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RESEARCH

Scalar Field Cosmology with Powerlaw and Hybrid Expansion Law in Symmetric Teleparallel Gravity

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Abstract

Recently, scalar field approaches have been widely considered in cosmological models due to their potential for useful investigation of the cosmological evolution of the Universe. The use of a scalar field as a matter source in a cosmological model within symmetric teleparallel gravity is as interesting as many other modified gravity models sourced by a scalar field. In this paper, we investigate the repercussions of power law and hybrid expansion laws in symmetric teleparallel gravity using an associated scalar field as a total matter source. We find that for both models, a normal scalar field is required to provide a viable description of the Universe's evolution, as opposed to a phantom scalar field, from an ad-hoc introduction of these two expansion laws. To back up this method of implementing these two ad-hoc expansion laws we also examined the evolution of the Hubble parameter by solving the Raychaudhuri equation which yields power law solutions in early radiation and matter dominated epochs and de-Sitter like solution in late Universe. We analysed this evolution in relation to observed Hubble data and found that the dependence of the model parameters in reproducing various epochs of the Universe is non-minimal. Furthermore, to test the stability of our models, we looked into the evolution of the speed of sound squared, which indicates that our models are stable to density perturbations. Finally, we utilize the geometrical method of $\boldsymbol{Om}$ diagnostics to conclude that our model exhibits quintessence like behaviour.

Keywords $f(Q)$ gravity · Scalar field · Dark energy · Power law · Hybrid law

1 Introduction

The existence of the Big Bang singularity problem, as well as the flatness, horizon, and monopole puzzles in cosmology [1], led to the conclusion that the mainstream cosmolog-

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ical model based on general relativity (GR) and the standard model of particle physics is insufficient to represent the Universe at the UV regime. GR, on the other hand, is a classical theory that cannot function as a fundamental theory in context of a quantum account of spacetime and gravity. Because of these factors, as well as the lack of a definite quantum theory of gravity, a number of alternative theories of gravitation have been developed in an attempt to define at least in semi-classical level in which GR and its accomplishments may be reproduced. In inflationary cosmology, modified theories of gravity, for instance Starobinsky R^2-gravity [2] naturally provides the necessary dynamics of inflationary processes with exceptional closeness with observed cosmological data. Along with the dynamics of inflation, a modified $f(R)$-gravity model [3] explains the late time cosmic acceleration of the Universe (dark energy) and the effects of dark matter such as the flatness of galaxies' rotation curves [4] without the need to include an exotic form of matter. Some detailed reviews can be found in [5–7].

The curvature based approach has been prominent in describing the observed accelerated expansion of the Universe. The modification of the curvature sector of the Einstein field equation can effectively lead to accelerated behaviour of the Universe [8–10]. On this footing the well known $f(R)$ gravity models are an extension to GR where the usual Ricci scalar is replaced by a function of it, and these are the models with zero torsion and non-metricity. Apart from curvature, torsion and non-metricity of the spacetime may show significant effect on the accelerated expansion of the Universe. On this basis, another class of extended gravity models are formulated known as the $f(\mathcal{T})$ models [11, 12] where, $\mathcal{T}$ is the torsion scalar, and are generally termed as teleparallel equivalent of GR (TEGR). In these models, the gravitational properties are attributed to the non-zero torsion and vanishing non-metricity and curvature. Finally, another class of models with non-zero metricity and vanishing curvature and torsion is known as the symmetric teleparallel equivalent of GR (STEGR), commonly termed as $f(Q)$ gravity models [14], where Q is the non-metricity scalar. STEGR is based on the principles of Weyl geometry, which is a complete expansion of Riemannian geometry that acts as the mathematical foundation of GR. According to Weyl geometry, the emergence of gravitational effects does not result from a change in the direction of a vector experiencing parallel transport, but rather from a difference in the length of the vector itself. Weyl geometry, in effect, offers a paradigm for understanding and interpreting the behaviour of gravitational fields in a far more comprehensive and robust way than Riemannian geometry. In contrast, the investigation of non-linear Lagrangians is made possible between torsion and non-metricity because of the nature of the connection terms associated between both of them, without the need for higher-order terms to be included in the field equations. In particular, $f(\mathcal{T})$ gravity is more constrained than $f(Q)$ gravity since the former demands additional conditions to be met on the associated connections. As a result, compared to $f(\mathcal{T})$ gravity, $f(Q)$ gravity provides more independent equations. The $f(\mathcal{T})$ gravity field equations contain anti-symmetric terms, and the problem of local invariance adds complexities that affect the natural fulfilment of the Bianchi Identity. In contrast, these difficulties are not relevant when using the symmetric field equations of $f(Q)$ gravity. Furthermore, while $f(Q)$ theories are nearly indistinguishable from $f(\mathcal{T})$ theories at the background level, $f(\mathcal{T})$ gravity is plagued by the strong coupling problem [15] (and references therein), which arises due to the survival of two additional scalar degrees of freedom, which are shown to be absent in $f(Q)$ theories on general FLRW backgrounds. Yet, strong coupling concerns arise in maximally symmetric backgrounds such as deSitter or Minkowski geometry, although not to the same level as in $f(\mathcal{T})$ gravity, leaving room for intriguing possibilities.

