Control Theory Home Work 1

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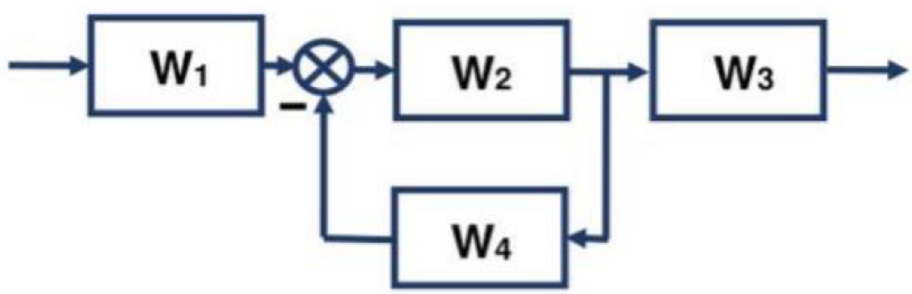
1 Git repo

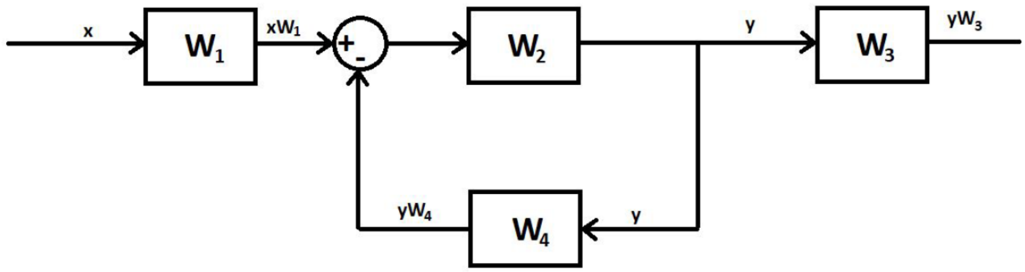
https://github.com/kalraUtkarsh/Control-Theory-Utkarsh-Kalra

2 TRANSFER FUNCTION CALCULATIONS



$W1 = \frac{2}{s+5}$
$W2 = \frac{s+1}{s+0.5}$
$W3 = \frac{1}{s+0.25}$
$W4 = \frac{1}{2s+3}$

2.1 2(A)



$(xW1 - yW4)W2 = y$

$xW1W2 - yW4W2$

$xW1W2 = y(1 + W4W2)$

$y = \frac{xW1W2}{1+W4W2}$

$$W = \frac{W1 * W2 * W3}{1 + W4W2}$$

$$W1 * W2 * W3 = \frac{2(s+1)}{(s+5)(s+0.5)(s+0.25)}$$

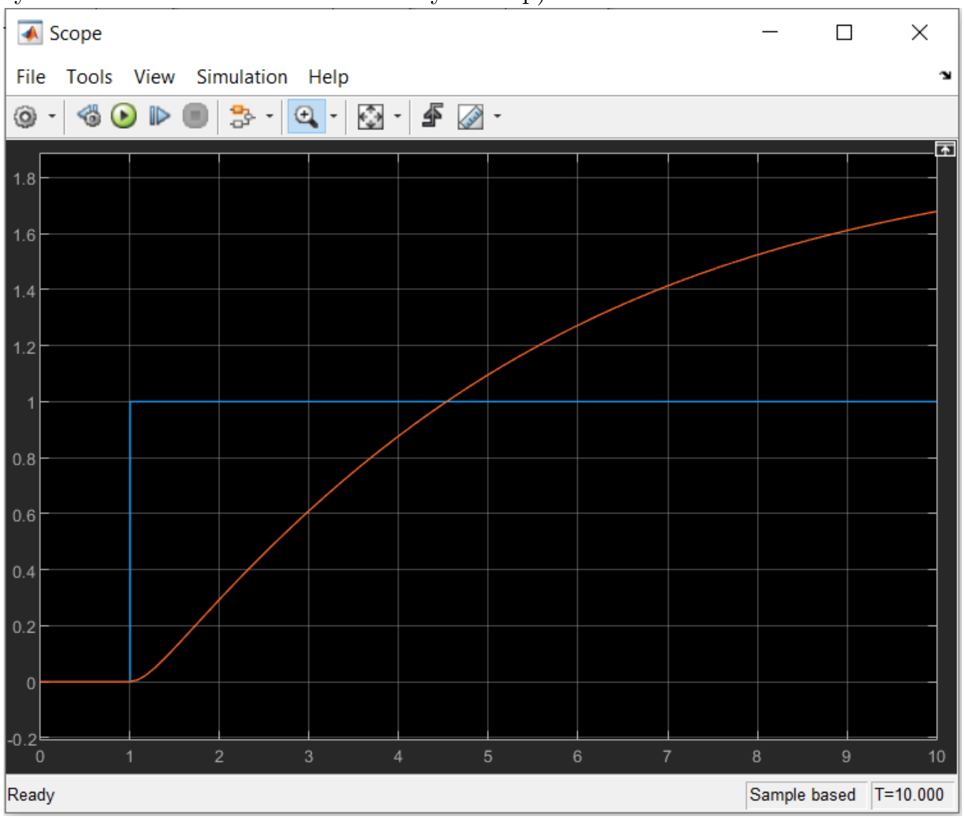
$$1 + W4W2 = \frac{s+1+(s+0.5)(2s+3)}{(s+0.5)(2s+3)}$$

