

Self-interacting dark matter model without dark energy in cosmology

Yixuan Zhu

Department of Astronomy, Beijing Normal University, Beijing 100875, PR China

ABSTRACT

1. Introduction

2. The basic equations in the IDM model

We assume that the total density of the cosmic fluid obeys the collisional Boltzmann equation

$$\dot{\rho} + 3H\rho + \kappa\rho^2 - 2\Psi = 0, \quad (1)$$

where ρ is the total energy-density of the cosmic fluid, containing dark matter, baryons, and any type of exotic energy, Ψ is the rate of creation of DM particle pairs, and the annihilation parameter $\kappa(\geq 0)$ is given by:

$$\kappa = \frac{\langle\sigma u\rangle}{M_x}, \quad (2)$$

where σ is the cross-section for annihilation, u is the mean particle velocity, and M_x is the mass of the DM particle. Compared to the usual fluid equation, the effective pressure term is

$$P = \frac{\kappa\rho^2 - \Psi}{3H}. \quad (3)$$

When $\kappa\rho^2 - \Psi < 0$, what means that the IDM particle creation term is larger than the annihilation item, IDM may serve as a negative pressure source in the global dynamics of the Universe, like the role of Dark Energy in the general cosmological models.

Basilakos & Plionis (2009) identified two functional forms for which the previous Boltzmann equation can be solved analytically. Referring to Appendix B in Basilakos & Plionis (2009), only one of these two is of interest because it provides a " $\propto a^{-3}$ " dependence of the scale factor, which is

$$\Psi(a) = aH(a)R(a) = C_1(n+3)a^nH(a) + \kappa C_1^2a^{2n}. \quad (4)$$

And the total energy density is

$$\rho(a) = C_1a^n + \frac{a^{-3}F(a)}{C_2 - \int_1^a x^{-3}f(x)F(x)dx}, \quad (5)$$

where $f(a) = -\kappa/[aH(a)]$, and the kernel function $F(a)$ has the form

$$F(a) = \exp\left[-2\kappa C_1 \int_1^a \frac{x^{n-1}}{H(x)}dx\right]. \quad (6)$$

The first term of Eq.(5) is the density corresponding to the residual matter creation that results from a possible disequilibrium between the particle creation and annihilation processes, while the second term can be viewed as the energy density of the self-IDM particles that are dominated by the annihilation process.

2.1. Model 1: relation to the Λ CDM model

If $n = 0$, the global density evolution can be transformed as

$$\rho(a) = C_1 + a^{-3} \frac{e^{-2\kappa C_1(t-t_0)}}{C_2 - \kappa Z(t)}, \quad (7)$$

where $Z(t) = \int_{t_0}^t a^{-3} e^{-2\kappa C_1(t'-t_0)} dt'$ (Basilakos & Plionis (2009)). Using the usual unit-less Ω -like parameterization, we obtain that

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{1,0} + \frac{\Omega_{1,0}\Omega_{2,0}a^{-3}e^{-2\kappa C_1(t-t_0)}}{\Omega_{1,0} + \kappa C_1\Omega_{2,0}Z(t)}, \quad (8)$$

where $\Omega_{1,0} = 8\pi G C_1/3H_0^2$ and $\Omega_{2,0} = 8\pi G/3H_0^2 C_2$, which related to Ω_Λ and Ω_m in the Λ CDM model, respectively. From Eq.(2), we can also give the mass of the DM particle related to the range of κC_1 (in the unit of Gyr^{-1})

$$M_x = \frac{3.325 \times 10^{-12}}{\kappa C_1} \frac{\langle\sigma u\rangle}{10^{-23}} h^2 (1 - \Omega_{2,0}) \text{ GeV}, \quad (9)$$

where $h \equiv H_0/[100\text{km/s/Mpc}]$.