The modified theory of $f(Q)$ gravity has gained significant attention in recent days due to its potential to address some of the important issues in cosmology and cosmography [15–24] (and references therein). Numerous studies have successfully tested $f(Q)$ gravity against various backgrounds and observational data. In particular, $f(Q)$ gravity has been shown to be consistent with the Cosmic Microwave Background (CMB) observations [25], Supernovae type Ia (SNIa) data [26], Baryonic Acoustic Oscillations (BAO) data [27]. The consistency of $f(Q)$ gravity with these various observational data suggests that it may challenge the standard ΛCDM scenario [28, 29]. This confrontation reveals that $f(Q)$ gravity can provide a more accurate and comprehensive description of the universe's evolution than the current model. Important relevant works in $f(Q)$ gravity are carried out by many authors, for instance Solanki et al [30] analysed a viscous fluid model in a cosmological background of some popular $f(Q)$ functions by taking into account of few different forms of bulk viscosity coefficients. Hassan et al [31] studied the effect of the Generalized Uncertainty Principle (GUP) in Casimir wormholes in $f(Q)$ gravity. Some authors also studied large scale structures (LSS) in $f(Q)$ gravity [32]. Koussour et al [33] performed an extensive analysis on late time behaviour using a powerlaw form of $f(Q)$ with a constant sound speed. Gadbail et al investigated a hybrid model comprising of bulk viscosity and generalized chaplygin gas in powerlaw $f(Q)$ gravity where they showed that a transition from decelerating to accelerating phase is possible with certain conditions [34]. Koussour et al suggested a new parametrization of Hubble parameter within $f(Q)$ gravity to explain accelerated expansion of the late Universe [47]. Many more works related to $f(Q)$ gravity can be found in [27, 35–37]

The paper is presented as follows : In Section 2 we present a brief formulation to $f(Q)$ gravity, in Section 3 we set up the necessary Friedmann equations relevant to the model. In Section 4 we shall describe the relevant scalar field dynamics to be utilized in the study. In Section 5 we shall investigate the behaviour of the physical paramters within a scalar field background in $f(Q)$ gravity. In Section 6 we shall obtain the scalar field potential for the models. In Section 7 we will make a detailed study of the evolution of the Hubble parameter using the Raychaudhuri equation with comparison to the observed Hubble data. In Section 8 we shall study the stability of the models. In Section 9 we perform the Om-diagnostic to classify whether the model shows like quintessence or phantom dark energy like behaviour. Finally, in Section 10 we conclude the study with our findings and future perspectives.

2 Formulation of f(Q) Gravity

The action in $f(Q)$ gravity is given by [13]

$$S = \frac{1}{2}\int d^4x\sqrt{-g}f(Q) + \int d^4x\sqrt{-g}L_m \tag{1}$$

where $f(Q)$ is an general function of the non-metricity scalar Q, g is the determinant of the metric tensor $g_{\mu\nu}$ and L_m is the matter Lagrangian.

The non-metricity tensor $Q_{\lambda\mu\nu}$ is the covariant derivative of the metric tensor and is defined as

$$Q_{\lambda\mu\nu} := \nabla_\lambda g_{\mu\nu}, \tag{2}$$

and its two traces Q_α and $\tilde{Q}_\alpha$ are

$$Q_\mu = Q_\mu{}^\alpha{}_\alpha, \quad \tilde{Q}^\mu = Q_\alpha^{\mu\alpha}, \tag{3}$$

The superpotential tensor is defined as

$$4P^\lambda_{\mu\nu} = -Q^\lambda_{\mu\nu} + 2Q_{(\mu^\lambda\nu)} + \left(Q^\lambda - \tilde{Q}^\lambda\right)g_{\mu\nu} - \delta^\mu_{(\lambda Q_\nu)}. \tag{4}$$

The non-metricity scalar is obtained by taking trace of non-metricity tensor and is obtained as

$$Q = -Q_{\lambda\mu\nu} P^{\lambda\mu\nu}. \tag{5}$$

The energy-momentum tensor (EMT) is defined as

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g} L_m\right)}{\delta g^{\mu\nu}} \tag{6}$$

where we define $f_Q = \frac{df}{dQ}$.

Now, varying the action (1) with respect to the metric $g_{\mu\nu}$, the gravitational field equation for $f(Q)$ gravity is obtained as

$$\frac{2}{\sqrt{-g}} \nabla_\lambda \left(\sqrt{-g} f_Q P^\lambda_{\mu\nu}\right) + \frac{1}{2} g_{\mu\nu} f + f_Q \left(P_{\mu\lambda\beta} Q_\nu^{\lambda\beta} - 2 Q_{\lambda\beta\mu} P^{\lambda\beta}_\nu\right) = -T_{\mu\nu} \tag{7}$$

Also, if we vary the action (1) with respect to the connection, we obtain

$$\nabla_\mu \nabla_\nu \left(\sqrt{-g} f_Q P_\lambda^{\mu\nu}\right) = 0 \tag{8}$$

3 Cosmological Field Equations in f(Q) Gravity

We adopt the spatially flat homogeneous and isotropic FLRW line element given by

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{9}$$

where $a(t)$ is the scale factor. The non-metricity scalar can be expressed in terms of the Hubble parameter as [14]

$$Q = 6H^2, \tag{10}$$

We consider the framework where the energy momentum tensor in the presence of a general fluid with energy density ρ and pressure p is defined by

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}, \tag{11}$$

where $u^\mu = (1, 0, 0, 0)$ is the four velocity of the fluid satisfying the condition $u^\mu u_\nu = -1$. The general fluid may be any combination of perfect fluid as well as well as a contribution from the non-metricity effects.

