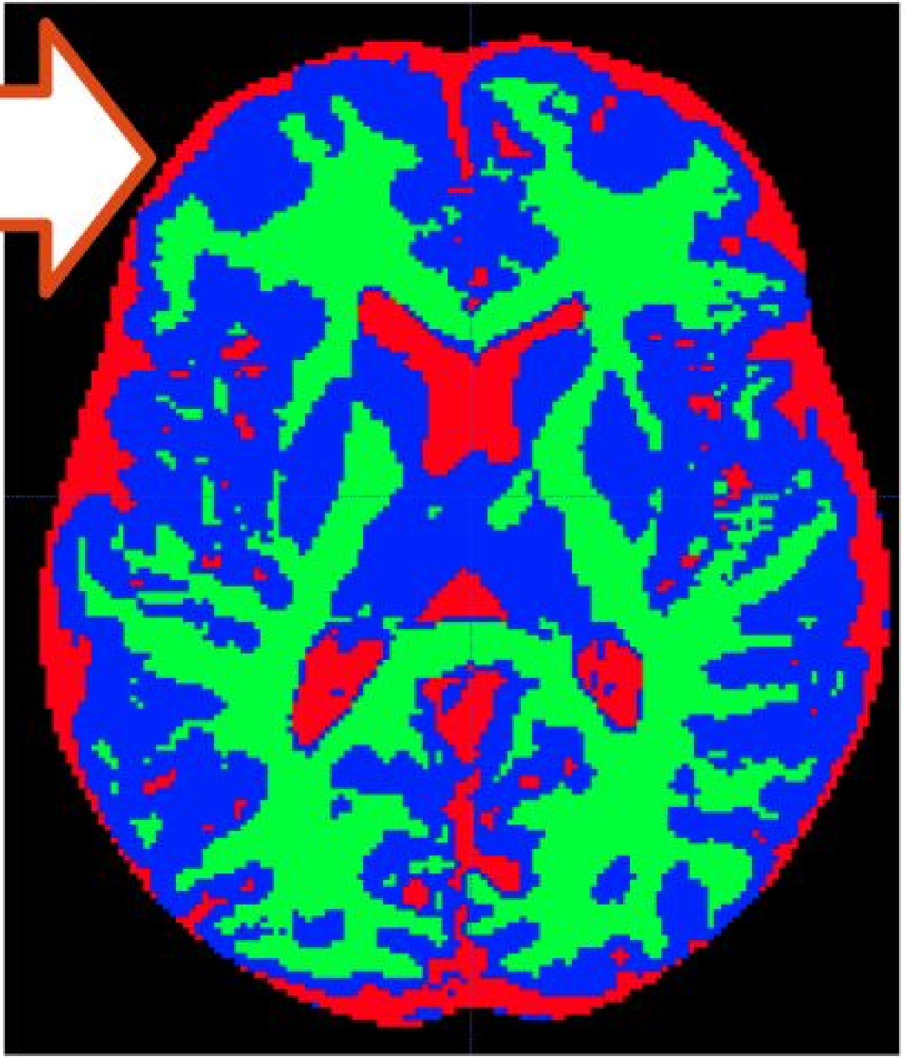
# Brain Image Segmentation

Kalpesh Patil (130040019)
Yash Bhalgat (13D070014)

