Revisiting Distance Based Ranking in Decomposition Based Evolutionary Algorithms

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1) Regular Problems: Next we demonstrate the performance of DBEA-DR on 13 unconstrained benchmark problems from DTLZ [1] and WFG [2]: DTLZ1-DTLZ4 and WFG1-WFG9. The performances delivered by the proposed approach is compared in terms of HV in Table I and IGD in Table II with several recent and popular algorithms. The performances delivered by the other algorithms are obtained from [3].

Based on mean HV statistics listed in Table I, DBEA-DR performs at par with the best performing algorithm in 3 objective and 5 objective instances of DTLZ1, DTLZ2, DTLZ4, WFG2 and WFG4-WFG9 problems i.e. 12 out of 13 problems. Therefore, it is very difficult to judge the performance of an algorithm based on HV alone. Hence, IGD values are also presented below for these problems.

For IGD computation of these problems, reference sets have been obtained from the authors of [3] and results of the listed algorithms are also from the same source [3]. Based on the mean IGD statistics, DBEA-DR performs best in 3 and 5 objective instances of DTLZ1-DTLZ4, WFG5 and WFG7 i.e. 6 out of 13 problems, and only 5 objective instances of WFG4, WFG6, WFG8 and WFG9 problems. However, the proposed approach delivers at par performance with the best performing algorithm in terms of mean IGD values in the context of other instances of other problems.

2) Minus Problems: Next we demonstrate the performance of DBEA-DR on 13 unconstrained Minus-problems introduced in [3]: DTLZ1^{-1}-DTLZ4^{-1} and WFG1^{-1}-WFG9^{-1}. These problems are well suited to demonstrate the efficiency of using both $\mathbf{W}^I$ and $\mathbf{W}^N$. The detailed description of the problems and the difficulties in solving them can be found in [3].

Based on mean HV statistics listed in Table III, DBEA-DR performs best in 3 and 5 objective instances of DTLZ2^{-1}-DTLZ4^{-1}, WFG2^{-1}, WFG4^{-1}, WFG5^{-1} and WFG9^{-1} and 3 objective instances of DTLZ1^{-1}, WFG1^{-1}, WFG6^{-1}, WFG7^{-1} and WFG8^{-1}. Also the proposed approach delivers at par performance with the best performing algorithm in the context of other objective instances of other problems. It is important to note that, performance of different algorithms based on HV varies significantly based on different reference points as seen in [3]. Therefore, it is very difficult to judge the performance of an algorithm based on HV alone. Hence, IGD values are also presented below for these problems.

For IGD computation of these problems, reference sets have been obtained from the authors of [3] and results of the listed algorithms are also from the same source [3]. Based on the mean IGD statistics listed in Table IV, DBEA-DR performs best in 3 and 5 objective instances of DTLZ2^{-1}-DTLZ4^{-1} and WFG9^{-1}, and 3 objective instances of DTLZ1^{-1}, WFG1^{-1}, WFG4^{-1}-WFG8^{-1} problems. However, the proposed approach delivers at par performance with the best performing algorithm in terms of mean IGD values in the context of other instances of other problems.

REFERENCES

[1] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable test problems for evolutionary multiobjective optimization," in *Proceedings of the International Conference on Evolutionary Multiobjective Optimization*, 2005, pp. 105–145.

[2] S. Huband, L. Barone, L. While, and P. Hingston, "A scalable multi-objective test problem toolkit," in *Proceedings of the International Conference on Evolutionary Multi-criterion Optimization*. Springer, 2005, pp. 280–295.

[3] H. Ishibuchi, S. Yu, M. Hiroyuki, and N. Yusuke, "Performance of decomposition-based many-objective algorithms strongly depends on Pareto front shapes," *IEEE Transactions on Evolutionary Computation*, 2016.