2.2. Model 2 : relation to the w CDM model

If $\kappa = 0$, the global density evolution can be written as

$$\rho(a) = \mathcal{D}a^{-3} + C_1a^n, \quad (10)$$

where $\mathcal{D} = C_2 - C - 1$. The conditions in which the current model acts as a quintessence cosmology are given by $\mathcal{D} > 0$, $C_1 > 0$, and $w_{\text{IDM}} = -1 - n/3$. This solution is mathematically equivalent to that of the gravitational matter creation model of(). The Hubble flow is now given by

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{2,0}a^{-3} + \Omega_{1,0}a^n, \quad (11)$$

where $\Omega_{2,0} = 8\pi G\mathcal{D}/3H_0^2$ and $\Omega_{1,0} = 8\pi G C_1/3H_0^2$, respectively.(Basilakos & Plionis (2009))

3. Dataset

To constrain the relevant IDM models (Basilakos & Plionis (2009)), we use the newly revised observational $H(z)$ data (OHD)(Zhang et al. (2014); Simon et al. (2005); Stern et al. (2010); Moresco et al. (2012); Moresco et al. (2016); Ratsimbazafy et al. (2017); Moresco (2015); Borghi et al. (2022); Jiao et al. (2023)), the Pantheon+ set of 1701 SNe Ia (Scolnic et al. (2022)), the BAO data from SDSS and DESI 2024.

3.1. The observational $H(z)$ data

It is widely known that the Hubble parameter $H(z)$ depends on the differential age as a function of redshift z in the form

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}, \quad (12)$$

which provides a direct measurement on $H(z)$ based on dz/dt . OHD measurements have recently been acquired mainly employing cosmic chronometers (CC). The CC method is used to provide 33 observational data points, which are taken in the redshift range $[0.07, 1.965]$. The Table 1 lists the OHD dataset used in this analysis. In this case, χ^2 can be defined as

$$\chi_{\text{OHD}}^2 = \sum_i^{33} \frac{(H_{\text{th}} - H_{\text{data}})^2}{\sigma_i^2}. \quad (13)$$

Table 1. The OHD dataset

z	$H(z)$	Reference
0.07	69 ± 19.6	Zhang et al. (2014)
0.09	69 ± 12	Simon et al. (2005)
0.12	68.6 ± 26.2	Zhang et al. (2014)
0.17	83 ± 8	Simon et al. (2005)
0.179	75 ± 4	Moresco et al. (2012)
0.199	75 ± 5	Moresco et al. (2012)
0.2	72.9 ± 29.6	Zhang et al. (2014)
0.27	77 ± 14	Simon et al. (2005)
0.28	88.8 ± 36.6	Zhang et al. (2014)
0.352	83 ± 14	Moresco et al. (2012)
0.3802	83 ± 13.5	Moresco et al. (2016)
0.4	95 ± 17	Simon et al. (2005)
0.4004	77 ± 10.2	Moresco et al. (2016)
0.4247	87.1 ± 11.2	Moresco et al. (2016)
0.4497	92.8 ± 12.9	Moresco et al. (2016)
0.47	89 ± 34	Ratsimbazafy et al. (2017)
0.4783	80.9 ± 9	Moresco et al. (2016)
0.48	97 ± 62	Stern et al. (2010)
0.593	104 ± 13	Moresco et al. (2012)
0.68	92 ± 8	Moresco et al. (2012)
0.75	98.8 ± 33.6	Borghi et al. (2022)
0.781	105 ± 12	Moresco et al. (2012)
0.8	113.1 ± 15.1	Jiao et al. (2023)
0.875	125 ± 17	Moresco et al. (2012)
0.88	90 ± 40	Stern et al. (2010)
0.9	117 ± 23	Simon et al. (2005)
1.037	154 ± 20	Moresco et al. (2012)
1.3	168 ± 17	Simon et al. (2005)
1.363	160 ± 33.6	Moresco (2015)
1.43	177 ± 18	Simon et al. (2005)
1.53	140 ± 14	Simon et al. (2005)
1.75	202 ± 40	Simon et al. (2005)
1.965	186.5 ± 50.4	Moresco (2015)

3.2. Type Ia supernovae

SNe Ia have long been used as "standard candles" to give a direct measurement of their luminosity distance, and provides strong constraints on cosmological parameters. We use the latest Pantheon+ data set of 1701 SNe Ia samples (Scolnic et al. (2022)), which covers the redshift range $[0, 2.26]$.

