Unveiling the Tensor Charge of the Nucleon at Jefferson Lab

Kalyan Allada*
MIT

Jian-Ping Chen†
Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

Zhong-Bo Kang‡
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

Nobuo Sato§
Theory Center, Jefferson Lab, 12000 Jefferson Avenue, Newport News, VA 23606, USA

Alexei Prokudin¶
Science Division, Penn State University Berks, Reading, Pennsylvania 19610, USA

Peng Sun**
MSU

Zhihong Ye††
Physics Division, Medium Energy Group, Argonne National Lab
9700 S. Cass Ave, Physics Division Bldg 203, Lemont, IL 60439

Feng Yuan‡‡
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

We present detailed estimates of how well future Jefferson Lab 12 experiments, in particular CLAS and SOLID, could constrain transversity ditribution and tensor charge of the nucleon. Our study is performed based on the global QCD extraction of transversity from available experimental data and on a robust Bayesian re-weighting technique that allows to use realistic pseudo-data generated for future measurements of Jefferson Lab.

PACS numbers: 12.38.Bx, 12.39.St, 13.85.Hd, 13.88.+e

I. INTRODUCTION

Nucleon tensor charge is one of the fundamental properties of the proton and its determination is among the main goals of existing and future experimental facilities [1–6]. It also plays an important role in constraining the nuclear physics aspects for probing new physics beyond the standard model, and has been an active subject from lattice QCD calculations [7, 8]. In terms of the partonic structure of the nucleon, the tensor charge, δq, for a particular quark type q is constructed from the quark transversity distribution, $h_1(x, Q^2)$, one of the three leading-twist quark distributions that describe completely spin-1/2 nucleon [1–3, 5]:

$$\delta q\left(Q^2\right) \equiv \int_0^1 dx\ \left(h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2)\right)\ . \qquad (1)$$

However, the experimental exploration of the quark transversity distribution in high energy scattering is difficult because of its odd chirality [2].

It is extremely important to extend the experimental study of the quark transversity distribution to both large and small x_B to constrain the total tensor charge contributions. The Jefferson Lab 12 GeV program [4] is going to explore the region of relatively high-x dominated by valence quarks. The planned Electron Ion Collider [5, 6, 9] is going to extend the range of explored x to lower values and thus provide a possibility to study anti-quark transversity.

In this paper we will present a detailed analysis of impact of future Jefferson Lab 12 data and in particular the data from CLAS and SOLID to determination of tensor charge and transversity distributions.

We will base our studies on a QCD global fit of the available Semi-Inclusive Deep Inelastic Scattering (SIDIS) data and e^+e^- annihilation into hadron pairs

*Electronic address: kalyan@jlab.org
†Electronic address: chen@jlab.org
‡Electronic address: zkang@lanl.gov
§Electronic address: nsato@jlab.org
¶Electronic address: prokudin@jlab.org
**Electronic address: psun@msu.edu
††Electronic address: sassot@df.uba.ar
‡‡Electronic address: fyuan@lbl.gov

performed in Ref. [10]. Using the best fit of transversity distributions of Ref. [10] we will generate pseu-data for SOLID and CLAS experiments and estimate experimental uncertainties of future data assuming transversity distributions of Ref. [10].

The current available experimental data suggest that anti-quark transversity is very small and does not constrain anti-quark transversity. In this study we will assume that anti-quark transversity are negligible and does not contribute to tensor charge. The analysis of impact of future data including anti-quark transversity and data from future Electron-Ion Collider will be published elsewhere.

In order to analyze the impact of future data we will utilize Bayesian reweighting technique method [11] that will allow us to reliably estimate impact of Jefferson Lab 12 data on both quark transversity and its contribution to tensor charge.

This study will also provide information on contribution of tensor charge from kinematical region of Jefferson Lab 12 and will serve as guidance to planning of future experiments.

