Maximum Likelihood Reconstruction for Emission Tomography

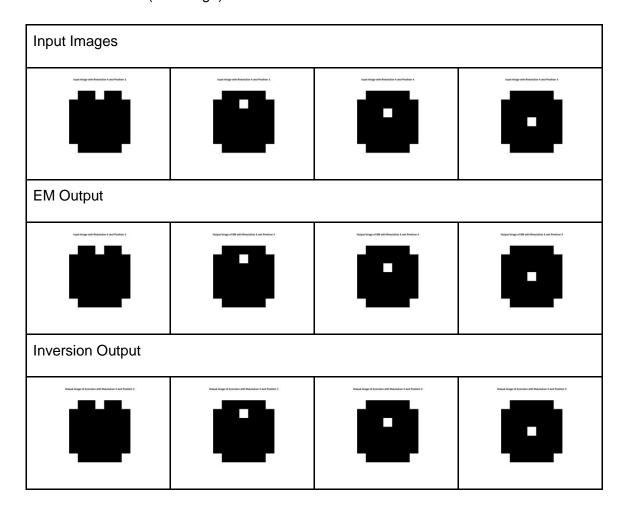
Team Members:

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- 2. Kalpesh Patil (130040019)

Results:

Number of detectors = 40

- Reconstruction of Single Pixel Impulse
 - 1. Resolution = 4 (9*9 image)

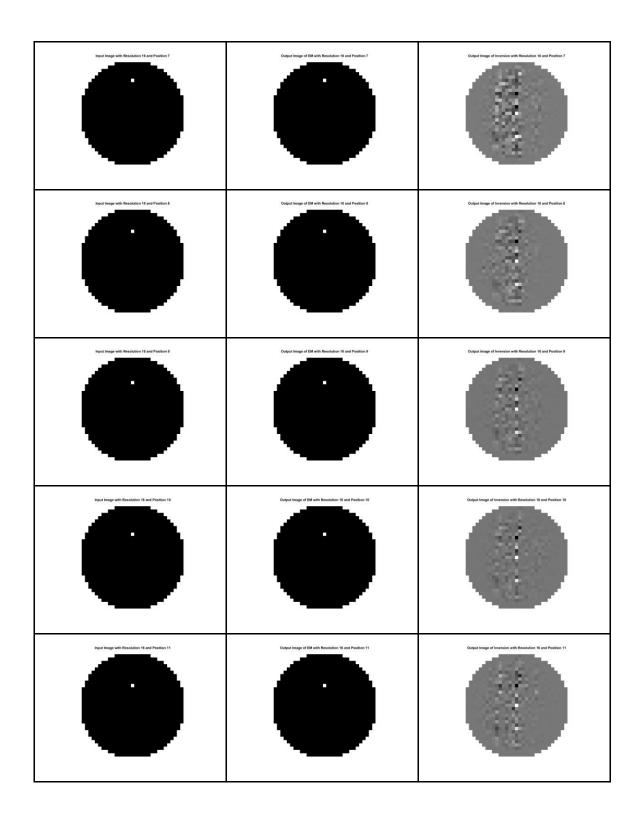


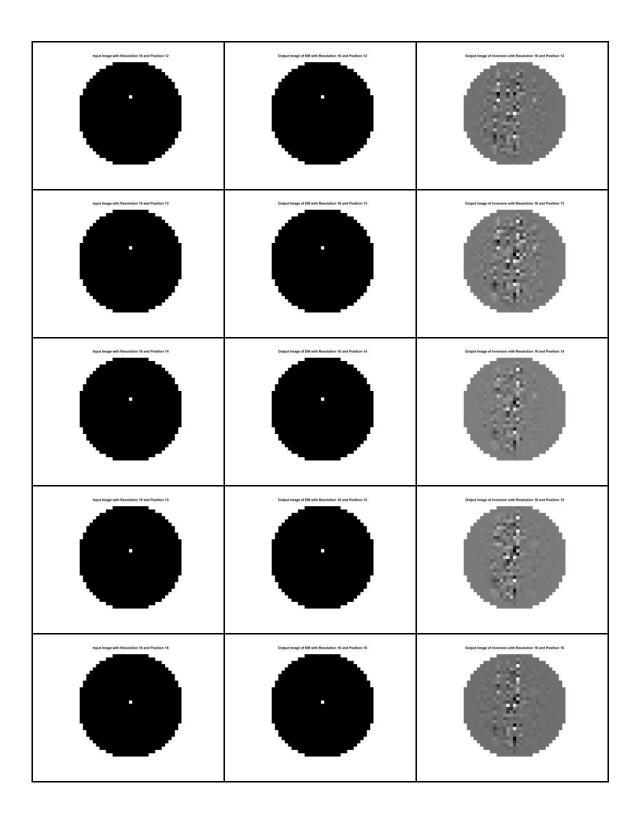
2. Resolution = 8 (17*17 image)

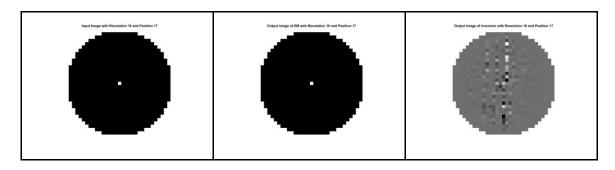
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3. Resolution = 16 (33*33 image)

