Segmentation of Brain MR Images

EE638 Course project

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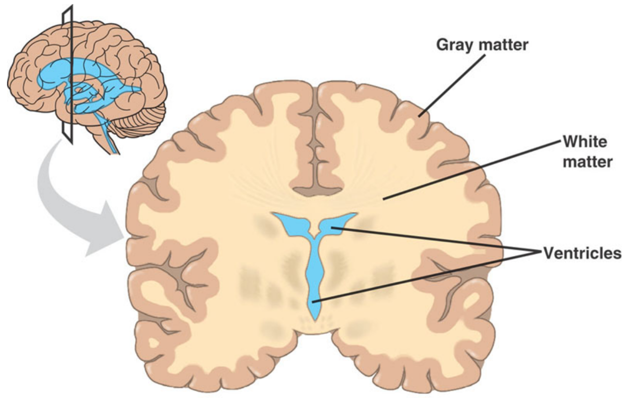


Figure 1: 3 major segments of the brain

Problem Statement

To segment raw noisy MR image of brain into 3 segments (gray matter, white matter and cerebrospinal fluid) using Expectation Maximization (EM) algorithm which relies on Gaussian Mixture Model (GMM) for pixel intensities and assumes Markov Random Field (MRF) prior on pixel values.

0.1 Background and motivation

Human brain can be divided into 3 main categories: White matter, gray matter and cerebrospinal fluid.

Brain image histrogram:

The intensity histogram gives information about those 3 tissues of the brain in addition to a bias field, which is caused by the imperfect RF pulse. Bias field is multiplicative in nature. So, before passing the image to the algorithm, we need to remove the bias field. Fuzzy C-means clustering gives a membership for each of the pixels and then can be used to solve for the bias.

0.2 Algorithm

The histogram is modelled as a mixture of 3 gaussians with unknown means and variances. So, the histogram is basically modelled as a probability distribution:

$$p(x) = \sum_{k=1}^{K} w_k G(x; \mu_k, C_k)$$

We use the Expectation Maximisation algorithm to solve this as a maximum likelihood problem on μ and C:

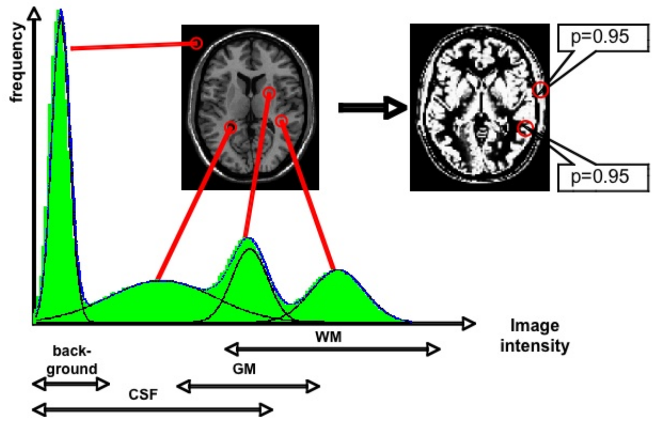


Figure 2: Bias field and brain image histogram

Step 1: E-step

$$Q(\theta;\theta^i) = E_{P(z|y,\theta^i)}[\log P(z,y|\theta)]$$

Step 2: M-step

$$\theta^{i+1} = \underset{\theta}{\operatorname{argmax}}\ Q(\theta;\theta^i)$$

$y = \{y_n\}$ *Observed data*

$z = \{z_n\}$ *class labels (hidden variables)*

After iterating these steps, we get a membership probability for every pixel:

$$\gamma^i_{nk} = P(z_n = k|y_n, \theta^i) = \frac{G(y_n|\mu^i_k, C^i_k)w_k}{\sum_{k=1}^{K} G(y_n|\mu^i_k, C^i_k)w_k}$$

where the parameter updates are given by:

$$\mu^{i+1}_k = \frac{\sum \gamma^i_{nk} y_n}{\sum \gamma^i_{nk}}$$

$$C^{i+1}_k = \frac{\sum \gamma^i_{nk}(y_n - \mu^i_k)(y_n - \mu^i_k)^{-1}}{\sum \gamma^i_{nk}}$$

0.3 Introduction of MRF prior

The problem with GMM+EM segmentation is that it doesn't enforce spatial smoothness constraint for segmentation of the image. It will directly predict the ML estimator. To avoid this, an MRF smoothness prior can be used.