The Friedmann equations for the model can be obtained by using the metric (9) in the field (7) as [14]

$$3H^2 = \frac{1}{2} f_Q \left(-\rho + \frac{f}{2}\right), \tag{12}$$

$$\dot{H} + 3H^2 + \frac{\dot{f}_Q}{f_Q} H = \frac{1}{2 f_Q} \left(p + \frac{f}{2}\right), \tag{13}$$

where the overdot represents differentiation over time and f_Q represents differentiation with respect to Q. The equation of state parameter w in this model may be defined by

$$w = \frac{p}{\rho} \tag{14}$$

such that $w = 0, 1/3, -1$ represents that of pressure-less dust, radiation and cosmological constant respectively. We note that the terms incorporated along with energy density and pressure may be responsible for causing accelerated expansion of the Universe. Thus, we define the combination as effective energy density and pressure as

$$\rho_{eff} = \frac{1}{f_Q}\left(\rho - \frac{f}{2}\right), \tag{15}$$

$$p_{eff} = -2H\frac{\dot{f}_Q}{f_Q} + \frac{1}{f_Q}\left(p + \frac{f}{2}\right) \tag{16}$$

By this definition we can express the Friedmann (7) and (8) in a form analogous to those obtained from GR

$$3H^2 = -\frac{1}{2}\rho_{eff}, \tag{17}$$

$$\dot{H} + 3H^2 = \frac{p_{eff}}{2}. \tag{18}$$

In this regard, the contribution from the non-metricity terms can lead to the onset of accelerated expansion of the Universe, without the need to explicitly impose any exotic fluids like dark energy. However, the possible accelerated expansion produced due to the presence of non-metricity (which is a geometric approach) can be studied equivalently by an associated scalar field.

4 Scalar Field Dynamics

Scalar field cosmology has proven to be extremely useful in understanding the dynamics of the Universe's evolution. Scalar fields' unique characteristics might offer explanations for key cosmological ideas like inflation, dark energy, and dark matter. In particular, perturbations in the scalar field, known to as "inflaton" in the framework of inflationary cosmology [1], serve as seeds to the formation of the Universe's large-scale structure. In the domain of late-time cosmology, scalar field models such as quintessence, phantom, and tachyonic dark energy have received more attention lately [38]. In $f(Q)$ gravity, it has been shown that it has a pressing advantage by not requiring the equivalence principle to be as an initial assumption and the theory is free from strong coupling problems because of the absence of the additional scalar modes [39] (and relevant references therein) in contrast to $f(\mathcal{T})$ theories. The extension from Q to $f(Q)$, in analogy to $f(R)$ theories in essence may lead to interesting dynamics of scalar field cosmology through a conformal transformation from Jordan to Einstein frame. Some other important perspectives applied to $f(Q)$ gravity and conformal degrees of freedom have been discussed widely in [40–43]. For instance, in a work by Capozziello and Shokri [39], a discussion on the consequences of $f(Q)$ gravity based on conformal transformations from Jordan to Einstein frame is presented in the context of slow-roll inflation where a scalar field and a corresponding potential is utilised, which indeed is an interesting method to study the role of scalar fields in modified gravity models in different eras of the Universe. However, in this paper we are not interested in the conformal transformation technique instead we follow the approach where the scalar field is the total matter content by itself to investigate the late-time Universe. In other words, the energy-momentum tensor reflects the involvement of the scalar field and its corresponding self-interacting potential. One of the work done by following this latter method can be found in [44], where the authors discussed the possibility of scalar field as total matter content. The same approach has also been implemented in the case of $f(R)$ gravity by Bairagi [45]. Inspired by these works, we have tried to implement the method of incorporating the effect of the scalar field in the matter sector in $f(Q)$ gravity, as we have seen this possibility has not yet been investigated till date to the best of our knowledge. However, we note that we find no sufficient explanations yet for how the approach we are following is better or worse than the conformal transformation approach. To study the role of the scalar field as a total matter content, we start from the energy-momentum tensor corresponding to a scalar field. The energy momentum tensor associated with a scalar field is

$$T^{(\phi)}_{\mu\nu} := \varepsilon\partial_\mu\phi\partial_\nu\phi - \mathcal{L}, \tag{19}$$

where $\mathcal{L} = \frac{1}{2}\varepsilon\partial_\mu\phi\partial^\nu\phi - V(\phi)$ is the Lagrangian density of the scalar field. ε keeps track of whether the field is of a 'normal' ($\varepsilon = +1$) or 'phantom' ($\varepsilon = -1$) type, The '00' and 'ii' ($i = 1, 2, 3$) components of $T^{(\phi)}_{\mu\nu}$ yields

$$\rho = \frac{1}{2}\varepsilon\dot{\phi}^2 + V(\phi), \tag{20}$$

and

$$p = \frac{1}{2}\varepsilon\dot{\phi}^2 - V(\phi), \tag{21}$$

respectively. The equation of state parameter is given by

$$w = \frac{\frac{1}{2}\varepsilon\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\varepsilon\dot{\phi}^2 + V(\phi)} \tag{22}$$

It is critical to emphasise that in this study, the scalar field will be treated as an autonomous entity from the overall matter content rather than as a part of the total matter content. As a result, the scalar field has been considered to be an equal representation of the total matter content. In other words the scalar field is considered as the only matter source. This theoretical setting suggests that a careful study of the scalar field's dynamics alone should be sufficient to determine the evolution of the Universe.

It is not realistic to establish the entire evolutionary dynamics of the Universe by merely defining a scalar field as the total matter content in the absence of a corresponding potential. Traditionally, the potential is determined employing particle physics or gravitational theories, which can be cumbersome due to the intricacy of the existing potential forms. As a result, various suggestions have been made to adopt potentials resulting from a specific evolutionary history. Ellis and Madsen [46], for instance, established several kinds of scalar potentials for inflationary evolution without relying on the slow-roll approximation. As a consequence, determining the general potential form for the scalar field that can adequately depict the development of the Universe is vital.

To investigate the time evolution of energy density and pressure as specified by (12) and (13), a function $f(Q)$ and the Hubble parameter H must be introduced. To that purpose, we will utilize both the powerlaw and hybrid expansion laws and conduct a comparative study between them. In addition, we will implement an $f(Q)$ function that have received a lot of attention in the literature namely $f(Q) = \alpha Q^\beta$.