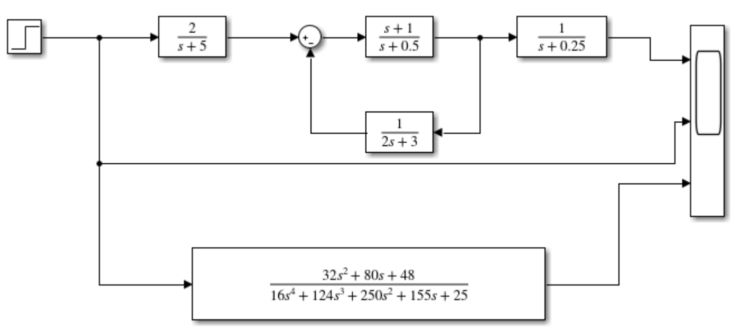
$$finalW = \frac{32s^2 + 80s + 48}{16s^4 + 124s^3 + 250s^2 + 155s + 25}$$

2.2 2(B)

2.2.1 Plot with the step

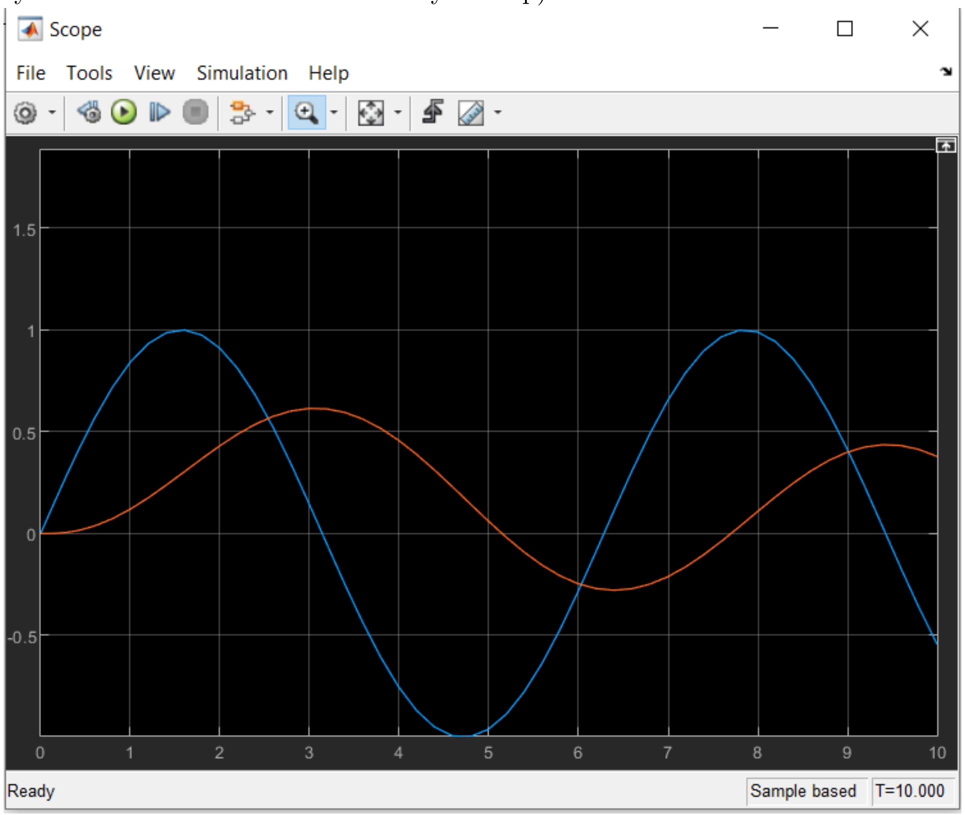
(There are 3 lines in the graph: blue for the input and yellow and red for the 2 systems of transfer functions but they overlap)

