# **Maximum Likelihood Reconstruction for Emission Tomography**

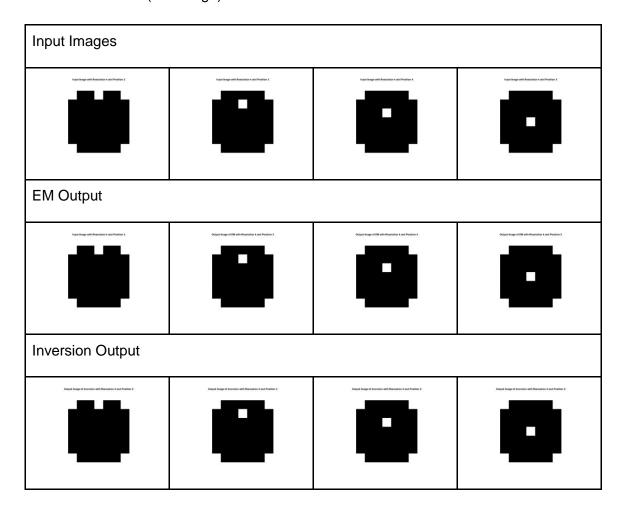
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- 2. Kalpesh Patil (130040019)

#### Results:

Number of detectors = 40

- Reconstruction of Single Pixel Impulse
  - 1. Resolution = 4 (9\*9 image)

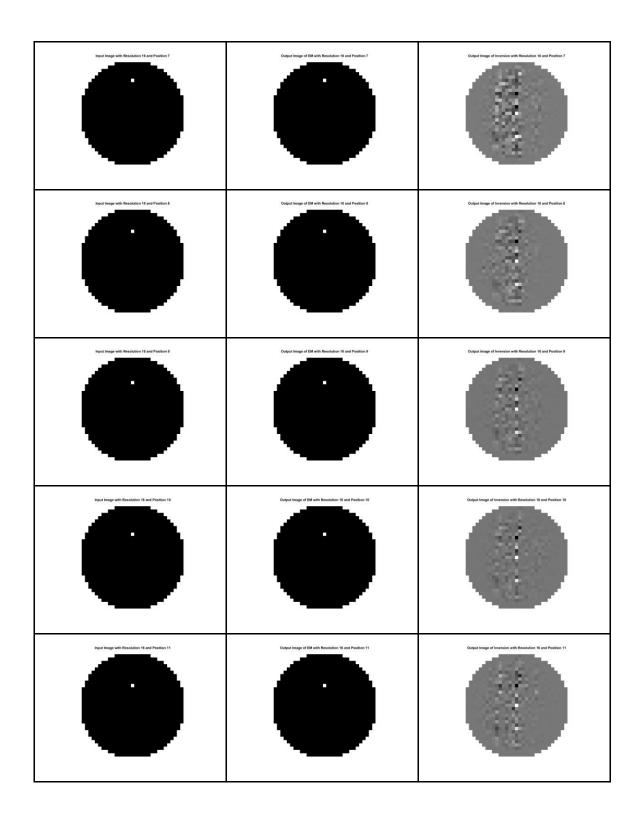


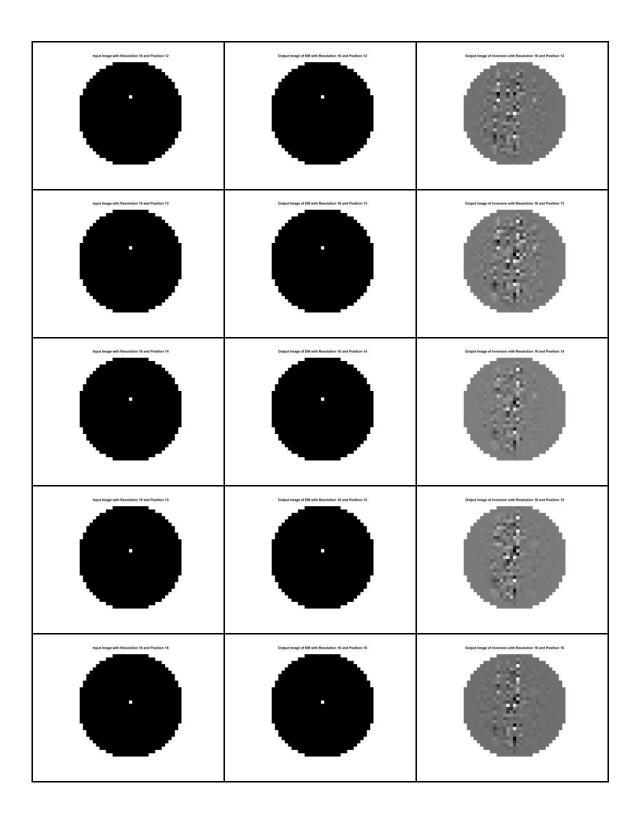
### 2. Resolution = 8 (17\*17 image)