5 Behaviour of Physical Parameters

To study the behaviour of the physical parameters of the model, we need to implement a viable form of scale factor within a $f(Q)$ model considering the functional form of an $f(Q)$ given by [33, 34]

$$f(Q) = \alpha Q^{\beta}, \quad \alpha, \beta > 0 \tag{23}$$

where α and β are the model parameters. In this work, we shall consider two types of scale factor viz the powerlaw and hybrid expansion law.

5.1 Case 1: Powerlaw Expansion

The powerlaw scale factor [48, 49] is given by

$$a(t) = nt^{m}, \quad n, m > 0 \tag{24}$$

where m and n are positive constants. For $0 < m < 1$ the model accommodates a constant deceleration parameter $q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{1}{m}$ and for $m > 1$, accelerated expansion. For the powerlaw model (24) the Hubble parameter, energy density ρ, pressure p, the kinetic term $\dot{\phi}^2$ and potential $V(t)$ are calculated as

$$H = \frac{m}{t}, \tag{25}$$

$$\rho = \alpha\left(-2^{\beta-1}\right)3^{\beta}(2\beta - 1)\left(\frac{m^2}{t^2}\right)^{\beta}, \tag{26}$$

$$p = \frac{\alpha 6^{\beta-1}(2\beta - 1)(3m - 2\beta)\left(\frac{m^2}{t^2}\right)^{\beta}}{m}, \tag{27}$$

$$\dot{\phi}^2 = -\varepsilon\frac{\alpha 2^{\beta}3^{\beta-1}\beta(2\beta - 1)\left(\frac{m^2}{t^2}\right)^{\beta}}{m}, \tag{28}$$

$$V(t) = \frac{6^{\beta}\alpha(m(0.5 - \beta) + (0.33\beta - 0.167)\beta)\left(\frac{m^2}{t^2}\right)^{\beta}}{m}, \tag{29}$$

In Fig. 1, the graphical representation of the evolutionary behaviour of the physical parameters in the power law case is displayed. The total energy density ρ, which depends on the model parameters α and β, is numerically studied to observe its evolution for various values of these parameters. It is seen that the energy density decreases with time and becomes asymptotically constant at later times. This fall in energy density may be attributed to the expansion of the Universe, as the volume of the Universe increases with expansion. This also is in conformity with the fact that the scalar field under consideration is a normal scalar field as in the case of a normal scalar field, the energy density decreases with time, which is opposite to the case of a phantom scalar field where the energy density grows indefinitely with time without bound. The dependence of ρ on α and β is however found to be relatively insensitive, and the variation do not exhibit significant deviation.

To evaluate the physical validity of the model, the variation of the kinetic term $\dot{\phi}^2$ was analyzed. A physically acceptable situation requires the scalar field to be real, necessitating $\dot{\phi}^2 > 0$. As demonstrated in Fig. 1, the kinetic term evolves within positive values across different values of the model parameters, suggesting that the nature of the scalar field is real.

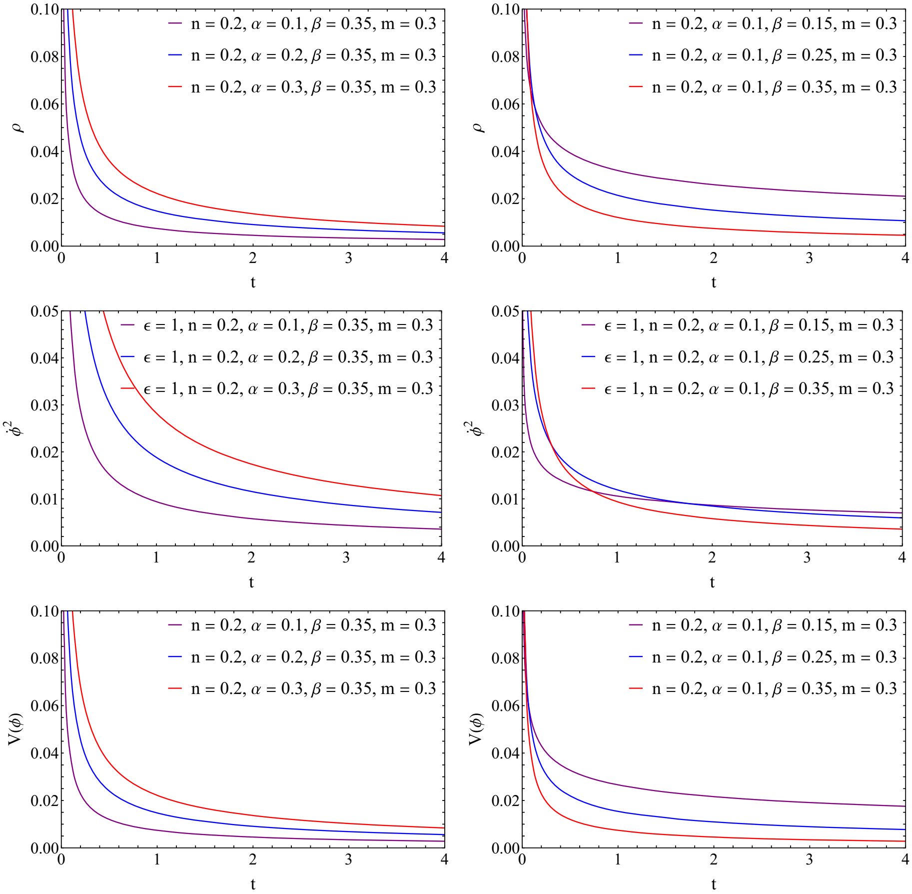


Fig. 1 The plots of ρ, $\dot{\phi}^2$ and V for different values of model parameters α and β for the powerlaw case

Thus power law expansion under this framework of $f(Q)$ is physical provided the scalar field is a real scalar field.