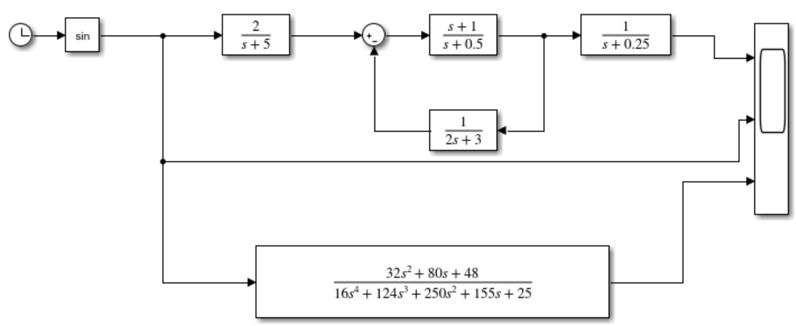




2.2.2 Plot with the frequency

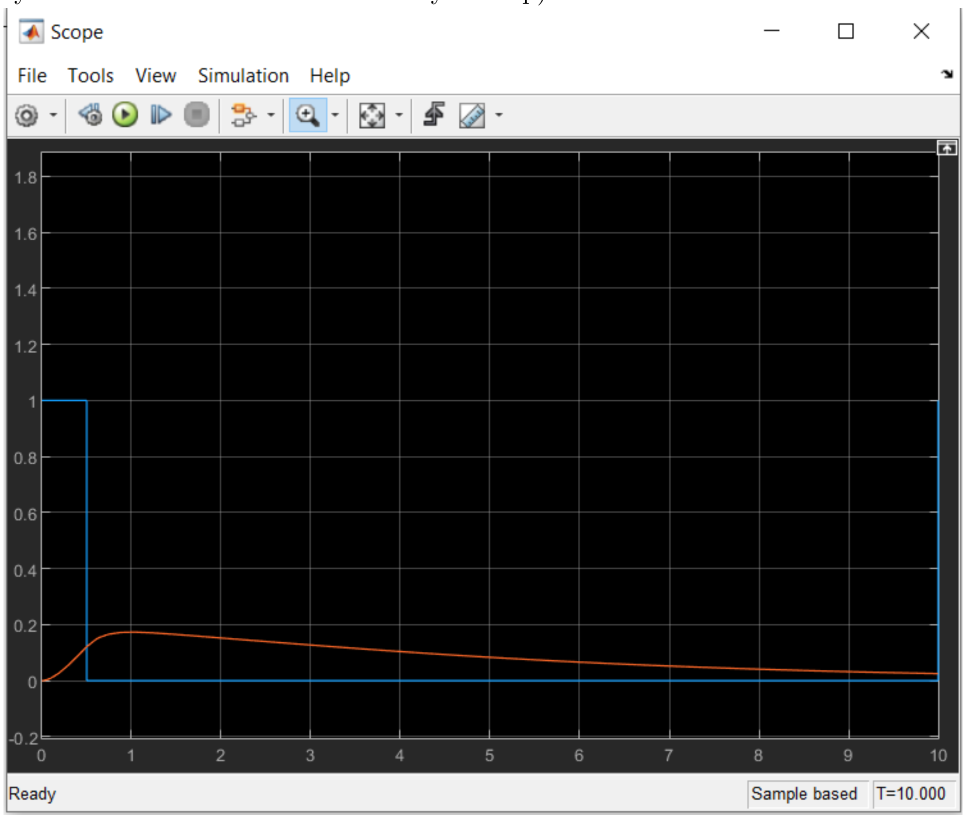
(There are 3 lines in the graph: blue for the input and yellow and red for the 2 systems of transfer functions but they overlap)

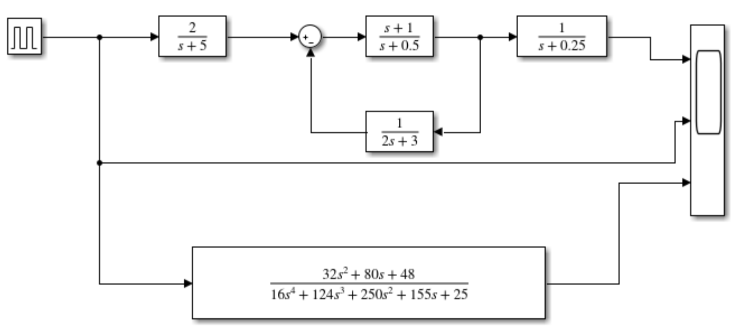




2.2.3 Plot with Impulse

(There are 3 lines in the graph: blue for the input and yellow and red for the 2 systems of transfer functions but they overlap)

