Gate EE - 18

EE23BTECH11216 - P.kalyan

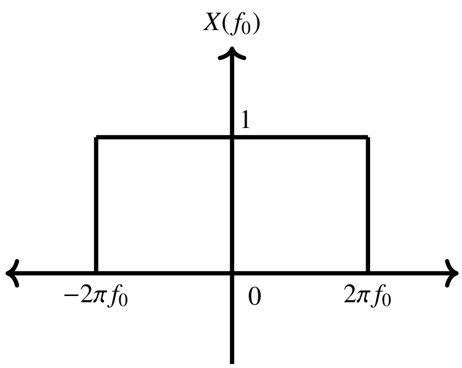
Question

The Fourier transform x(ω) of the signal x(t) is given by

$$X(\omega) = \begin{cases} 1, \text{for } |\omega| < \omega_0 \\ 0, \text{for } |\omega| > \omega_0 \end{cases}$$

(A) x(t) tends to be an impulse as $W_0 \to \infty$.
(B) x(0) decreases as W_0 increases.
(C) At t = $\frac{\pi}{2W_0}$, x(t) = -$\frac{1}{\pi}$.
(D) At t = $\frac{\pi}{2W_0}$, x(t) = $\frac{1}{\pi}$. (GATE EE 2023)

solution



By taking inverse Fourier transform,

$$x(t) = \frac{\sin(t)}{\pi t} \tag{1}$$

$$x\left(\frac{\pi}{2(2\pi f_0)}\right) = \frac{2(2\pi f_0)}{\pi^2} \tag{2}$$

So, option (C) and (D) are wrong.

$$x(0) = \lim_{t \to 0} \frac{\sin(2\pi f_0)t}{\pi t} = \frac{2\pi f_0}{\pi} \tag{3}$$

So, $x(0) \propto f_0 \Rightarrow$ Option (B) is wrong.

When $f_0 \to \infty$, $X(f_0)$ will be a D.C signal and inverse Fourier transform of a D.C signal will be impulse signal

So, option (A) is correct

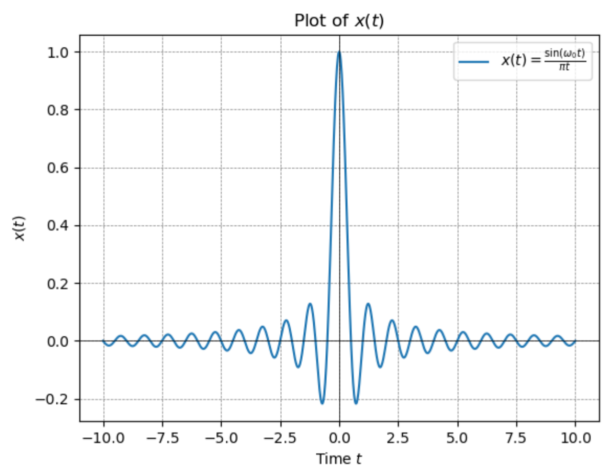


Fig. 0. plot of X(t)