

EE 779: Computing Assignment 1

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Qusetion 1: Impulse response of single formant resonator

Poles of the transfer function can be obtained as follows

$$r = e^{-\frac{\pi B}{f_s}} \quad \text{and} \quad \theta = \frac{2\pi f}{f_s}$$

B = bandwidth of the filter

f = formant frequency

f_s = sampling frequency

This would give us locations of poles in complex plane. The transfer function can be written as

$$H(z) = \frac{k}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})}$$

- Code

```
clear all
```

```
//Function to calculate filter response using difference equation
```

```
function [y]=time_response(x, num, den, n_samples)
```

```
    y = zeros(n_samples,1)
```

```
    //numerator is constant (all pole filter)
```

```
    y(1) = num(1)*x(1)
```

```
    //response by taking coefficients for denominator
```

```
    for ii=2:n_samples
```

```
        temp = k*x(ii)
```

```
        for jj = 1:min(ii-1,length(den)-1)
```

```
            temp = temp - den(jj+1)*y(ii - jj)
```

```
        end
```

```
        y(ii) = temp
```

```
    end
```

```
endfunction
```

```
//specifying parameters
```

```
F1 = 1000
```

```
B1 = 200
```

```
Fs = 16000
```

```
//numerator of filter assumed to be 1
```

```
k = 1
```

```

//number of samples
n_samples = 200
//number of points for fft
n_fft = n_samples*10

//computing poles
r = exp(-B1*%pi/Fs);
theta = 2*%pi*F1/Fs

//specifying coefficients of discrete time filter
num = k
den = [1 -2*r*cos(theta) r^2]

//impulse input
delta_n = zeros(n_samples,1)
delta_n(1) = 1

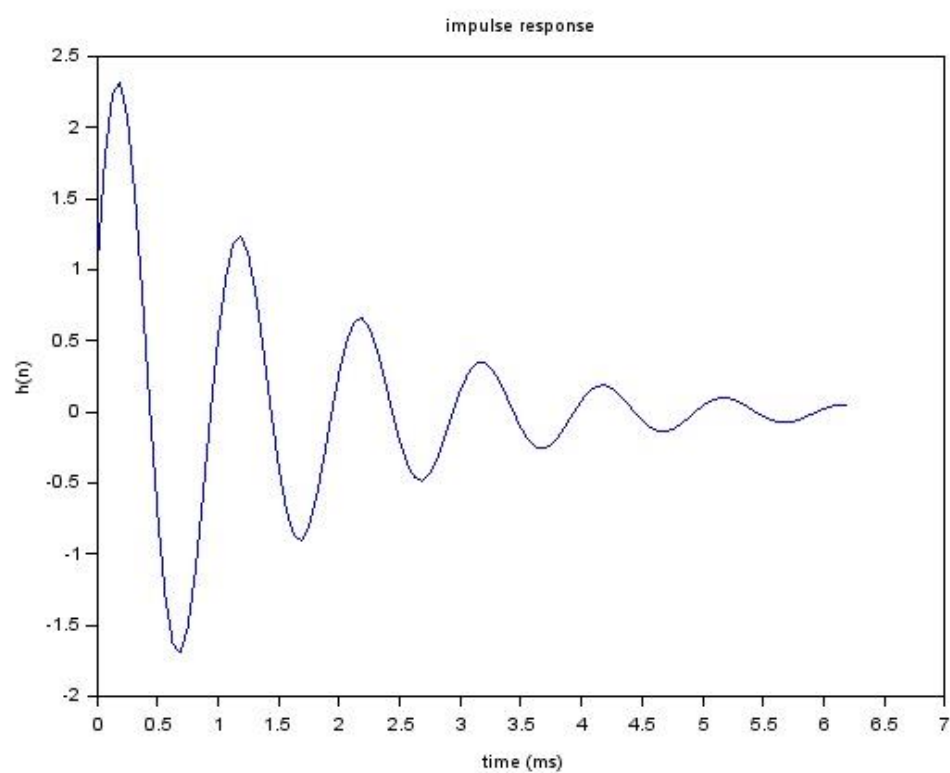
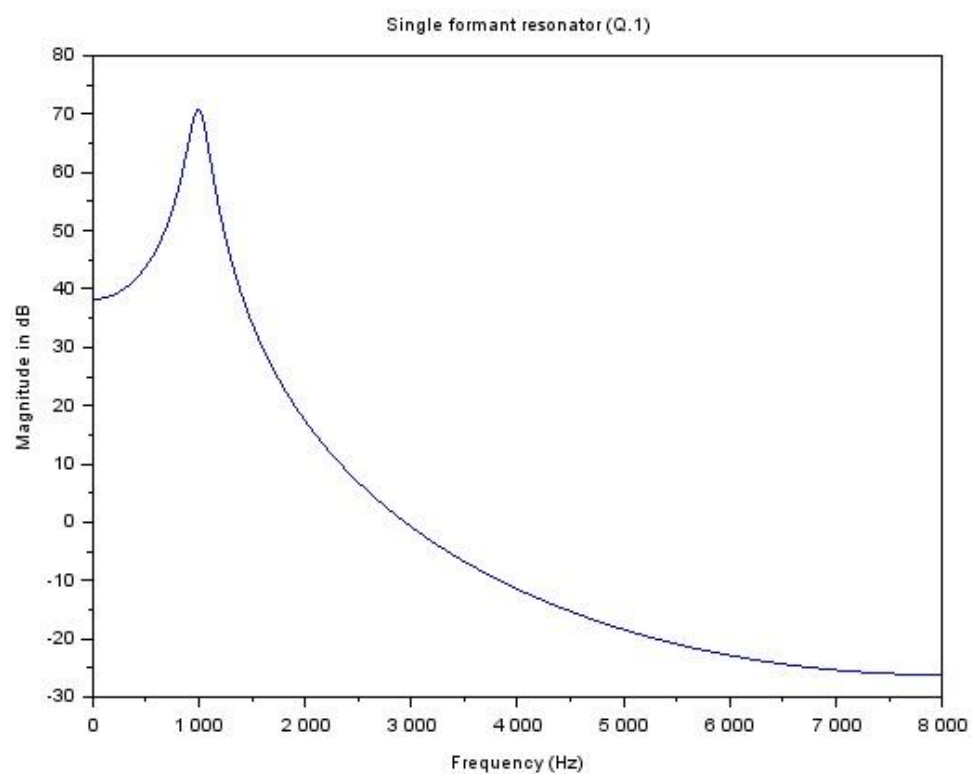
//impulse response
h = time_response(delta_n,num,den,n_samples)
h_padded = zeros(n_fft,1)
h_padded(1:length(h)) = h
n_time_samples = 100
time_array = [0:n_time_samples-1]*Fs/1000
fig = scf()
plot(time_array, h(1:n_time_samples))
xlabel('impulse response','time (ms)','h(n)')
xs2jpg(gcf(), '../plots/Q1/impulse response.jpg');

//Frequency response of the filter
fig = scf()
H = fftshift(fft(h_padded))
H_mag = abs(H(n_fft/2 +1:n_fft))
freq_array = linspace(0,Fs/2,n_fft/2)
plot(freq_array,20*log(H_mag))
xlabel('Single formant resonator (Q.1)','Frequency (Hz)','Magnitude in dB')
xs2jpg(gcf(), '../plots/Q1/single_formant_resonator_magnitude_plot.jpg');

```

- Results & Discussion

Plots for impulse response in time domain and frequency response of the filter are attached below. We can observe that peak in frequency response is around 1000Hz which was formant frequency. Also in the plot of impulse response in time domain, peaks are occurring after the interval of 1ms, which signifies 1000Hz as formant frequency.



Question 2: Exciting the filter in question 1 with periodic source excitation and compute output

As mentioned in the problem statement, source signal was approximated with narrow triangular pulse train. Following difference equation was implemented, which is obtained using filter described in question 1.

$$y[n] = x[n] + 2r\cos(\theta)y[n-1] + r^2y[n-2]$$

System was assumed to be causal and hence $y[n] = 0$ for $n < 0$.

- Code

```
clear all
```

```
//Function to calculate filter response using difference equation
```

```
function [y]=time_response(x, num, den, n_samples)
```

```
    y = zeros(n_samples,1)
```

```
    //numerator is constant (all pole filter)
```

```
    y(1) = num(1)*x(1)
```

```
    //response by taking coefficients for denominator
```

```
    for ii=2:n_samples
```

```
        temp = k*x(ii)
```

```
        for jj = 1:min(ii-1,length(den)-1)
```

```
            temp = temp - den(jj+1)*y(ii - jj)
```

```
        end
```

```
        y(ii) = temp
```

```
    end
```

```
endfunction
```

```
//specifying parameters
```

```
F1 = 1000
```

```
B1 = 200
```

```
Fs = 16000
```

```
F0 = 150
```

```
t_duration = 0.5
```

```
//numerator of filter assumed to be 1
```

```
k = 1
```

```
//specifying input signal
```

```
n_samples = t_duration*Fs + 1
```

```
n_fft = n_samples*10
```

```
x = zeros(n_samples,1)
```

```
//impulse is approximated by narrow triangular pulse as described in
```

```
//problem statement
```

```
for i = 2:t_duration*F0
```

```
    x(floor((i-1)*Fs/F0))= 1
```

```

x(floor((i-1)*Fs/F0 + 1))= 0.75
x(floor((i-1)*Fs/F0 + 2))= 0.5
x(floor((i-1)*Fs/F0 + 3))= 0.25
x(floor((i-1)*Fs/F0 - 1))= 0.75
x(floor((i-1)*Fs/F0 - 2))= 0.5
x(floor((i-1)*Fs/F0 - 3))= 0.25
end
//computing poles
r = exp(-B1*%pi/Fs);
theta = 2*%pi*F1/Fs

//specifying coefficients of discrete time filter
num = k
den = [1 -2*r*cos(theta) r^2]

//output from filter
y = time_response(x,num,den,n_samples)

//plotting time domain for response
t_obs = 25 //we will observe signal for 25ms
n_time_samples = floor(t_obs*Fs/1000)
time_array = linspace(0,t_obs,n_time_samples)
temp = find(y~=0)
init = temp(1) //capture n_time_samples here onwards
fig = scf()
plot(time_array, y(init:init+n_time_samples-1))
xlabel('Time domain output','time (ms)','Fiter output')
xs2jpg(gcf(), '../plots/Q2/time_domain_output.jpg');

//converting to y to sound by limiting amplitude in [-1,1]
//as required by the wavewrite
y_snd = y/max(y)
playsnd(y_snd,Fs);

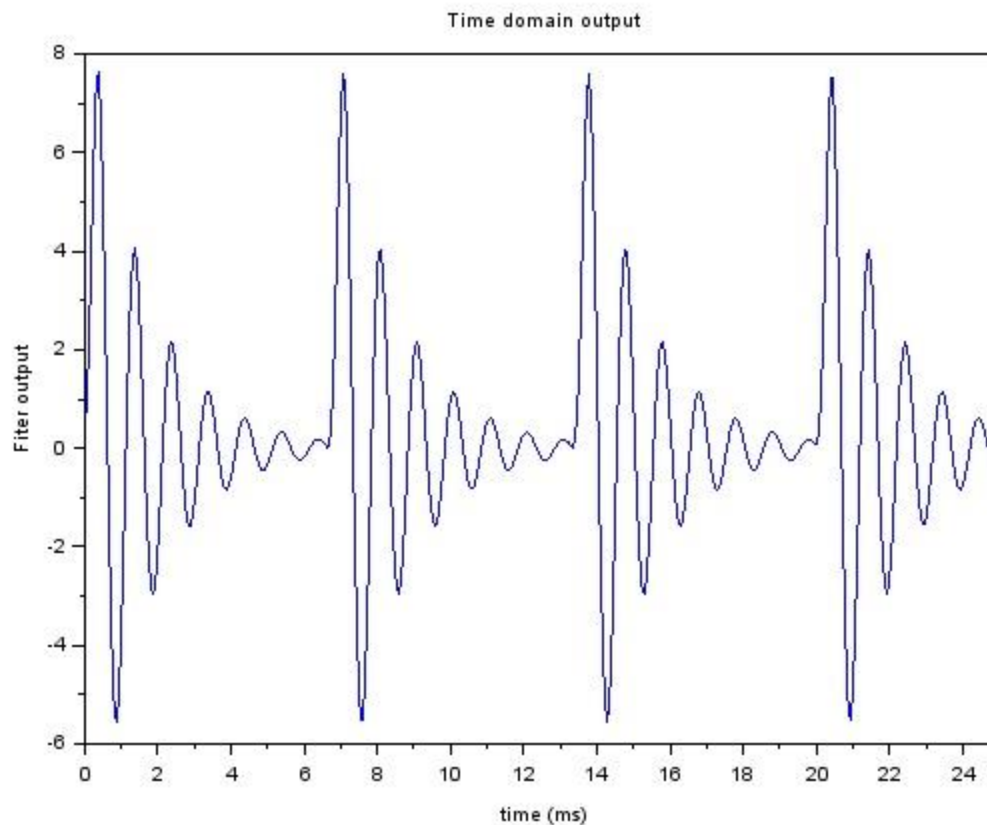
wavfile = './wav_files/Q2/output_signal_F0_150_F1_1000.wav';
wavwrite(y_snd, Fs, wavfile);