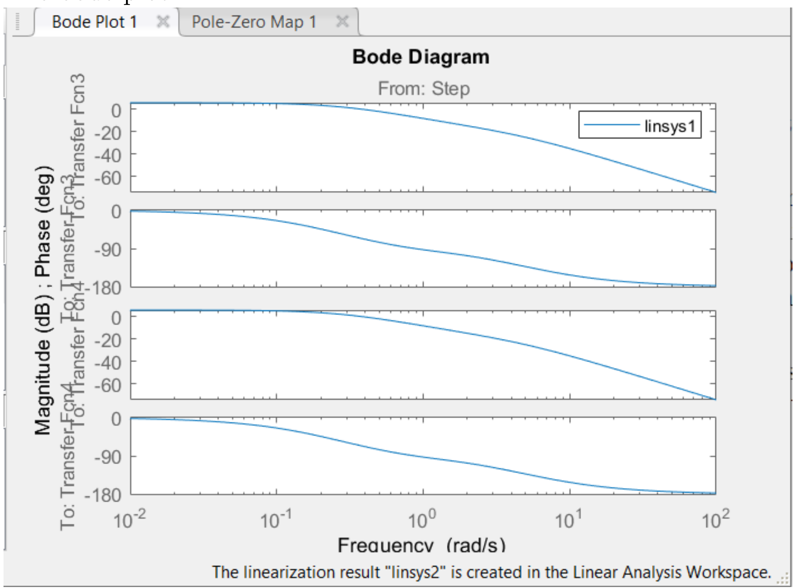




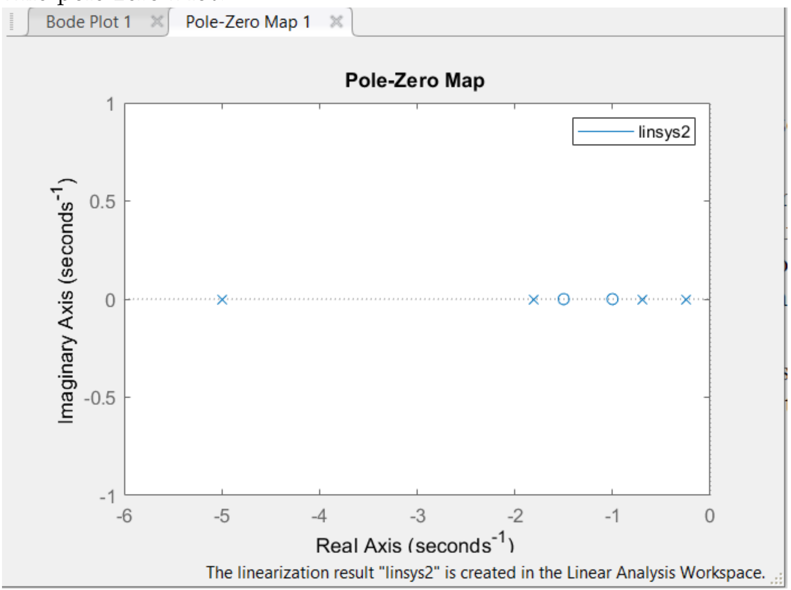
2.3 2(C)

2.3.1 Taking the step input and generating the bode plot and pole zero plot.

The bode plot:



The pole zero Plot:



The given System is Stable as the bode plot is converging and the Phase margins are positive.

2.4 2(D): Analyzing the Bode plot, calculating the asymptotes and frequency breaks

our transfer function: $\dfrac{xW1W2}{1+W4W2}$

$$= \frac{\frac{2}{s+5}\frac{s+1}{s+0.5}\frac{1}{s+0.25}}{1+\frac{s+1}{s+0.5}\frac{1}{2s+3}} = \frac{48(s+1)(\frac{s}{1.5}+1)}{25(\frac{s}{5}+1)(\frac{s}{0.25}+1)(\frac{s}{\frac{5-\sqrt{5}}{4}+1})(\frac{s}{\frac{5+\sqrt{5}}{4}+1})}$$

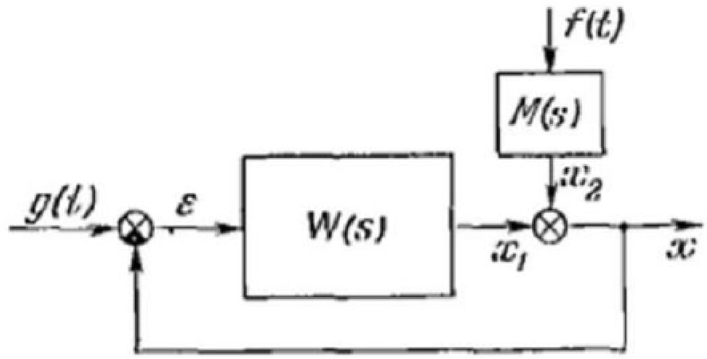
This shows that we have 2 zeroes at frequencies 1.5 and 1 and 4 poles at frequencies 5,0.25,$\frac{5-\sqrt{5}}{4}$ and $\frac{5-\sqrt{5}}{4}$ Therefore there are 6 break frequencies. Asymptote 1 is a horizontal line through magnitude $A_0 = 20log(48/25) \approx 5.57db$ As we know that we can state that the next asymptotes slopes increases or decreases 20db/dec in comparison to the previous ones. And using the formula Anext = slope * log($\frac{Wnext}{Wprev}$) + Aprev, we have the table of asymptotes for magnitude plot as below.

