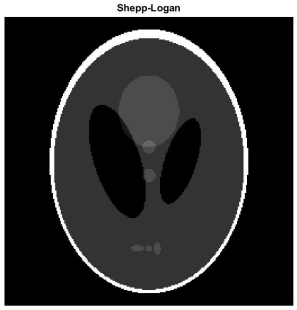
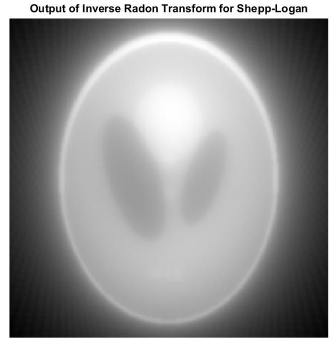
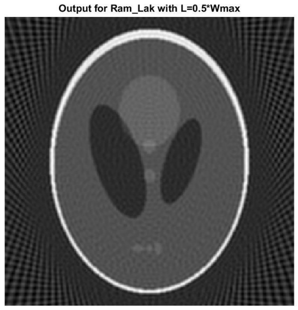
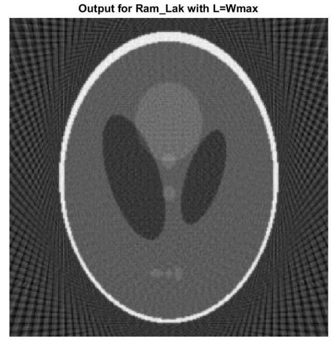
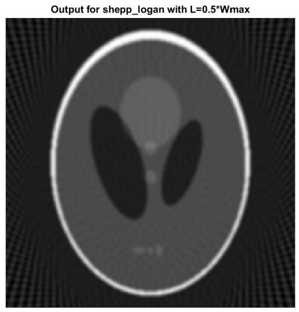
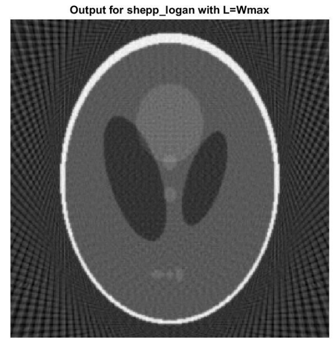
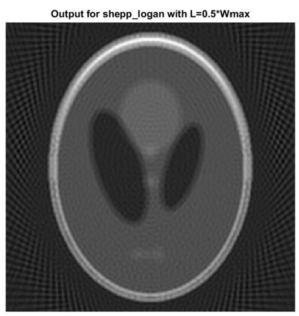
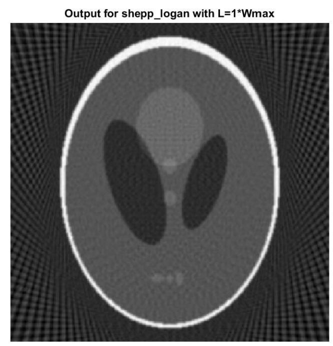
Assignment 1 : Q2: Report

Question 2 *X-Ray Computed Tomography: Filtered Backprojection*

Part a

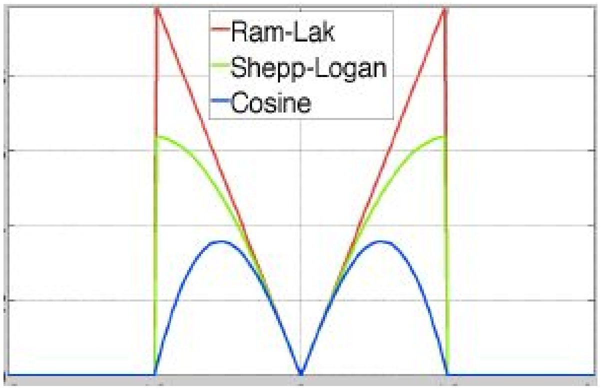
Following contain reconstructed images for different filters and cut off frequency settings.
Detailed high resolution images can be found in "..\images\Question2_a\"