```

- Results & Discussion

Time domain waveform over few periods is plotted below. It is observed that pattern of waveform is repeating after 6.6 ms, which corresponds to 151.5 Hz. The pitch of the input given was 150 Hz. Hence we were able to observe this. Also the difference between two consecutive decaying peaks was observed to be 1ms which corresponds to 1000 Hz formant frequency used in the input.

Quality of the sound clip played for 0.5 sec was not good. It was a bit rough and artificial (not signifying any human voice component). Roughness can be attributed to abrupt changes in waveform.



Question 3: Vary the parameters and observe differences in waveform and sound quality

Following code implements task of varying parameters and plotting and storing waveforms.

- Code

```
clear all
```

```
//Function to calculate filter response using difference equation
```

```
function [y]=time_response(x, num, den, n_samples)
```

```
    y = zeros(n_samples,1)
```

```
    //numerator is constant (all pole filter)
```

```
    y(1) = num(1)*x(1)
```

```
    //response by taking coefficients for denominator
```

```
    for ii=2:n_samples
```

```
        temp = k*x(ii)
```

```
        for jj = 1:min(ii-1,length(den)-1)
```

```

        temp = temp - den(jj+1)*y(ii - jj)
    end
    y(ii) = temp
end
endfunction

F0_list = [120 120 180]
F1_list = [300 1200 300]
B1_list = [100 200 100]

for iter = 1:length(F0_list)
    //specifying parameters
    F1 = F1_list(iter)
    B1 = B1_list(iter)
    Fs = 16000
    F0 = F0_list(iter)
    t_duration = 0.5
    //numerator of filter assumed to be 1
    k = 1
    //specifying input signal
    n_samples = t_duration*Fs + 1
    n_fft = n_samples*10

    x = zeros(n_samples,1)

    //impulse is approximated by narrow triangular pulse
    for i = 2:t_duration*F0
        x(floor((i-1)*Fs/F0))= 1
        x(floor((i-1)*Fs/F0 + 1))= 0.75
        x(floor((i-1)*Fs/F0 + 2))= 0.5
        x(floor((i-1)*Fs/F0 + 3))= 0.25
        x(floor((i-1)*Fs/F0 - 1))= 0.75
        x(floor((i-1)*Fs/F0 - 2))= 0.5
        x(floor((i-1)*Fs/F0 - 3))= 0.25
    end

    //computing poles
    r = exp(-B1*%pi/Fs);
    theta = 2*%pi*F1/Fs

    //specifying coefficients of discrete time filter
    num = k
    den = [1 -2*r*cos(theta) r^2]

    //output from filter
    y = time_response(x,num,den,n_samples)

    //plotting time domain for response
    t_obs = 25 //we will observe signal for 25ms
    n_time_samples = floor(t_obs*Fs/1000)
    time_array = linspace(0,t_obs,n_time_samples)

```

```

temp = find(y~=0)
init = temp(1) //capture n_time_samples here onwards
fig = scf()
plot(time_array, y(init:init+n_time_samples-1))
plot_title = strcat(['Output signal for F0 = ',string(F0),'Hz,F1 = ',string(F1),'Hz,B1 = ',string(B1),'Hz'])
xlabel(plot_title,'time (ms)','y(n)')
xs2jpg(gcf(), strcat(['../plots/Q3/',plot_title,'.jpg']));

//converting to y to sound by limiting amplitude in [-1,1]
//as required by the wavewrite
y_snd = y'/max(y')
playsnd(y_snd,Fs);

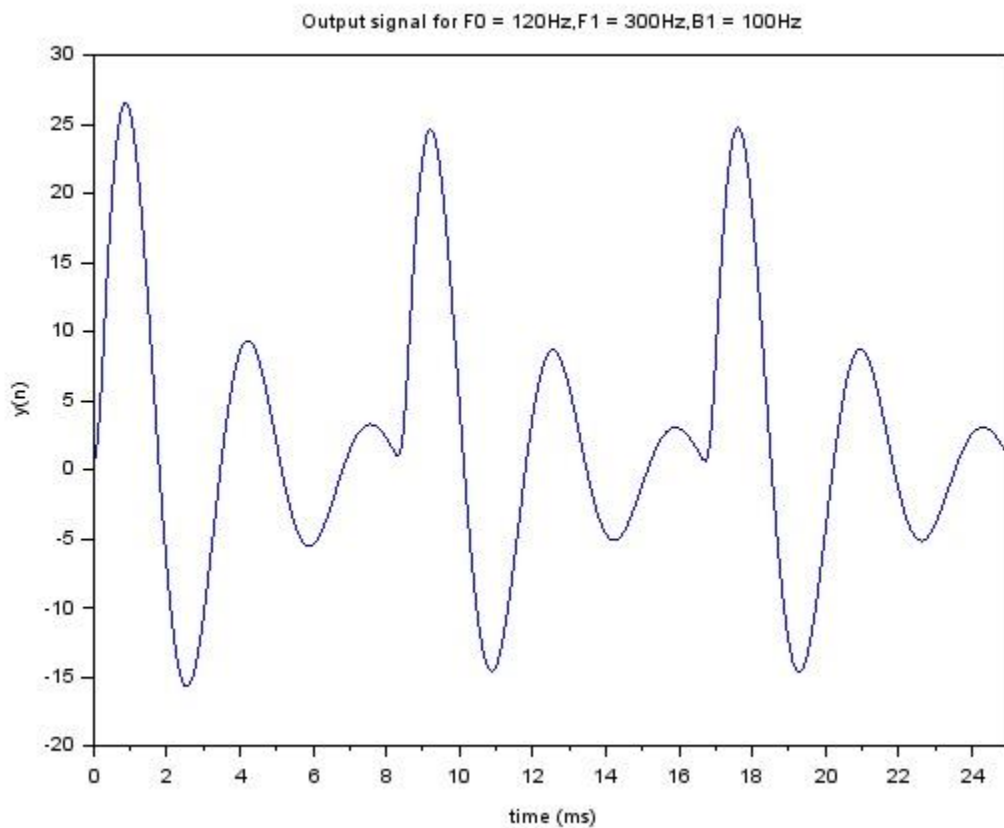
wavfile = strcat(['../wav_files/Q3/',plot_title,'.wav']);
wavwrite(y_snd, Fs, wavfile);

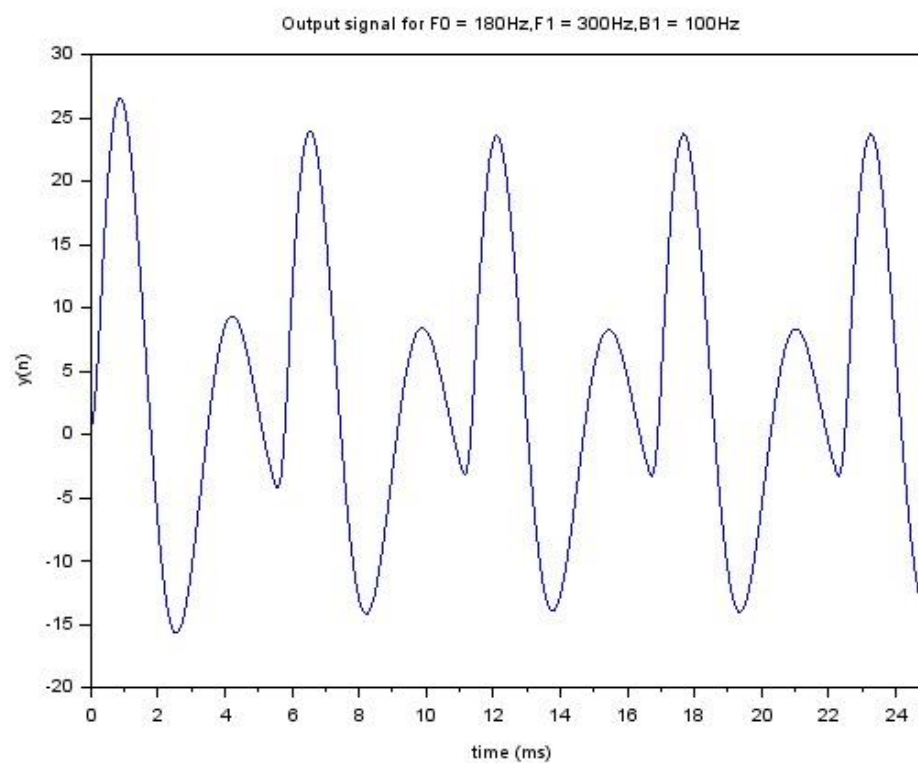
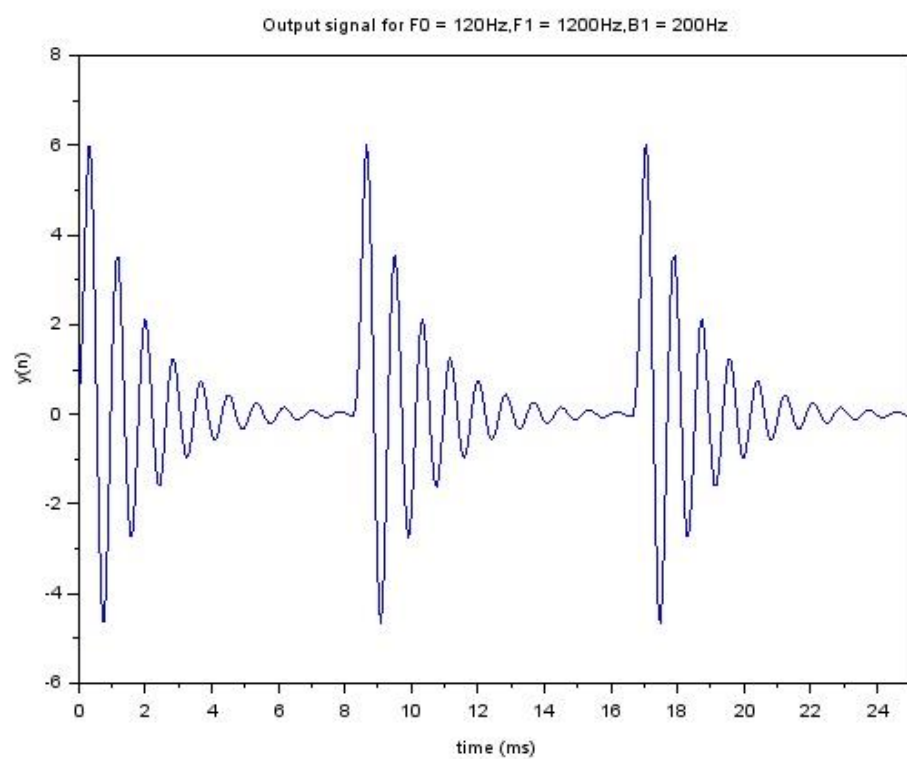
end