From the potential-time plot in Fig. 1, we see that the potential decreases with time and flattens out at late times. This is important in a sense that, because of the presence of a flat region of the potential, the model can accommodate slow-roll dynamics of late time accelerated expansion. We observe from the figure that smaller values of α provides a flatter potential, whereas β does not seem to affect the flatness of the potential significantly.

5.2 Case 2: Hybrid Law Expansion

The hybrid law scale factor [50–53] is given by

$$a(t) = t^m \exp(nt), \quad m, n > 0 \tag{30}$$

The hybrid law is a cosmological model that may exhibit both power-law and de-Sitter-like expansion behaviour, making it a more generic model. This model specifically represents the cosmic history of the Universe as governed by the power-law term (t^m) in early time and the exponential term (exp(nt) in later times. Notably, the model acts like a power-law model when $n = 0$, and like a de-Sitter Universe when $m = 0$. As a result, the hybrid scale factor allows the Universe to transition from an early decelerated phase to a late-time accelerated phase.

For the hybrid law model (24) the Hubble parameter, energy density ρ, pressure p, the kinetic term $\dot{\phi}^2$ and potential $V(t)$ are calculated as

$$H = t^{-m}e^{-nt}\left(mt^{m-1}e^{nt} + nt^m e^{nt}\right), \tag{31}$$

$$\rho = \alpha\left(-2^{\beta-1}\right)3^{\beta}(2\beta-1)\left(\frac{(m+nt)^2}{t^2}\right)^{\beta}, \tag{32}$$

$$p = \frac{\alpha 6^{\beta-1}(2\beta-1)\left(\frac{(m+nt)^2}{t^2}\right)^{\beta-1}\left(3(m+nt)^2 - 2\beta m\right)}{t^2}, \tag{33}$$

$$\dot{\phi}^2 = -\varepsilon\frac{\alpha 2^{\beta}3^{\beta-1}\beta(2\beta-1)m\left(\frac{(m+nt)^2}{t^2}\right)^{\beta-1}}{t^2}, \tag{34}$$

$$V(t) = \frac{\alpha 6^{\beta}\left(\frac{(m+nt)^2}{t^2}\right)^{\beta-1}}{t^2}\left[m^2(0.5-\beta) + mn(1-2\beta)t \right. \\ \left. + n^2(0.5-\beta)t^2 + (0.33\beta - 0.167)\beta m\right], \tag{35}$$

In the case of the hybrid model, the physical parameters exhibit analogous behavior to that observed in the power law model as evident from Fig. 2. Specifically, we observe that the energy density falls with time, eventually approaching a constant value at late times. As such, the hybrid model demands that the scalar field in the model must be real valued. In terms of the model's validity, we find that $\dot{\phi}^2$ maintains positive values throughout its evolution, thus confirming the viability of the hybrid model as a possible explanation for the evolution of the Universe under $f(Q)$ theory.

With regard to the shape of the potential for the hybrid model, we observe that it is flat at late times, which may be suitable for slow-roll dynamics and the production of accelerated expansion. However, we note that the slope of the potential appears to be minimally affected by variations in the values of α and β.

6 Scalar Field Potential

As mentioned in the previous section, construction of a scalar field potential is quite crucial in providing a viable explanation of cosmological evolution of the Universe. To that matter, we have obtained a numerical form of the scalar field potential as the analytic form of the potential is hard to obtain in a closed form due to the complicated structure of the potential functions (29) and (35). The qualitative form of the potentials for both the models are depicted in Fig. 3. For the plots, we have considered $n = 0.2, m = 0.3, \alpha = 0.2, \beta = 0.25$. In a qualitative sense, because of the presence of the sufficiently flat region of the potential, the form of the potential might show slow-roll dynamics at the late times.

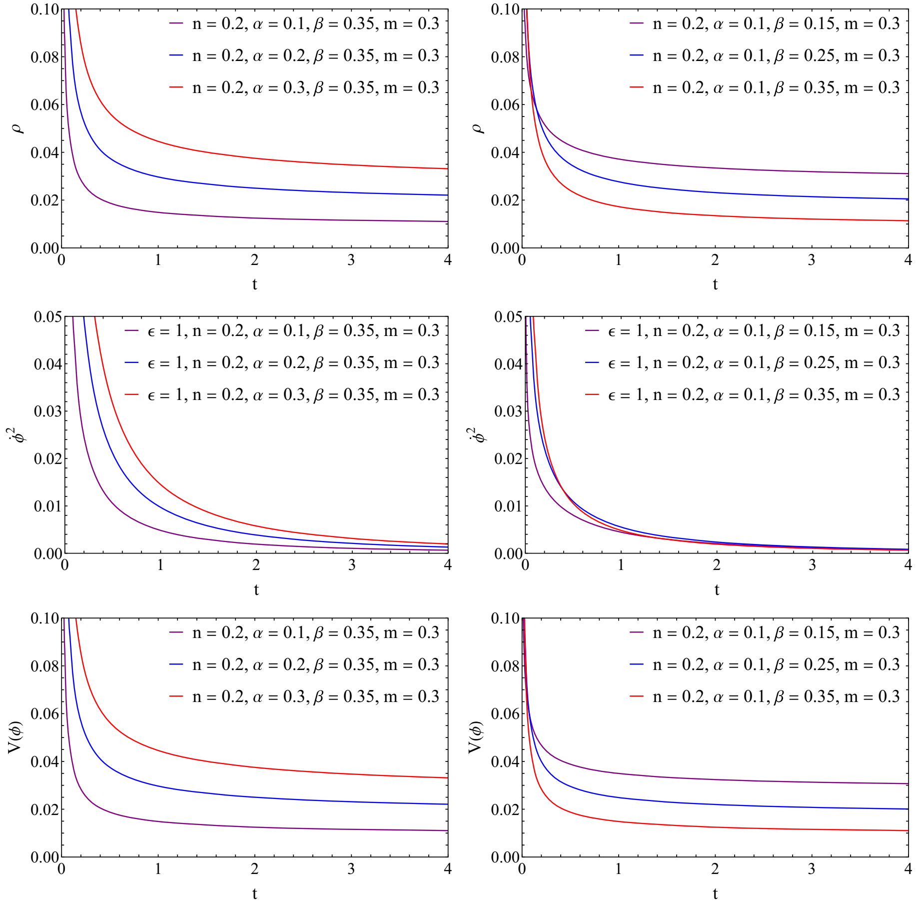
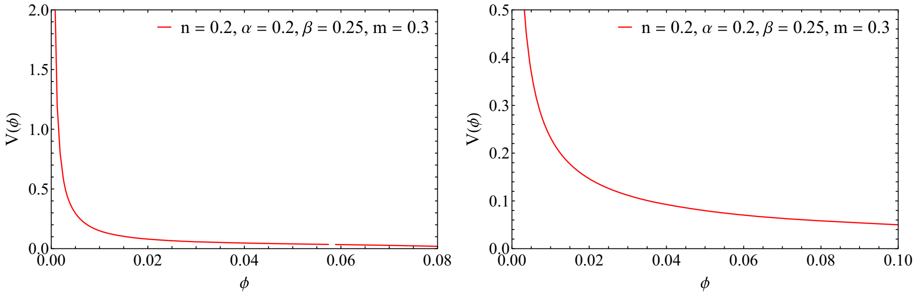


