**EE 779 Advanced Topics in Signal Processing**
**Assignment 2**
**Assigned: 24/08/16, Due: 1/09/16**
**Indian Institute of Technology Bombay**

## Note

- Submit the starred ($*$) problems and **all** simulations.
- For the simulation problems, you can use the Matlab functions provided by S & M [1] or Hayes [2]. A copy of the files is in moodle.

## Problems

1. [$*$]The estimated autocorrelation sequence of a random process $x(n)$ is

$$r_x(k) = 2, 1, 1, 0.5, 0; \text{ for lags } k = 0, 1, 2, 3, 4;$$

   Estimate the power spectrum of $x(n)$ for each of the following cases.

   (a) $x(n)$ is an AR(2) process.

   (b) $x(n)$ is an MA(2) process.

   (c) $x(n)$ is an ARMA(1,1) process.

   **Note:** Take a look at a short reference from Hayes in moodle (or) refer to page number 191 in Hayes book, for one way to solve the ARMA process.

2. Given the autocorrelation sequence

$$r_x(k) = 1, 0.8, 0.5, 0.1; \text{ for lags } k = 0, 1, 2, 3;$$

   find the reflection coefficients, $\Gamma_j$, the model parameters, $a_j(k)$ and the modeling errors, $\epsilon_j$, for $j = 1, 2, 3$. Use Levinson Durbin's algorithm.

3. [$*$]Determine whether the following statements are True or False. Justify.

   (a) If $r_x(k)$ is an autocorrelation sequence with $r_x(k) = 0$ for $|k| > p$, then $\Gamma_k = 0$ for $|k| > p$.

   (b) Given an autocorrelation sequence, $r_x(k)$ for $k = 0, \ldots, p$, if the $(p+1) \times (p+1)$ Toeplitz matrix

$$\mathbf{R}_p = Toep\{r_x(0), r_x(1), \ldots, r_p(p)\}$$

   is positive definite, then

$$\mathbf{r}_x = [r_x(0), r_x(1), \ldots, r_x(p), 0, 0, \ldots]^T$$

   will always be a valid autocorrelation sequence, i.e., extending $r_x(k)$ with zeros is a valid autocorrelation extension. Note: A Toeplitz matrix is described by one of its column or row.

   (c) If $r_x(k)$ is periodic, then $\Gamma_j$ will be periodic with the same period.

4. A random process may be classified in terms of the properties of the prediction error sequence $\epsilon_k$ that is produced when fitting an all–pole model to the process. Listed below are different classifications for the error sequence:

   (a) $\epsilon_k = c > 0$ for all $k \geq 0$.

   (b) $\epsilon_k = c > 0$ for all $k \geq k_0$ for some $k_0 > 0$.

   (c) $\epsilon_k \to c$ as $k \to \infty$ where $c > 0$.

   (d) $\epsilon_k \to 0$ as $k \to \infty$

(e) $\epsilon = 0$ for all $k \geq k_0$ for some $k_0 > 0$.

For each of these classifications, describe as completely as possible the characteristics that may be attributed to the process and its power spectrum.

5. [∗]Estimates are made of the correlation function of a particular signal and the values obtained are: $r_x(0) = 7.24, r_x(1) = 3.6$. Determine the parameters of the MA(1) model:

$$H(z) = b_0 + b_1 z^{-1},$$

which matches these correlation values using:

(a) Direct solution of the Yule-Walker MA equations.

(b) By spectral factorization.

Sketch the power spectral estimate obtained using this MA model. Fit a AR(1) model for the given correlation data and sketch the resulting spectral estimate. Is this estimate better than that obtained using the MA model ?

**Reference**

1. Petre Stoica and Randolph Moses, "Spectral analysis of signals", Prentice Hall, 2005. (Indian edition available)

2. Monson H. Hayes, "Statistical signal processing and modeling", Wiley India Pvt. Ltd., 2002. (Indian edition available)

3. Charles W. Therrien, "Discrete random signals and statistical signal processing", Charles W. Therrien, 2004.