	Input	Ouput
Back Projection	Shepp-Logan 	Output of Inverse Radon Transform for Shepp-Logan 
Filter	L= $\omega_{max}/2$	L= ω_{max}
Ram Lak Filter	Output for Ram_Lak with L=0.5*Wmax 	Output for Ram_Lak with L=Wmax 

Shepp-Logan Filter		
Cosine Filter		

Observations: Firstly, using plane back projection blurs image too much as shown in first row of above table. So we use filter to enhance high frequency components to have edge enhancement. For every filter, transition from L=$\omega_{max}/2$ to L=ω_{max} leads to enhancement of edges as higher frequency components get amplified, but this transition lead also to noise enhancement (SNR is low in high frequency region so Noise amplification prevails here). Same phenomenon is observed in practise as well, along each row image for column L=ω_{max} is more sharper but at the same time more noisy than L=$\omega_{max}/2$.
Now let's focus on variation across different filtering schemes. We can group them on exactly same trade off of edge enhancement versus noise amplification as we did earlier.

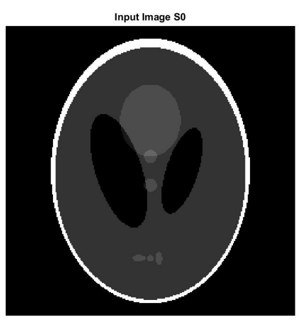
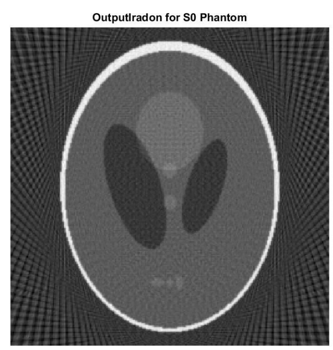
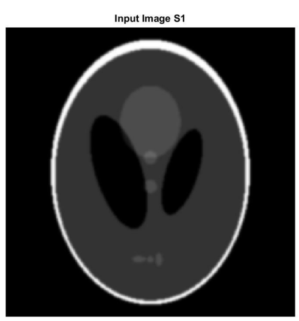
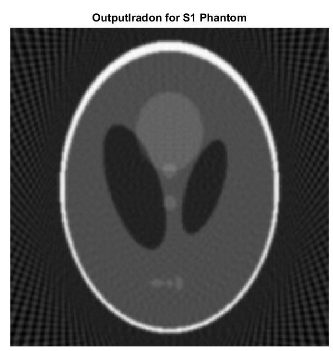
Following image shows shape of different filters. From this we clearly see that Ram-Lak amplifies high frequency component most, then comes Shepp Logan and at the end Cosine. So it is expected that Ram Lak filters will sharpen edges and enhance noise and these characteristics will decrease as we go to cosine filter. This theoretical viewpoint is also reflected in practical. For any choice of 'L' Ram Lak filter images were sharp and noisy than Shepp Logan and Shepp Logan were more sharp and noisy than Cosine.



Clearly Ram-Lak provide greater high frequency amplification, then comes Shepp Logan and at the end Cosine as shown in figure beside

Part b

Following table shows result for S0, S1, S5. Detailed high resolution images can be found in "..\images\Question2_b\"