II. PRESENT STATUS OF EXTRACTION OF TRANSVERSITY FROM EXPERIMENTAL DATA

An important channel to investigate the quark transversity distribution is to measure the Collins azimuthal spin asymmetries in semi-inclusive hadron production in deep inelastic scattering (SIDIS) [12]. Measurements have been made by the HERMES Collaboration [13, 14], the COMPASS Colaboration [15], and JLab HALL A [16] experiments. However, the extraction of the quark transversity distributions requires the knowledge of the Collins fragmentation functions [12], which are different from the usual unpolarized fragmentation functions. It was further suggested to measure the Collins fragmentation functions from the azimuthal angular asymmetries of two back-to-back hadron productions in e^+e^- annihilations [17]. Recently both BELLE and *BABAR* Collaborations have studied these asymmetries at the B-factories at center of mass energy around $\sqrt{s} \simeq 10.6$ GeV [18–20]. Analogous data [21], newly released by the BESIII collaboration, on e^+e^- annihilations into pion pair at the much lower $\sqrt{s} \simeq 3.6$ GeV, offer the ideal ground for a study on the sensitivity of these azimuthal correlations on TMD evolution effects.

The data from two processes, SIDIS and e^+e^-, can be combined in global QCD analysis due to the universality of the Collins fragmentation functions [22]. The effort to extract the transversity distributions and Collins fragmentation functions has been carried out by the Torino-Cagliari-JLab group extensively in the last few years [23–26]. Transversity coupled to the so-called dihadron interference fragmentation functions is employed to study transversity in its collinear version in Ref. [27].

These results have demonstrated the powerful capability of the Collins asymmetry measurements in constraining the quark transversity distributions and hence the nucleon tensor charge in high energy scattering experiments.

The first extraction of the transversity distribution and Collins fragmentation functions with TMD evolution was performed in Ref. [10]. It was demonstrated that the TMD evolution can describe the experimental data and constrain the nucleon tensor charge with improved theoretical accuracy. To achieve that, the most recent developments from both theory and phenomenology sides [28–37] were used, and the TMD evolution at NLL order within the Collins-Soper-Sterman (CSS) [38, 39] formalism was applied to the data.

As our study is dedicated to Jefferson Lab 12 that will utilize SIDIS for experimental studies, let us discuss the origin of Collins asymmetries in SIDIS.

Collins asymmetries in SIDIS are generated by the convolution of the transversity function h_1 and the Collins TMD FF $H_1^\perp$.

The relevant contributions to the SIDIS cross-sections are

$$\frac{d^5\sigma(S_\perp)}{dx_B dy dz_h d^2P_{h\perp}} = \sigma_0(x_B, y, Q^2)\Big[F_{UU}+ \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + ...\Big]\,. \quad (2)$$

The polarized structure function $F_{UT}^{\sin(\phi_h+\phi_s)}$ contains the convolution of transversity with the Collins function, $h_1\otimes H_1^\perp$.

Experimentally measured Collins asymmetry is then

$$A_{UT}^{\sin(\phi_h+\phi_s)}(x_B, y, z_h, P_{h\perp}) = \frac{2(1-y)}{1+(1-y)^2}\,\frac{F_{UT}^{\sin(\phi_h+\phi_s)}}{F_{UU}} \quad (3)$$

Applying the TMD evolution, F_{UU} and $F_{UT}^{\sin(\phi_h+\phi_s)}$ can be written as [31, 38–40]

$$\begin{aligned} F_{UU} &= \frac{1}{z_h^2}\int \frac{db\, b}{2\pi} J_0\Big(\frac{P_{h\perp}b}{z_h}\Big)\, e^{-S_{\rm PT}(Q,b_*)-S_{\rm NP}^{\rm (SIDIS)}(Q,b)} \\ &\times\ C_{q\leftarrow i}\otimes f_1^i(x_B,\mu_b)\ \hat{C}_{j\leftarrow q}^{\rm (SIDIS)}\otimes \hat{D}_{h/j}(z_h,\mu_b), \end{aligned} \quad (4)$$

$$\begin{aligned} F_{UT}^{\sin(\phi_h+\phi_s)} &= -\frac{1}{2z_h^3}\int \frac{db\, b^2}{2\pi} J_1\Big(\frac{P_{h\perp}b}{z_h}\Big)\, e^{-S_{\rm PT}(Q,b_*)-S_{\rm NP\,coll}^{\rm (SIDIS)}(Q,b)} \\ &\times\ \delta C_{q\leftarrow i}\otimes h_1^i(x_B,\mu_b)\ \delta\hat{C}_{j\leftarrow q}^{\rm (SIDIS)}\otimes \hat{H}_{h/j}^{(3)}(z_h,\mu_b), \end{aligned} \quad (5)$$