		Corner(point passed through)	
Sr.	Slope(db/dec)	Frequency(rad/sec)	Magnitude(db)
1	0	0.25	5.57
1	-20	0.25	5.57
2	-40	0.69	-3.25
3	-20	1	-9.7
4	0	1.5	-13.22
5	-20	1.81	-13.22
6	-40	5	-22.05

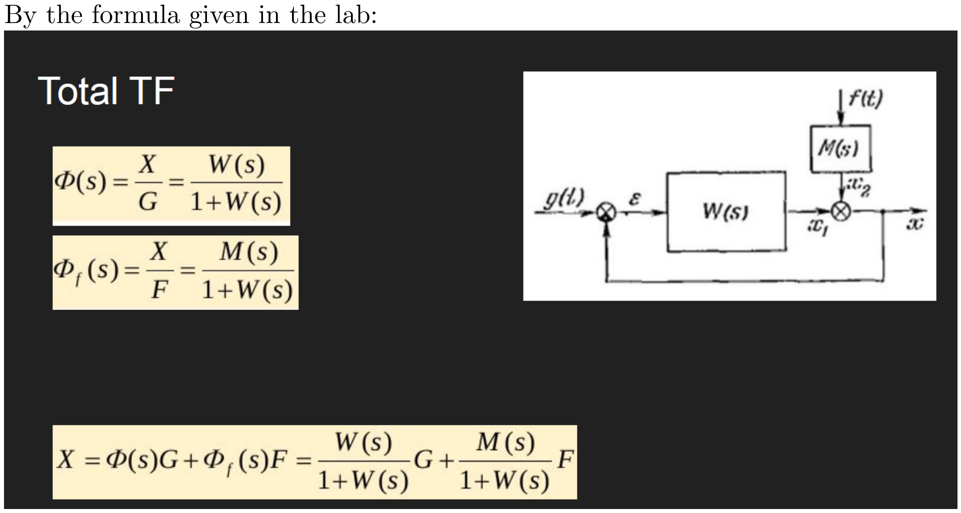
As from the bode plot previously, we got gain crossover frequency of the system as 3.777rad/sec. So, the magnitude plot and frequency axis intersect at (3.777,0).

3 Total transfer function of the given loop

$W(s) = \dfrac{2}{s^2+2}$
$M(s) = \dfrac{s+2}{2s+3}$



By the formula given in the lab:



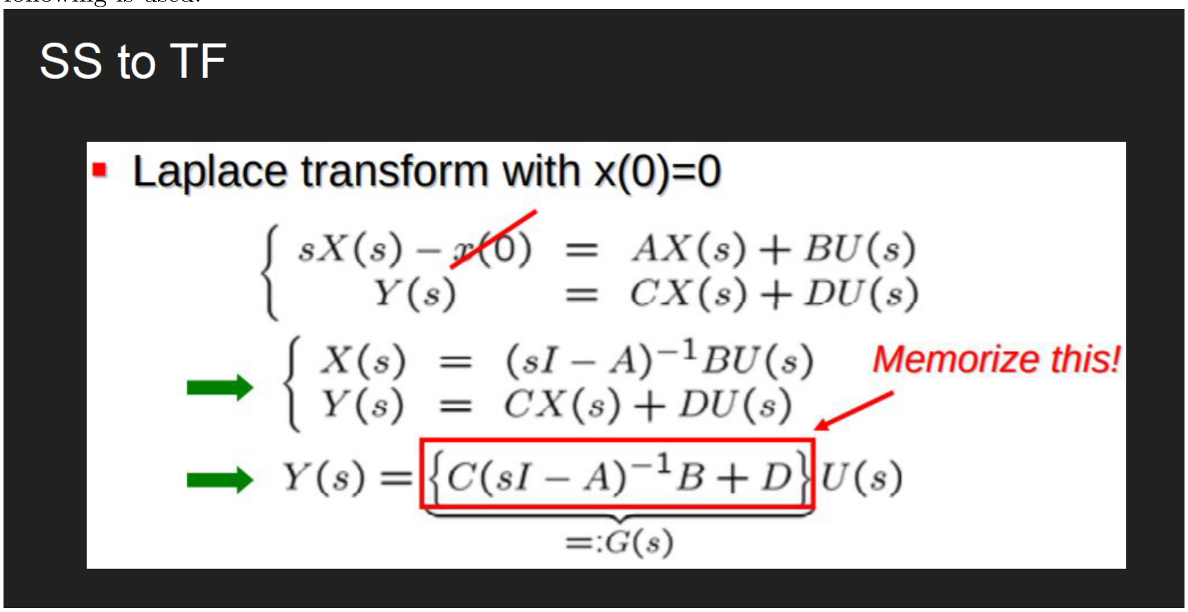
$$x = \frac{g(t)W(s) + f(t)M(s)}{1+W(s)}$$