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TABLE I
MEAN HV STATISTICS FOR DTLZ AND WFG PROBLEMS

Problem	M	DBEA-DR	NSGA-III	θ-DEA	MOEA/DD	MOEA/D-PBI	MOEA/D-Tch	MOEA/D-WS	MOEA/D-IPBI	NSGA-II
DTLZ1	3	1.11657	1.11508	1.11767	**1.11913**	1.11711	1.06842	0.39572	0.48149	1.07411
	5	1.56068	1.57677	1.57767	**1.57794**	1.57768	1.51186	0.50052	0.02284	0.00000
	8	NaN	2.13770	2.13788	2.13730	2.13620	2.05463	0.96246	1.44289	0.00000
	10	NaN	2.59280	2.59272	2.59260	2.59220	2.51973	1.07913	1.90272	0.00000
DTLZ2	3	0.74018	0.74336	0.74390	**0.74445**	0.74418	0.70168	0.33187	0.33100	0.69708
	5	1.29193	1.30317	1.30679	**1.30778**	1.30728	1.14598	0.61944	0.27191	0.67442
	8	NaN	1.96916	1.97785	1.97862	1.97817	1.35469	0.68315	0.54410	0.00004
	10	NaN	2.50878	2.51416	2.51509	2.51500	1.69045	0.83883	0.64925	0.00000
DTLZ3	3	**0.74358**	0.73300	0.73642	0.73944	0.73654	0.69553	0.33026	0.31397	0.69959
	5	1.26280	1.29894	1.30376	**1.30638**	1.30398	1.14475	0.60143	0.00750	0.00000
	8	NaN	1.95007	1.96849	1.97162	1.74240	1.33166	0.66684	0.29765	0.00000
	10	NaN	2.50727	2.51279	2.51445	2.50933	1.69956	0.80348	0.52362	0.00000
DTLZ4	3	0.74138	0.73221	0.71077	**0.74484**	0.48232	0.45889	0.17191	0.23377	0.70481
	5	1.28523	1.30839	1.30878	**1.30876**	1.20680	1.00426	0.42941	0.33457	1.00881
	8	NaN	1.98025	1.98078	1.98083	1.86439	1.35100	0.71296	0.53303	0.00000
	10	NaN	2.51524	2.51539	2.51532	2.43536	1.56890	0.95488	0.64498	0.00000
WFG1	3	0.50294	0.65088	0.70151	0.69393	0.67291	**0.92204**	0.73804	0.81622	0.75944
	5	0.68580	0.85608	1.14844	1.23809	1.34797	**1.51824**	1.36724	1.36241	1.03120
	8	NaN	1.36206	1.88297	1.91925	1.73875	2.05117	1.85604	1.75472	1.51083
	10	NaN	2.22078	2.38349	2.37705	1.78435	2.46470	2.27031	2.18237	2.38032
WFG2	3	1.22597	1.22359	**1.22945**	1.22193	1.11888	1.12990	1.12266	1.16687	1.20760
	5	1.57110	1.59770	**1.59708**	1.55672	1.52205	1.58417	1.42821	1.42081	1.58790
	8	NaN	2.13629	2.12442	2.04619	2.01678	2.13569	2.11651	2.11529	2.13214
	10	NaN	2.58890	2.57778	2.48332	2.45715	2.58891	2.57478	2.57367	2.58882
WFG3	3	**0.98265**	0.81929	0.81556	0.77295	0.75364	0.80041	0.48971	0.74146	0.82967
	5	**1.38769**	1.01000	1.02782	0.95386	0.89357	0.88322	0.71619	0.93099	1.06314
	8	NaN	1.21146	1.11348	1.15306	0.74674	1.27479	0.92248	1.41331	1.41857
	10	NaN	1.55771	1.55919	1.37737	0.55186	1.69917	1.13233	1.72878	1.76576
WFG4	3	0.70356	0.72867	**0.72949**	0.72031	0.68710	0.66650	0.34131	0.63483	0.67605
	5	1.19012	1.28496	**1.28736**	1.26067	1.15695	1.01300	0.71180	1.04810	1.07969
	8	NaN	1.96402	1.96426	1.83751	1.19841	1.33398	0.95883	1.45141	1.40330
	10	NaN	2.50322	2.50376	2.22383	1.43393	1.49165	1.20197	1.74551	1.70402
WFG5	3	0.67828	0.68658	**0.68706**	0.67698	0.65668	0.61681	0.27764	0.58174	0.65059
	5	1.17570	1.22187	**1.22209**	1.18965	1.11627	0.93276	0.58164	0.96542	1.06695
	8	NaN	1.84995	1.85027	1.71196	1.27483	1.18970	0.96591	1.33675	1.39529
	10	NaN	2.34640	2.34644	2.07711	1.53615	1.35553	1.18471	1.57386	1.61976
WFG6	3	0.66788	0.68696	**0.68698**	0.67923	0.65655	0.62307	0.28542	0.58469	0.64111
	5	1.13737	1.21978	**1.22284**	1.19424	1.04043	0.93460	0.55026	0.97587	1.01175
	8	NaN	1.84625	1.84330	1.69055	0.71742	1.17924	0.63171	1.21597	1.27938
	10	NaN	2.32660	2.32759	2.01837	0.82027	1.44519	0.77606	1.48368	1.59677
WFG7	3	0.72079	0.72894	**0.73099**	0.72126	0.61145	0.66659	0.33309	0.62859	0.68591
	5	1.23202	1.29190	**1.29548**	1.25983	1.07723	1.01449	0.63899	1.04794	0.97811
	8	NaN	1.97138	1.97353	1.82024	0.83439	1.30773	0.71170	1.45307	1.22911
	10	NaN	2.50754	2.50858	2.25713	0.95972	1.59993	0.97177	1.73385	1.59601
WFG8	3	0.63543	0.66560	**0.66687**	0.65741	0.62986	0.61394	0.24450	0.26792	0.61230
	5	1.07350	1.18225	**1.18354**	1.15376	0.95660	0.60364	0.46673	0.82273	0.96648
	8	NaN	1.75970	1.76647	1.70621	0.30471	1.20786	0.67808	1.24044	1.28486
	10	NaN	2.28203	2.28502	2.10729	0.27470	1.60952	0.82704	1.57781	1.69433
WFG9	3	0.66696	0.67519	**0.67978**	0.67146	0.57864	0.62177	0.25170	0.51403	0.62199
	5	1.15289	1.21058	**1.22122**	1.15493	1.02426	0.78608	0.53143	0.94420	0.92841
	8	NaN	1.80911	1.83678	1.60407	0.97800	1.23897	0.72454	1.18318	1.07824
	10	NaN	2.34332	2.36516	1.92977	1.15138	1.59168	0.86178	1.49927	1.42611