where b is the Fourier conjugate variable to the measured final hadron momentum $P_{h\perp}$, J_1 is the Bessel function, $\mu_b = c_0/b_*$ with $c_0 \simeq 1.12$, and the symbol $\otimes$ represents the usual convolution in momentum fractions. The sum over quark flavors q weighted with quark charge, $\sum_q e_q^2$, and the sum over $i, j = q, \bar{q}, g$, are implicit in all formulas for the structure functions. C, $\hat{C}$ and δC, $\delta\hat{C}$

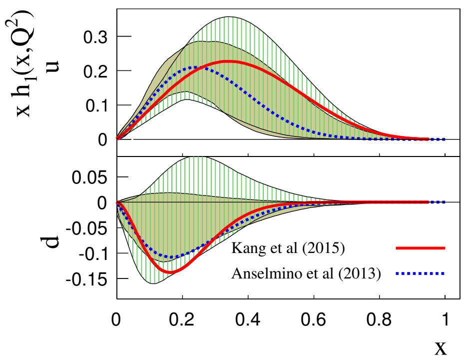

FIG. 1: [color online] Transversity distribution for up and down quarks comparison of extraction in Ref. [10] and [25]. The band corresponds to the uncertainty of the extraction.

are the coefficient functions for the unpolarized distribution and fragmentation functions, and for transversity and Collins FF, that can be calculated perturbatively. The usual b_*-prescription that allows for smooth transition from perturbative to non perturbative physics was used in Ref. [10]. $S_{\rm NP}^{\rm (SIDIS)}$ and $S_{\rm NP\,coll}^{\rm (SIDIS)}$ are non perturbative factors contain information on the initial conditions of evolution:

$$S_{\rm NP}^{\rm (SIDIS)} = g_2 \ln\left(\frac{b}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right) + \left(g_q + \frac{g_h}{z_h^2}\right) b^2 \ , \quad (6)$$

$$S_{\rm NP\,collins}^{\rm SIDIS} = g_2 \ln\left(\frac{b}{b_*}\right) \ln\left(\frac{Q}{Q_0}\right) + \left(g_q + \frac{g_h - g_c}{z_h^2}\right) b^2 \ , \quad (7)$$

where $Q_0^2 = 2.4$ GeV^2, for the spin-averaged contribution. In the above parameterization, the parameters $g_q = g_1/2 = 0.106$, $g_2 = 0.84$, $g_h = 0.042$ (GeV^2) have been determined from the analysis of SIDIS and Drell-Yan processes in Ref. [37].

The existing experimental data does not allow to determine precisely shapes of all polarised distributions in coordinate space, the parameter g_c allowed for the Collins fragmentation function to modify its shape with respect to unpolarised fragmentation distributions.

The Collins fragmentation function $H_{1\,h/j}^{\perp}(z_h, p_\perp)$ [12] at small values of b can be reated to the so called twist-three fragmentation function $\hat{H}_{h/q}^{(3)}(z_h)$ [29] via Operator Product Expansion (OPE).

Three important ingredients have to be included to achieve the NLL formalism for the above structure functions and asymmetries. First of all, the perturbative Sudakov form factor [41],

$$S_{\rm PT}(Q, b_*) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln \frac{Q^2}{\mu^2} + B\right] \ , \quad (8)$$

with perturbative coefficients $A^{(1,2)} \sim \alpha_s^{(1,2)}$ and $B^{(1)} \sim \alpha_s^1$ [41, 42]. Then, the scale evolutions of the quark transversity distribution and of the Collins fragmentation functions up to the scale of μ_b.

The global fit of SIDIS and e^+e^- was performed in Ref. [10]. The quark transversity distributions was parametrized as as

$$h_1^q(x, Q_0) = N_q^h x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{a_q + b_q}}{a_q^{a_q} b_q^{b_q}} \times \frac{1}{2} \left(f_1^q(x, Q_0) + g_1^q(x, Q_0)\right) \ , \quad (9)$$

at the initial scale Q_0, for up and down quarks $q = u, d$, respectively, where f_1^q are the unpolarized CT10 NLO quark distributions [43] and g_1^q are the NLO DSSV quark helicity distributions [44]. The available SIDIS data does not allow for extraction of anti-quark transversity distributions and anti-quark transversity was assumed to be zero, $h_1^{\bar q} = 0$.