$$x = \frac{g(t)\frac{2}{s^2+2} + f(t)\frac{s+2}{2s+3}}{1+\frac{2}{s^2+2}}$$

4 Finding the Transfer Function from the State Space Representation

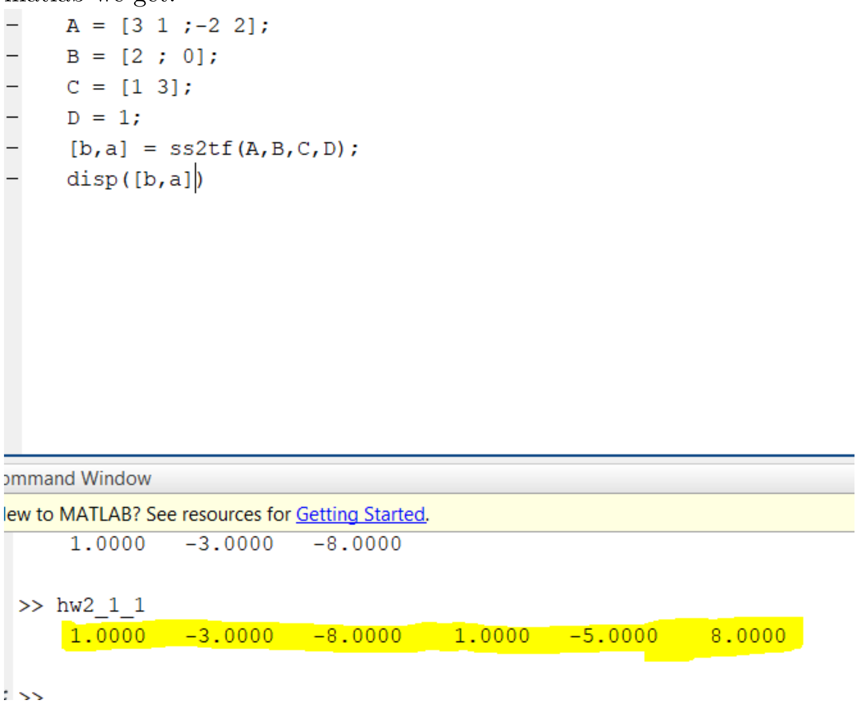
(g) $A = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 3 \end{pmatrix}, D = \begin{pmatrix} 1 \end{pmatrix}$

As given in the lab for converting from the State Space to Transfer function the following is used:



so for our given Matrices:

$\begin{bmatrix} 1 & 3 \end{bmatrix}$ ($\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$ - $\begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$ $)^{-1}$ * $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ + $\begin{bmatrix} 1 \end{bmatrix}$ By using the ss2tf function in matlab we get:



Therefore the Transfer funtion is:

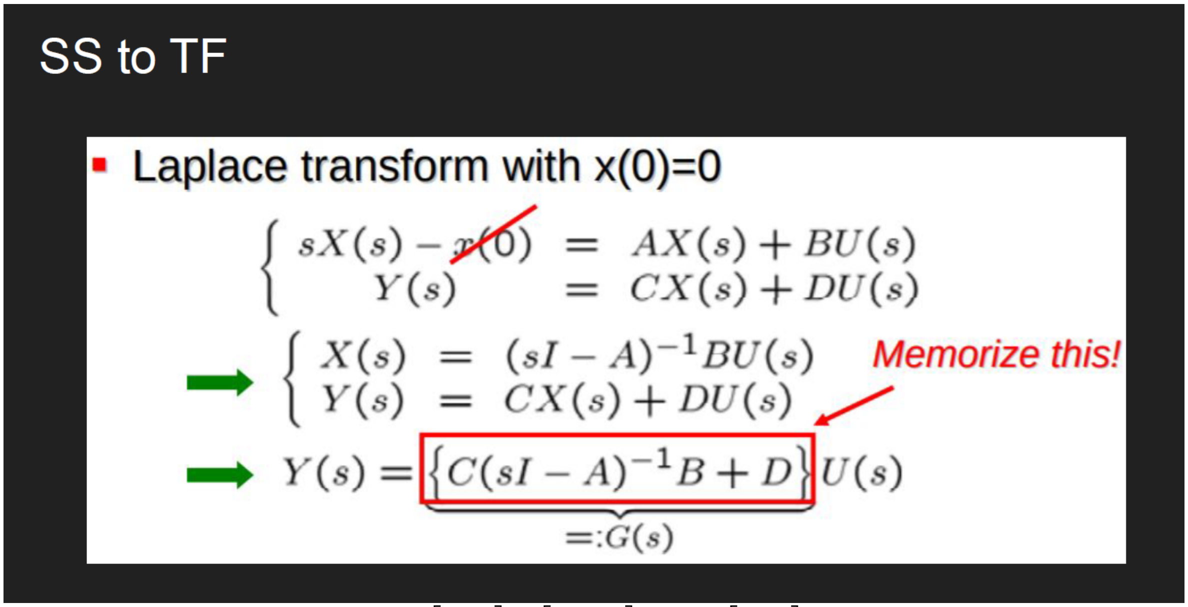
$$TF = \frac{s^2 - 3s - 8}{s^2 - 5s + 8}$$

