Maximum Likelihood Reconstruction for Emission Tomography

Team Members:

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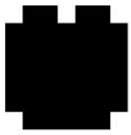
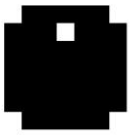
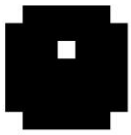
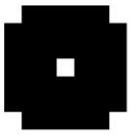
2. Kalpesh Patil (130040019)

Results:

Number of detectors = 40

- Reconstruction of Single Pixel Impulse

  1. Resolution = 4 (9*9 image)

Input Images			
			
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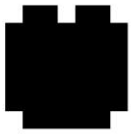
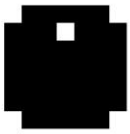
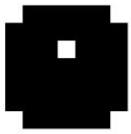
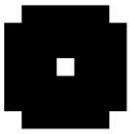
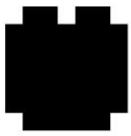
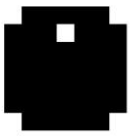
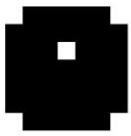
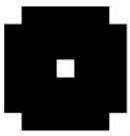
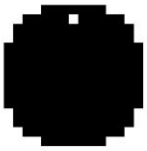
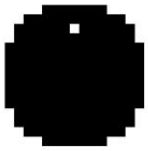
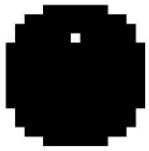
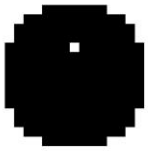
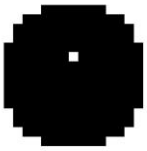
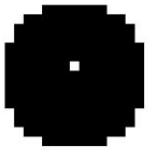
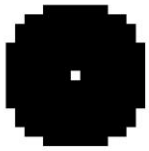
			