Fig. 2 The plots of ρ, $\dot{\phi}^2$ and V for different values of model parameters α and β for the hybrid law case



(a) Powerlaw (b) Hybrid Law

Fig. 3 The scalar field potential $V(\phi)$ in terms of the field ϕ is shown for both powerlaw and hybrid law. Here we have considered $n = 0.2$, $m = 0.3$, $\alpha = 0.2$, $\beta = 0.25$

7 Evolution of the Hubble Parameter

Until now we studied the behaviour of the model by implementing ad-hoc expansion laws, viz power law and hybrid law. In this section we want to see whether a general top-down approach of solving the Raychaudhuri equation can lead us to obtaining these forms of expansion laws.

To study the evolution of the Hubble parameter, we use the function (23). From (12) we can obtain the energy density ρ as

$$\rho = 6^{\beta}\left(\frac{1}{2} - \beta\right)\alpha H^{2\beta} \tag{36}$$

Equation (13) can be expressed in the following form by using (23) as

$$\dot{H} + \frac{3H^2}{2\beta} - \frac{6^{1-\beta}p}{2\alpha\beta(2\beta-1)}H^{2(1-\beta)} = 0, \tag{37}$$

Now using the EoS (14) and (36), (37) can be expressed fully in terms of the Hubble parameter H as

$$\dot{H} + \frac{3}{2\beta}(1+w)H^2 = 0 \tag{38}$$

To change the variables from time to scale factor a we replace $\frac{d}{dt}$ to $H\frac{d}{d\ln a}$ and integrate to obtain the expression for Hubble parameter in terms of the scale factor a

$$H(a) = H_0 a^{-\frac{3(1+w)}{2\beta}}, \tag{39}$$

Replacing $a = (1+z)^{-1}$ we can express (39) in terms of the redshift z as

$$H(z) = H_0(1+z)^{\frac{3(1+w)}{2\beta}} \tag{40}$$

It is clear that $H(z)$ is independent of the model paramter α but depends on β. The Hubble constant H_0 for the model in comparison to the ΛCDM is presented in Table 1. The current rate of expansion of the Universe is determined using observable Hubble data (OHD). Irrespective of cosmological models, OHD observations can determine the Hubble parameter $H(z)$ as a function of redshift z. OHD measurements have recently been acquired employing cosmic chronometers (CC) and baryon acoustic oscillation (BAO) observations. Our dataset has 36 discrete data points. The CC method is used to produce 31 of these observational data points, while the remaining three were obtained from the radial baryon acoustic oscillation signal detected in galaxy distributions. Two of the data points (at $z = 2.34$ and 2.36 redshifts, respectively) were taken solely from the BAO signal detected in the Lyα forest distribution or by cross-correlation with quasi-stellar objects (QSOs). We make use these unique data points, which are taken in the redshift range [0, 2.36], to assess the validity of our model. The Table 2 lists the dataset used in this analysis. We clearly observe from Fig. 4 that the $f(Q)$ model shows better fitting at some values than the ΛCDM corresponding to the model parameter β at $\beta = 0.663$. The best fit parameters are obtained on the basis of Markov Chain