TABLE II
MEAN IGD STATISTICS FOR DTLZ AND WFG PROBLEMS

Problem	M	DBEA-DR	NSGA-III	θ-DEA	MOEA/DD	MOEA/D-PBI	MOEA/D-Tch	MOEA/D-WS	MOEA/D-IPBI	NSGA-II
DTLZ1	3	**0.04085**	0.04362	0.04170	0.04138	0.04175	0.06082	0.50173	0.42397	0.06481
	5	**0.10797**	0.11308	0.11125	0.11110	0.11128	0.22189	0.73685	6.52117	19.87954
	8	NaN	0.17984	0.17513	0.17541	0.17601	0.23603	0.72480	0.52039	75.18619
	10	NaN	0.19094	0.18527	0.18552	0.18611	0.23786	0.78417	0.49928	77.22337
DTLZ2	3	**0.05410**	0.05799	0.05804	0.05801	0.05800	0.07318	0.54279	0.54641	0.07182
	5	**0.16794**	0.19403	0.19363	0.19368	0.19368	0.32648	0.69062	0.93890	0.31393
	8	NaN	0.40062	0.39802	0.39575	0.39572	0.46026	0.94291	0.99204	1.90946
	10	NaN	0.46752	0.46462	0.46145	0.46120	0.53319	1.00370	1.05344	2.15108
DTLZ3	3	**0.05351**	0.06261	0.05908	0.05824	0.05848	0.07349	0.54419	0.54800	0.07194
	5	**0.18360**	0.19601	0.19496	0.19384	0.19400	0.32551	0.70566	40.98681	116.19480
	8	NaN	0.41225	0.40224	0.39694	0.46660	0.47438	0.94647	1.23378	348.09573
	10	NaN	0.46843	0.46545	0.46165	0.46321	0.53973	1.01331	1.12693	308.79409
DTLZ4	3	**0.05408**	0.07550	0.10791	0.05800	0.45495	0.47158	0.83789	0.71489	0.07012
	5	**0.17325**	0.19378	0.19373	0.19372	0.33507	0.45264	0.82880	0.89434	0.22875
	8	NaN	0.39672	0.39597	0.39534	0.53322	0.64479	0.95178	1.00074	2.11783
	10	NaN	0.46302	0.46191	0.46074	0.56608	0.61814	0.99026	1.05641	2.33543
WFG1	3	0.37862	0.21258	0.18074	0.18377	0.20233	**0.07600**	0.20087	0.15597	0.16604
	5	0.38576	0.29117	0.20606	0.17134	0.19663	**0.08683**	0.18288	0.18297	0.26815
	8	NaN	0.16839	0.07692	0.06678	0.08509	0.08045	0.10808	0.12427	0.33417
	10	NaN	0.08868	0.09112	0.07619	0.14610	0.10095	0.10556	0.11972	0.23599
WFG2	3	0.31750	0.04072	**0.03577**	0.04866	0.08872	0.08739	0.17910	0.12579	0.05805
	5	0.07906	0.05691	**0.05685**	0.08325	0.10423	0.15136	0.21243	0.20765	0.12767
	8	NaN	0.07015	0.08495	0.09183	0.09860	0.11937	0.13764	0.13030	0.19386
	10	NaN	0.05969	0.08920	0.09114	0.09578	0.11840	0.13169	0.12416	0.19704
WFG3	3	0.11423	0.15399	0.28832	0.05425	**0.03745**	0.04070	0.20844	0.19232	0.05006
