Maximum Likelihood Reconstruction for Emission Tomography
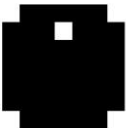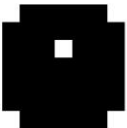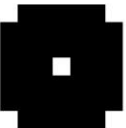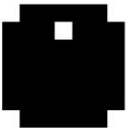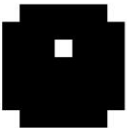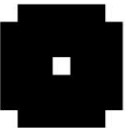
Team Members:
1. Sanket Barhate (120070010)
2. Kalpesh Patil (130040019)
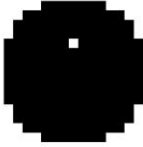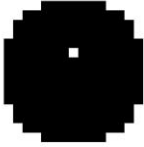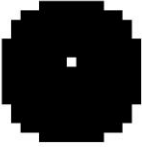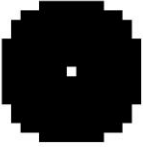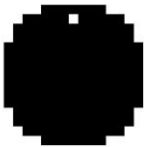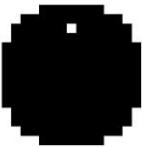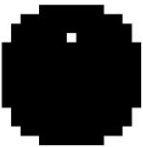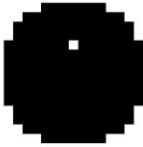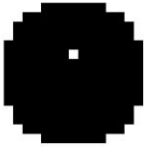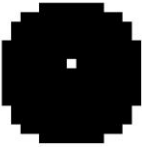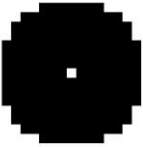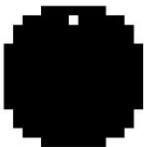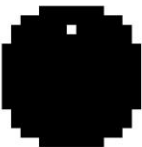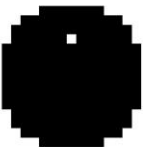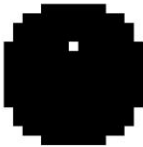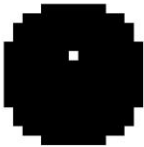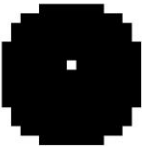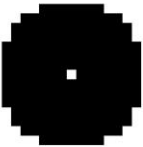
Results:

Number of detectors = 40
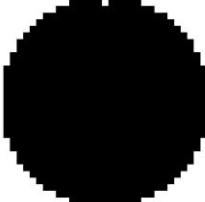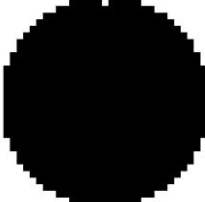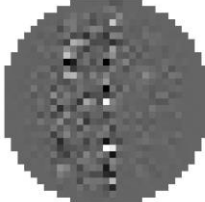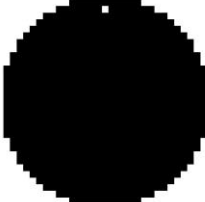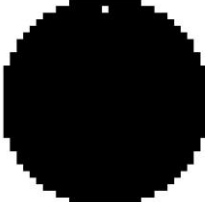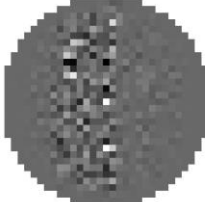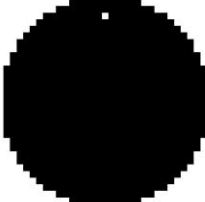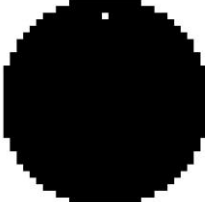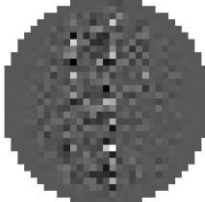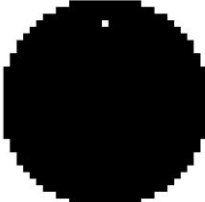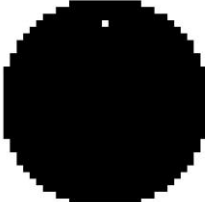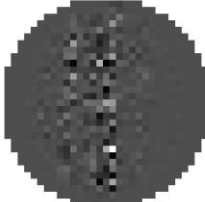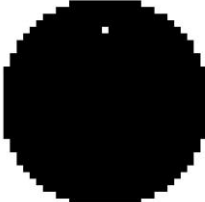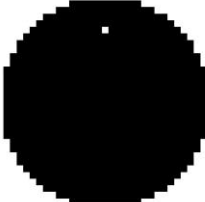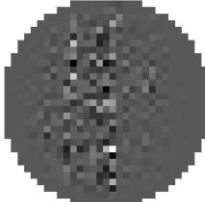
- Reconstruction of Single Pixel Impulse
  1. Resolution = 4 (9*9 image)

