NCERT Maths 10.5.3 Q14

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Question: Find the sum of odd numbers in 1 to 2001.
Solution:

tables/table.tex

TABLE 0
GIVEN PARAMETERS

Last term of the given sequence is 2001.

$$x(n) = (2n + 1)u(n) \tag{1}$$
$$\therefore (2n + 1) = 2001 \tag{2}$$
$$\implies n = 1000 \tag{3}$$

Applying Z transform From equation (**??**):

$$X(z) = \frac{1 + z^{-1}}{(1 - z^{-1})^2}, \quad |z| > |1| \tag{4}$$

For AP, the sum of first n+1 terms can be written as

$$y(n) = x(n) * u(n) \tag{5}$$
$$Y(z) = X(z)U(z) \tag{6}$$
$$= \frac{1}{(1 - z^{-1})^2} + \frac{2z^{-1}}{(1 - z^{-1})^3}, \quad |z| > |1| \tag{7}$$

Using contour integration to find inverse Z transform

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z) z^{n-1} dz \tag{8}$$
$$y(1000) = \frac{1}{2\pi j} \int Y(z) z^{999} dz \tag{9}$$
$$= \frac{1}{2\pi j} \int \frac{1.z^{1001}}{(z-1)^2} dz - \frac{1}{2\pi j} \int \frac{2.z^{1001}}{(z-1)^3} dz \tag{10}$$
$$\because R = \frac{1}{(m-1)!} \lim_{z \to a} \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \tag{11}$$

For R1 , $m = 2$, where m corresponds to number of repeated poles

$$R_1 = \frac{1}{1!} \lim_{z \to 1} \frac{d}{dz} \left((z-1)^2 . \frac{1.z^{1001}}{(z-1)^2}\right) \tag{12}$$
$$= 1001 \tag{13}$$

For R2 , $m = 3$

$$R_2 = \frac{1}{2!} \lim_{z \to 1} \frac{d^2}{dz^2} \left((z-1)^3 . \frac{2.z^{1001}}{(z-1)^3}\right) \tag{14}$$
$$= 1001000 \tag{15}$$
$$\implies y(1000) = R_1 + R_2 \tag{16}$$
$$= 1002001 \tag{17}$$

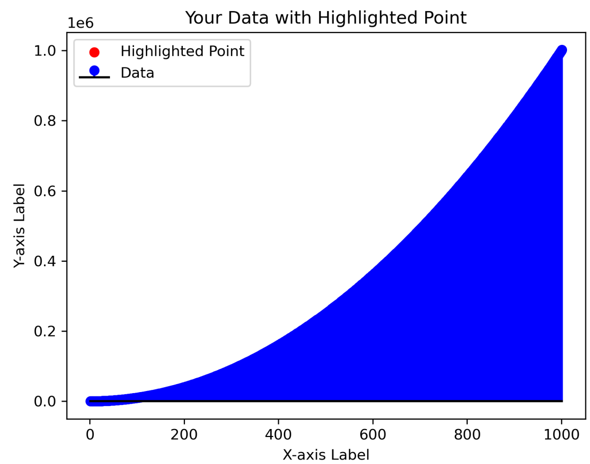

Fig. 0. Combination of stem and scatter plot of y(n)