	5	0.14467	0.09697	0.12176	0.12018	**0.08618**	0.15235	0.34998	0.28723	0.10195
	8	NaN	0.23351	0.56029	0.14305	0.22451	0.33536	0.56095	0.43524	0.15998
	10	NaN	0.16754	0.41979	0.15640	0.31725	0.39634	0.57148	0.55067	0.16206
WFG4	3	0.05942	0.05818	0.05823	**0.07217**	0.07700	0.09484	0.52334	0.25250	0.07274
	5	**0.16924**	0.19213	0.19223	0.26733	0.30864	0.41147	0.63375	0.42761	0.18244
	8	NaN	0.39954	0.39905	0.51790	0.72445	0.51843	0.85709	0.59237	0.37909
	10	NaN	0.46687	0.46624	0.66822	0.84257	0.58032	0.92412	0.70445	0.45848
WFG5	3	**0.06080**	0.06216	0.06212	0.07543	0.07569	0.10004	0.52875	0.24320	0.07718
	5	**0.16315**	0.18937	0.18935	0.25529	0.29036	0.40381	0.65914	0.41589	0.18139
	8	NaN	0.39141	0.39123	0.51273	0.67067	0.51038	0.81440	0.48871	0.36793
	10	NaN	0.45671	0.45638	0.65521	0.80237	0.56802	0.88882	0.55651	0.45670
WFG6	3	0.06614	0.06237	**0.06236**	0.07542	0.08158	0.09964	0.53091	0.24512	0.08111
	5	**0.17112**	0.18939	0.18942	0.26168	0.32816	0.40693	0.67423	0.41625	0.19635
	8	NaN	0.39279	0.39211	0.52623	0.84861	0.52593	0.92164	0.70887	0.40164
	10	NaN	0.45856	0.45750	0.66364	0.95099	0.57914	0.97505	0.81882	0.46819
WFG7	3	**0.05736**	0.05858	0.05843	0.07272	0.10435	0.09461	0.53919	0.25365	0.07482
	5	**0.16835**	0.19302	0.19308	0.26131	0.34346	0.40967	0.67685	0.42667	0.22350
	8	NaN	0.39970	0.39841	0.50986	0.81487	0.52613	0.92975	0.61293	0.43800
	10	NaN	0.46668	0.46543	0.63276	0.94246	0.59069	0.97643	0.65444	0.49155
WFG8	3	0.07706	0.06858	**0.06826**	0.07974	0.08798	0.10758	0.53692	0.50862	0.09200
	5	**0.18385**	0.19572	0.19568	0.27004	0.31288	0.51613	0.70712	0.51826	0.21824
	8	NaN	0.41691	0.41495	0.49936	0.80811	0.54876	0.92428	0.79070	0.43170
	10	NaN	0.50584	0.49280	0.64259	0.92544	0.62707	1.00382	0.86101	0.48245
WFG9	3	0.06389	0.06403	**0.06323**	0.07385	0.10025	0.09920	0.50142	0.26204	0.08311
	5	**0.16523**	0.18615	0.18634	0.24683	0.29613	0.47733	0.66154	0.44104	0.21086
	8	NaN	0.39688	0.39539	0.51814	0.71655	0.53759	0.85700	0.67375	0.45885
	10	NaN	0.46273	0.46209	0.66553	0.83358	0.60033	0.92832	0.73585	0.50534