Inversion Output			
			

2. Resolution = 8 (17*17 image)

Input Images			
			
			
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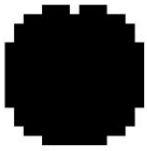
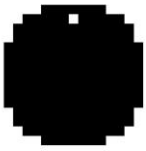
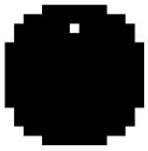
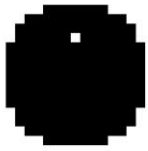
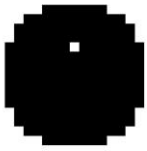
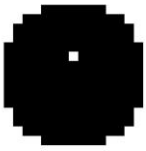
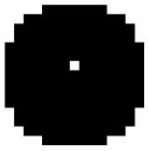
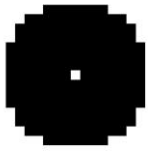
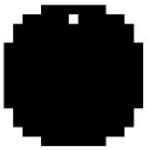
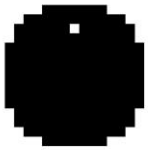
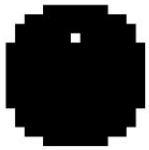
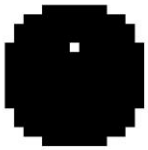
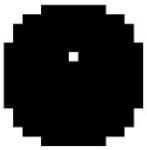
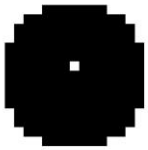
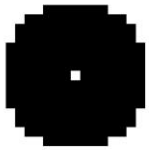
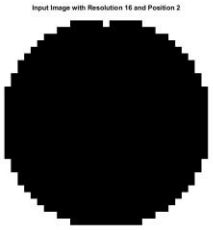
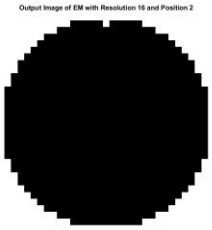
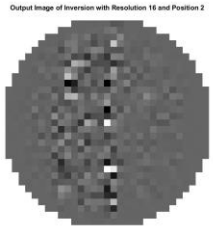
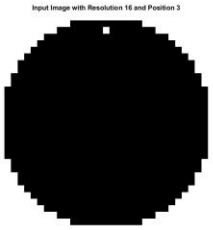
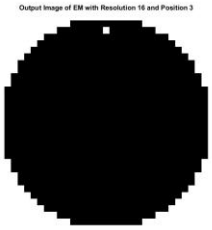
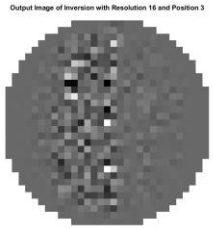
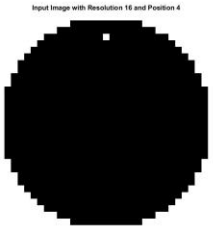
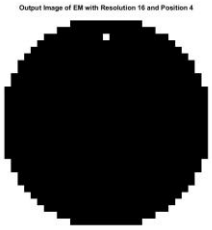