Table 1 Best fit value of H_0 for the model $f(Q) = \alpha Q^{\beta}$

Model	$H_0 = H(0)$
ΛCDM	67. 72
$f(Q) = \alpha Q^{\beta}$, $\beta = 0.663$	68.07

Table 2 The table shows the observed Hubble parameter data points

z	$H(z)$	Method	Reference
0.07	69 ± 19.6	CC	[54]
0.09	69 ± 12	CC	[55]
0.12	68.6 ± 26.2	CC	[54]
0.17	83 ± 8	CC	[55]
0.179	75 ± 4	CC	[56]
0.199	75 ± 5	CC	[56]
0.2	72.9 ± 29.6	CC	[54]
0.27	77 ± 14	CC	[55]
0.28	88.8 ± 36.6	CC	[54]
0.352	83 ± 14	CC	[56]
0.38	81.9 ± 1.9	BAO	[57]
0.3802	83 ± 13.5	CC	[58]
0.4	95 ± 17	CC	[55]
0.4004	77 ± 10.2	CC	[58]
0.4247	87.1 ± 11.2	CC	[58]
0.4497	92.8 ± 12.9	CC	[58]
0.47	89 ± 50	CC	[59]
0.4783	80.9 ± 9	CC	[58]
0.48	97 ± 62	CC	[59]
0.51	90.8 ± 1.9	BAO	[57]
0.593	104 ± 13	CC	[56]
0.61	97.8 ± 2.1	BAO	[57]
0.68	92 ± 8	CC	[56]
0.781	105 ± 12	CC	[56]
0.875	125 ± 17	CC	[56]
0.88	90 ± 40	CC	[59]
0.9	117 ± 23	CC	[55]
1.037	154 ± 20	CC	[56]
1.3	168 ± 17	CC	[55]
1.363	160 ± 33.6	CC	[60]
1.43	177 ± 18	CC	[55]
1.53	140 ± 14	CC	[55]
1.75	202 ± 40	CC	[55]
1.965	186.5 ± 50.4	CC	[60]
2.34	223 ± 7	BAO-Lyα/QSOs	[61]
2.36	227 ± 8	BAO-Lyα/QSOs	[62]

Here CC represents Cosmic Chronometers, BAO represents Baryon Acoustic Oscillations in galaxy distribution and BAO-Lyα/QSOs represents BAO signal in Lyα forest distribution alone or cross-correlated with QSOs

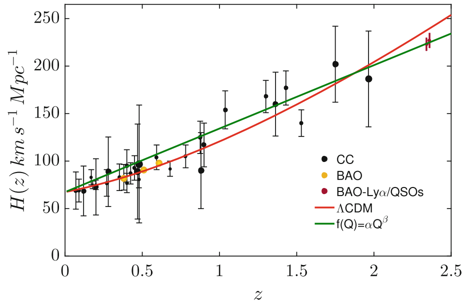


Fig. 4 The plot of $H(z)$ vs z

Monte Carlo (MCMC) and Planck 18 data as $H_0 = 68.12 \pm 0.40$ and $\beta = 0.665 \pm 0.003$ (Fig. 5).

7.1 Scale Factors

From (39) we may have

$$da = H_0 a^{\frac{2\beta - 3(1+w)}{2\beta}} dt \tag{41}$$

This equation can be solved to obtain the scale factors for different cases -

7.1.1 Case 1: Radiation Dominated Era ($w = 1/3$)

Setting $w = 1/3$ in (41) and integrating gives the scale factor for the radiation dominated era

$$a(t) = \left(\frac{H_0}{\beta}\right)^{\frac{1}{\beta}} t^{\frac{1}{\beta}}, \tag{42}$$

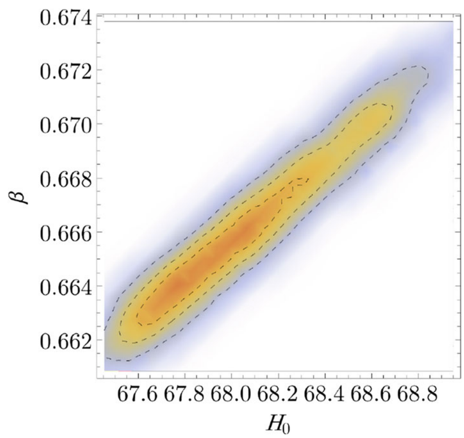


Fig. 5 Constraints on the parameters β and H_0 on the basis of Planck data

7.1.2 Case 2: Matter Dominated Era ($w = 0$)

Setting $w = 0$ in (41) and integrating gives the scale factor for the matter dominated era

$$a(t) = \left(\frac{H_0}{2\beta}\right)^{\frac{1}{2\beta}} t^{\frac{1}{2\beta}}, \tag{43}$$

7.1.3 Case 3: Dark Energy Dominated Era ($w = -1$)

Setting $w = -1$ in (41) and integrating gives the scale factor for the dark energy dominated era

$$a(t) = a_0 \exp(H_0 t), \tag{44}$$

The study of the temporal evolution of the universe has shown that the scale factor displays power-law behaviour in the early phases of radiation and matter dominance. The scale factor, on the other hand, follows an exponential course in the latter dark energy-dominated epoch. Furthermore, as shown by (42) and (43), the model parameter β has been found to have a noticeable impact on the temporal evolution of the Universe in the radiation and matter-dominated phases. Equation (44) shows that this dependence is minimal throughout the late-time era. It is clear that the (42) and (43) yield the scale factors of $a(t) \propto t^{2/3}$ and $a(t) \propto t^{1/2}$, respectively, for $\beta = \frac{3}{2}$ and $\beta = 1$. These values correspond to the scale factors of the radiation and matter dominated eras in the standard ΛCDM model. As such, we infer that our model mimics the ΛCDM model for these particular values of β.

8 Stability Analysis

The stability analysis of the model is performed on the basis of the speed of sound squared c_s^2 [63, 64]. A non-negative value of c_s^2 (real value of c_s) indicates a stable propagation mode for density perturbation and a negative value of c_s^2 (imaginary value of c_s) indicates a unstable propagation of density perturbation. The allowed range for squared sound speed for a physically viable model is $0 < c_s^2 < c^2$. In natural unit system ($c = 1$), the allowed range is given by $0 < c_s^2 < 1$. This means that sound speed for a viable model should not exceed the speed of light.

In the case of the power law model we obtain the value of c_s^2 to be

$$c_s^2 = \frac{3m}{2\beta - 3m}, \tag{45}$$

which is found to be independent of time. For stable propagation of density perturbation for this case we require $\beta > 3m$ in order to maintain values in the range $0 < c_s^2 < 1$.