	Input	Ouput
S0		
S1		

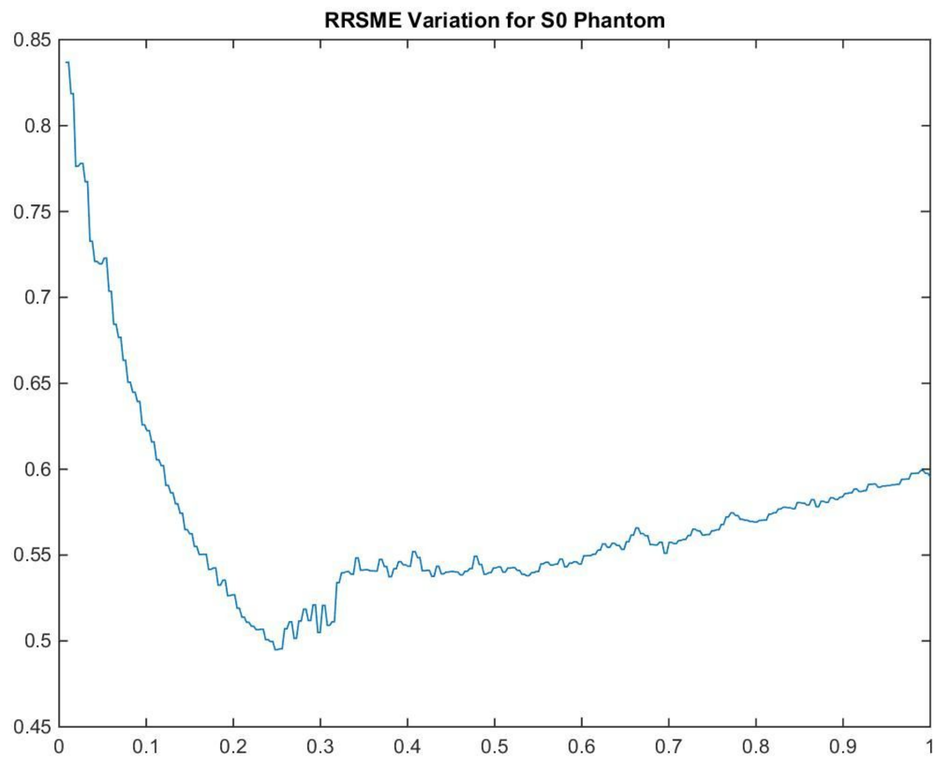
S5		

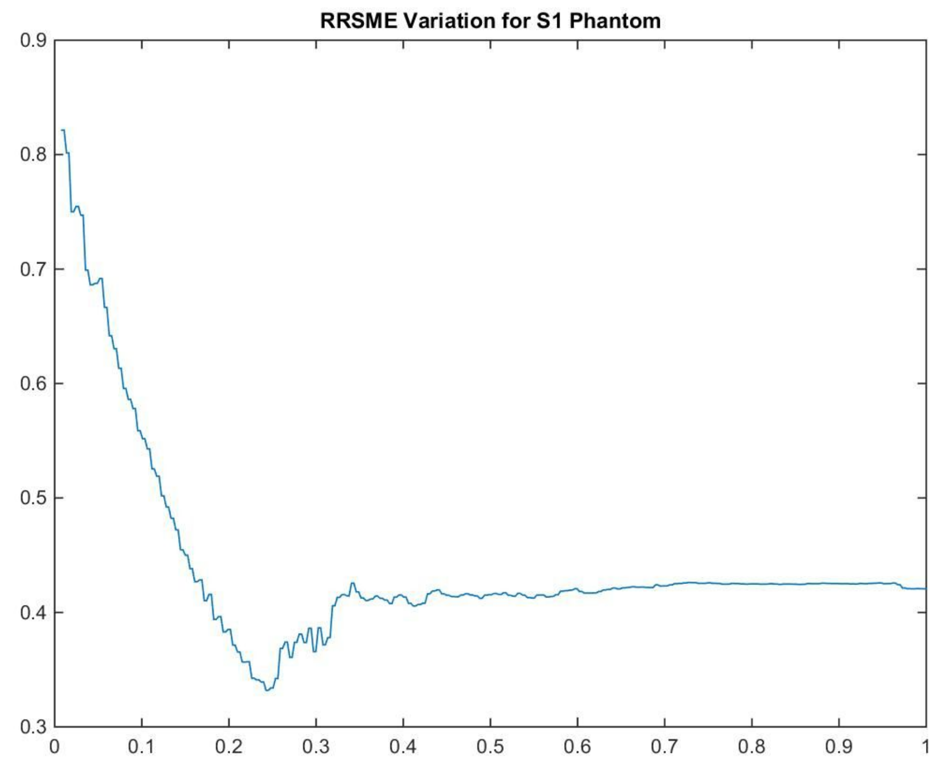
RRMSE for S0 = 0.81518 ; RRMSE for S1 = 0.52661; RRMSE for S5 = 0.06634