```

- Results

Plots for the given combinations are as follows. All the plots are for a sample of output captured for 25ms of duration.





- Discussion
 1. F0 is same for waveform 1 and waveform 2. But due to changing other parameters we can observe that there is hardly any interference adjacent impulse responses in waveform 2, while lot of inference is present in waveform 1. Waveform 2 decays much faster compared to waveform 1.
 2. Pitch of the output depends upon F0. For a fixed time frame (25 ms), first two waveforms had less number of impulse responses (major peaks), while 3rd waveforms had 5 responses.
 3. Sound of waveform 2 seems rough compared to other two sounds, because it has huge abrupt changes compared to others. Sound of waveform 3 appears to be the most pleasant amongst all.

Question 4: Generate given vowels

Cascade of multiple single formant resonators were used to generate a given vowel. Transformation function of cascade system is product of individual transformation functions.

- Code

`clear all`

//Function to calculate filter response using difference equation

```
function [y]=time_response(x, num, den, n_samples)
    y = zeros(n_samples,1)
    //numerator is constant (all pole filter)
    y(1) = num(1)*x(1)
    //response by taking coefficients for denominator
    for ii =2:n_samples
        temp = k*x(ii)
        for jj = 1:min(ii-1,length(den)-1)
            temp = temp - den(jj+1)*y(ii - jj)
        end
        y(ii) = temp
    end
endfunction
```

//function to find discrete coefficients of cascade of multiple filters

```
function [num, den]=find_cascade_filter(F_list, B_list, Fs)
    num = 1
    den = [1]
    for iter = 1:length(F_list)
        F = F_list(iter)
        B = B_list(iter)
        //finding poles for current iteration
        r = exp(-B*%pi/Fs)
        theta = 2*%pi*F/Fs
        //current filter
        num_curr = 1
```

```

    den_curr = [1 -2*r*cos(theta) r^2]
    //multiplication of polynomials can be computed using their
    //convolution. Numerator is constant(1). Hence only
    //denominator needs to be multiplied
    den = conv(den,den_curr)
end
endfunction

//plots frequency response given impulse response
function plot_frequency_response(h, n_fft, vowel)
    h_padded = zeros(n_fft,1)
    h_padded(1:length(h)) = h
    //Frequency response of the filter
    fig = scf()
    H = fftshift(fft(h_padded))
    H_mag = abs(H(n_fft/2 + 1:n_fft))
    freq_array = linspace(0,Fs/2,n_fft/2)
    plot(freq_array,20*log(H_mag))
    plot_title = strcat(['Frequency response of vowel ',vowel])
    xtitle(plot_title,'Frequency (Hz)','Magnitude in dB')
    xs2jpg(gcf(), strcat(['../plots/Q4/',plot_title,'.jpg']));
endfunction

//Plots output signal, plays and store the sound
function plot_y(y, F0, Fs, vowel)
    //plotting time domain for response
    t_obs = 25 //we will observe signal for 25ms
    n_time_samples = floor(t_obs*Fs/1000)
    time_array = linspace(0,t_obs,n_time_samples)
    temp = find(y~=0)
    init = temp(1) //capture n_time_samples here onwards
    fig = scf()
    plot(time_array, y(init:init+n_time_samples-1))
    plot_title = strcat(['Output signal of vowel ',vowel,' for F0 = ',string(F0)])
    xtitle(plot_title,'time (ms)','y(n)')
    xs2jpg(gcf(), strcat(['../plots/Q4/',plot_title,'.jpg']));

    //converting to y to sound by limiting amplitude in [-1,1]
    //as required by the wavewrite
    y_snd = y'/max(y')
    playsnd(y_snd,Fs);
    wavfile = strcat(['../wav_files/Q4/',plot_title,'.wav'])
    wavwrite(y_snd, Fs, wavfile);
endfunction

F0_list = [120,220]
vowel_frequency_matrix = [730, 1090, 2440; //F1,F2,F3 for vowel a
    270, 2290, 3010; //F1,F2,F3 for vowel i
    300, 870, 2240] //F1,F2,F3 for vowel u

vowel_list = ['a','i','u']