Twist-3 Collins fragmentation functions were parametrized in terms of the unpolarized fragmentation functions,

$$\hat{H}_{fav}^{(3)}(z, Q_0) = N_u^c z^{\alpha_u} (1-z)^{\beta_u} D_{\pi^+/u}(z, Q_0) \ , \quad (10)$$

$$\hat{H}_{unf}^{(3)}(z, Q_0) = N_d^c z^{\alpha_d} (1-z)^{\beta_d} D_{\pi^+/d}(z, Q_0) \ , \quad (11)$$

which correspond to the favored and unfavored Collins fragmentation functions, respectively. The newest NLO extraction of fragmentation functions [45] was used for unpolarised FF, D.

Therefore, Ref. [10] introduced total of 13 parameters in the global fit: N_u^h, N_d^h, a_u, a_d, b_u, b_d, N_u^c, N_d^c, α_u, α_d, β_d, β_u, g_c (GeV^2).

The global fit of SIDIS and e^+e^- data resulted in the total $\chi^2 = 218.407$, $n_{d.o.f.} = 249$, and $\chi^2/n_{d.o.f} = 0.88$. The description of the data was equally good for SIDIS and e^+e^-: $\chi^2_{SIDIS}/n_{SIDIS} = 0.93$, $\chi^2_{e^+e^-}/n_{e^+e^-} = 0.72$.

The resulting parameters of Ref. [10] after minimization procedure are presented in Table I.

$N_u^h = 0.85 \pm 0.09$	$a_u = 0.69 \pm 0.04$	$b_u = 0.05 \pm 0.04$
$N_d^h = -1.0 \pm 0.13$	$a_d = 1.79 \pm 0.32$	$b_d = 7.00 \pm 2.65$
$N_u^c = -0.262 \pm 0.025$	$\alpha_u = 1.69 \pm 0.01$	$\beta_u = 0.00 \pm 0.54$
$N_d^c = 0.195 \pm 0.007$	$\alpha_d = 0.32 \pm 0.04$	$\beta_d = 0.00 \pm 0.79$
$g_c = 0.0236 \pm 0.0007$ (GeV^2)		
$\chi^2_{min} = 218.407$	$\chi^2_{min}/n.d.o.f = 0.88$	

TABLE I: Fitted parameters of the transversity quark distributions for u and d and Collins fragmentation functions.

Fig. 1 shows transversity distributions for u and d quarks results obtained in Ref. [10] and compared to results of Ref. [25].

Since the experimental data has only probed the limited region $0.0065 < x_B < 0.35$, the following partial contribution to the tensor charge was defined

$$\delta q^{[x_{\rm min}, x_{\rm max}]}(Q^2) \equiv \int_{x_{\rm min}}^{x_{\rm max}} dx\, h_1^q(x, Q^2) \ . \quad (12)$$

