Here 'b' denote sites, There are 'B' sites in total
'd' denote detector tube and there are 'D' tubes in total.
$P(b,d)$ is the probability that emission in 'b' is detected in tube 'd'. The emission at site 'b' is assumed to be poisson with mean $\lambda(b)$

$n^*(d)$ is the total number of emission detected in tube 'd' during acquisition (real data)

$n(b,d)$ denote the number of emission in 'b' detected in 'd'
$n(b,d)$ are mutually independent poisson random variable, across b, across d. (Split of poisson process independent)

$n(b,d) \sim \text{Poisson}(\lambda(b,d))$ $\qquad \sum_{d=1}^{D} P(b,d) = 1$

$\lambda(b,d) = \lambda(b) \times P(b,d)$

$$n^*(d) = \sum_{b=1}^{B} n(b,d) \Rightarrow \lambda^*(d) = E[n^*(d)] = \sum_{b=1}^{B} \lambda(b) P(b,d)$$

$\Rightarrow$ Best estimate of $\lambda^*(d) = n^*(d)$

$$\Rightarrow n^*(d) = \sum_{b=1}^{B} \lambda(b) P(b,d) \quad \text{for } d = 1, \dots D.$$

Above set of linear equations form the basis for inversion reconstruction.

Now let's move on to expectation maximisation. If we have $n(b,d)$ as hidden random variable then estimation of $\lambda(b)$ becomes very easy.

So X = Hidden Random Variable $= n(b,d)$

Y = observed data $= n^*(d)$

$$\text{Likelihood} = P(X,Y) = \prod_{\substack{b=1\ldots B \\ d=1\ldots D}} e^{-\lambda(b,d)} \frac{(\lambda(b,d))^{n(b,d)}}{n(b,d)!}$$ ($\because$ each $n(b,d)$ poisson process is independent)

$$\left(\sum_{b=1}^{B} n(b,d) = n^*(d) \ \forall d \right)$$

$$\text{log likelihood} = \sum_{d=1}^{D} \sum_{b=1}^{B} -\lambda(b,d) + n(b,d)\log(\lambda(b,d)) - \log(n(b,d)!)$$

$\uparrow$ of no use as does not contain parameter $\lambda(b,d)$

$$E[n(b,d) / n^*(d)] = \frac{n^*(d)\, \lambda^i(b,d)}{\sum_{b'=1}^{B} \lambda^i(b',d)}$$ $\lambda^i(b,d)$ = estimate at i^{th} iteration

$$\Rightarrow E[\text{log likelihood}] = \sum_{d=1}^{D} \sum_{b=1}^{B} -\lambda(b,d) + \frac{n^*(d)\,\lambda^i(b,d)}{\sum_{b'=1}^{B} \lambda^i(b',d)} \log(\lambda(b,d)) = Q(\theta^i \theta^i)$$

Differentiating above with respect to $\lambda(b,d)$ to get maxima

$$\Rightarrow \lambda^{i+1}(b,d) = \frac{n^*(d)\, \lambda^i(b,d)}{\sum_{b'=1}^{B} \lambda^i(b',d)}$$ $\lambda^i(b,d) = \lambda^i(b)\, P(b,d)$

$$\lambda(b) = \sum_{d=1}^{D} \lambda(b,d)$$

$$\Rightarrow \lambda^{new}(b) = \lambda^{old}(b) \sum_{d=1}^{D} \frac{n^*(d)\, P(b,d)}{\sum_{b'=1}^{B} \lambda_{old}(b')\, P(b',d)}$$

Final formula for update is given below

$$\boxed{\lambda^{new}(b) = \lambda^{old}(b) \sum_{d=1}^{D} \frac{n^*(d)\, P(b,d)}{\sum_{b'=1}^{B} \hat{\lambda}_{old}(b')\, P(b',d)}}$$