# Background and Motivation

# Problem Statement

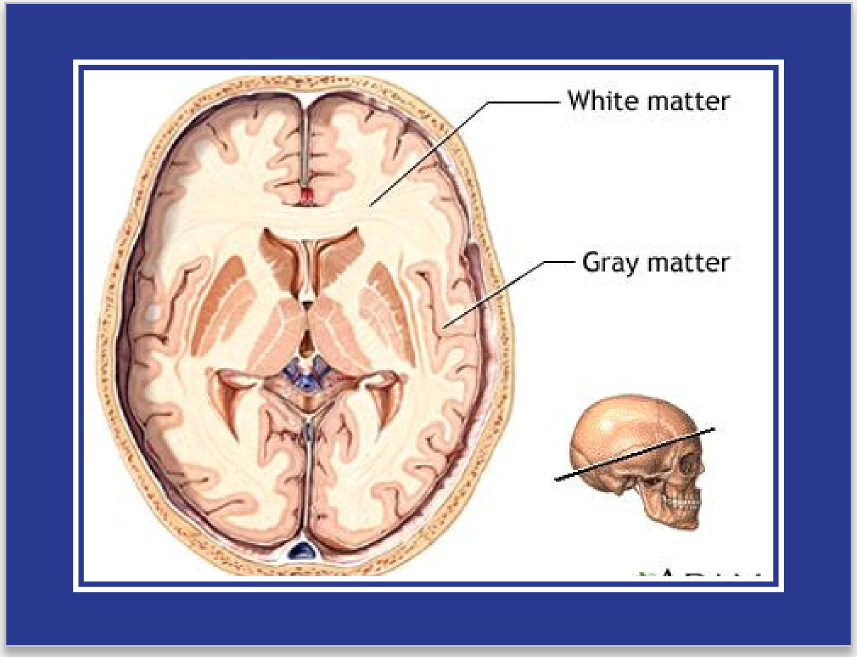
To segment raw noisy MR image of brain into 3 segments (gray matter, white matter and cerebrospinal fluid) using Expectation Maximization (EM) algorithm which relies on Gaussian Mixture Model (GMM) for pixel intensities and assumes Markov Random Field (MRF) prior on pixel values