2. Resolution = 8 (17*17 image)

Input Images

EM Output

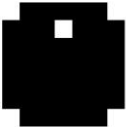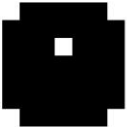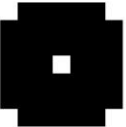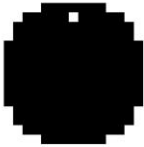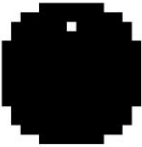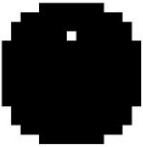

Inversion Output

3. Resolution = 16 (33*33 image)
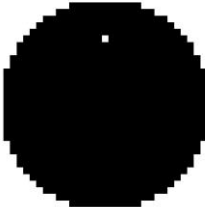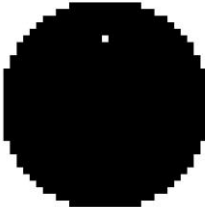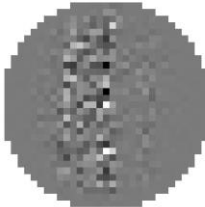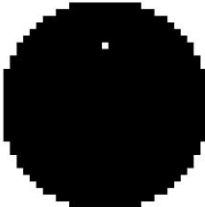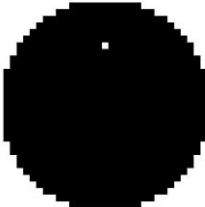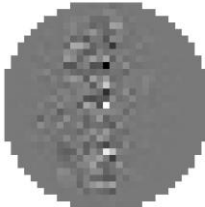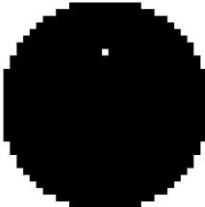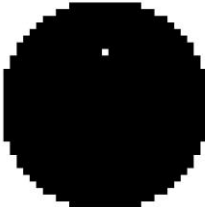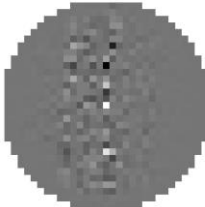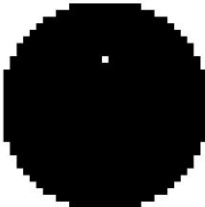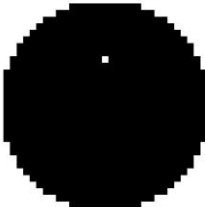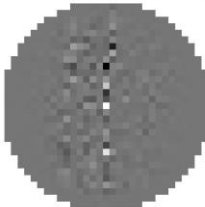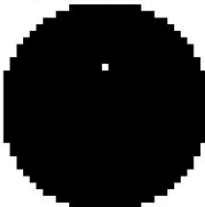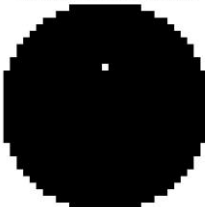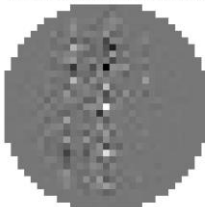
Input Image	EM Output	Inversion
Input Image with Resolution 16 and Position 2	Output Image of EM with Resolution 16 and Position 2	Output Image of Inversion with Resolution 16 and Position 2
Input Image with Resolution 16 and Position 3	Output Image of EM with Resolution 16 and Position 3	Output Image of Inversion with Resolution 16 and Position 3
Input Image with Resolution 16 and Position 4	Output Image of EM with Resolution 16 and Position 4	Output Image of Inversion with Resolution 16 and Position 4
Input Image with Resolution 16 and Position 5	Output Image of EM with Resolution 16 and Position 5	Output Image of Inversion with Resolution 16 and Position 5
Input Image with Resolution 16 and Position 6	Output Image of EM with Resolution 16 and Position 6	Output Image of Inversion with Resolution 16 and Position 6