0.3.1 Markov Random Fields

Markovianity: Pixel value is conditionally independent of values at non-neighboring pixels if values at neighboring pixels are given.

$$P(X_i|X_{S-\{i\}}) = P(X_i|X_{N_i})$$

Homogeneous Markov Random field: Probability Xi given its neighbors is independent of location of Xi, i.e. independent of i .

0.3.2 MRF+GMM+EM

Instead of finding the update on entire dataset, we will try to to maximize it at every point conditioned on its neighbours (ICM algorithm).

$$MAP \;\; segmentation$$

$$\max_{Z} \; P(Z|y,\theta)$$

$$Z = \{z_l \;\; (label \;\; of \;\; y_l)\}$$

$$y = \{y_n \;\; (observed \;\; input)\}$$

After calculation and an approximation in E-step, we obtain:

$$Q(\theta;\theta^i) = \sum_{n=1}^{N}\sum_{k=1}^{K} P(z_n = k \mid x_{N_n}^{MAP}, y, \theta^i) \log P(y^n|z_n = k, \theta)$$

and the membership values as:

$$\gamma_{nk} = \frac{G(y_n|\mu_k,\sigma_k)P(x_n=k|x_{N_n}^{MAP})}{\sum_{k=1}^{K} G(y_n|\mu_k,\sigma_k)P(x_n=k|x_{N_n}^{MAP})}$$

Hammersley–Clifford theorem: An MRF can equivalently be characterized by a Gibbs distribution.
Hence, we have the following potential function:

$$P(z_n|z_{N_n}) = \frac{\exp(-\sum_{a\in A_n} V_a(z_a)}{\sum \exp(-\sum_{a\in A_n} V_a(z_a)}$$

In our implementation, we used 4 neighborhood in image and define V(L1,L2) = 0 if L1 = L2 else V(L1,L2) = β. β is a tunable parameter which determines strength of the prior.

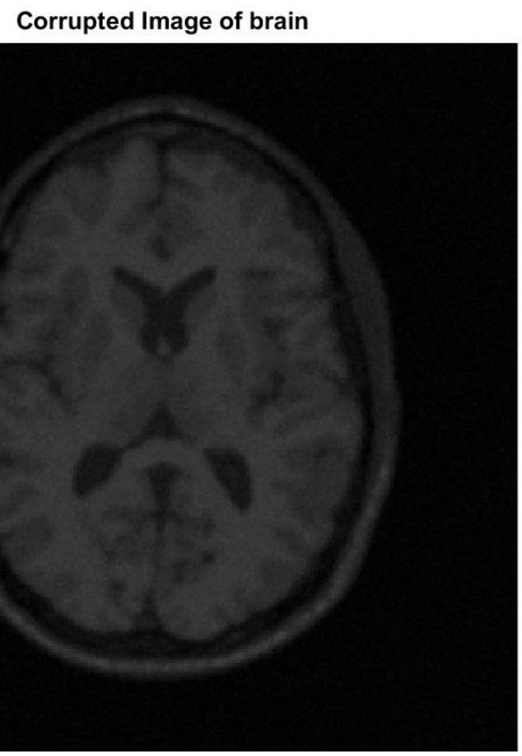
0.4 Simulation

0.4.1 Raw corrupted input image

An MR image of the brain is taken as an input to the system

Algorithm 0.1 MRF+GMM+EM algorithm

Preprocess raw input image to remove bias field etc.
Initialize parameters (means, covariances etc)
E - step:
Compute MAP label image, given parameters
Evaluate memborship
M - step:
Update covariances and means
Repeat E and M step until convergence