TABLE III
MEAN HV STATISTICS FOR DTLZ^{-1} AND WFG^{-1} PROBLEMS

Problem	M	DBEA-DR	NSGA-III	θ-DEA	MOEA/DD	MOEA/D-PBI	MOEA/D-Tch	MOEA/D-WS	MOEA/D-IPBI	NSGA-II
DTLZ1^{-1}	3	**0.29207**	0.27258	0.25057	0.24887	0.26146	0.27141	0.03935	0.17744	0.26905
	5	0.01678	0.01265	0.00898	0.00972	**0.01739**	0.01208	0.00083	0.00671	0.01520
	8	NaN	5.227E-05	4.499E-05	0.881E-05	0.598E-05	3.215E-05	0.139E-05	2.855E-05	3.568E-05
	10	NaN	1.185E-06	0.451E-06	0.100E-06	0.079E-06	0.620E-06	0.025E-06	0.567E-06	0.765E-06
DTLZ2^{-1}	3	**0.71081**	0.68986	0.69303	0.68912	0.69439	0.68780	0.70652	0.70650	0.68187
	5	**0.20760**	0.13957	0.13496	0.08794	0.15984	0.15556	0.14930	0.14910	0.17147
	8	NaN	4.454E-03	3.406E-03	2.690E-03	5.978E-03	0.459E-03	1.560E-03	1.560E-03	4.585E-03
	10	NaN	6.308E-04	5.541E-04	1.836E-04	5.199E-04	0.052E-04	0.640E-04	0.639E-04	3.797E-04
DTLZ3^{-1}	3	**0.70917**	0.69251	0.69468	0.68990	0.69609	0.68667	0.70650	0.70650	0.68267
	5	**0.20422**	0.12951	0.13273	0.08190	0.15902	0.15199	0.14891	0.14886	0.16472
	8	NaN	0.00414	0.00401	0.00255	0.00596	0.00050	0.00156	0.00156	0.00390
	10	NaN	0.00054	0.00059	0.00018	0.00052	0.00001	0.00006	0.00006	0.00033
DTLZ4^{-1}	3	**0.71168**	0.69397	0.69546	0.68942	0.59319	0.68049	0.70650	0.64625	0.68358
	5	**0.19195**	0.12326	0.11428	0.07242	0.12296	0.14878	0.14881	0.13995	0.16970
	8	NaN	4.582E-03	3.921E-03	2.198E-03	2.020E-03	0.485E-03	1.563E-03	1.340E-03	3.886E-03
	10	NaN	6.065E-04	6.409E-04	2.569E-04	2.333E-04	0.043E-04	0.642E-04	0.649E-04	3.006E-04
WFG1^{-1}	3	**0.10958**	0.10955	0.08936	0.08475	0.03944	0.07838	0.04427	0.06037	0.12500
	5	0.00243	0.00221	0.00155	0.00094	0.00033	0.00174	0.00089	0.00113	**0.00296**
	8	NaN	1.835E-06	1.401E-06	1.028E-06	0.126E-06	3.015E-06	1.767E-06	1.798E-06	3.640E-06
	10	NaN	1.891E-08	1.524E-08	0.962E-08	0.149E-08	4.755E-08	2.414E-08	2.533E-08	4.974E-08
WFG2^{-1}	3	**0.38567**	0.38373	0.38347	0.38123	0.37769	0.37505	0.20617	0.31447	0.36889
	5	**0.01160**	0.01067	0.00805	0.00611	0.00500	0.01143	0.00398	0.00443	0.01055
	8	NaN	0.784E-05	0.638E-05	0.383E-05	0.368E-05	1.585E-05	0.690E-05	0.730E-05	1.290E-05
	10	NaN	0.795E-07	0.569E-07	0.441E-07	0.378E-07	2.304E-07	0.885E-07	0.977E-07	1.787E-07