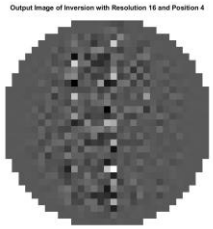
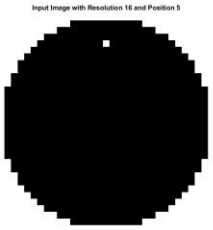
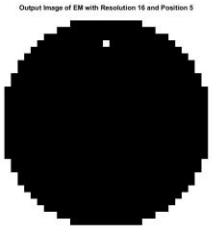
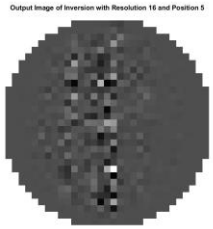
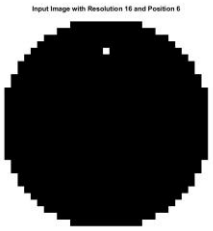
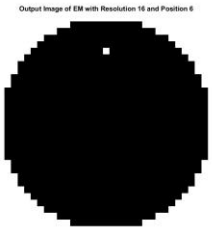
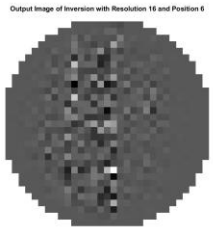
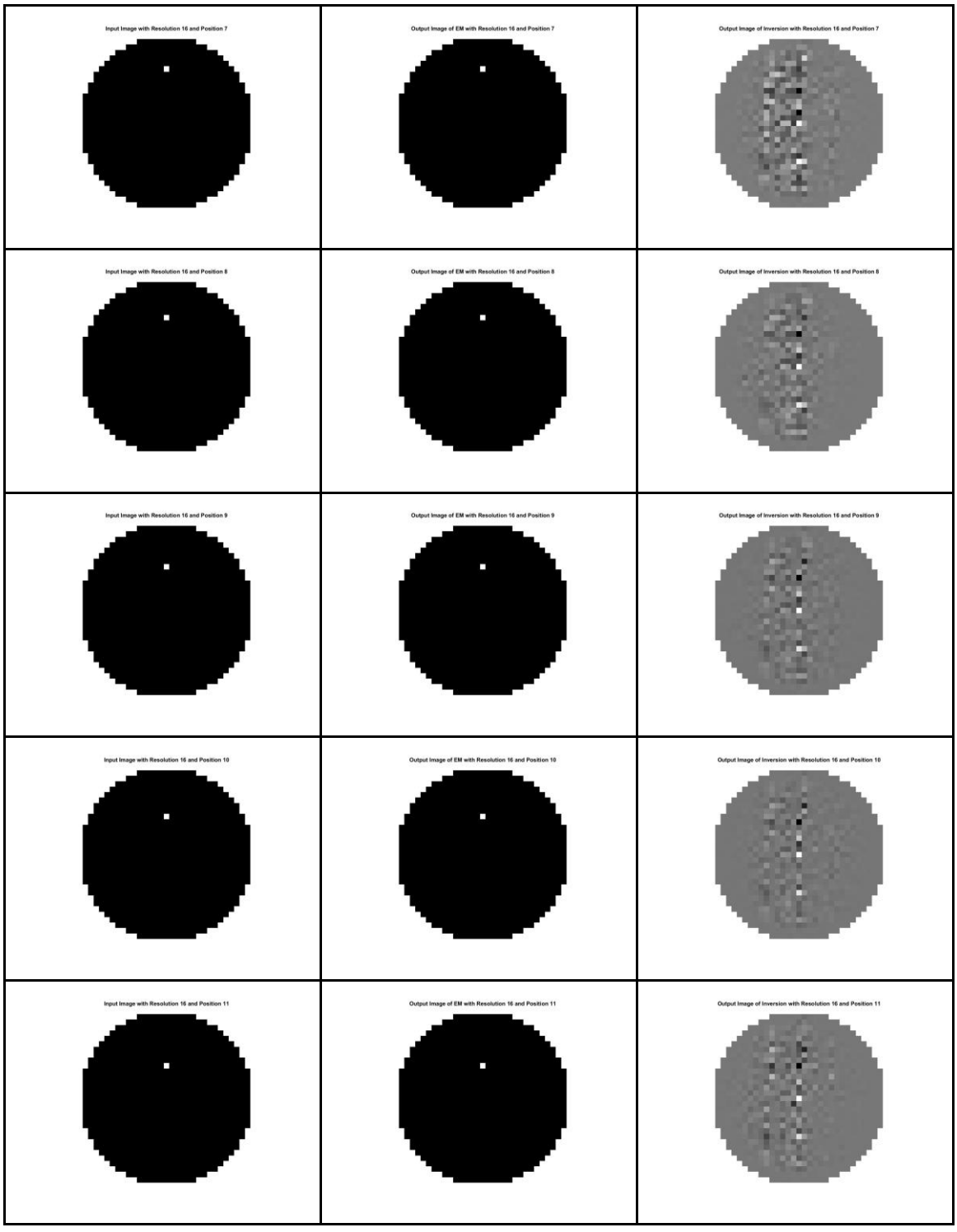
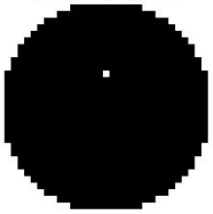
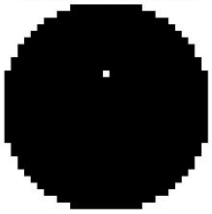
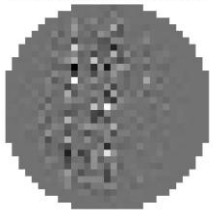
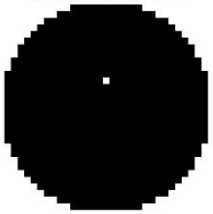
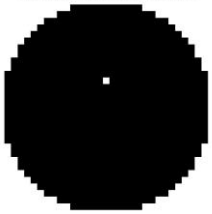
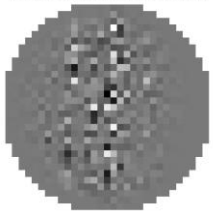
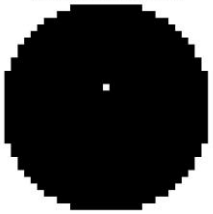
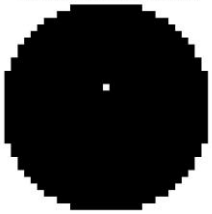
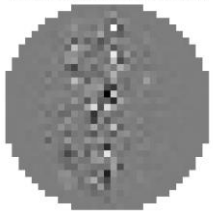
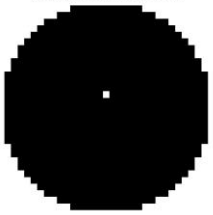
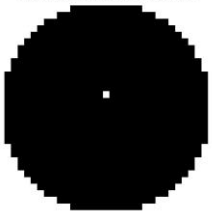
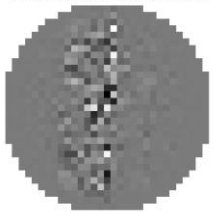
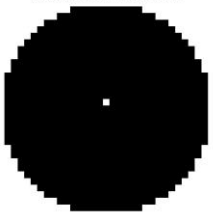
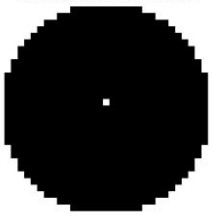
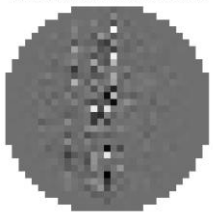
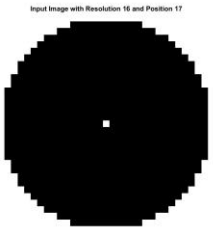
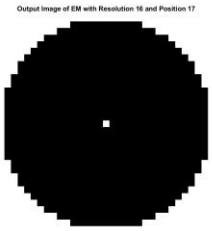
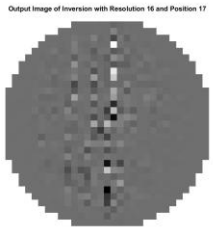
			