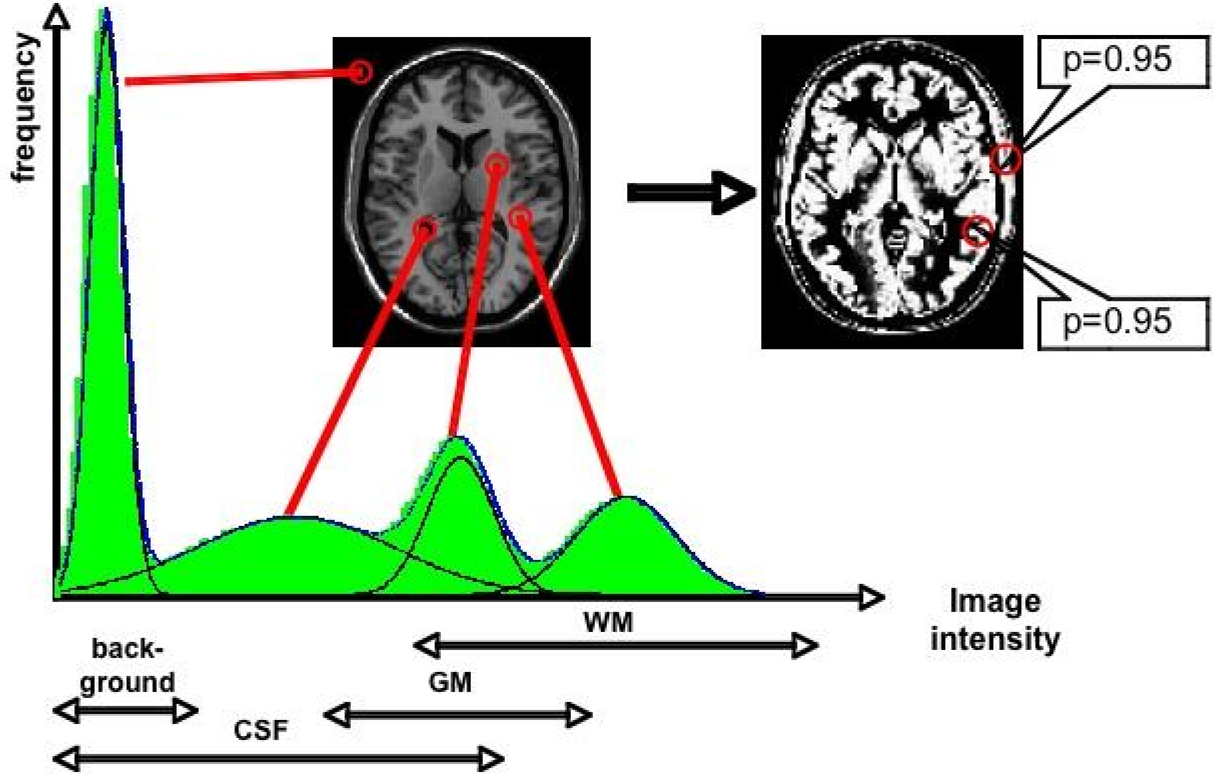


# Background and Motivation

Human brain can be mainly divided into three major segments

1. White matter
2. Gray matter
3. Cerebrospinal fluid






Brain
Image
Histogram

# Algorithm

# GMM

$$p(x) = \sum_{k=1}^{K} w_k G(x; \mu_k, C_k)$$

Probability as sum of Gaussians

In our case K = 3

Maximizing likelihood of parameters

$$\max_{\theta} L(\theta|y) = \max_{\theta} \sum_{n=1}^{N} \log\left( \sum_{k=1}^{K} w_k G(x; \mu_k, C_k) \right)$$

# Expectation Maximization

$$Step\ \ 1:\ \ E\text{-}step$$

$$Q(\theta;\theta^{i}) \ = \ E_{P(z|y,\theta^{i})}[\log P(z,y|\theta)]$$

$$Step\ \ 2:\ \ M\text{-}step$$

$$\theta^{i+1} \ = \ \underset{\theta}{argmax}\ \ Q(\theta;\theta^{i})$$

$$y \ = \ \{y_n\}\ \ Observed\ \ data$$

$$z \ = \ \{z_n\}\ \ class\ \ labels\ \ (hidden\ \ variables)$$

# Expectation Maximization Contd..

Membership of yn to the class k

$$\gamma^{i}_{nk} = P(z_n = k | y_n, \theta^i) = \frac{G(y_n | \mu^i_k, C^i_k) w_k}{\sum_{k=1}^{K} G(y_n | \mu^i_k, C^i_k) w_k}$$

Parameter updates

$$\mu^{i+1}_k = \frac{\sum \gamma^{i}_{nk} y_n}{\sum \gamma^{i}_{nk}} \qquad C^{i+1}_k = \frac{\sum \gamma^{i}_{nk} (y_n - \mu^i_k)(y_n - \mu^i_k)^{-1}}{\sum \gamma^{i}_{nk}}$$

# GMM + EM Segmentation

- **Problem:** It doesn't enforce spatial smoothness constraint for segmentation of the image. It will directly predict the ML estimator

- **Solution:** Use MRF (Markov Random FIeld) prior on label images which enforces smoothness constraint and gives MAP estimate

# Markov Random Field (MRF)

- **Markovianity:**
  Pixel value is conditionally independent of values at non-neighboring pixels if values at neighboring pixels are given

$$P(X_i | X_{S-\{i\}}) = P(X_i | X_{N_i})$$

- **Homogeneous Markov Random field**
  Probability Xi given its neighbors is independent of location of Xi, i.e. independent of i

# MRF + GMM + EM

$$MAP \ \ segmentation$$

$$\max_{Z} \ P(Z|y,\theta)$$

$$Z = \{z_l \ \ (label \ \ of \ \ y_l)\}$$

$$y = \{y_n \ \ (observed \ \ input)\}$$

Instead of finding the update on entire dataset, we will try to to maximize it at every point conditioned on its neighbours (ICM algorithm).

# MRF + GMM + EM

After calculation and an approximation in E-step, we obtain

$$Q(\theta;\theta^i) = \sum_{n=1}^{N}\sum_{k=1}^{K} P(z_n = k \mid x_{N_n}^{MAP}, y, \theta^i) \log P(y^n \mid z_n = k, \theta)$$

Membership values

$$\gamma_{nk} = \frac{G(y_n \mid \mu_k, \sigma_k) P(z_n = k \mid z_{N_n}^{MAP})}{\sum_{k=1}^{K} G(y_n \mid \mu_k, \sigma_k) P(z_n = k \mid z_{N_n}^{MAP})}$$

# MRF + GMM + EM

By Hammersley–Clifford theorem, an MRF can equivalently be characterized by a Gibbs distribution. Hence we have following potential function

$$P(z_n|z_{N_n}) = \frac{exp(-\sum_{a \in A_n} V_a(z_a))}{\sum exp(-\sum_{a \in A_n} V_a(z_a))}$$