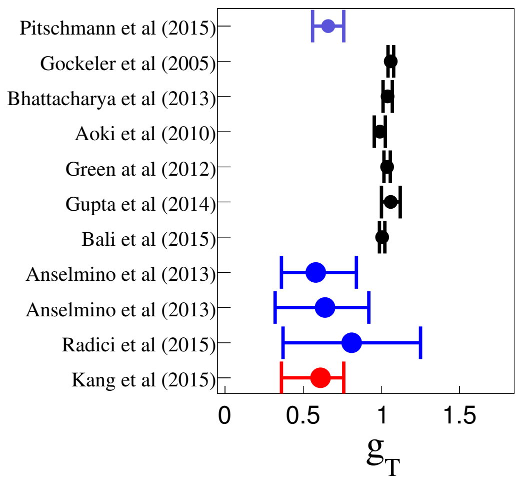

FIG. 2: [color online] Comparison of the isovector nucleon tensor charge g_T from this paper at 68% C.L. (Kang et al 2015) at $Q^2 = 10$ GeV^2 and result from Ref. [27] (Radici et al 2015) at 68% CL and $Q^2 = 4$ GeV^2, and Ref. [25] at 95% CL standard and polynomial fit (Anselmino et al 2013) at $Q^2 = 0.8$ GeV^2. Other points are lattice computation at $Q^2 = 4$ GeV^2 of Bali et al Ref. [47], Gupta et al Ref. [48], Green et al Ref. [8], Aoki et al Ref. [49], Bhattacharya et al ref. [50], Gockeler et al Ref. [51]. Pitschmann et al is DSE calculation at $Q^2 = 4$ GeV^2 Ref. [52].

and the following partial tensor charges were calculated [46]

$$\delta u^{[0.0065,0.35]} = +0.30^{+0.08}_{-0.12}\,, \tag{13}$$
$$\delta d^{[0.0065,0.35]} = -0.20^{+0.28}_{-0.11}\,, \tag{14}$$

at 90% C.L. at $Q^2 = 10$ GeV^2.

The tensor charge calculated over the whole kinematical region $\delta q^{[0,1]}$ is:

$$\delta u^{[0,1]} = +0.39^{+0.16}_{-0.20}\,, \tag{15}$$
$$\delta d^{[0,1]} = -0.22^{+0.31}_{-0.10}\,, \tag{16}$$

at 90% C.L.

One can see that the kinematical region not covered by experimental data can potentially give $\sim 25\%$ contribution to the tensor charge.

Apart from that the assumption that anti-quark transversity is exactly equal to zer may not be true in future and maximal contribution from see quarks can be .... for $\bar{u}$ and $\bar{d}$ quarks respectively.

The isoscalar nucleon tensor charge $g_T^0 = \delta u + \delta d$ is

$$g_T^0 = +0.17^{+0.47}_{-0.30}\,, \tag{17}$$

at 90% C.L. at $Q^2 = 10$ GeV^2.

The isoscalar tensor charge is presented in Fig. 2. One can see that lattice QCD calculations have much better precision compared to extractions from experimental data.

Future Jefferson Lab 12 data is going to allow for much more precise extraction of tensor charge that will compete with ab-initio calculations.

III. BAYESIAN RE-WEIGHTING TECHNIQUE

Nobuo, could you write description of the re-weighting procedure, please? Bayes theorem allows to incorporate information from new data by applying re-weighting of probability densities for model parameters. The details of application of re-weighting are explained in Ref. [11].

Probability density function for model parameters $\boldsymbol{\alpha}$, $\mathcal{P}(\boldsymbol{\alpha})$, is going to be modified in presence of new data and the Bayes theorem states that

$$\mathcal{P}(\boldsymbol{\alpha}|D) = \frac{\mathcal{P}(D|\boldsymbol{\alpha})}{\mathcal{P}(D)}\mathcal{P}(\boldsymbol{\alpha}), \tag{18}$$

where $\mathcal{P}(\boldsymbol{\alpha}|D)$ is the so-called *posterior* density, is the updated pdf from the *prior* density $\mathcal{P}(\boldsymbol{\alpha})$.

The quantity $\mathcal{P}(D|\boldsymbol{\alpha})$ called the *likelihood* function, represents the conditional probability for a data set D given the parameters $\boldsymbol{\alpha}$ of the model. The quantity $\mathcal{P}(D)$ ensures the normalization of the posterior density to unity.

For a particular observable $\mathcal{O}$ one can write the expectation value with the new data as,

$$\begin{aligned}\mathrm{E}[\mathcal{O}] &= \int d^n\alpha \mathcal{P}(\boldsymbol{\alpha}|D)\mathcal{O}(\boldsymbol{\alpha}) \\ &= \int d^n\alpha \frac{\mathcal{P}(D|\boldsymbol{\alpha})}{\mathcal{P}(D)}\mathcal{P}(\boldsymbol{\alpha})\mathcal{O}(\boldsymbol{\alpha}) \\ &= \frac{1}{N}\sum_k w_k \mathcal{O}(\boldsymbol{\alpha}_k).\end{aligned} \tag{19}$$

In the last line a Monte Carlo approximation of the integral is used. Similarly, the variance is of observable $\mathcal{O}$ is given by

$$\mathrm{Var}[\mathcal{O}] = \frac{1}{N}\sum_k w_k(\mathcal{O}(\boldsymbol{\alpha}_k) - \mathrm{E}[\mathcal{O}])^2. \tag{20}$$

The quantities $\{w_k\}$ are called *weights* and are proportional to $\mathcal{P}(D|\boldsymbol{\alpha}_k)$. Their normalization is fixed by demanding $\mathrm{E}[1] = 1$, that is, $\sum_k w_k = N$.