WFG3^{-1}	3	0.24752	**0.26507**	0.24959	0.23184	0.25481	0.25408	0.03245	0.11691	0.26451
	5	0.01170	0.01279	0.00912	0.00388	0.00459	0.01082	0.00053	0.00286	**0.01312**
	8	NaN	3.666E-05	1.415E-05	0.262E-05	0.417E-05	1.598E-05	0.083E-05	0.300E-05	2.035E-05
	10	NaN	6.673E-07	2.511E-07	0.250E-07	0.483E-07	2.704E-07	0.106E-07	0.499E-07	4.847E-07
WFG4^{-1}	3	**0.70957**	0.66343	0.68880	0.66140	0.68582	0.66881	0.68655	0.69140	0.66561
	5	**0.15227**	0.12711	0.14416	0.10758	0.13711	0.08523	0.10288	0.11997	0.14780
	8	NaN	5.007E-03	5.123E-03	0.255E-03	0.602E-03	0.548E-03	2.351E-03	1.914E-03	2.758E-03
	10	NaN	5.475E-04	2.537E-04	0.039E-04	0.239E-04	0.171E-04	1.539E-04	1.151E-04	1.951E-04
WFG5^{-1}	3	**0.70099**	0.66841	0.68748	0.67405	0.68567	0.67011	0.68645	0.69118	0.67184
	5	**0.14506**	0.12789	0.12399	0.12320	0.13919	0.08783	0.10558	0.12259	0.16091
	8	NaN	0.00421	0.00436	0.00062	0.00080	0.00050	0.00237	0.00195	0.00250
	10	NaN	0.00046	0.00025	0.00002	0.00003	0.00001	0.00016	0.00011	0.00015
WFG6^{-1}	3	**0.70518**	0.68331	0.69235	0.67553	0.68534	0.66845	0.68665	0.69144	0.68281
	5	0.13073	0.13628	0.12549	0.12332	0.13846	0.08150	0.10292	0.11987	**0.16948**
	8	NaN	0.00450	0.00382	0.00075	0.00076	0.00043	0.00236	0.00194	0.00248
	10	NaN	0.00053	0.00022	0.00002	0.00003	0.00001	0.00016	0.00011	0.00020
WFG7^{-1}	3	**0.70742**	0.65101	0.68135	0.65126	0.67742	0.65881	0.68664	0.69143	0.65047
	5	0.13430	0.11727	0.11857	0.11268	0.13727	0.08508	0.10297	0.11996	**0.14742**
	8	NaN	0.00441	0.00382	0.00049	0.00054	0.00050	0.00237	0.00192	0.00340
	10	NaN	0.00047	0.00023	0.00002	0.00002	0.00001	0.00015	0.00011	0.00032
WFG8^{-1}	3	**0.71138**	0.68958	0.69311	0.67910	0.68517	0.66818	0.68660	0.69143	0.68535
	5	0.16025	0.13845	0.12755	0.12962	0.13872	0.08272	0.10293	0.11978	**0.17643**
	8	NaN	0.00460	0.00405	0.00129	0.00090	0.00038	0.00237	0.00195	0.00381
	10	NaN	0.00055	0.00023	0.00005	0.00003	0.00001	0.00016	0.00012	0.00034
WFG9^{-1}	3	**0.70013**	0.67193	0.68446	0.64574	0.66636	0.65325	0.68255	0.68630	0.66060
	5	**0.16604**	0.13747	0.12627	0.11905	0.13411	0.09712	0.10808	0.12487	0.15893
	8	NaN	0.00478	0.00431	0.00088	0.00073	0.00075	0.00222	0.00181	0.00380
	10	NaN	0.00048	0.00026	0.00003	0.00003	0.00003	0.00014	0.00010	0.00040