```

```

Fs = 16000

for i_f0 = 1:length(F0_list)
    F0 = F0_list(i_f0)
    t_duration = 0.5
    //numerator of filter assumed to be 1
    k = 1

    //specifying input signal
    n_samples = t_duration*Fs + 1
    n_fft = n_samples*10;
    x = zeros(n_samples,1);
    //impulse is approximated by narrow triangular pulse
    for i = 2:t_duration*F0
        x(floor((i-1)*Fs/F0))= 1
        x(floor((i-1)*Fs/F0 + 1))= 0.75
        x(floor((i-1)*Fs/F0 + 2))= 0.5
        x(floor((i-1)*Fs/F0 + 3))= 0.25
        x(floor((i-1)*Fs/F0 - 1))= 0.75
        x(floor((i-1)*Fs/F0 - 2))= 0.5
        x(floor((i-1)*Fs/F0 - 3))= 0.25
    end

    //Create delta input for finding impulse response
    delta_n = zeros(n_samples,1)
    delta_n(1) = 1

    for j = 1:length(vowel_frequency_matrix(1,:))
        F_list = vowel_frequency_matrix(j,:)
        //Bandwidth is constant for all formants
        B_list = [100,100,100]

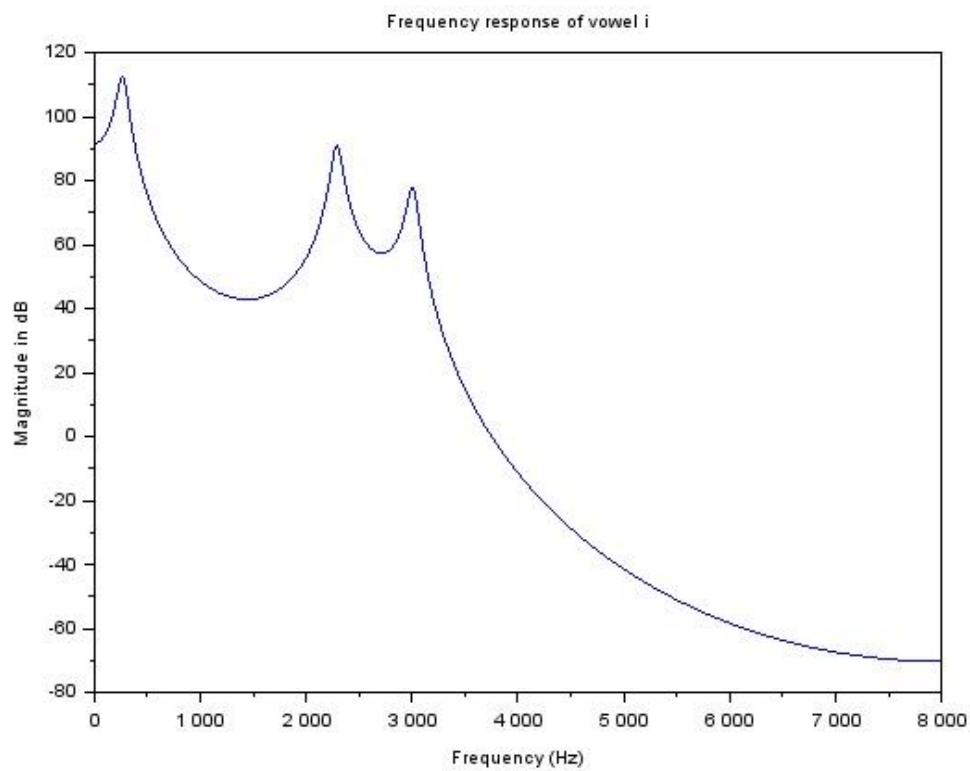
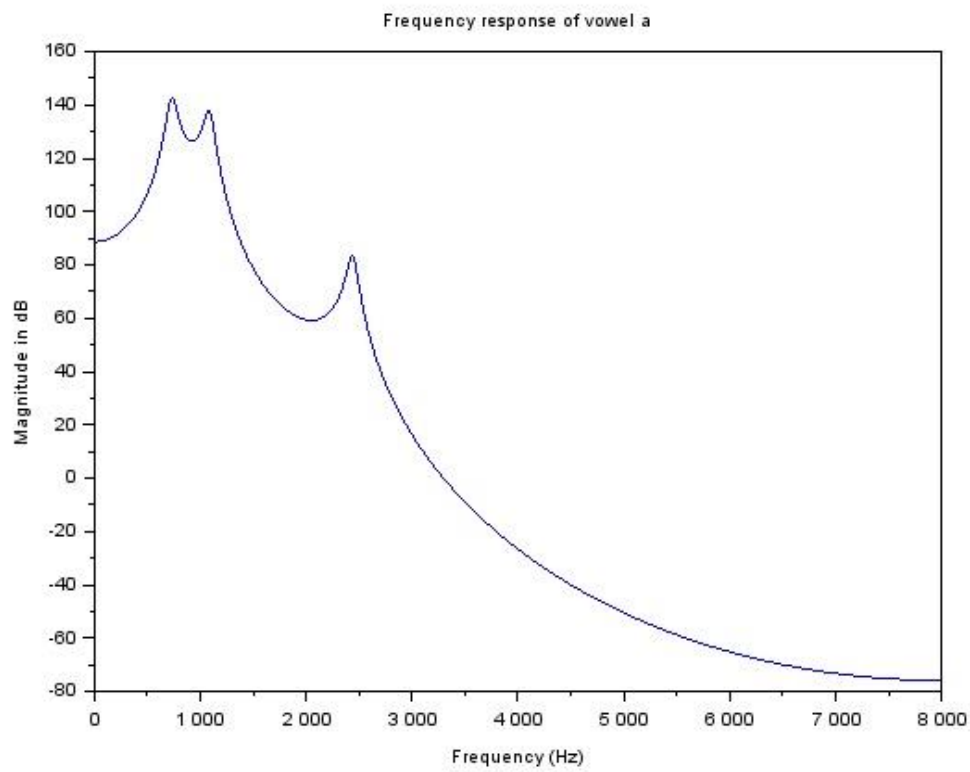
        //find output of cascade filter
        [num,den] = find_cascade_filter(F_list,B_list,Fs)
        h = time_response(delta_n,num,den)
        y = time_response(x,num,den)

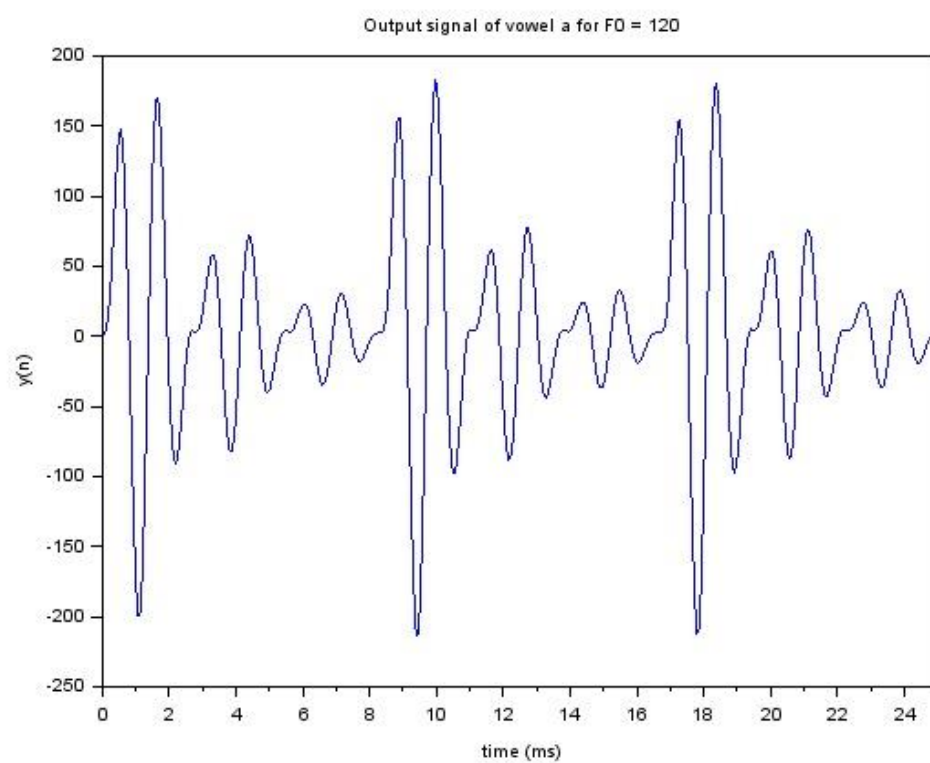
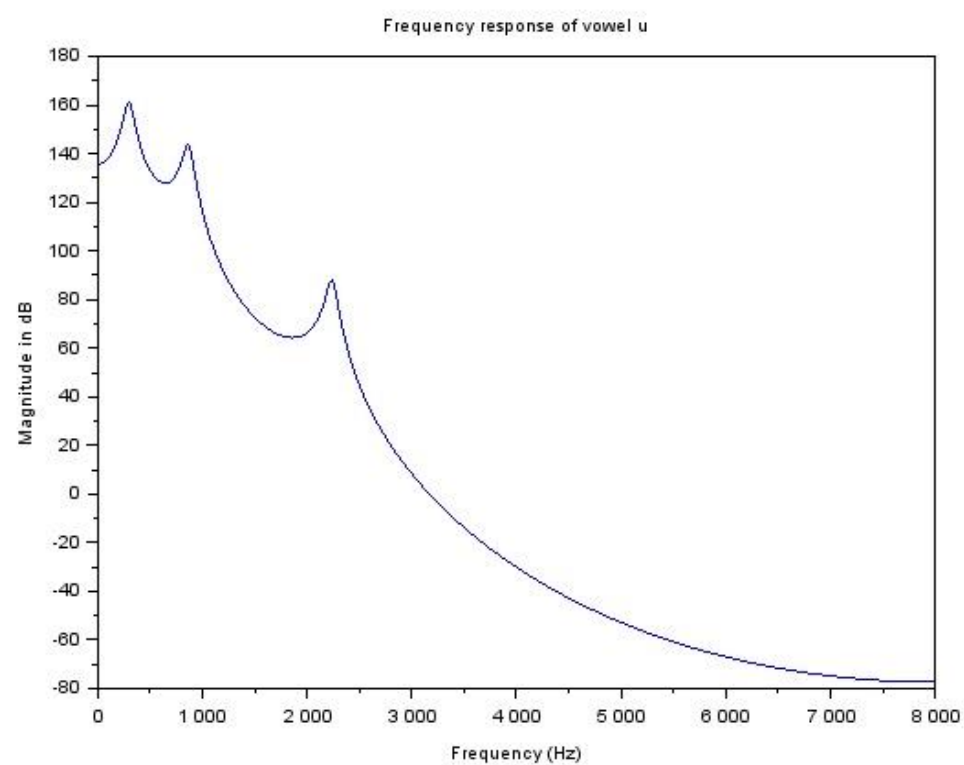
        //plot responses and save sound files
        plot_frequency_response(h,n_fft,vowel_list(j))
        plot_y(y,F0,Fs,vowel_list(j))
    end
end

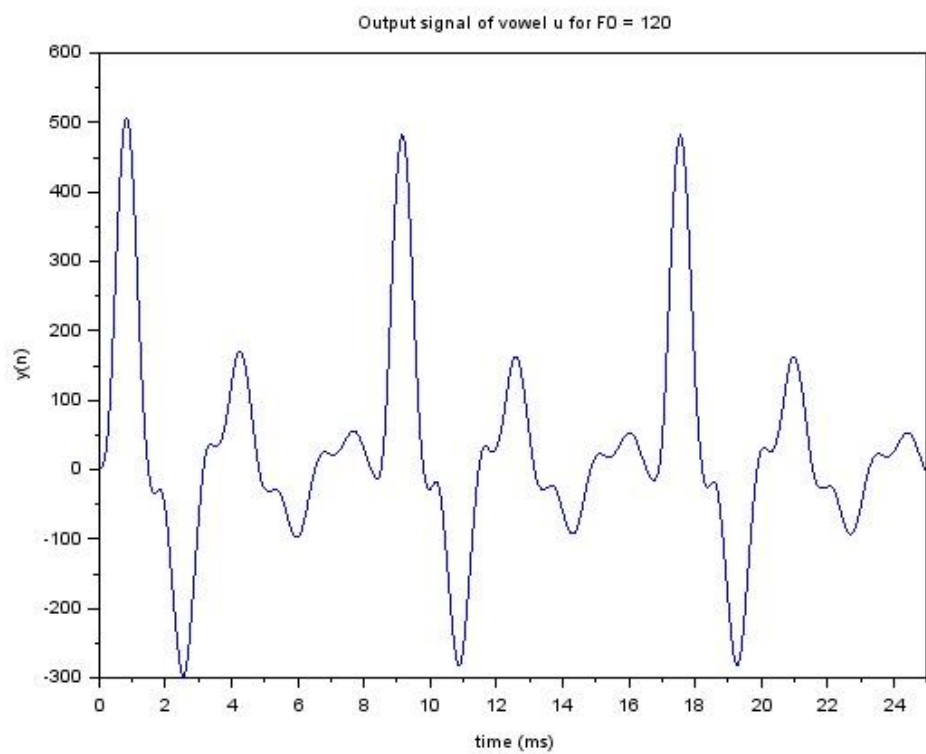
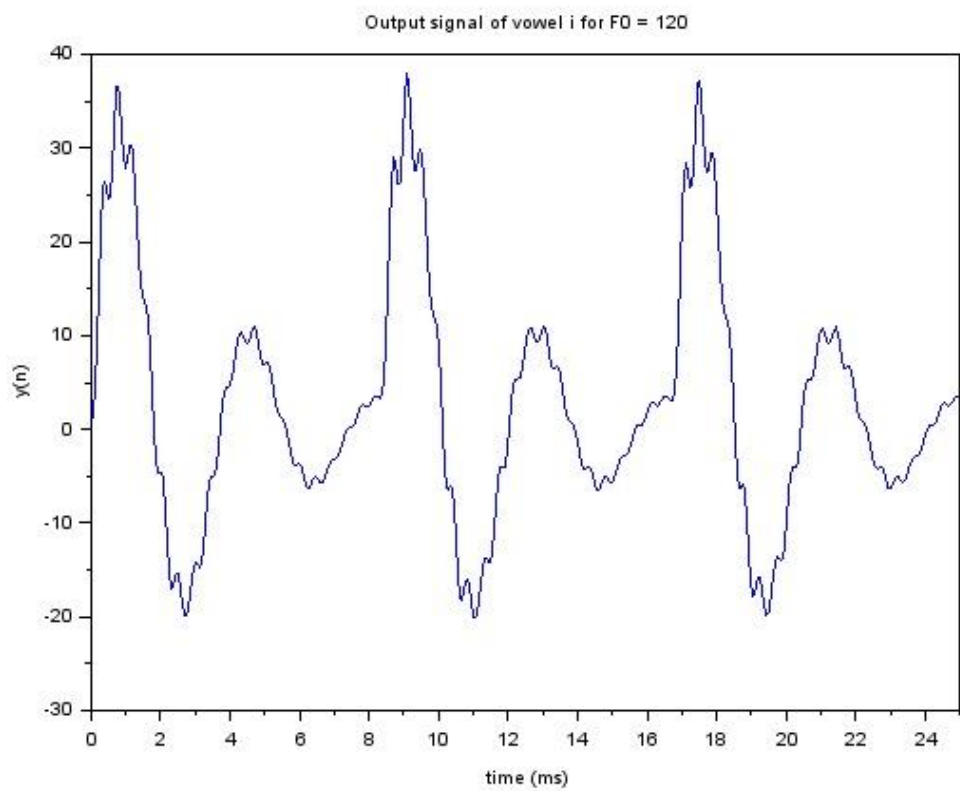
```