5 Finding the Transfer function from the given State Space Representation

$$\text{(g)}\quad A = \begin{pmatrix} 5 & 1 \\ 0 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 2 \\ 2 & 3 \end{pmatrix}, \qquad C = \begin{pmatrix} 1 & 1 \end{pmatrix}, D = \begin{pmatrix} 1 & 6 \end{pmatrix}$$

As the given D Matrix has two columns this means that there will be Two tranfer functions as there are Two inputs

And to convert SS to TF the formula is:



So for our given matrices: $\begin{bmatrix} 1 & 1 \end{bmatrix}$ ($\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$ - $\begin{bmatrix} 5 & 1 \\ 0 & -2 \end{bmatrix}$)$^{-1}$ * $\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$ + $\begin{bmatrix} 1 & 6 \end{bmatrix}$

By using the ss2tf function in matlab we get:
For first Input:

```
Editor - C:\Users\Asus\Documents\hw2_1_1.m
hw2_3.m  hw2_1_1.m
A = [5 1 ;0 -2];
B = [0 2 ; 2 3];
C = [1 1];
D = [1 6];
[b,a] = ss2tf(A,B,C,D,1);
disp([b,a])
```

```
Command Window
New to MATLAB? See resources for Getting Started.
     6    -13    -68      1     -3    -10

>> hw2_1_1
    1.0000   -1.0000  -18.0000    1.0000   -3.0000  -10.0000
```

So the TF for this input is:

$$TF = \frac{s^2 - s - 18}{s^2 - 3s - 10}$$

For Second input:

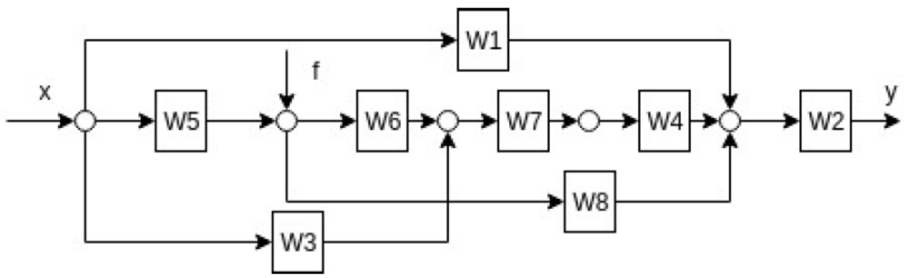
```
Editor - C:\Users\Asus\Documents\hw2_1_1.m
hw2_3.m  hw2_1_1.m
A = [5 1 ;0 -2];
B = [0 2 ; 2 3];
C = [1 1];
D = [1 6];
[b,a] = ss2tf(A,B,C,D,2);
disp([b,a])
```

```
Command Window
New to MATLAB? See resources for Getting Started.
    1.0000   -1.0000  -18.0000    1.0000   -3.0000  -10

>> hw2_1_1
     6    -13    -68      1     -3    -10
```

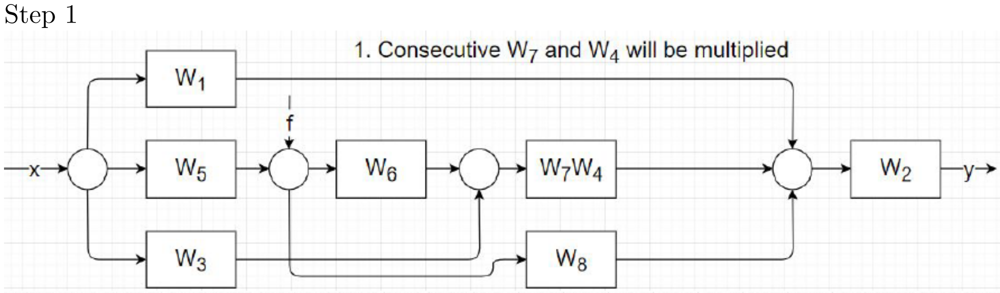
So the TF for this input is: $TF = \dfrac{6s^2 - 13s - 68}{s^2 - 3s - 10}$

