

# NCERT Maths 10.5.3 Q14

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Symbol	Value	Description
$x(0)$	1	first term of AP
$d$	2	common difference
$x(n)$	$(1 + 2n)u(n)$	$n$ -th term of AP

TABLE 0  
GIVEN PARAMETERS

**Question:** Find the sum of odd numbers in 1 to 2001.

**Solution:**

Last term of the given sequence is 2001.

$$x(n) = (2n + 1)u(n) \quad (1)$$

$$\therefore (2n + 1) = 2001 \quad (2)$$

$$\Rightarrow n = 1000 \quad (3)$$

Applying Z transform From equation (??):

$$X(z) = \frac{1 + z^{-1}}{(1 - z^{-1})^2}, \quad |z| > |1| \quad (4)$$

For AP, the sum of first  $n+1$  terms can be written as

$$y(n) = x(n) * u(n) \quad (5)$$

$$Y(z) = X(z)U(z) \quad (6)$$

$$= \frac{1}{(1 - z^{-1})^2} + \frac{2z^{-1}}{(1 - z^{-1})^3}, \quad |z| > |1| \quad (7)$$

Using contour integration to find inverse Z transform

$$y(n) = \frac{1}{2\pi j} \oint_C Y(z)z^{n-1} dz \quad (8)$$

$$y(1000) = \frac{1}{2\pi j} \int Y(z)z^{999} dz \quad (9)$$

$$= \frac{1}{2\pi j} \int \frac{1 \cdot z^{1001}}{(z-1)^2} dz - \frac{1}{2\pi j} \int \frac{2 \cdot z^{1001}}{(z-1)^3} dz \quad (10)$$

$$\therefore R = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z)) \quad (11)$$

For  $R_1$ ,  $m = 2$ , where  $m$  corresponds to number of repeated poles

$$R_1 = \frac{1}{1!} \lim_{z \rightarrow 1} \frac{d}{dz} \left( (z-1)^2 \cdot \frac{1 \cdot z^{1001}}{(z-1)^2} \right) \quad (12)$$

$$= 1001 \quad (13)$$

For  $R_2$ ,  $m = 3$

$$R_2 = \frac{1}{2!} \lim_{z \rightarrow 1} \frac{d^2}{dz^2} \left( (z-1)^3 \cdot \frac{2 \cdot z^{1001}}{(z-1)^3} \right) \quad (14)$$

$$= 1001000 \quad (15)$$

$$\Rightarrow y(1000) = R_1 + R_2 \quad (16)$$

$$= 1002001 \quad (17)$$

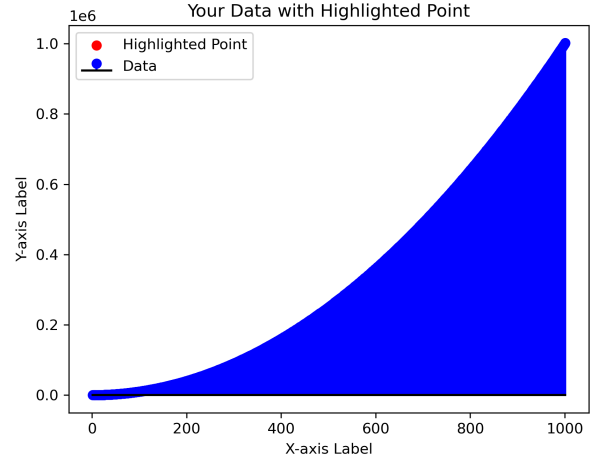


Fig. 0. Combination of stem and scatter plot of  $y(n)$