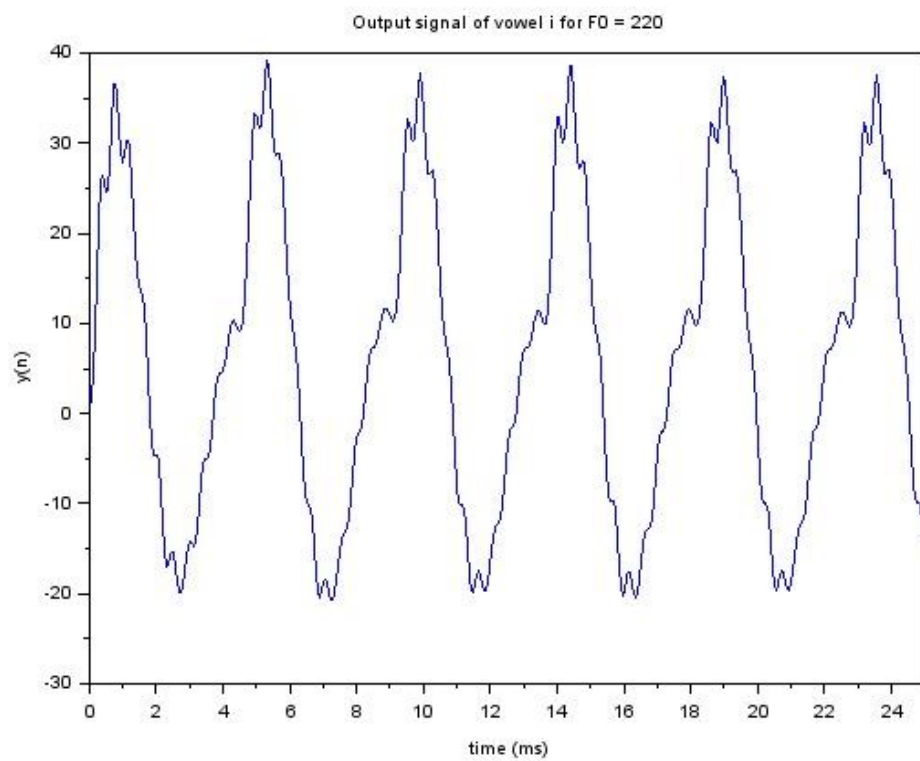
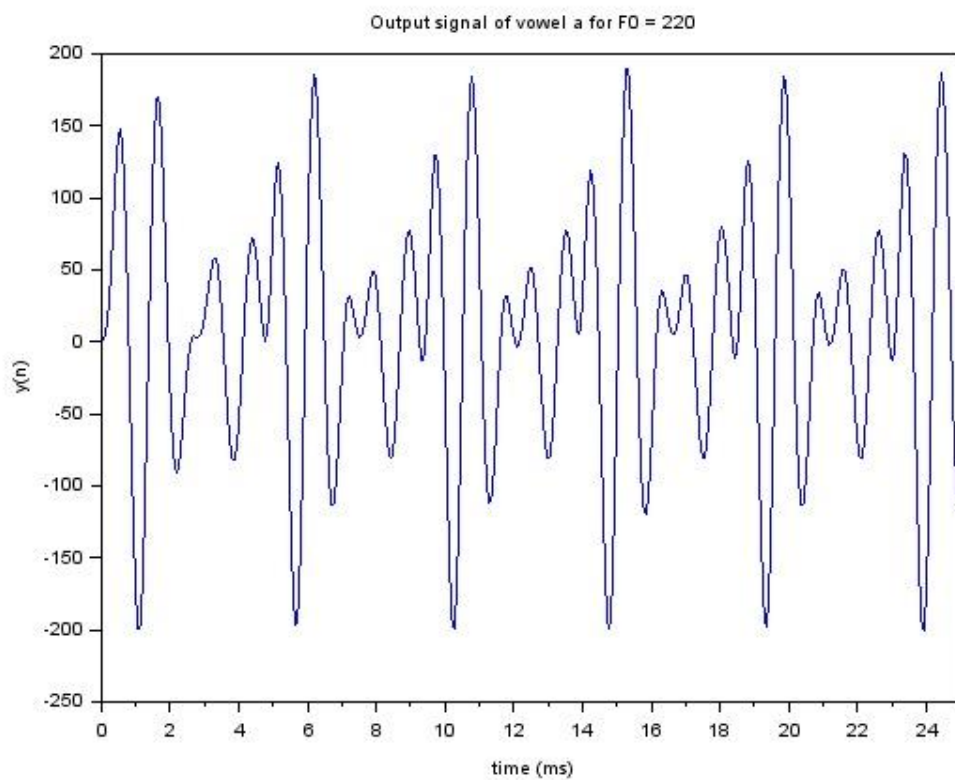
- Results

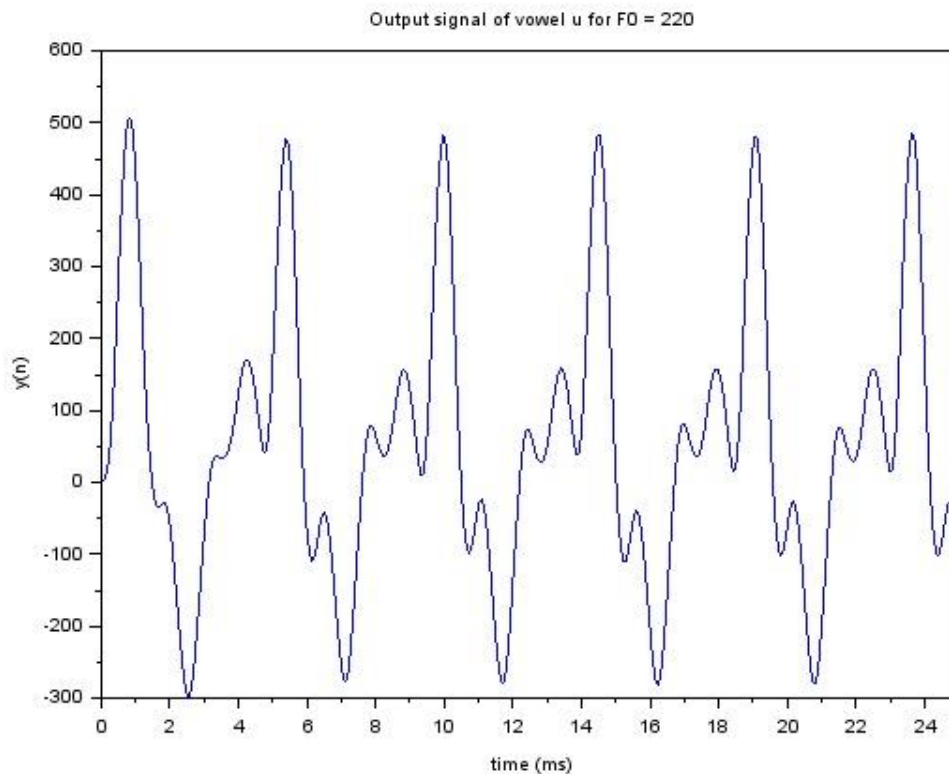
The plots for time domain output at different pitches and for different vowels are shown below. Also the frequency response of the cascade filter used is plotted.











- Discussion

The sound for all three vowels is coming properly. The sounds can be better observed if listen to them one after the other. As F0 changes, number of repetitive patterns in the given time frame (25 ms) will change. This is observed. Also the peak frequencies observed in frequency response are matching with the formant frequencies of the cascaded filters.

Question 5: Signal analysis using different windows

The wavfiles of different vowels generated were loaded. Windowed DFT was calculated for different windows and time durations

- Code

```
clear all
//function to read previously generated wavfiles
function y=read_vowel(vowel, F0)
    wavefile = strcat(['Output signal of vowel ',vowel,' for F0 = ',string(F0),'.wav'])
```

```

y = loadwave(strcat(['../wav_files/Q4/',wavefile]));
endfunction

function [X_mag]=find_windowed_FFT(x, window_type, N, n_fft)

    //we can put this window anywhere on the signal. We will put it
    //at the centre

    x_init = (length(x)+1)/2 - (N-1)/2
    select window_type
    case 'hamming' then
        win_hamming = window('hm', N);
        windowed_x = x(x_init: x_init + N - 1).*win_hamming;
    case 'rect' then
        windowed_x = x(x_init: x_init + N - 1)
    end

    //zero padding
    pad = zeros(1,ceil((n_fft - N)/2));
    padded_x = [pad windowed_x pad]
    //finding fft
    freq_x = fftshift(fft(padded_x))
    X_mag = abs(freq_x(n_fft/2 + 1:n_fft))
endfunction

window_times = [5, 10, 20, 40] //time in ms
Fs = 16000
//finding window lengths
window_lengths = window_times*Fs/1000 + 1
n_fft = max(window_lengths)*10

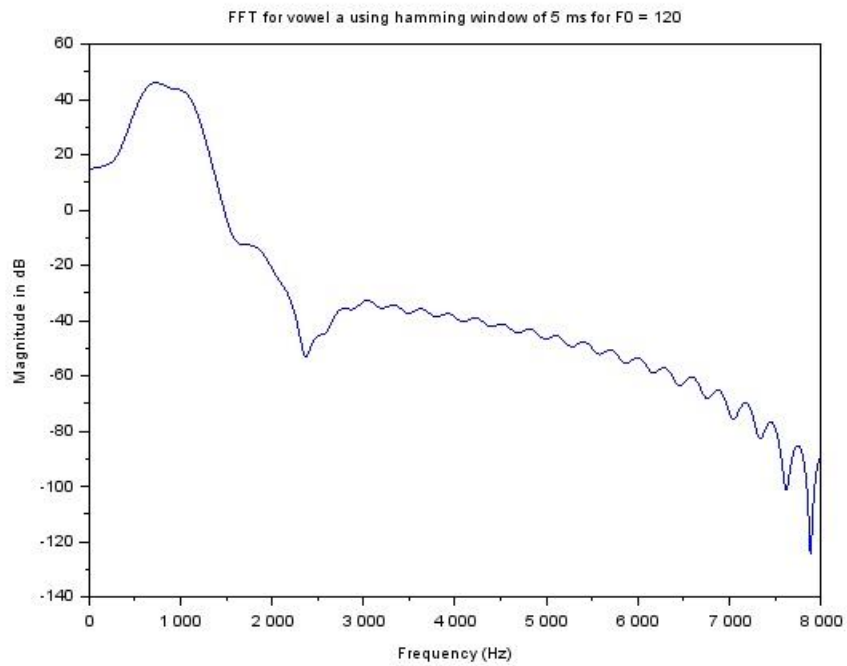
vowel_list = ['a','i','u','a','i','u']
F0_list = [120 120 120 220 220 220]
for p = 1:length(F0_list)
    vowel = vowel_list(p)
    F0 = F0_list(p)
    for window_type = ['hamming','rect']
        for i = 1:length(window_times)
            N = window_lengths(i)
            x = read_vowel(vowel,F0)
            [X_mag] = find_windowed_FFT(x,window_type,N,n_fft)
            freq_array = linspace(0,Fs/2,n_fft/2)
            fig = scf()
            plot(freq_array,20*log(X_mag))
            plot_title = strcat(['FFT for vowel ',vowel,' using ',window_type,' window of
',string(window_times(i)), ' ms',' for F0 = ',string(F0)])
            xtitle(plot_title,'Frequency (Hz)','Magnitude in dB')
            xs2jpg(gcf(), strcat(['../plots/Q5/',plot_title,'.jpg']));
        end
    end
end
end