			
Inversion Output			
			
			

3. Resolution = 16 (33*33 image)

Input Image	EM Output	Inversion
		
		
		
		
		

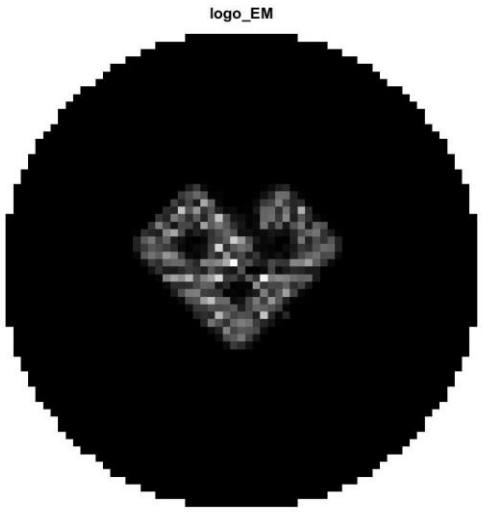
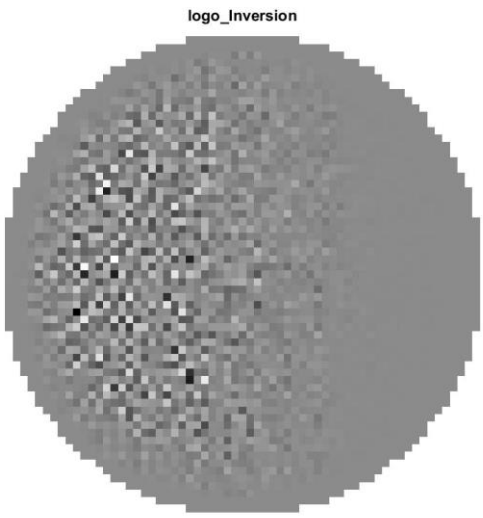

Input Image with Resolution 16 and Position 7
Output Image of EM with Resolution 16 and Position 7
Output Image of Inversion with Resolution 16 and Position 7
Input Image with Resolution 16 and Position 8
Output Image of EM with Resolution 16 and Position 8
Output Image of Inversion with Resolution 16 and Position 8
Input Image with Resolution 16 and Position 9
Output Image of EM with Resolution 16 and Position 9
Output Image of Inversion with Resolution 16 and Position 9
Input Image with Resolution 16 and Position 10
Output Image of EM with Resolution 16 and Position 10
Output Image of Inversion with Resolution 16 and Position 10
Input Image with Resolution 16 and Position 11
Output Image of EM with Resolution 16 and Position 11
Output Image of Inversion with Resolution 16 and Position 11

Input Image with Resolution 16 and Position 12 	Output Image of EM with Resolution 16 and Position 12 	Output Image of Inversion with Resolution 16 and Position 12 
Input Image with Resolution 16 and Position 13 	Output Image of EM with Resolution 16 and Position 13 	Output Image of Inversion with Resolution 16 and Position 13 
Input Image with Resolution 16 and Position 14 	Output Image of EM with Resolution 16 and Position 14 	Output Image of Inversion with Resolution 16 and Position 14 
Input Image with Resolution 16 and Position 15 	Output Image of EM with Resolution 16 and Position 15 	Output Image of Inversion with Resolution 16 and Position 15 
Input Image with Resolution 16 and Position 16 	Output Image of EM with Resolution 16 and Position 16 	Output Image of Inversion with Resolution 16 and Position 16 