The reweighting procedure depends on the form assumed for the likelihood function. We use χ^2 minimization in the fits and thus weights have the following form:

$$w_k \propto \exp\left(-\frac{1}{2}\chi^2\right), \tag{21}$$

IV. SIMULATED DATA FOR JEFFERSON LAB

Zhihong, Kalyan, could you write description of CLAS, SOLID and how data were generated, please?

TABLE II: Pseudo-data generated and kinematical limits of SOLID.

FIG. 3: [color online] Kinematical plane with Jefferson Lab data.

V. TENSOR CHARGE AND TRANSVERSITY FROM CLAS

VI. TENSOR CHARGE AND TRANSVERSITY FROM SOLID

VII. SUMMARY AND CONCLUSIONS

Acknowledgments

We are grateful to This work was partially supported by the U.S. Department of Energy under Contract No. .

[1] J. P. Ralston and D. E. Soper, Nucl. Phys. **B152**, 109 (1979).
[2] R. L. Jaffe and X. Ji, Phys. Rev. Lett. **67**, 552 (1991).
[3] V. Barone, A. Drago, and P. G. Ratcliffe, Phys. Rept. **359**, 1 (2002), hep-ph/0104283.
[4] J. Dudek, R. Ent, R. Essig, K. Kumar, C. Meyer, et al., Eur.Phys.J. **A48**, 187 (2012), 1208.1244.
[5] D. Boer, M. Diehl, R. Milner, R. Venugopalan, W. Vogelsang, et al. (2011), 1108.1713.
[6] A. Accardi, J. Albacete, M. Anselmino, N. Armesto, E. Aschenauer, et al. (2012), 1212.1701.
[7] T. Bhattacharya, V. Cirigliano, S. D. Cohen, A. Filipuzzi, M. Gonzalez-Alonso, et al., Phys.Rev. **D85**, 054512 (2012), 1110.6448.
[8] J. Green, J. Negele, A. Pochinsky, S. Syritsyn, M. Engelhardt, et al., Phys.Rev. **D86**, 114509 (2012), 1206.4527.
[9] E.-C. Aschenauer, I. Balitsky, L. Bland, S. J. Brodsky, M. Burkardt, et al. (2014), 1410.8831.
[10] Z.-B. Kang, A. Prokudin, P. Sun, and F. Yuan (2015), 1505.05589.
[11] N. Sato, J. F. Owens, and H. Prosper, Phys. Rev. **D89**, 114020 (2014), 1310.1089.
[12] J. C. Collins, Nucl.Phys. **B396**, 161 (1993), hep-ph/9208213.
[13] A. Airapetian et al. (HERMES), Phys. Rev. Lett. **94**, 012002 (2005), hep-ex/0408013.
[14] A. Airapetian et al. (HERMES), Phys. Lett. **B693**, 11 (2010), 1006.4221.
[15] C. Adolph et al. (COMPASS Collaboration), Phys.Lett. **B717**, 376 (2012), 1205.5121.
[16] X. Qian et al. (Jefferson Lab Hall A Collaboration), Phys.Rev.Lett. **107**, 072003 (2011), 1106.0363.
[17] D. Boer, R. Jakob, and P. J. Mulders, Nucl. Phys. **B504**, 345 (1997), hep-ph/9702281.
[18] K. Abe et al. (Belle), Phys. Rev. Lett. **96**, 232002 (2006), hep-ex/0507063.
[19] R. Seidl et al. (Belle), Phys. Rev. **D78**, 032011 (2008), 0805.2975.
[20] I. Garzia (BaBar), PoS **ICHEP2012**, 272 (2013), 1211.5293.
[21] M. Ablikim et al. (BESIII Collaboration) (2015), 1507.06824.
[22] A. Metz, Phys. Lett. **B549**, 139 (2002), hep-ph/0209054.
[23] M. Anselmino et al., Phys. Rev. **D75**, 054032 (2007), hep-ph/0701006.
[24] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, and S. Melis, Nucl. Phys. Proc. Suppl. **191**, 98 (2009), 0812.4366.
[25] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, et al., Phys.Rev. **D87**, 094019 (2013), 1303.3822.
[26] M. Anselmino, M. Boglione, U. D'Alesio, J. O. Gonzalez Hernandez, S. Melis, F. Murgia, and A. Prokudin, Phys. Rev. **D92**, 114023 (2015), 1510.05389.
[27] M. Radici, A. Courtoy, A. Bacchetta, and M. Guagnelli (2015), 1503.03495.
[28] J. Collins, *Foundations of Perturbative QCD*, , Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology (Cambridge University Press, 2011), ISBN 9780521855334.
[29] F. Yuan and J. Zhou, Phys. Rev. Lett. **103**, 052001 (2009), 0903.4680.
[30] Z.-B. Kang, Phys.Rev. **D83**, 036006 (2011), 1012.3419.
[31] Z.-B. Kang, B.-W. Xiao, and F. Yuan, Phys.Rev.Lett. **107**, 152002 (2011), 1106.0266.
[32] M. G. Echevarra, A. Idilbi, and I. Scimemi, Phys.Lett. **B726**, 795 (2013), 1211.1947.
[33] A. Bacchetta and A. Prokudin, Nucl.Phys. **B875**, 536 (2013), 1303.2129.
[34] P. Sun and F. Yuan, Phys.Rev. **D88**, 114012 (2013), 1308.5003.
[35] M. G. Echevarria, A. Idilbi, Z.-B. Kang, and I. Vitev, Phys.Rev. **D89**, 074013 (2014), 1401.5078.
[36] M. G. Echevarria, A. Idilbi, and I. Scimemi, Phys.Rev. **D90**, 014003 (2014), 1402.0869.
[37] P. Sun, J. Isaacson, C. P. Yuan, and F. Yuan (2014), 1406.3073.
[38] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981).
[39] J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys.

