Control Theory Home Work 3

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Varient g

1 Git repo

https://github.com/kalraUtkarsh/Control-Theory-Utkarsh-Kalra

2

2.1 Write equations of motion of the system in manipulator form

The normal equations we get from the system we have:
$(M+m)x'' - mlcos(\theta)\theta'' + mlsin(\theta)\theta^2$
$l\theta'' - cos(\theta)x'' - gsin(\theta) = 0$

and Manipulator form is given by:
$M(q)q'' + n(q,q') = Bu$
$u = F$
$q = \begin{bmatrix} x & \theta \end{bmatrix}^T$

So by co-relating both the given forms, we get the following equations:
$\begin{bmatrix} M+m & -mlcos(\theta) \end{bmatrix} \begin{bmatrix} x'' \\ \theta'' \end{bmatrix} + (mlsin(sin(\theta))\theta'^2) = [1]u$

This is in the manipulator form but we should also consider the second equation of the system so the above system transforms to:

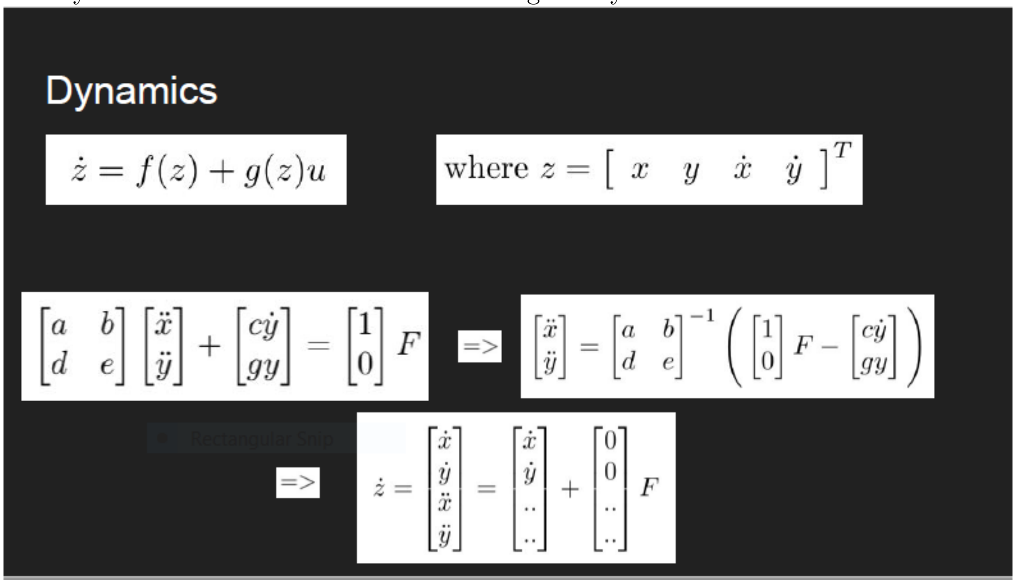
$$\begin{bmatrix} M+m & -mlcos(\theta) \\ -cos(\theta) & l \end{bmatrix} \begin{bmatrix} x'' \\ \theta'' \end{bmatrix} + \begin{bmatrix} mlsin(\theta)\theta'^2 \\ -gsin(\theta) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$AndformyVarient\mathbf{g}, givenm = 3.6, M = 3.6, l = 1.01 theequationbecomes :,$

$$\begin{bmatrix} 10.2 & -3.636 * cos(\theta) \\ -cos(\theta) & 1.01 \end{bmatrix} \begin{bmatrix} x'' \\ \theta'' \end{bmatrix} + \begin{bmatrix} 3.636 * sin(\theta)\theta'^2 \\ -9.81 sin(\theta) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

2.2 Write dynamics of the system in control affine non-linear form

The dynamic control affine non-liner form is given by:



This is taken from lab9, only in our case y is replaced by θ
and representing x' and θ' through

$$\begin{bmatrix} x' \\ \theta' \end{bmatrix} = \begin{bmatrix} x' \\ \theta' \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} F$$

And the equations of our system are same as in the previous task
So all we have to do is represent x" and θ'' as x,x',θ, θ''

$$x'' = \frac{l\theta'' - gsin(\theta)}{cos(\theta)}$$

$$\theta'' = \frac{(M+m)x'' + mlsin(\theta)\theta'^2 - F}{mlcos(\theta)}$$

$$\theta'' = \frac{(M+m)(l\theta'')}{mlcos^2(\theta)} - \frac{(M+m)gsin(\theta)}{mlcos^2(\theta)} + tg(\theta)\theta'^2 - \frac{F}{mlcos(\theta)}$$

$$\theta'' = \frac{-(M+m)gsin(\theta) + mlcos(\theta)sin(\theta)\theta'^2 - Fcos(\theta)}{mlcos^2(\theta) - (M+m)l}$$

$$\theta'' = \frac{-(M+m)gsin(\theta) + mlcos(\theta)sin(\theta)\theta'^2}{mlcos^2(\theta) - (M+m)l} + \frac{-Fcos(\theta)}{mlcos^2(\theta) - (M+m)l}$$