RRMSE was highest in case of S0 while lowest in case of S5. This happens because S5 was smoothest image in all of them while S0 was without any smoothing.
We know that inverse radon transform without any filtering results in blurring so we employ frequency domain filters to enhance edges but as a setback it also lead to noise enhancement. So if the original image doesn't have too many sharp edges and less noise then we expect reconstruction to be more or less accurate. This Gaussian smoothing does exactly the same, it blurs edges as well denoise it. So in case of S5 where maximum smoothing has happened we expect drastic change in RRMSE and hence is has RRMSE as low as 0.06634 while for S1 where reasonable smoothing has happened RRMSE is 0.52661 which is lower than RRMSE for S0 (0.81548) where absolutely no smoothing has happened.

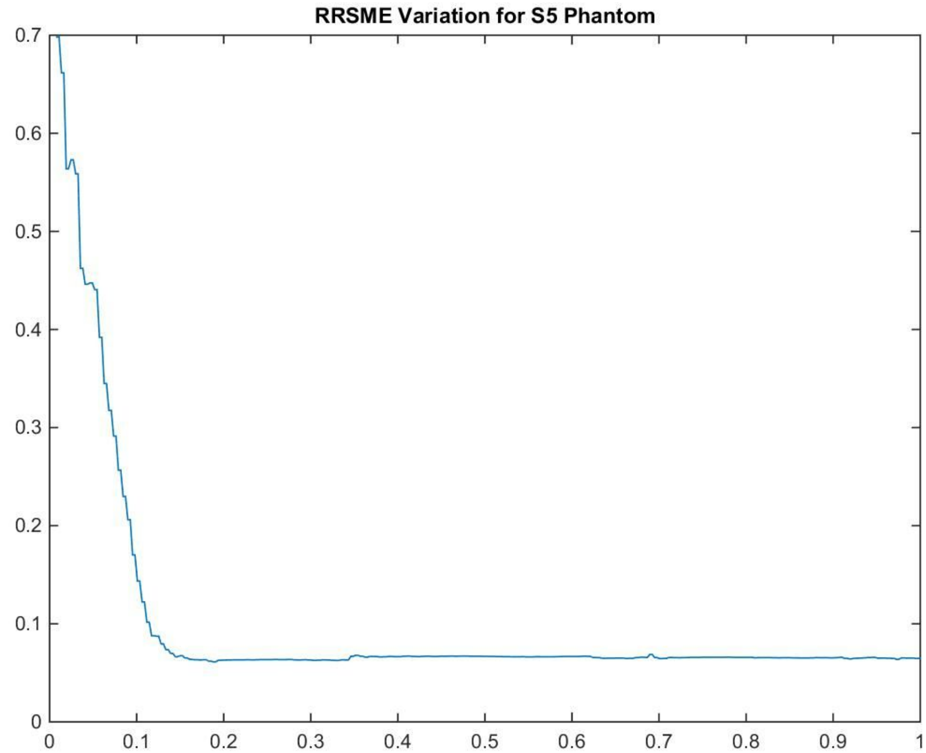
Part c

RRMSE variation plots Detailed high resolution plots can be found in "..\images\Question2_c\"

Image Type	RRMSE variation across Normalized cut off frequency (L)
S0	

S1	

RRSME Variation for S1 Phantom
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

S5	

Observations: All plots have some minima while minima is pronounced in case of S0 and is almost inevident in case of S5. Secondly error plot of S5 tend to saturate after reaching minima of around 0.06. Minimum value of S0>Minimum value of S1>Minimum value of S5
Justification: There are two factor playing role, increasing cut off frequency does sharpen edges and hence leads to reduction in RRMSE but at the same time it also enhances noise and hence increses RRMSE. Strength of these two coercion decides net change in RRMSE. So there opposing forces are expected to stabilise at some point and hence give us minima. So this minima is observed in all three plots. But it is not equally pronounced in all of them. Reason being smoothing tend to affect strength of each force as it denoise and smoothen image so minimas are less evident in case of smoothed image. Secondly we have already argued that smoothing lead to reduction in RRMSE in question 2_b same phenomenon is also evident through this plot by looking at trend of minimum value across S0, S1, S5.