```

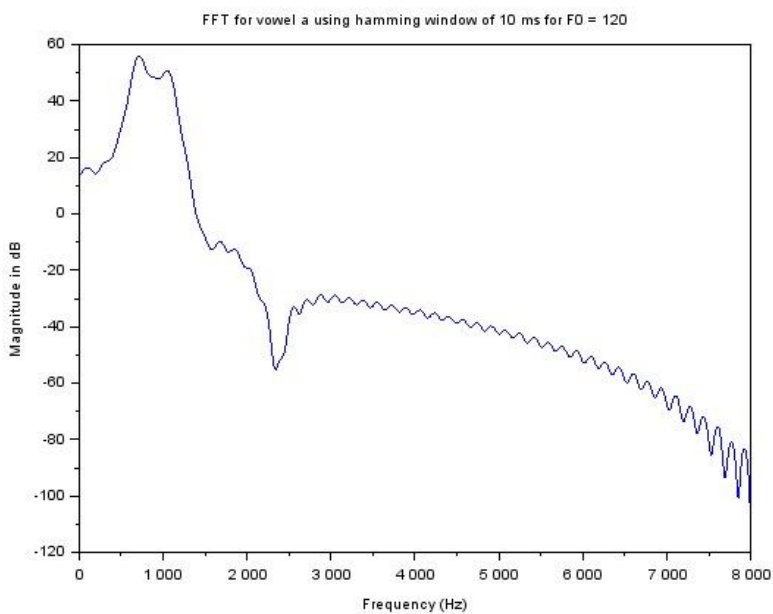
- Results

Vowel /a/ using hamming window at $F_0 = 120$ Hz

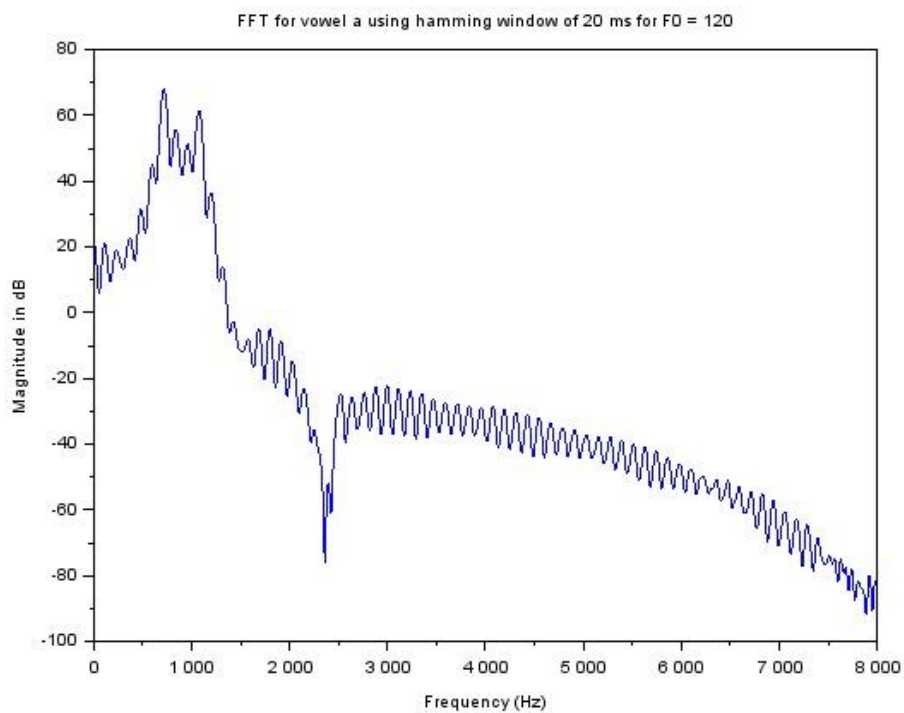
$t = 5$ ms



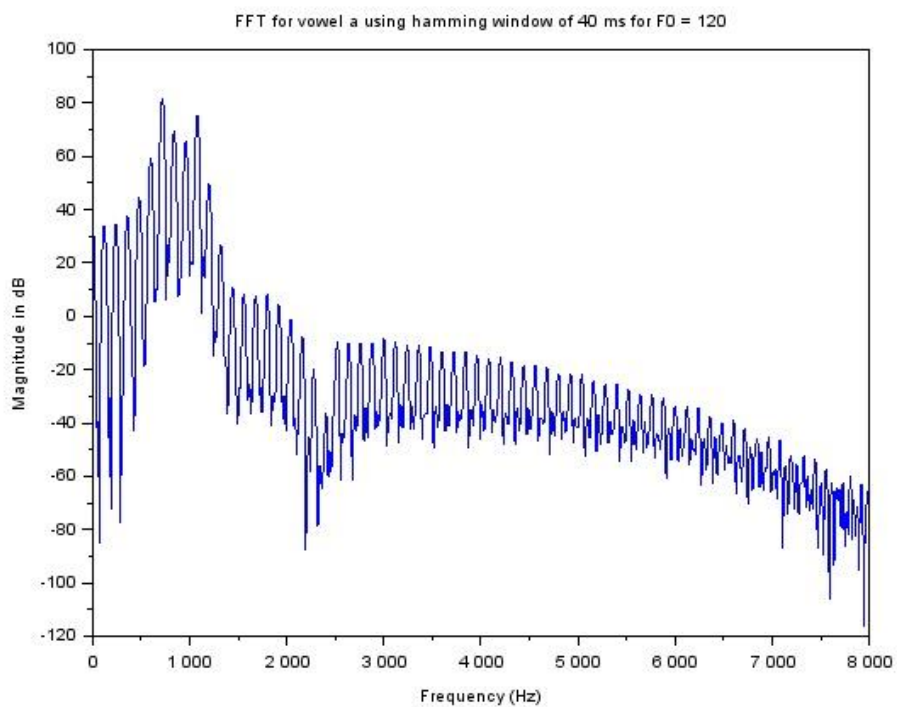
$t = 10$ ms



$t = 20$ ms

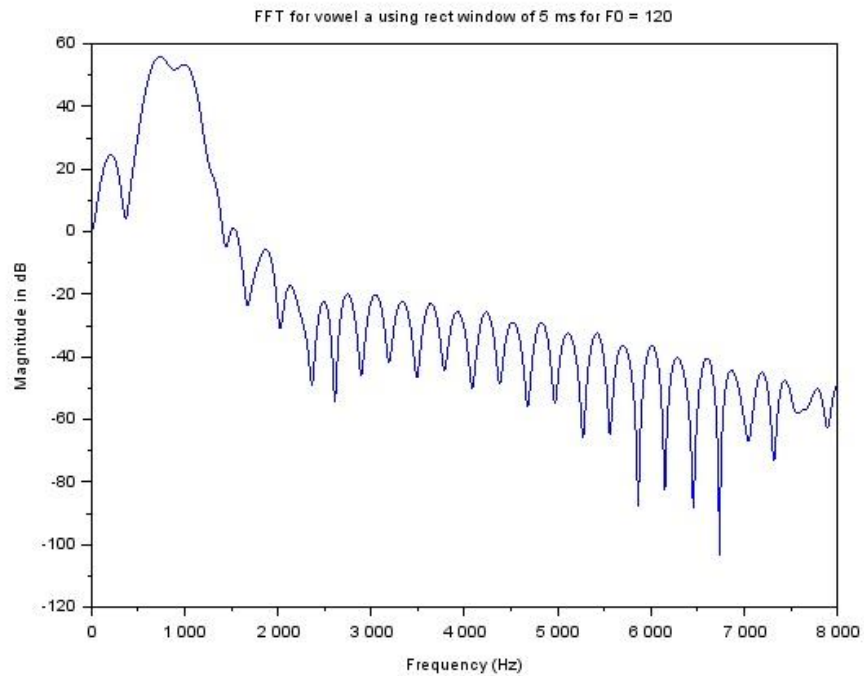


$t = 40$ ms

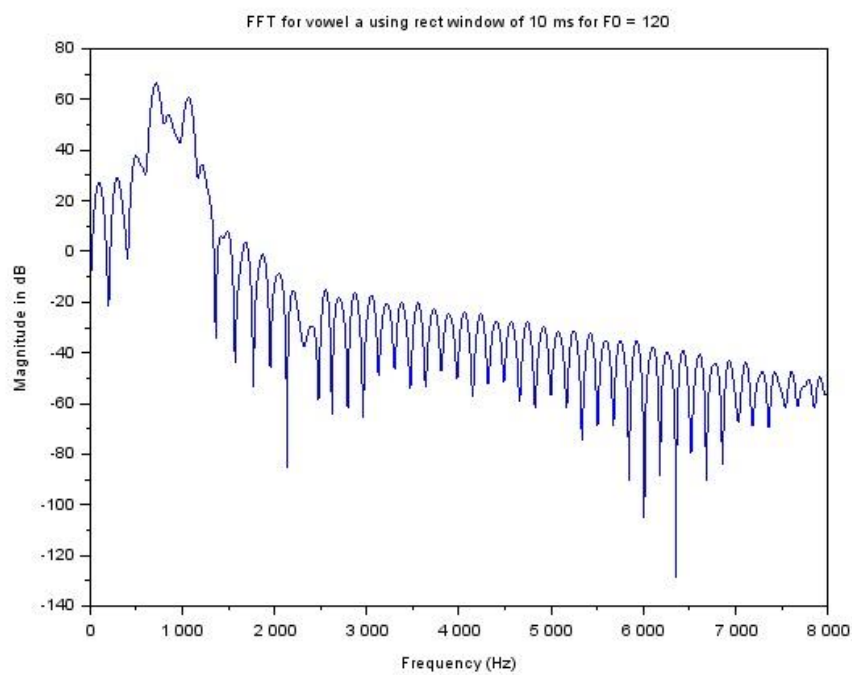


Vowel /a/ using rectangular window at $F_0 = 120$ Hz

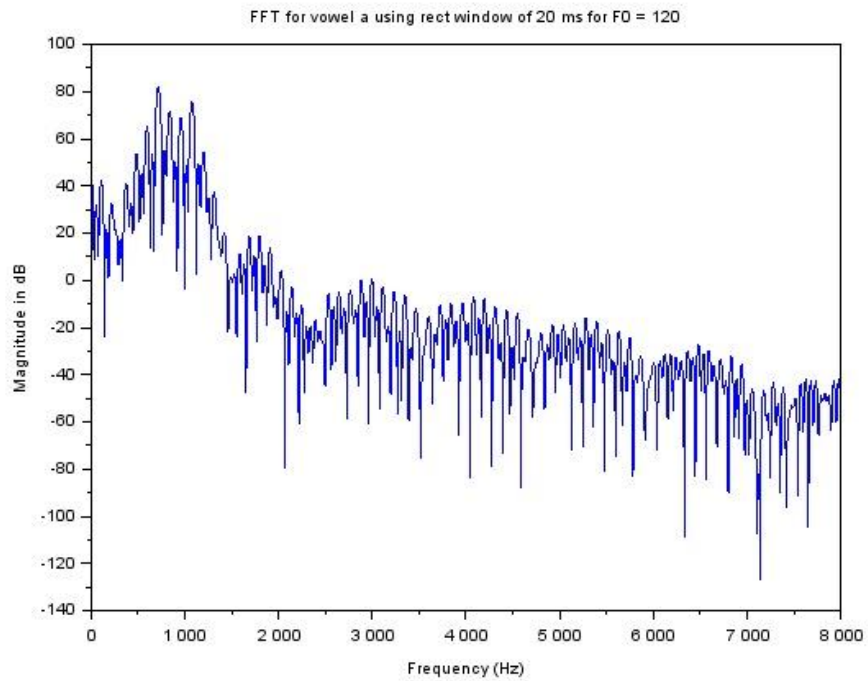
$t = 5$ ms



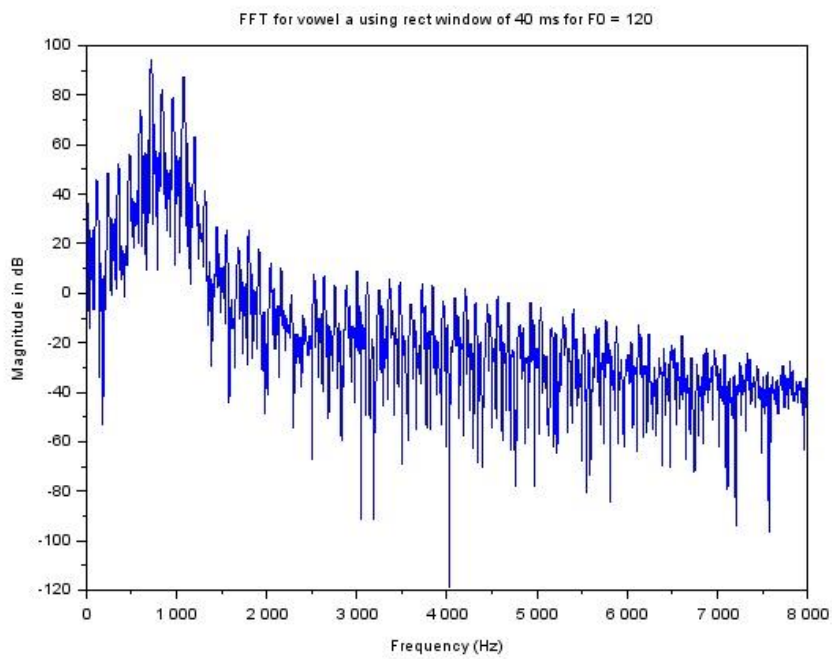
$t = 10$ ms



t = 20 ms

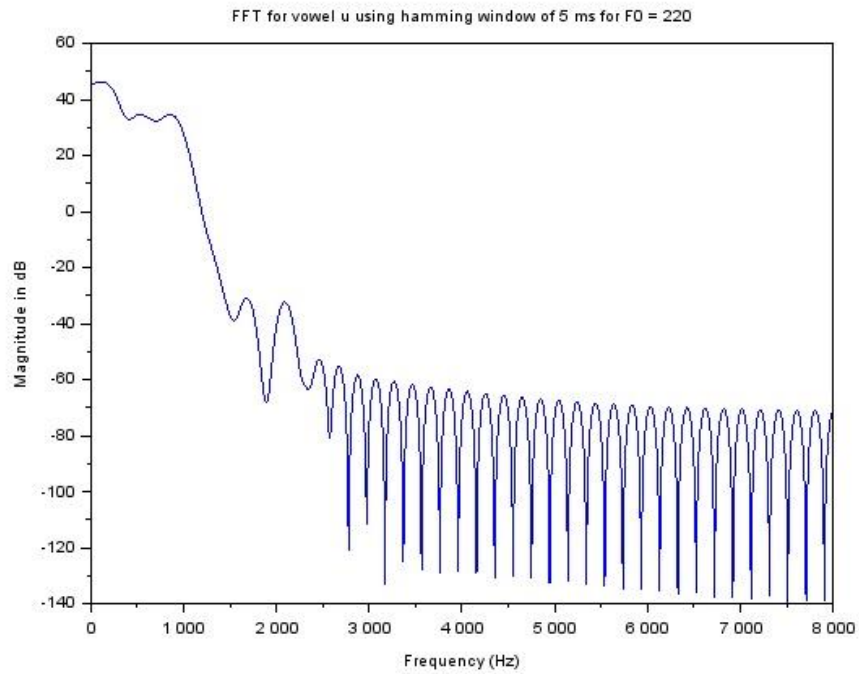


t = 40 ms

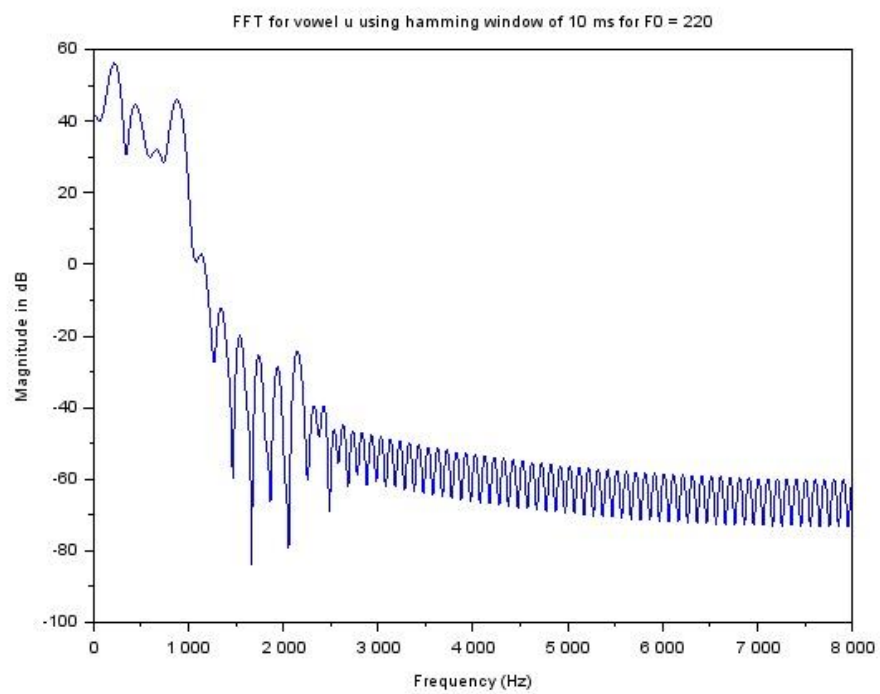


Vowel /u/ using hamming window at F0 = 220 Hz

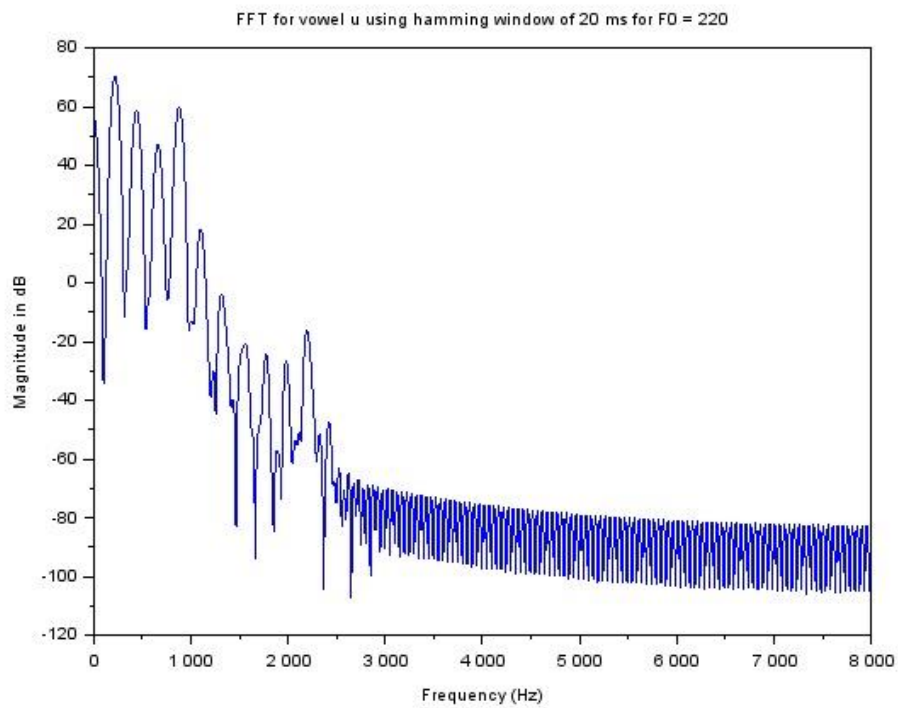
t = 5 ms



t = 10 ms

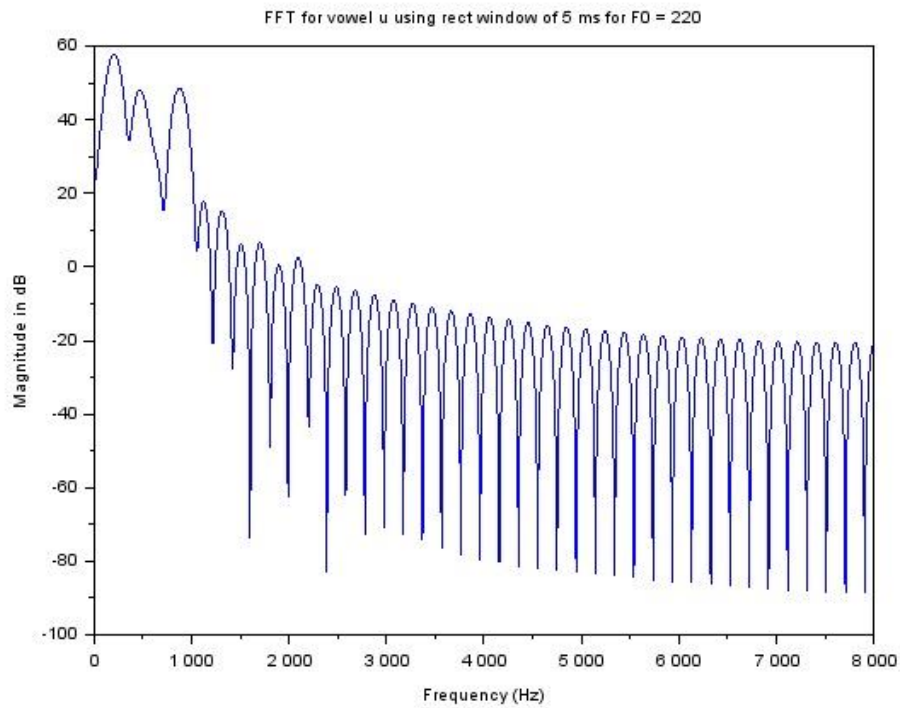


t = 20 ms

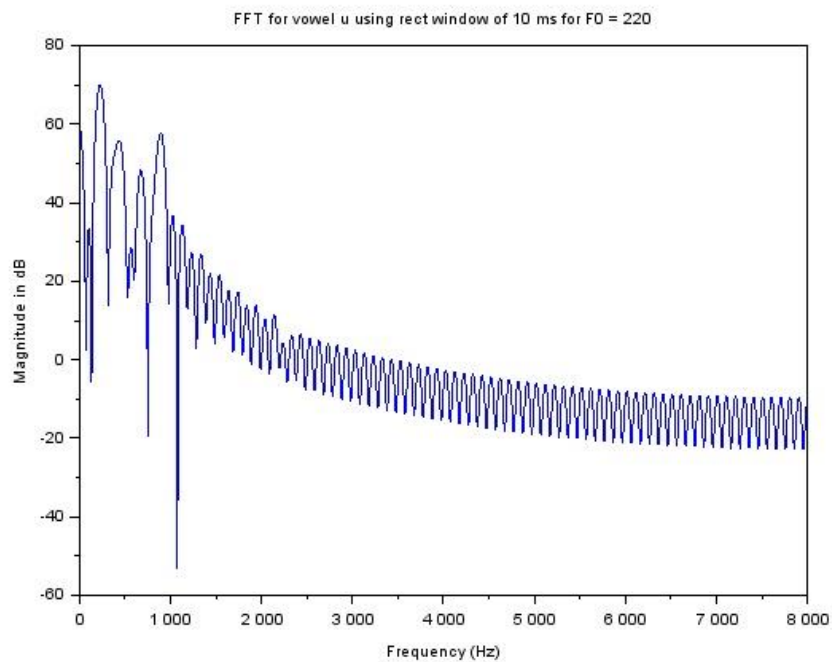


Vowel /u/ using rectangular window at $F_0 = 220$ Hz

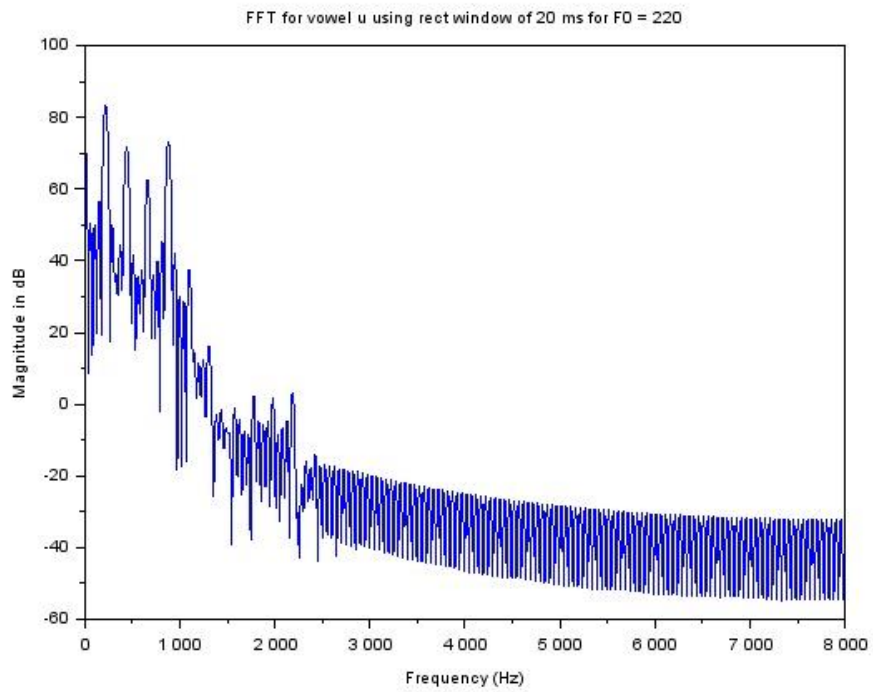
$t = 5$ ms



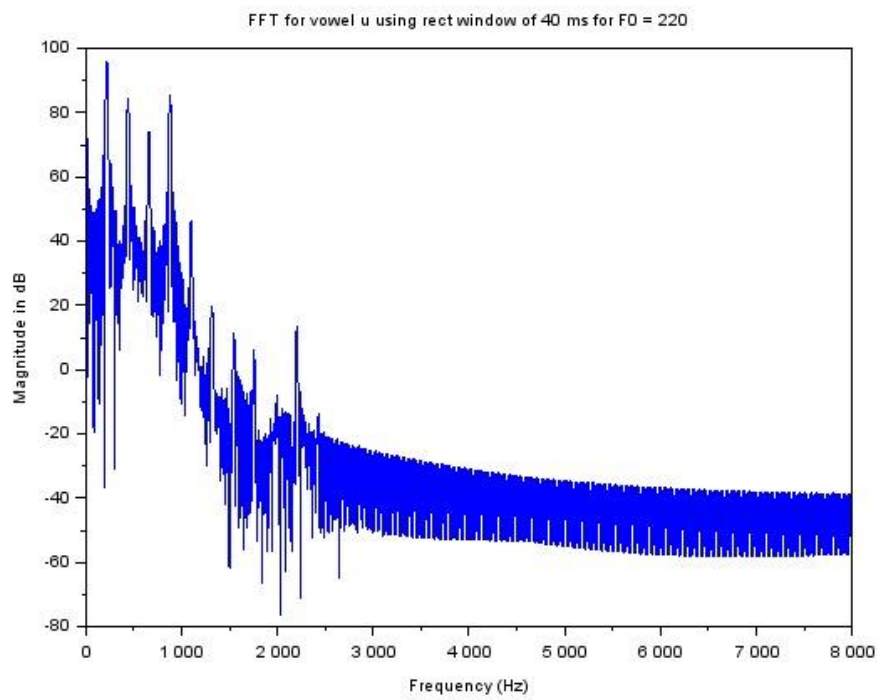
$t = 10$ ms



$t = 20 \text{ ms}$



$t = 40 \text{ ms}$



- Discussion

1. **Similarities and differences between different spectra**

There are mainly two things changing for a given vowel, window type and length of window.

- a. Effect of window size on spectrum