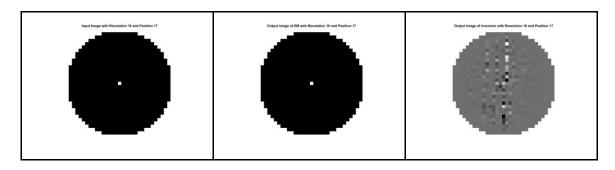
| Input Images   |  |  |  |  |
|--|--|--|--|--|
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| Hapel Brager alth Streeding & And Publish S.                   | hyper brougs with Passandine 6 and Provider 7              | haped broage with Private Annual State Sta | hyper broage with Resembles 6 and Provides 6             |  |
| EM Output  |  |  |  |  |
| Output map of SN with Resounders 6 and Prostoco 2              | Ougset loogs of the with femalishes a seef Product 3       | Ougs I trapp of Diff with Resolution 6 and Problem 5   | Ougsel toage of the with femaleshin 1 and Product 1      |  |
| Corpus major of DW with Remodelles & and Printers &            | Output Trough of Eller with Securities 6 and Publish 7     | Output Trough of Elle with Translation 6 and Provides 6  | Output Trapp of Eller with Seculation 6 and Publisher 6  |  |
| Inversion Output   |  |  |  |  |
| Output larger of housester with Recordance is and Parallel II. | Output Brogge of Transvisor with Stitute of and Frankiso 3 | Output longer of traversion with Managister & and Parallelo 4.   | Output longs of housekins with Monardon & and Frankins & |  |
| Output temps of homeston with Resolution 6 and Profiles 6      | Original Visualities with Record and State Processor T     | Chapter longs of boundary with Remarket at and Publics &   | Stripe long of housine will fine date of and Pacifice S. |  |

## 3. Resolution = 16 (33\*33 image)

| Input Image                                    | EM Output   | Inversion  |
|--|---|--|
| legal Image with Resolution 16 and Position 2  | Output Image of EM with Resolution 16 and Position 2  | Output Image of Inversion with Resolution 16 and Position 2  |
| tegat Image with Resolution 16 and Position 3  | Output Image of EM with Resolution 16 and Position 3  | Output Image of Inversion with Resolution 15 and Position 3  |
| Inquel Image with Resolution 16 and Position 4 | Output Image of EM with Resolution 16 and Position 6  | Output Inlage of Invertion with Resolution 16 and Position 4 |
| teput Image with Resolution 16 and Position 5  | Output Image of EM with Resolution 16 and Position 5  | Output Image of Invernion with Resolution 15 and Physics 5   |
| legad Image with Resolution 16 and Position 6  | Output Invage of EM with Resolution 16 and Position 6 | Output Image of Inverzion with Resolution 16 and Publicon 6  |