Input Image with Resolution 16 and Position 7 — Output Image of EM with Resolution 16 and Position 7 — Output Image of Inversion with Resolution 16 and Position 7

Input Image with Resolution 16 and Position 8 — Output Image of EM with Resolution 16 and Position 8 — Output Image of Inversion with Resolution 16 and Position 8

Input Image with Resolution 16 and Position 9 — Output Image of EM with Resolution 16 and Position 9 — Output Image of Inversion with Resolution 16 and Position 9

Input Image with Resolution 16 and Position 10 — Output Image of EM with Resolution 16 and Position 10 — Output Image of Inversion with Resolution 16 and Position 10

Input Image with Resolution 16 and Position 11 — Output Image of EM with Resolution 16 and Position 11 — Output Image of Inversion with Resolution 16 and Position 11

Input Image with Resolution 16 and Position 12   Output Image of EM with Resolution 16 and Position 12   Output Image of Inversion with Resolution 16 and Position 12

Input Image with Resolution 16 and Position 13   Output Image of EM with Resolution 16 and Position 13   Output Image of Inversion with Resolution 16 and Position 13

Input Image with Resolution 16 and Position 14   Output Image of EM with Resolution 16 and Position 14   Output Image of Inversion with Resolution 16 and Position 14

Input Image with Resolution 16 and Position 15   Output Image of EM with Resolution 16 and Position 15   Output Image of Inversion with Resolution 16 and Position 15

Input Image with Resolution 16 and Position 16   Output Image of EM with Resolution 16 and Position 16   Output Image of Inversion with Resolution 16 and Position 16
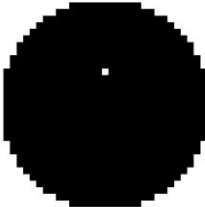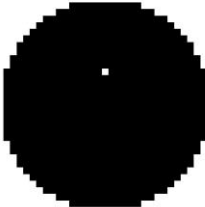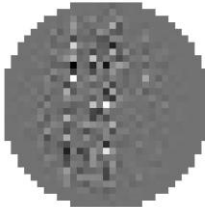
Input Image with Resolution 16 and Position 17 | Output Image of EM with Resolution 16 and Position 17 | Output Image of Inversion with Resolution 16 and Position 17
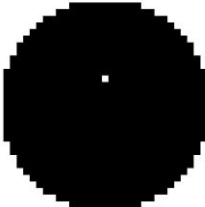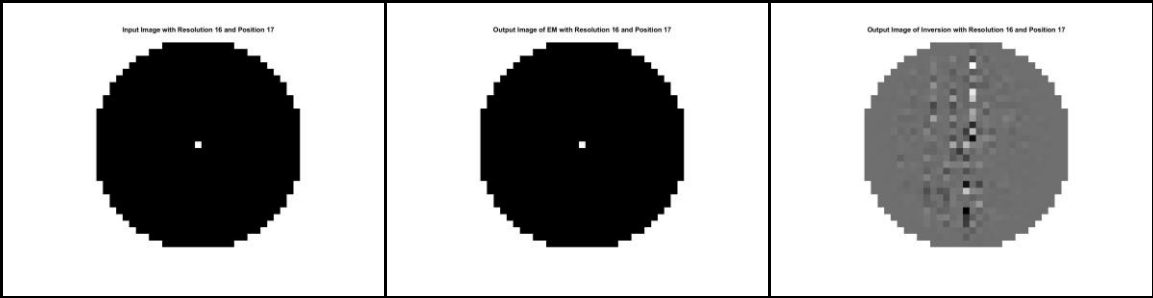
- Reconstruction of Binary Image
  Number of detectors = 80
  Resolution = 32 (65*65 image)

Images	Description
logo_input	Input Binary Image (logo)

logo_EM	EM Output
logo_Inversion	Inversion Output

Theory Concerning Project

Here $'b'$ denote sites. There are $'B'$ sites in total. $'d'$ denotes detector tube and there are $'D'$ tubes in total. $P(b,d)$ is the probability that emission in $'b'$ is detected in tube $'d'$. The emission at site $'b'$ is assumed to be Poisson with mean $\lambda(b)$.