B250, 199 (1985).
[40] D. Boer, Nucl. Phys. **B603**, 195 (2001), hep-ph/0102071.
[41] Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. **B744**, 59 (2006), hep-ph/0602188.
[42] P. M. Nadolsky, D. Stump, and C. Yuan, Phys.Rev. **D61**, 014003 (2000), hep-ph/9906280.
[43] H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. M. Nadolsky, et al., Phys.Rev. **D82**, 074024 (2010), 1007.2241.
[44] D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang, Phys.Rev. **D80**, 034030 (2009), 0904.3821.
[45] D. de Florian, R. Sassot, M. Epele, R. J. Hernndez-Pinto, and M. Stratmann, Phys.Rev. **D91**, 014035 (2015), 1410.6027.
[46] Z.-B. Kang, A. Prokudin, P. Sun, and F. Yuan, Phys.Rev. **D91**, 071501 (2015), 1410.4877.
[47] G. S. Bali, S. Collins, B. Glssle, M. Gckeler, J. Najjar, et al., Phys.Rev. **D91**, 054501 (2015), 1412.7336.
[48] R. Gupta, T. Bhattacharya, A. Joseph, H.-W. Lin, and B. Yoon, PoS **Lattice2014**, 152 (2014), 1501.07639.
[49] Y. Aoki, T. Blum, H.-W. Lin, S. Ohta, S. Sasaki, et al., Phys.Rev. **D82**, 014501 (2010), 1003.3387.
[50] T. Bhattacharya, S. D. Cohen, R. Gupta, A. Joseph, H.-W. Lin, et al., Phys.Rev. **D89**, 094502 (2014), 1306.5435.
[51] M. Gockeler et al. (QCDSF, UKQCD), Phys.Lett. **B627**, 113 (2005), hep-lat/0507001.
[52] M. Pitschmann, C.-Y. Seng, C. D. Roberts, and S. M. Schmidt, Phys.Rev. **D91**, 074004 (2015), 1411.2052.