6 Simplifying the system step by step for both the Inputs x and f

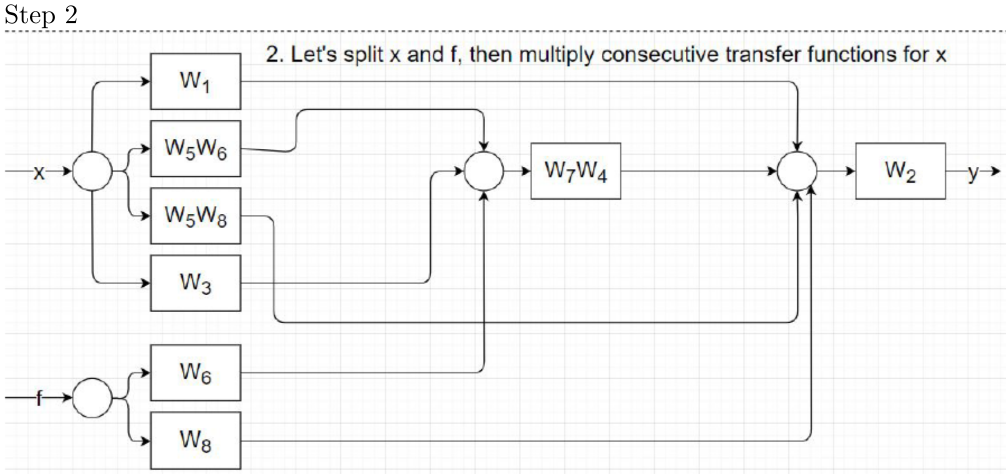


(g)

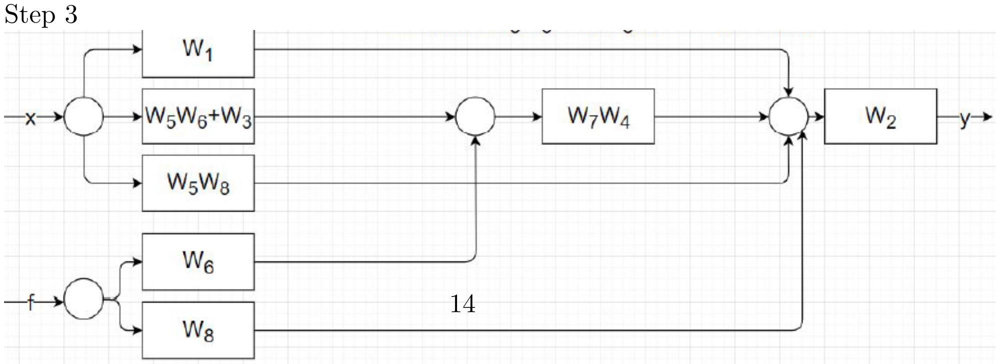
Step 1



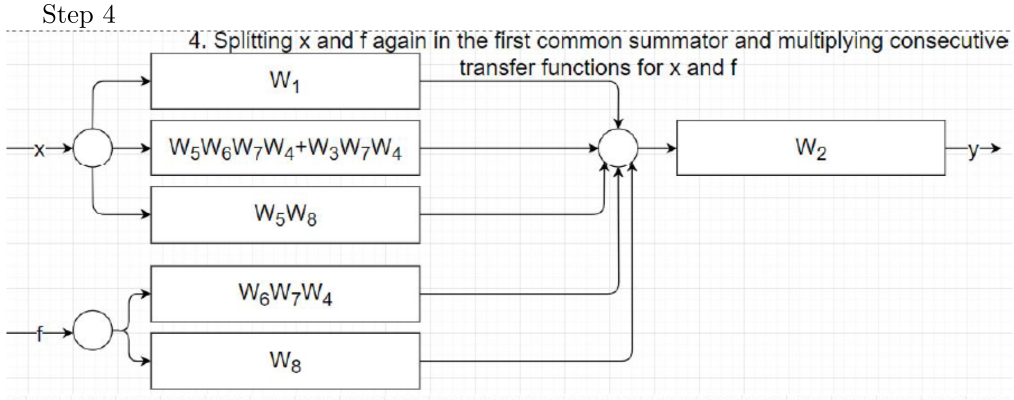
Step 2



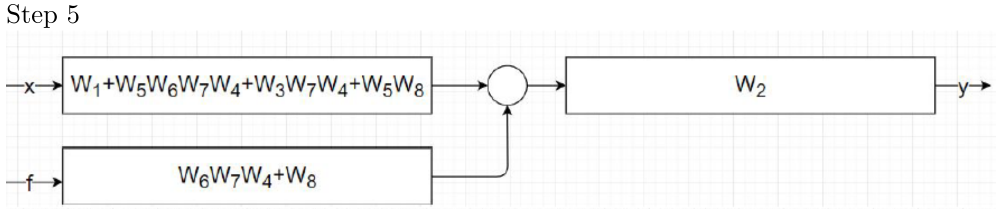
Step 3



Step 4



Step 5



Final answers: