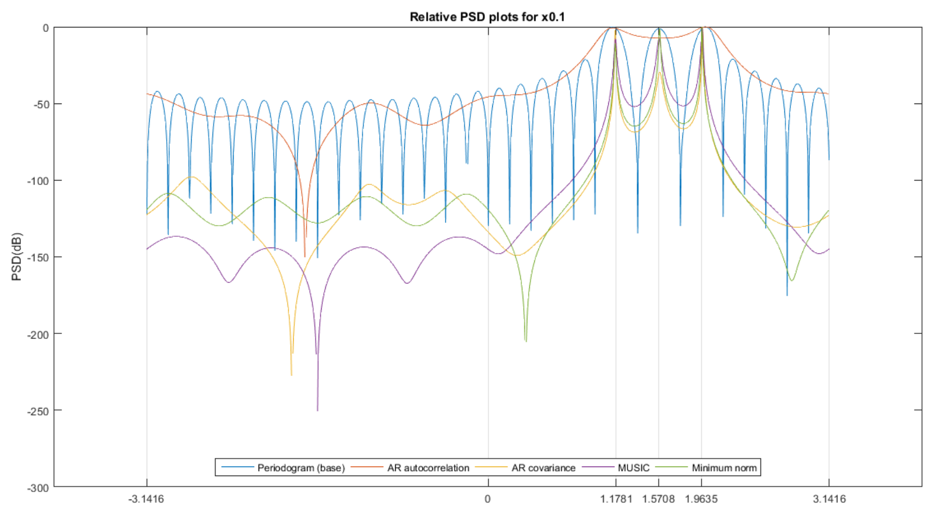
All plots combined

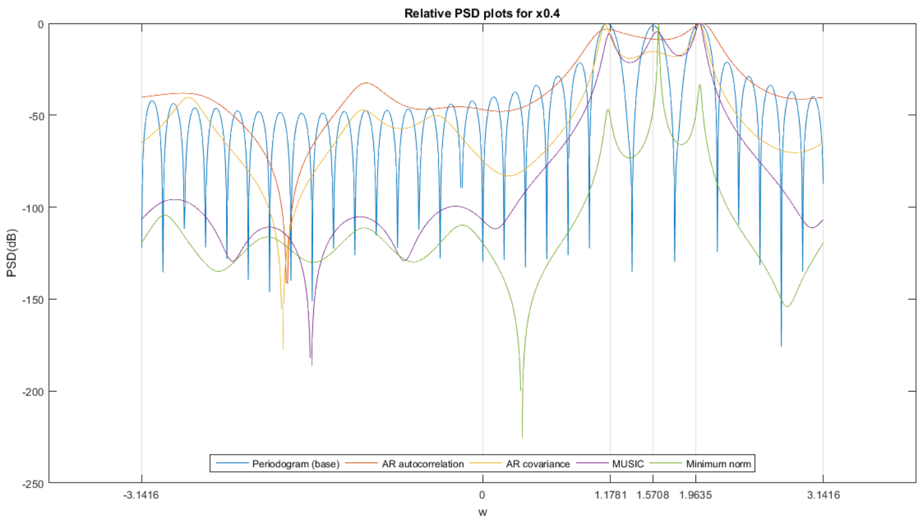
Here all the plots are scaled for comparison.

```
for k = 1:length(k_list)
    x = x_list(:,k);

    [AR_autocorr_psd,freq] =
 AR_autocorrelation_spectrum(x,7,k_list(k));
    AR_autocorr_psd= (AR_autocorr_psd-min(AR_autocorr_psd))./
(max(AR_autocorr_psd)-min(AR_autocorr_psd));
    [AR_covar_psd,freq] = AR_covariance_spectrum(x,7,k_list(k));
    AR_covar_psd = (AR_covar_psd-min(AR_covar_psd))./
(max(AR_covar_psd)-min(AR_covar_psd));
    [MUSIC_psd,freq] = MUSIC_spectrum(x,7,3,k_list(k));
    MUSIC_psd = (MUSIC_psd-min(MUSIC_psd))./(max(MUSIC_psd)-
min(MUSIC_psd));
    [minimum_norm_psd,freq] = minimum_norm_spectrum(x,7,3,k_list(k));
    minimum_norm_psd = (minimum_norm_psd-min(minimum_norm_psd))./
(max(minimum_norm_psd)-min(minimum_norm_psd));

    prdgrm_psd_x01 = (prdgrm_psd_x01 - min(prdgrm_psd_x01))./
(max(prdgrm_psd_x01)-min(prdgrm_psd_x01));

    all_plots =
 10*log([prdgrm_psd_x01,AR_autocorr_psd,AR_covar_psd,MUSIC_psd,minimum_norm_psd]);
    fig = figure;
    plot(freq,all_plots);
    title(['Relative PSD plots for x',num2str(k_list(k))]);
    xlabel('w');
    ylabel('PSD(dB)');
    lgnd = legend('Periodogram (base)','AR autocorrelation','AR
 covariance','MUSIC','Minimum norm');
    lgnd.Orientation = 'horizontal';
```

```
    lgnd.Location = 'south';
    set(gcf, 'Position', get(0, 'Screensize'));
    set(gca,'xtick',[-pi,0,3*pi/8,pi/2,5*pi/8,pi]);
    set(gca,'xgrid','on');
    saveas(fig,['../results/Q2/','Relative PSD plots for
 x',num2str(k_list(k))],'jpg');

end
```







Comparison of different methods

- AR autocorrelation method is not able to capture all three frequencies for any value of noise. It is not suitable for distinguishing closely spaced frequencies.
- For the lesser values of noise (0.01), AR covariance, MUSIC and minimum norm all three work well and produce peaks at given frequencies
- For the larger values of noise (0.4), AR covarianc is not able to capture the middle peak. Also the quality of peaks produced MUSIC method decreases. Minimum norm method prodcues better peaks compared to others for higher values of noise
- We also observe deviation of peaks from ideal input frequencies (which are displayed as vertical lines in PSD plots). The deviation is observed to increase as noise increases. Deviation from actual frequencies: Minimum norm < MUSIC < AR covariance < AR autocorrelation
- Energy in sidelobes (bands other other than peaks) can also be compared to observe quality of peaks. Quality of peaks degrade with amount of noise. Also it depends on different methods. Energy in sidelobes: Minimum norm < MUSIC < AR covariance < AR autocorrelation
- Overall performance for frequency estimation in line spectrum is: Minimum norm > MUSIC > AR covariance > AR autocorrelation

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