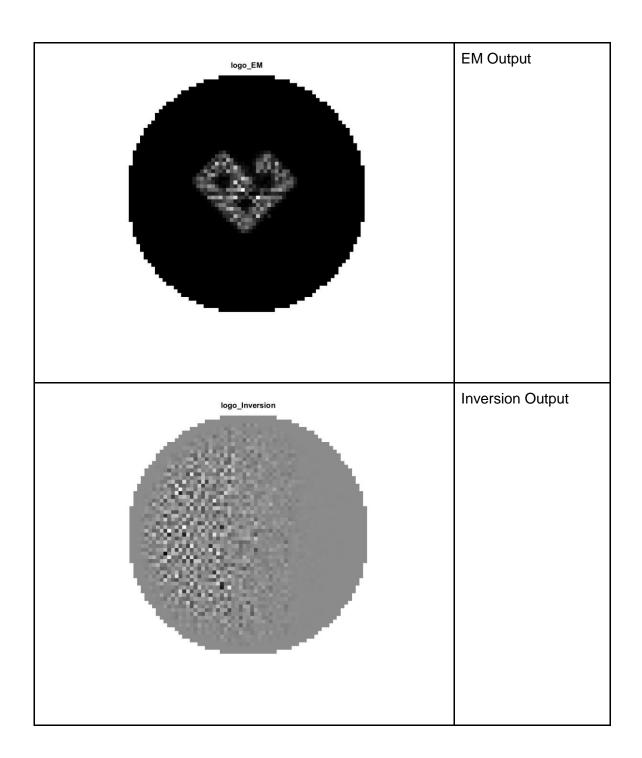






 Reconstruction of Binary Image Number of detectors = 80 Resolution = 32 (65\*65 image)

| Images     | Description               |
|------------|---------------------------|
| logo_input | Input Binary Image (logo) |



### Theory Concerning Project

Here 'b' denote sites. There are 'B' sites in total. 'd' denotes detector tube and there are 'D' tubes in total. P(b,d) is the probability that emission in 'b' is detected in tube 'd'. The emission at site 'b' is assumed to be Poisson with mean  $\lambda(b)$ .

 $n^*(d)$  is the total number of emissions detected in tube d during acquisition (real data).

n(b,d) denotes the number of emissions in 'b' detected in 'd'. n(b,d) are mutually independent Poisson random variable, across 'b', across 'd' (split of Poisson process independent).

$$n(b,d) \sim Poisson (\lambda(b,d))$$

$$\lambda(b,d) = \lambda(b)p(b,d)$$

$$\sum_{d=1}^{D} p(b,d) = 1$$

$$n^*(d) = \sum_{b=1}^{B} n(b,d)$$

$$\lambda^*(b,d) = E[n^*(d)] = \sum_{b=1}^{B} \lambda(b)p(b,d)$$

Best estimate of  $\lambda^*(d)$  is  $n^*(d)$ 

$$n^*(d) = \sum_{b=1}^{B} \lambda(b) p(b, d)$$
 for  $d = 1, 2, ... D$ 

Above sets of linear equations form the basis for inversion reconstruction.

Now let's move onto expectation maximization. If we have n(b,d) as hidden random variable then estimation of  $\lambda(b)$  becomes very easy.

So,

$$X = hidden \ variable = n(b,d) = x$$
 
$$Y = observed \ data = p^*(d) = y$$
 
$$Likelihood = p(x,y) = \prod_{b=1,2...B} \frac{e^{-\lambda(b,d)} \big(\lambda(b,d)\big)^{n(b,d)}}{n(b,d)!}$$

Each n(b, d) poisson process is independent

$$\sum_{b=1}^{B} n(b,d) = n^*(d) \quad \text{for all 'd'}$$

$$\log(Likelihood) = \sum_{d=1}^{D} \sum_{b=1}^{B} -\lambda(b,d) + n(b,d)\log(\lambda(b,d)) - \log(n(b,d)!)$$

 $\log(n(b,d)!)$  of no use since it doesn't contain parameter  $\lambda(b,d)$ 

$$E[n(b,d)|n^*(d)] = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b=1}^B \lambda^i(b,d)} \qquad \lambda^i(b,d) = \text{estimate at iteration } i$$

$$E[\log(Likelihood)] = \sum_{d=1}^{D} \sum_{b=1}^{B} -\lambda(b,d) + \frac{n^*(d)\lambda^i(b,d)\log(\lambda(b,d))}{\sum_{b=1}^{B} \lambda^i(b,d)} = Q(\theta,\theta^i)$$

Differentiating above expression respect to  $\lambda(b,d)$  to get maxima

$$\lambda^{i+1} = \frac{n^*(d)\lambda^i(b,d)}{\sum_{b'=1}^B \lambda^i(b',d)}$$

$$\lambda^i(b,d) = \lambda^i(b)p(b,d) \quad and \quad \lambda(b) = \sum_{d=1}^D \lambda(b,d)$$

$$\lambda^{new}(b) = \lambda^{old}(b) \sum_{d=1}^{D} \frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B} \lambda^{old}(b')p(b',d)}$$

Final formula for update is given below

$$\lambda^{new}(b) = \lambda^{old}(b) \sum_{d=1}^{D} \frac{n^*(d)p(b,d)}{\sum_{b'=1}^{B} \lambda^{old}(b')p(b',d)}$$