0.4.2 Preprocessing

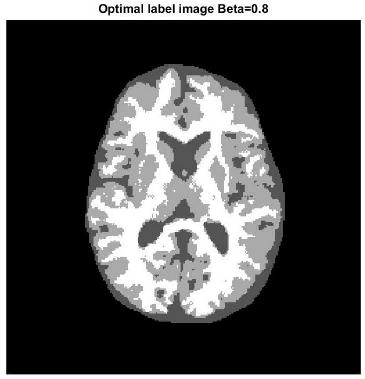
Bias field is removed. Initial parameters (means and class labels) are computed by using Fuzzy C-Means (FCM) segmentation. This image is passed to the EM algorithm:

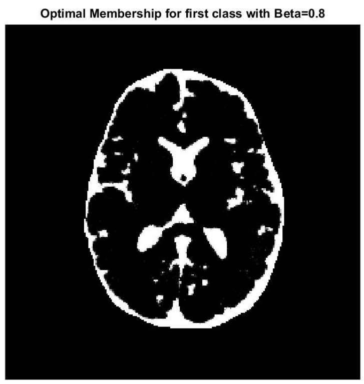
Initial Guess of Labels using question 1

0.4.3 Output image

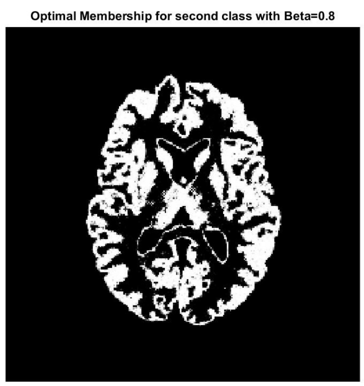
It can be clearly seen that segmented image is smooth compared to input image and classes can be distinguished unambiguously. The trade off between smooth vs representative of data is made through a tuning parameter beta, which determine the strength of the prior.

Optimal label image Beta=0.8

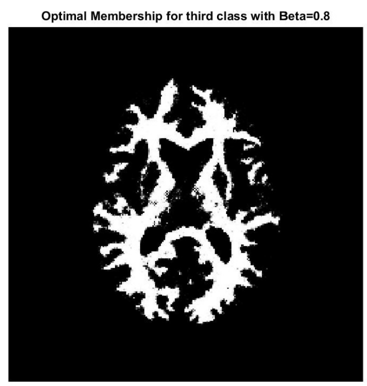




(a) Cerebrospinal fluid



(b) Gray matter



(c) White matter

Figure 3: Classes of the MR scan

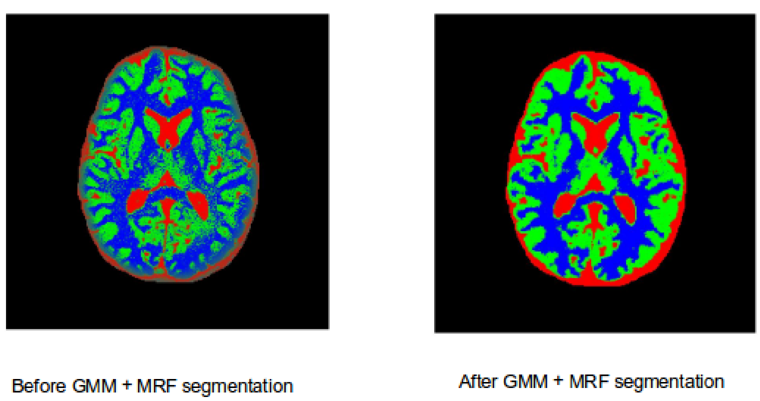


Figure 4: RGB representation of the classes

Bibliography

[1] Zhang, Y., Brady, M., & Smith, S. (2001). Segmentation of brain MR images through a hidden Markov random field model and the expectation-maximization algorithm. IEEE transactions on medical imaging, 20(1), 45-57.

[2] Shah, S. A., & Chauhan, N. C. (2015). An Automated Approach for Segmentation of Brain MR Images using Gaussian Mixture Model based Hidden Markov Random Field with Expectation Maximization. Journal of Biomedical Engineering and Medical Imaging, 2(4), 57