- Reconstruction of Binary Image
  Number of detectors = 80
  Resolution = 32 (65*65 image)

Images	Description
	Input Binary Image (logo)

	EM Output
	Inversion Output

Theory Concerning Project

Here $'b'$ denote sites. There are $'B'$ sites in total. $'d'$ denotes detector tube and there are $'D'$ tubes in total. $P(b,d)$ is the probability that emission in $'b'$ is detected in tube $'d'$. The emission at site $'b'$ is assumed to be Poisson with mean $\lambda(b)$.

$n^*(d)$ is the total number of emissions detected in tube d during acquisition (real data).

$n(b,d)$ denotes the number of emissions in $'b'$ detected in $'d'$. $n(b,d)$ are mutually independent Poisson random variable, across $'b'$, across $'d'$ (split of Poisson process independent).

$$n(b,d) \sim Poisson\ (\lambda(b,d))$$

$$\lambda(b,d) = \lambda(b)p(b,d)$$

$$\sum_{d=1}^{D} p(b,d) = 1$$

$$n^*(d) = \sum_{b=1}^{B} n(b,d)$$

$$\lambda^*(b,d) = E[n^*(d)] = \sum_{b=1}^{B} \lambda(b)p(b,d)$$

Best estimate of $\lambda^*(d)$ is $n^*(d)$

$$n^*(d) = \sum_{b=1}^{B} \lambda(b)p(b,d) \qquad for\ d = 1,2,\dots D$$

Above sets of linear equations form the basis for inversion reconstruction.

Now let's move onto expectation maximization. If we have $n(b,d)$ as hidden random variable then estimation of $\lambda(b)$ becomes very easy.

So,

$$X = hidden\ variable = n(b,d) = x$$

$$Y = observed\ data = p^*(d) = y$$

$$Likelihood = p(x,y) = \prod_{\substack{b=1,2\dots B \\ d=1,2\dots D}} \frac{e^{-\lambda(b,d)}\left(\lambda(b,d)\right)^{n(b,d)}}{n(b,d)!}$$

Each $n(b,d)$ poisson process is independent

$$\sum_{b=1}^{B} n(b,d) = n^*(d) \qquad for\ all\ 'd'$$

$$\log(Likelihood) = \sum_{d=1}^{D}\sum_{b=1}^{B} -\lambda(b,d) + n(b,d)\log(\lambda(b,d)) - \log(n(b,d)!)$$

$\log(n(b,d)!)$ of no use since it doesn't contain parameter $\lambda(b,d)$

$$E[n(b,d)|\, n^*(d)] = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b=1}^{B}\lambda^i(b,d)} \qquad \lambda^i(b,d) = estimate\ at\ iteration\ i$$

$$E[\log(Likelihood)] = \sum_{d=1}^{D}\sum_{b=1}^{B} -\lambda(b,d) + \frac{n^*(d)\lambda^i(b,d)\log(\lambda(b,d))}{\sum_{b=1}^{B}\lambda^i(b,d)} = Q(\theta,\theta^i)$$

Differentiating above expression respect to $\lambda(b,d)$ to get maxima

$$\lambda^{i+1} = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b'=1}^{B}\lambda^i(b',d)}$$

$$\lambda^i(b,d) = \lambda^i(b)p(b,d) \quad and \quad \lambda(b) = \sum_{d=1}^{D}\lambda(b,d)$$

$$\lambda^{new}(b) = \lambda^{old}(b)\sum_{d=1}^{D}\frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B}\lambda^{old}(b')p(b',d)}$$

Final formula for update is given below

$$\lambda^{new}(b) = \lambda^{old}(b)\sum_{d=1}^{D}\frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B}\lambda^{old}(b')p(b',d)}$$