Here $'b'$ denote sites. There are $'B'$ sites in total. $'d'$ denotes detector tube and there are $'D'$ tubes in total. $P(b,d)$ is the probability that emission in $'b'$ is detected in tube $'d'$. The emission at site $'b'$ is assumed to be Poisson with mean $\lambda(b)$.

$n^*(d)$ is the total number of emissions detected in tube d during acquisition (real data).

$n(b,d)$ denotes the number of emissions in $'b'$ detected in $'d'$. $n(b,d)$ are mutually independent Poisson random variable, across $'b'$, across $'d'$ (split of Poisson process independent).

$$n(b,d) \sim Poisson\ (\lambda(b,d))$$

$$\lambda(b,d) = \lambda(b)p(b,d)$$

$$\sum_{d=1}^{D} p(b,d) = 1$$

$$n^*(d) = \sum_{b=1}^{B} n(b,d)$$

$$\lambda^*(b,d) = E[n^*(d)] = \sum_{b=1}^{B} \lambda(b)p(b,d)$$

Best estimate of $\lambda^*(d)$ is $n^*(d)$

$$n^*(d) = \sum_{b=1}^{B} \lambda(b)p(b,d) \qquad for\ d = 1,2, \dots D$$

Above sets of linear equations form the basis for inversion reconstruction.

Now let's move onto expectation maximization. If we have $n(b,d)$ as hidden random variable then estimation of $\lambda(b)$ becomes very easy.

So,

$$X = hidden\ variable = n(b,d) = x$$

$$Y = observed\ data = p^*(d) = y$$

$$Likelihood = p(x,y) = \prod_{\substack{b=1,2 \dots B \\ d=1,2 \dots D}} \frac{e^{-\lambda(b,d)}\left(\lambda(b,d)\right)^{n(b,d)}}{n(b,d)!}$$

Each $n(b,d)$ poisson process is independent

$$\sum_{b=1}^{B} n(b,d) = n^*(d) \qquad for\ all\ 'd'$$

$$\log(Likelihood) = \sum_{d=1}^{D}\sum_{b=1}^{B} -\lambda(b,d) + n(b,d)\log(\lambda(b,d)) - \log(n(b,d)!)$$

$\log(n(b,d)!)$ of no use since it doesn't contain parameter $\lambda(b,d)$

$$E[n(b,d)|\, n^*(d)] = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b=1}^{B}\lambda^i(b,d)} \qquad \lambda^i(b,d) = estimate\ at\ iteration\ i$$

$$E[\log(Likelihood)] = \sum_{d=1}^{D}\sum_{b=1}^{B} -\lambda(b,d) + \frac{n^*(d)\lambda^i(b,d)\log(\lambda(b,d))}{\sum_{b=1}^{B}\lambda^i(b,d)} = Q(\theta,\theta^i)$$

Differentiating above expression respect to $\lambda(b,d)$ to get maxima

$$\lambda^{i+1} = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b'=1}^{B}\lambda^i(b',d)}$$

$$\lambda^i(b,d) = \lambda^i(b)p(b,d) \quad and \quad \lambda(b) = \sum_{d=1}^{D}\lambda(b,d)$$

$$\lambda^{new}(b) = \lambda^{old}(b)\sum_{d=1}^{D}\frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B}\lambda^{old}(b')p(b',d)}$$

Final formula for update is given below

$$\lambda^{new}(b) = \lambda^{old}(b)\sum_{d=1}^{D}\frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B}\lambda^{old}(b')p(b',d)}$$