We use the fiducial SN Ia magnitude (M_b) determined from SH0ES 2021 Cepheid host distances (Riess et al. (2022)), which gives the μ_{data} and constrains H_0 in advance. To eliminate the influence of M_b , we give the likelihood function as

$$\chi_{\text{SNe}}^2 = A - \frac{B^2}{C} + \ln\left(\frac{C}{2\pi}\right), \quad (14)$$

where $A = \sum_{i=1}^{1701} (\mu_{\text{th}} - \mu_{\text{data}})^2 / \sigma_i^2$, $B = \sum_{i=1}^{1701} (\mu_{\text{th}} - \mu_{\text{data}}) / \sigma_i^2$, $C = \sum_{i=1}^{1701} 1 / \sigma_i^2$, the distance modulus is $\mu = 5 \log_{10}(d_L / \text{Mpc}) + 25$, and the luminosity distance d_L can be given as a function of redshift z

$$d_L = (1+z) \int_0^z \frac{cdz'}{H(z')}. \quad (15)$$

However, the Eq.(9) just need the H_0 to calculate the M_x and we would still use the simple likelihood function as

$$\tilde{\chi}_{\text{SNe}}^2 = A = \sum_{i=1}^{1701} \frac{(\mu_{\text{th}} - \mu_{\text{data}})^2}{\sigma_i^2}. \quad (16)$$

Apparently these two functions give the same constraints in $\Omega_{2,0}$ and κC_1 .

3.3. Baryon acoustic oscillation

The Baryon acoustic oscillation method (BAO) provides a key cosmological probe sensitive to the cosmic expansion history with well-controlled systematics. We use two BAO data sets from the SDSS (Alam et al. (2021)) and DESI 2024 (Collaboration et al. (2024)), which are given at Table 2 and Table 3, respectively.

Table 2. The BAO-only dataset from SDSS

z_{eff}	D_M/r_d	D_H/r_d	D_V/r_d
0.15			4.47 ± 0.17
0.38	10.23 ± 0.17	25 ± 0.76	
0.51	13.36 ± 0.21	22.33 ± 0.58	
0.7	17.86 ± 0.33	19.33 ± 0.53	
0.85			$18.33^{+0.57}_{-0.62}$
1.48	30.69 ± 0.8	13.26 ± 0.55	
2.33	37.6 ± 1.9	8.93 ± 0.28	
2.33	37.3 ± 1.7	9.08 ± 0.34	

Table 3. The BAO dataset from DESI 2024

z_{eff}	D_M/r_d	D_H/r_d	D_V/r_d
0.295			7.93 ± 0.15
0.51	13.62 ± 0.25	20.98 ± 0.61	
0.706	16.85 ± 0.32	20.08 ± 0.6	
0.93	21.71 ± 0.28	17.88 ± 0.35	
1.317	27.79 ± 0.69	13.82 ± 0.42	
1.491			26.07 ± 0.67
2.33	39.71 ± 0.94	8.52 ± 0.17	

4. Constraint results

5. Conclusions

References

Alam, S., Aubert, M., Avila, S., et al. 2021, Phys. Rev. D, 103, 083533

- Basilakos, S. & Plionis, M. 2009, *A&A*, 507, 47
- Borghi, N., Moresco, M., & Cimatti, A. 2022, *The Astrophysical Journal Letters*, 928, L4
- Collaboration, D., Adame, A. G., Aguilar, J., et al. 2024, DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations
- Jiao, K., Borghi, N., Moresco, M., & Zhang, T.-J. 2023, *The Astrophysical Journal Supplement Series*, 265, 48
- Moresco, M. 2015, *Monthly Notices of the Royal Astronomical Society: Letters*, 450, L16
- Moresco, M., Cimatti, A., Jimenez, R., et al. 2012, *Journal of Cosmology and Astroparticle Physics*, 2012, 006
- Moresco, M., Pozzetti, L., Cimatti, A., et al. 2016, *Journal of Cosmology and Astroparticle Physics*, 2016, 014
- Ratsimbazafy, A. L., Loubser, S. I., Crawford, S. M., et al. 2017, *Monthly Notices of the Royal Astronomical Society*, 467, 3239
- Riess, A. G., Yuan, W., Macri, L. M., et al. 2022, *The Astrophysical Journal Letters*, 934, L7
- Scolnic, D., Brout, D., Carr, A., et al. 2022, *The Astrophysical Journal*, 938, 113
- Simon, J., Verde, L., & Jimenez, R. 2005, *Phys. Rev. D*, 71, 123001
- Stern, D., Jimenez, R., Verde, L., Kamionkowski, M., & Stanford, S. A. 2010, *Journal of Cosmology and Astroparticle Physics*, 2010, 008
- Zhang, C., Zhang, H., Yuan, S., et al. 2014, *Research in Astronomy and Astrophysics*, 14, 1221