It is observed that spectra with low window size (windows of shorter duration) are smoother compared to that of longer durations. This phenomenon was observed for both Hamming as well as rectangular window. This is expected because for a given window type shorter length in time domain implies more spread in frequency domain. Original spectrum when convolved with this window having more spread in frequency domain, small variations will be removed and we will obtain a smoother window. This destroys pitch information (since multiple peaks will be merged together). But we can't use longer windows in real life scenarios, because real signals are not stationary over longer duration.

- b. Effect of window type on spectrum

Main lobe width of Hamming window is more than that of rectangular window. Therefore Hamming window spreads the spikes in original signal more than rectangular window. We can observe that peaks in case of Hamming window are broader compared to rectangular window. But sidelobes are more significant in case of rectangular window than Hamming window. Therefore detecting formants is more difficult in case rectangular window due to high energy sidelobes. Therefore hamming window should be preferred for detecting formants. But to find the exact frequency of the detected formant, we should use rectangular window, since peaks in that case will be sharper.

2. **Signal Parameters**

- a. Formant frequencies

Formant frequencies for all combinations both the vowels were found out manually by zooming into the spectrum

For vowel /a/ at F0 = 120 Hz

Window	Formant 1	Formant 2	Formant 3
Real value (ground truth from Q4)	730	1090	2440
Hamming 5 ms	718	988	2792
Hamming 10 ms	707	1058	2550
Hamming 20 ms	726	1077	2527
Hamming 40 ms	716	1076	2517
Rectangular 5 ms	724	992	2721
Rectangular 10 ms	721	1068	2312
Rectangular 20 ms	718	1073	Not clear
Rectangular 40 ms	718	1079	Not clear

For vowel /u/ at F0 = 220

Window	Formant 1	Formant 2	Formant 3
Real value (ground truth from Q4)	300	870	2240
Hamming 5 ms	150	860	2098
Hamming 10 ms	225	875	2142
Hamming 20 ms	222	877	2192
Hamming 40 ms	224	878	2209
Rectangular 5 ms	200	868	2090
Rectangular 10 ms	225	894	2135
Rectangular 20 ms	213	874	2184
Rectangular 40 ms	217	878	2202

b. Bandwidth

Bandwidth is calculated by considering envelope of the spectrum and finding a frequency difference between the points 3dB below the formant frequencies. For generation we simplistically assumed $B = 100$ Hz for all formants for all vowels. But when we try to measure it using the method mentioned above, there are lots of parameters involved. Hence bandwidth estimate isn't much reliable as compared formant.

c. Pitch

Pitch is estimated by frequency difference between two points divided by number of peaks in between. Here we assume that every spike in signal is captured as a peak in estimated spectrum. But that is not the case always. Hence estimated pitch was varying between 100 Hz to 200 Hz for 120 Hz vowels.