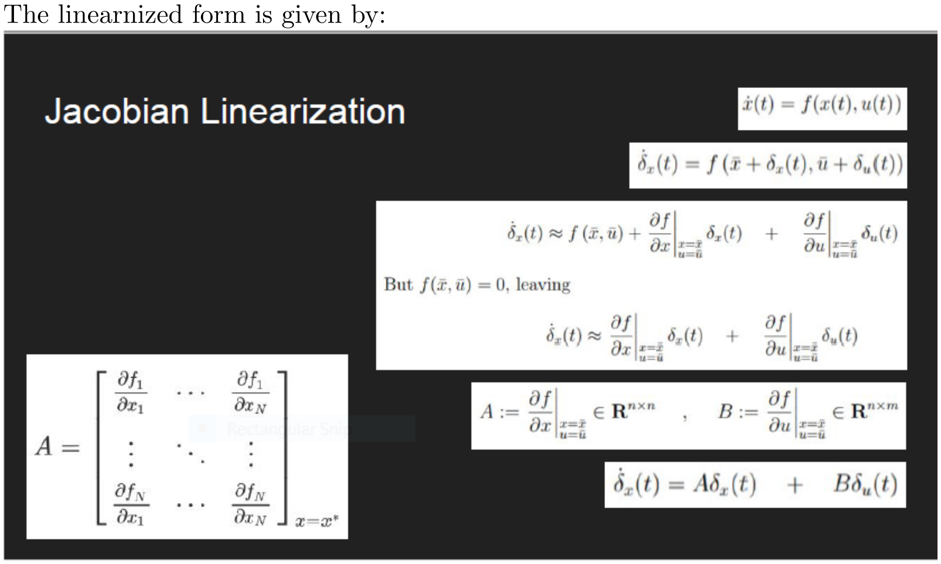
Now that we have the equation for θ'' we can substitute it to the equation

of x".
$x'' = \frac{-mlsin(\theta)\theta'^2 + mgsin(\theta)cos(\theta)}{(M+m) - mcos^2(\theta)} + \frac{F}{(M+m) - mcos^2(\theta)}$
so in control affine non-linear form can be written as:
$\begin{bmatrix} x' \\ \theta' \\ x'' \\ \theta'' \end{bmatrix} = \begin{bmatrix} x' \\ \theta' \\ \frac{-mlsin(\theta)\theta'^2+mgsin(\theta)cos(\theta)}{(M+m)-mcos^2(\theta)} \\ \frac{-(M+m)gsin(\theta)+mlcos(\theta)sin(\theta)\theta'^2}{mlcos^2(\theta)-(M+m)l} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{(M+m)-mcos^2(\theta)} \\ \frac{cos(\theta)}{(M+m)-mcos^2(\theta)} \end{bmatrix}$
And substituting the values for my varient:
$\begin{bmatrix} x' \\ \theta' \\ x'' \\ \theta'' \end{bmatrix} = \begin{bmatrix} x' \\ \theta' \\ \frac{-3.636sin(\theta)\theta'^2+35.316sin(\theta)cos(\theta)}{(7.2)-3.6cos^2(\theta)} \\ \frac{-70.632sin(\theta)+3.636cos(\theta)sin(\theta)\theta'^2}{3.636cos^2(\theta)-7.272} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{(7.2)-3.6cos^2(\theta)} \\ \frac{cos(\theta)}{(7.2)-3.6cos^2(\theta)} \end{bmatrix}$

2.3 Linearize nonlinear dynamics of the systems around equilibrium point $\vec{z} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$

0 0 0 0 bmatrix$typeoutnfss@catcodes$

The linearnized form is given by:



For $\vec{z}$ to be an equilibrium point:
$x' = 0$
$\theta' = 0$
$x'' = 0 + \frac{1}{(7.2) - 3.6cos^2(\theta)} u_e = 0$
$\theta'' = 0 + \frac{cos(\theta)}{(7.2) - 3.6cos^2(\theta)} = 0$
This is true if u_e is 0. so our system is like:

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_3' \end{bmatrix} = \begin{bmatrix} f_1(z,u) \\ f_2(z,u) \\ f_3(z,u) \\ f_4(z,u) \end{bmatrix}$$

And for linearizing we need A and B and the equations for finding A and B are given in the photo above.
f_1, f_2, f_3, f_4 can be taken from the previous part of the task.

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} x \\ \theta \\ x' \\ \theta' \end{bmatrix}$$

$f_1 = z_3$
$f_2 = z_4$

$$f_3 = \frac{-3.636sin(z_2)z_4^2 + 35.316sin(z_2)cos(z_2)}{(7.2) - 3.6cos^2(z_2)} + \frac{u}{(7.2) - 3.6cos^2(z_2)}$$

$$f_4 = \frac{-70.632sin(z_2) + 3.636cos(z_2)sin(z_2)z_4^2}{3.636cos^2(z_2) - 7.272} + \frac{cos(z_2)u}{(7.2) - 3.6cos^2(z_2)}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 9.81 & -0.2778 & 0 \\ 0 & -19.4257 & -0.275 & 0 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 0.2778 \\ 0.275 \end{bmatrix}$$

2.4 Stability checking

this can be simply done by calculating the eigen values of the A matrix we got in the prevoius part of the task.

```
 0.0000 + 0.0000i
 0.0694 + 4.4135i
 0.0694 - 4.4135i
-0.4167 + 0.0000i
```

eigenvalues:= *These are calculated in Matlab
So it turns out the there is positive eigenvalues of A so, the system is non-stable

2.5 Controllability checking

This can be done by making a matrix $\Gamma = \begin{bmatrix} B & AB & A^2B & A^3B \end{bmatrix}$

And if the rank of the Γ Matix is equal to that of A, then the system is controlable.

0	0.2778	-0.0772	2.7437
0	0.2775	-0.0771	-5.3692
0.2778	-0.0772	2.7437	-1.5185
0.2775	-0.0771	-5.3692	0.7361

$\Gamma =$

So the rank of Γ is 4 and the rank of A is 3. As you can see they both are different so the system is non-controlable.