We used 4 neighborhood in image and define V(L1,L2) = 0 if L1 = L2 else V(L1,L2) = beta

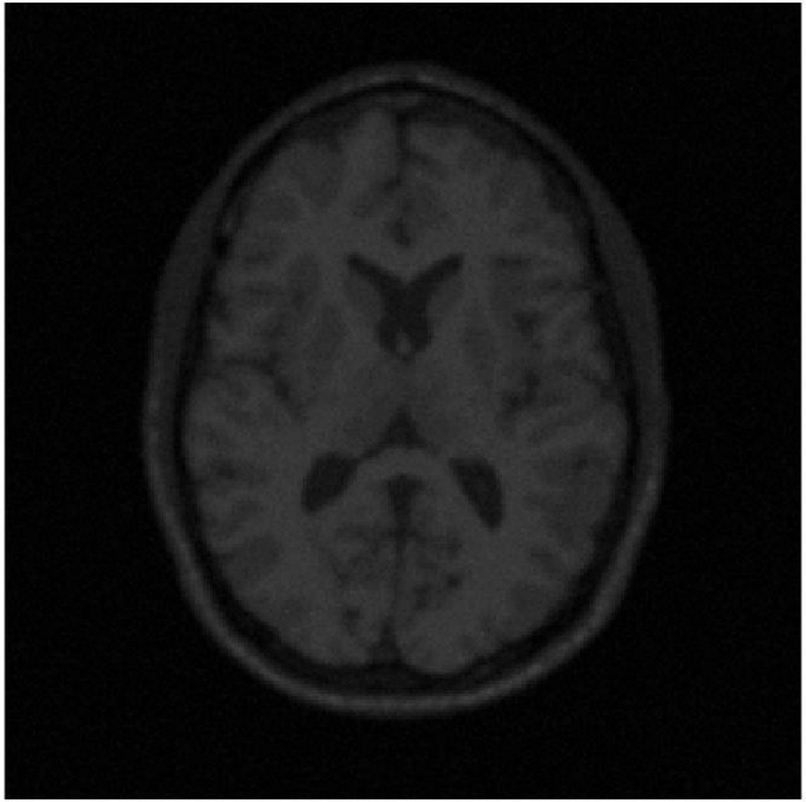
Here beta is a tunable parameter which determines strength of the prior

# Algorithm

- Preprocess raw input image to remove bias field etc.
- Initialize parameters (means, covariances etc)
- E - step
  - Compute MAP label image, given parameters
  - Evaluate membership
- M - step
  - Update covariances and means
- Repeat E and M step until convergence
- Output memberships (soft, spatially smooth)

# Simulation

Corrupted Image of brain

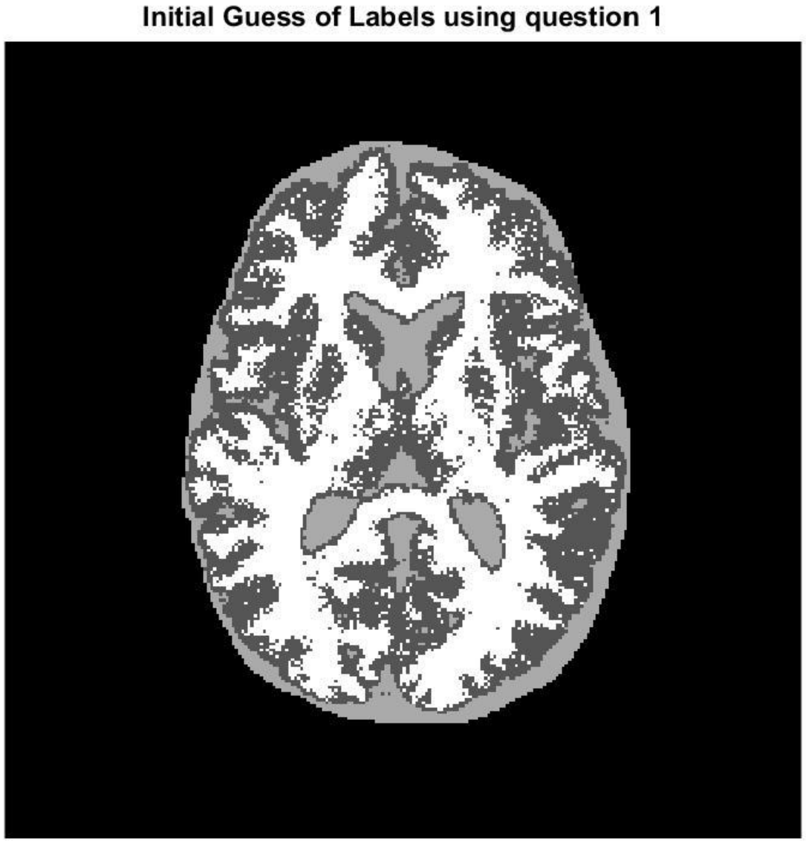


# Raw corrupted input image

An MR image of the brain is taken as an input to the system

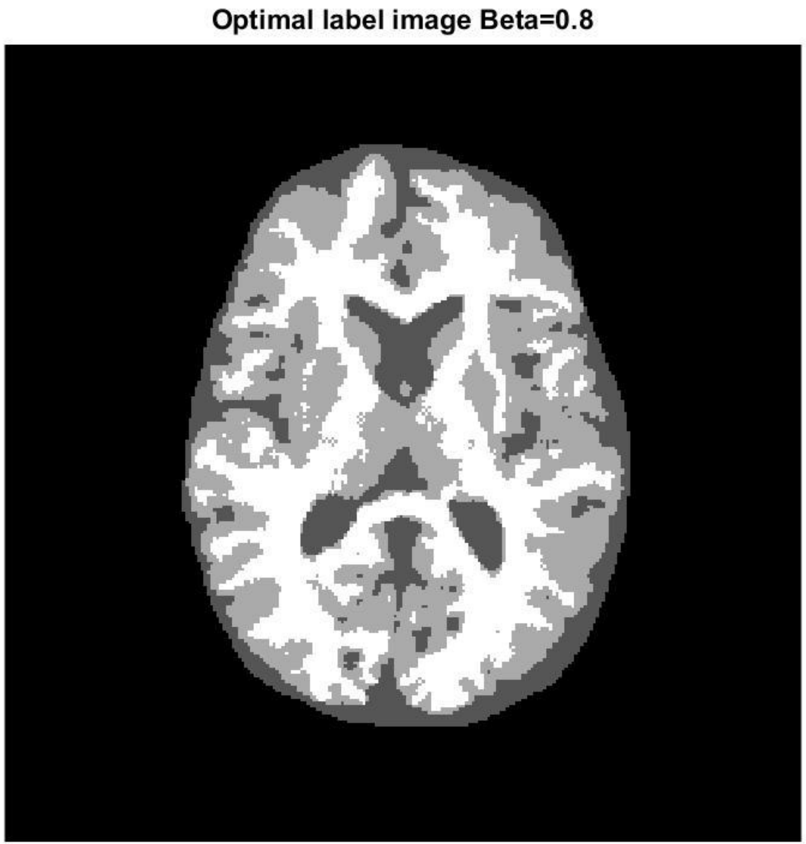
# After Preprocessing

- Bias field is removed
- Initial parameters (means and class labels) are computed by using Fuzzy C-Means (FCM) segmentation
- This image is passed to the EM algorithm



# Output Image

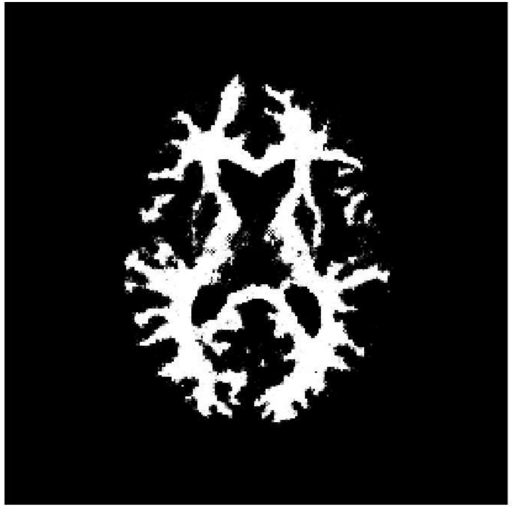
- It can be clearly seen that segmented image is smooth compared to input image and classes can be distinguished unambiguously
- The trade off between smooth vs representative of data is made through a tuning parameter beta, which determine the strength of the prior