Input Image	EM Output	Inversion
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trond troops with Resolution 16 and Position 5	Output Inveger of EM with Resolution 16 and Position 5	Output Image of Inversion with Resolution 16 and Position 5
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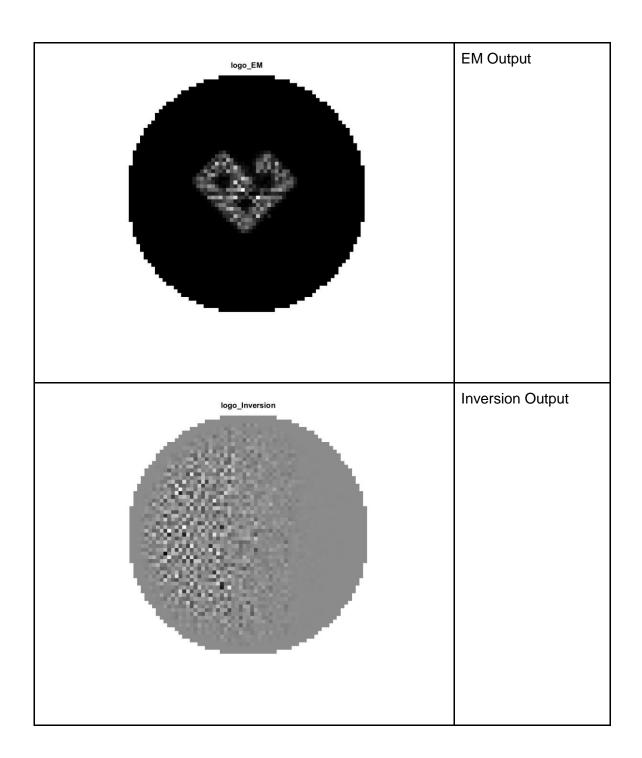






 Reconstruction of Binary Image Number of detectors = 80 Resolution = 32 (65*65 image)

Images	Description
logo_input	Input Binary Image (logo)



Theory Concerning Project

Here 'b' denote sites. There are 'B' sites in total. 'd' denotes detector tube and there are 'D' tubes in total. P(b,d) is the probability that emission in 'b' is detected in tube 'd'. The emission at site 'b' is assumed to be Poisson with mean $\lambda(b)$.

 $n^*(d)$ is the total number of emissions detected in tube d during acquisition (real data).

n(b,d) denotes the number of emissions in 'b' detected in 'd'. n(b,d) are mutually independent Poisson random variable, across 'b', across 'd' (split of Poisson process independent).

$$n(b,d) \sim Poisson (\lambda(b,d))$$

$$\lambda(b,d) = \lambda(b)p(b,d)$$

$$\sum_{d=1}^{D} p(b,d) = 1$$

$$n^*(d) = \sum_{b=1}^{B} n(b,d)$$

$$\lambda^*(d) = E[n^*(d)] = \sum_{b=1}^{B} \lambda(b)p(b,d)$$

Best estimate of $\lambda^*(d)$ is $n^*(d)$

$$n^*(d) = \sum_{b=1}^{B} \lambda(b)p(b,d)$$
 for $d = 1,2,...D$

Above sets of linear equations form the basis for inversion reconstruction.

Now let's move onto expectation maximization. If we have n(b,d) as hidden random variable then estimation of $\lambda(b)$ becomes very easy.

So,

$$X = hidden \ variable = n(b,d) = x$$

$$Y = observed \ data = n^*(d) = y$$

$$Likelihood = p(x,y) = \prod_{b=1,2...B} \frac{e^{-\lambda(b,d)} \big(\lambda(b,d)\big)^{n(b,d)}}{n(b,d)!}$$

Each n(b, d) poisson process is independent

$$\sum_{b=1}^{B} n(b,d) = n^*(d) \quad \text{for all 'd'}$$

$$\log(Likelihood) = \sum_{d=1}^{D} \sum_{b=1}^{B} -\lambda(b,d) + n(b,d)\log(\lambda(b,d)) - \log(n(b,d)!)$$

 $\log(n(b,d)!)$ of no use since it doesn't contain parameter $\lambda(b,d)$

$$E[n(b,d)|n^*(d)] = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b=1}^B \lambda^i(b,d)} \qquad \lambda^i(b,d) = \text{estimate at iteration } i$$

$$E[\log(Likelihood)] = \sum_{d=1}^{D} \sum_{b=1}^{B} -\lambda(b,d) + \frac{n^*(d)\lambda^i(b,d)\log(\lambda(b,d))}{\sum_{b'=1}^{B} \lambda^i(b',d)} = Q(\theta,\theta^i)$$

Differentiating above expression respect to $\lambda(b,d)$ to get maxima

$$\lambda^{i+1}(b,d) = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b'=1}^B \lambda^i(b',d)}$$

$$\lambda^i(b,d) = \lambda^i(b)p(b,d) \quad and \quad \lambda(b) = \sum_{d=1}^D \lambda(b,d)$$

$$\lambda^{new}(b) = \lambda^{old}(b) \sum_{d=1}^{D} \frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B} \lambda^{old}(b')p(b',d)}$$

Final formula for update is given below

$$\lambda^{new}(b) = \lambda^{old}(b) \sum_{d=1}^{D} \frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B} \lambda^{old}(b')p(b',d)}$$