$n^*(d)$ is the total number of emissions detected in tube d during acquisition (real data).
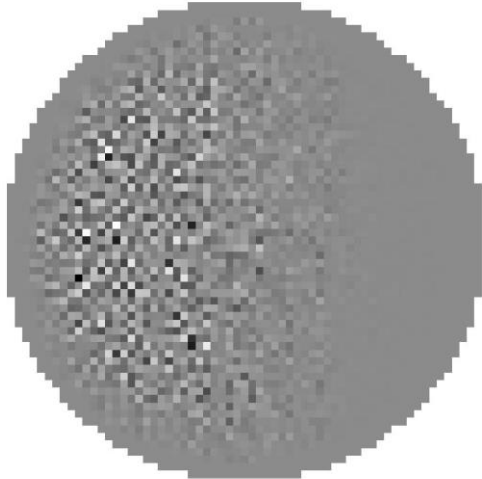
$n(b,d)$ denotes the number of emissions in $'b'$ detected in $'d'$. $n(b,d)$ are mutually independent Poisson random variable, across $'b'$, across $'d'$ (split of Poisson process independent).

$$n(b,d) \sim Poisson\ (\lambda(b,d))$$

$$\lambda(b,d) = \lambda(b)p(b,d)$$

$$\sum_{d=1}^{D} p(b,d) = 1$$

$$n^*(d) = \sum_{b=1}^{B} n(b,d)$$

$$\lambda^*(d) = E[n^*(d)] = \sum_{b=1}^{B} \lambda(b)p(b,d)$$

Best estimate of $\lambda^*(d)$ is $n^*(d)$

$$n^*(d) = \sum_{b=1}^{B} \lambda(b)p(b,d) \qquad for\ d = 1,2,\dots D$$

Above sets of linear equations form the basis for inversion reconstruction.

Now let's move onto expectation maximization. If we have $n(b,d)$ as hidden random variable then estimation of $\lambda(b)$ becomes very easy.

So,

$$X = hidden\ variable = n(b,d) = x$$

$$Y = observed\ data = n^*(d) = y$$

$$Likelihood = p(x,y) = \prod_{\substack{b=1,2\dots B \\ d=1,2\dots D}} \frac{e^{-\lambda(b,d)}\big(\lambda(b,d)\big)^{n(b,d)}}{n(b,d)!}$$

Each $n(b,d)$ poisson process is independent

$$\sum_{b=1}^{B} n(b,d) = n^*(d) \quad for\ all\ 'd'$$

$$\log(Likelihood) = \sum_{d=1}^{D}\sum_{b=1}^{B} -\lambda(b,d) + n(b,d)\log(\lambda(b,d)) - \log(n(b,d)!)$$

$\log(n(b,d)!)$ of no use since it doesn't contain parameter $\lambda(b,d)$

$$E[n(b,d)| n^*(d)] = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b=1}^{B}\lambda^i(b,d)} \qquad \lambda^i(b,d) = estimate\ at\ iteration\ i$$

$$E[\log(Likelihood)] = \sum_{d=1}^{D}\sum_{b=1}^{B} -\lambda(b,d) + \frac{n^*(d)\lambda^i(b,d)\log(\lambda(b,d))}{\sum_{b'=1}^{B}\lambda^i(b',d)} = Q(\theta,\theta^i)$$

Differentiating above expression respect to $\lambda(b,d)$ to get maxima

$$\lambda^{i+1}(b,d) = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b'=1}^{B}\lambda^i(b',d)}$$

$$\lambda^i(b,d) = \lambda^i(b)p(b,d) \qquad and \qquad \lambda(b) = \sum_{d=1}^{D}\lambda(b,d)$$

$$\lambda^{new}(b) = \lambda^{old}(b)\sum_{d=1}^{D}\frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B}\lambda^{old}(b')p(b',d)}$$

Final formula for update is given below

$$\lambda^{new}(b) = \lambda^{old}(b)\sum_{d=1}^{D}\frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B}\lambda^{old}(b')p(b',d)}$$