For the case of hybrid model, the speed of sound squared is found to be

$$c_s^2 = \frac{2\beta m - 3(m + nt)^2 + 2nt}{3(m + nt)^2} \tag{46}$$

The time evolution of c_s^2 is shown in Fig. 6. It is seen from figure that the speed of sound squared evolves with positive values at all times and its value is seen to be within the range $0 < c_s^2 < 1$ which implies the model is plausible. Thus the hybrid law is stable under density perturbation.

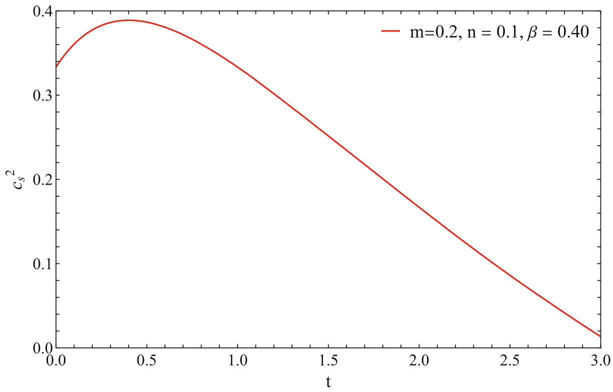


Fig. 6 The time evolution of speed of sound squared is shown for the hybrid model

9 Om Diagnostics

The Om diagnostic [65, 66] is a important geometrical technique used to differentiate different dark energy models. For a constant slope of the $Om(z)$ vs z graph, the model in consideration is ΛCDM ($w = -1$) and positive slope and negative slope represents a phantom dark energy model ($w < -1$) and quintessence dark energy model ($w > -1$) respectively. The expression for $Om(z)$ is given by

$$Om(z) = \frac{\left(\frac{H(z)}{H_0}\right)^2 - 1}{(1+z)^3 - 1}, \tag{47}$$

By using (40) in (47) we obtain

$$Om(z) = \frac{(z+1)^{3\beta(w+1)} - 1}{(z+1)^3 - 1}, \tag{48}$$

The $Om(z)$ vs z relation is plotted in the Fig. 7. From Fig. 7 we see that the slope of the graph is negative which indicates the behaviour of the model is of quintessence type.

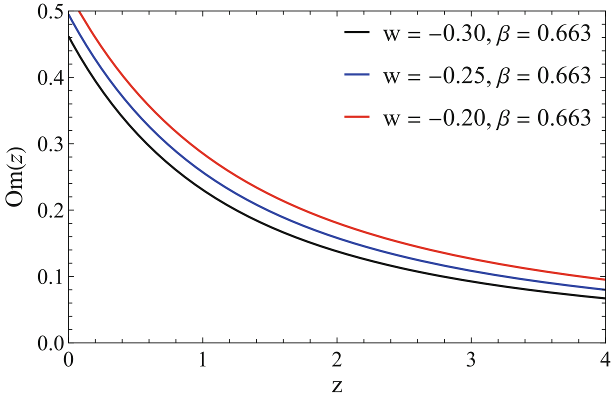


Fig. 7 The $Om(z)$ vs z relation is shown for three different values of w

10 Conclusion

Scalar field approaches are useful ways and are widely utilized in particle physics and modern cosmology. The present work is aimed at investigating the effectiveness of two expansion laws, namely power law and hybrid law, in explaining the evolutionary dynamics of the Universe through the inclusion of a scalar field within $f(Q)$ gravity. Ad-hoc forms of power-law and hybrid law scale factors were assumed in this work and the time evolution of those is studied in terms of the scalar field. Unlike many other approaches by different authors, where perfect fluid and scalar field are assumed to have independent contributions, this study considers a self-interacting scalar field, accompanied by a self-interaction potential, as the primary matter source. It is believed that the scalar field alone can account for the evolutionary dynamics of the Universe. Under the framework of $f(Q)$ gravity, the time evolution of the physical parameters of the models is studied for both power law and hybrid laws. Our results indicate that the models satisfy the validity test, and the potentials obtained in our models are suitable for slow-roll dynamics to account for the late time accelerated expansion of the Universe.

Furthermore, a top-down approach is followed to analyze the model, starting with the Friedmann equations in the $f(Q)$ model. The exact solutions for the Raychaudhuri equations are obtained, and it is found that the model can reproduce the three phases of evolution of the Universe i.e the radiation dominated phase, the matter dominated phase and the late time dark-energy phase. Furthermore, it was observed that the ΛCDM model can be replicated for certain values of the model parameter, β.

To examine the stability of our models, we verified the squared sound speed of density perturbation of our model, which we found to be stable for proper constrained values of the model parameter β. Finally, to infer the nature of the model as an accelerating cosmological model we performed the Om-diagnostics to conclude that the model behaves like a quintessence dark energy model.

As a future perspective, it will be interesting to investigate the consequences of the model with the inclusion of additional degrees of freedom like trace of energy momentum tensor, bulk viscous fluid etc. Moreover, it may also be interesting to take into account of the degrees of freedom arising from the modification in the non-metricity sector and how they can be influential in contrast to the scalar field degree of freedom and in what respect.

Author Contributions M.M.G contributed to the idea, implementation, typesetting and carried out the work in Sections 7, 8, 9. R.M. carried out the work contained in Section 5.2 and and inserted the references. S.T. carried out the work contained in Section 6. S.S.A carried out the calculations contained in the Section 5.1. S.P. prepared the figures contained in Section 5.1. K.B. contributed to the implementation of the MCMC sampling in the work and proof-reading of the manuscript. R.C contributed to the proof-reading of the manuscript. A.D. contributed to the proof-reading and a few reference insertions in the manuscript.

Data Availability Data sharing is not applicable to this article.

Declarations

Competing interests The authors declare no competing interests.

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