# Individual Class Images

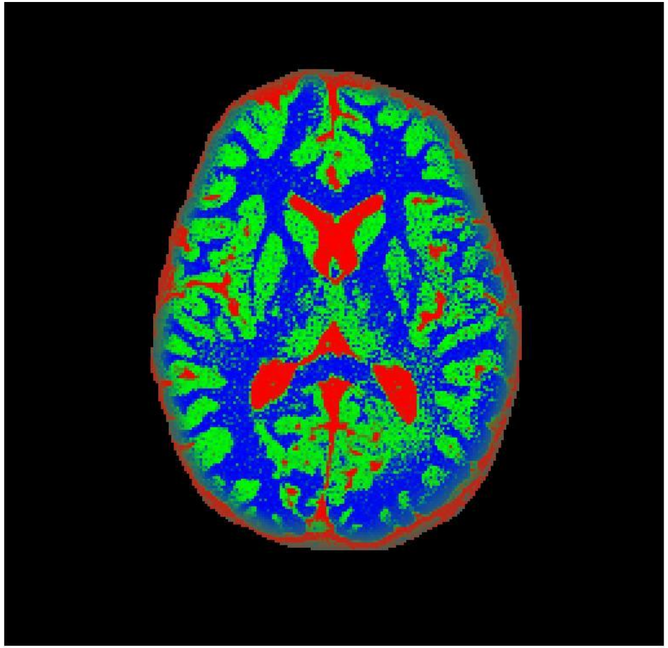
Class 1
Cerebrospinal fluid

Class 2
Grey Matter

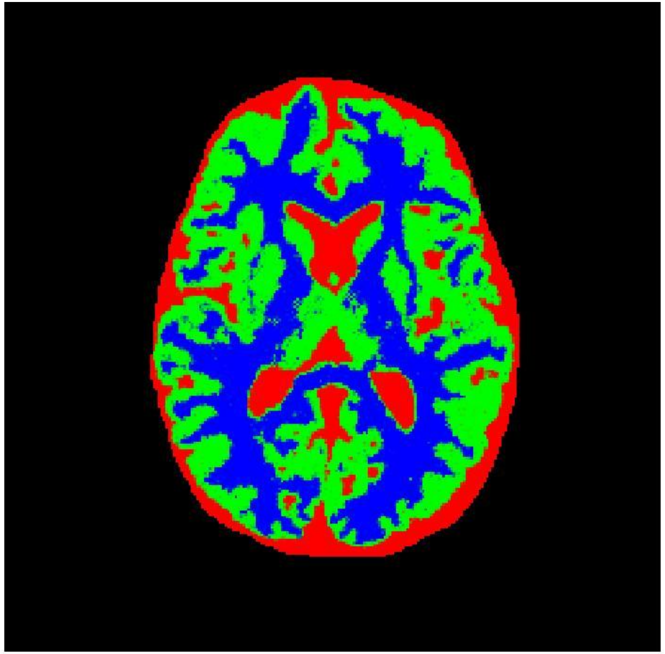


Class 3
White Matter

# RGB Representation of 3 Classes



Before GMM + MRF segmentation



After GMM + MRF segmentation

# References

[1] Zhang, Y., Brady, M., & Smith, S. (2001). Segmentation of brain MR images through a hidden Markov random field model and the expectation-maximization algorithm. *IEEE transactions on medical imaging*, *20*(1), 45-57.

[2] Shah, S. A., & Chauhan, N. C. (2015). An Automated Approach for Segmentation of Brain MR Images using Gaussian Mixture Model based Hidden Markov Random Field with Expectation Maximization. *Journal of Biomedical Engineering and Medical Imaging*, *2*(4), 57.

Thanks!