TABLE IV
MEAN IGD STATISTICS FOR DTLZ^{-1} AND WFG^{-1} PROBLEMS

Problem	M	DBEA-DR	NSGA-III	θ-DEA	MOEA/DD	MOEA/D-PBI	MOEA/D-Tch	MOEA/D-WS	MOEA/D-IPBI	NSGA-II
DTLZ1^{-1}	3	**0.04204**	0.06023	0.08080	0.07764	0.07235	0.06726	0.46615	0.15033	0.05772
	5	0.14258	0.15781	0.21539	0.18317	0.13134	0.17583	0.58701	0.24709	**0.12841**
	8	NaN	0.19939	0.22664	0.28573	0.42514	0.27490	0.67675	0.28280	0.21727
	10	NaN	0.19114	0.25461	0.28540	0.42793	0.30308	0.68698	0.30610	0.22753
DTLZ2^{-1}	3	**0.05529**	0.06849	0.07061	0.07231	0.06733	0.08081	0.05795	0.05797	0.07106
	5	**0.17814**	0.20140	0.22591	0.26158	0.20294	0.19062	0.19319	0.19338	0.17835
	8	NaN	0.39590	0.44648	0.44169	0.38813	0.46403	0.39536	0.39528	0.34390
	10	NaN	0.41607	0.45647	0.50964	0.44616	0.55239	0.46082	0.46084	0.38069
DTLZ3^{-1}	3	**0.05507**	0.06945	0.06923	0.07147	0.06640	0.08231	0.05799	0.05799	0.07117
	5	**0.17233**	0.20451	0.22735	0.26960	0.20317	0.19464	0.19361	0.19366	0.18317
	8	NaN	0.39347	0.43321	0.44242	0.38697	0.46253	0.39517	0.39519	0.34945
	10	NaN	0.41589	0.45076	0.50678	0.44444	0.55227	0.46063	0.46065	0.38427
DTLZ4^{-1}	3	**0.05532**	0.06933	0.06795	0.07172	0.14957	0.08734	0.05800	0.10622	0.07001
	5	**0.16489**	0.21479	0.24070	0.27921	0.27387	0.19831	0.19371	0.21271	0.17809
	8	NaN	0.36310	0.42714	0.45958	0.52122	0.46517	0.39528	0.43285	0.35118
	10	NaN	0.39219	0.43337	0.48691	0.52423	0.55814	0.46055	0.46365	0.39096
WFG1^{-1}	3	**0.02730**	0.05290	0.10325	0.11665	0.29294	0.15693	0.37597	0.29304	0.04386
	5	0.08794	0.11311	0.19147	0.32509	0.55926	0.17943	0.35539	0.31550	**0.06629**
	8	NaN	0.26898	0.34785	0.39702	0.99344	0.16412	0.32690	0.31295	0.10819
	10	NaN	0.29323	0.38999	0.43426	0.96296	0.14229	0.33064	0.31388	0.09226
WFG2^{-1}	3	0.05323	**0.04190**	0.04306	0.05567	0.05739	0.04694	0.33038	0.22170	0.06632
	5	0.12385	**0.07881**	0.14449	0.18688	0.24328	0.07440	0.37314	0.32020	0.09517
	8	NaN	0.20953	0.35963	0.33890	0.45393	0.11549	0.42160	0.38028	0.15819
	10	NaN	0.25544	0.43200	0.36370	0.50669	0.12588	0.46156	0.40894	0.15103
WFG3^{-1}	3	0.11100	0.06083	0.08114	0.08829	0.08214	0.08449	0.49054	0.24954	**0.05986**
	5	0.21589	0.16490	0.21009	0.26876	0.34333	0.19835	0.62850	0.37408	**0.13927**
	8	NaN	0.25089	0.31689	0.44622	0.47588	0.34387	0.72693	0.51896	0.25419
	10	NaN	0.25988	0.29982	0.46272	0.49450	0.36499	0.75953	0.50412	0.25219
WFG4^{-1}	3	**0.05509**	0.07067	0.06850	0.08952	0.08597	0.09036	0.07125	0.06708	0.07162
	5	0.22407	0.20365	0.21184	0.27358	0.25924	0.27079	0.25575	0.22894	**0.18955**
	8	NaN	0.38615	0.41425	0.63248	0.60677	0.48890	0.42937	0.41881	0.37240
	10	NaN	0.43182	0.49779	0.72355	0.66994	0.55916	0.47924	0.47420	0.41505
WFG5^{-1}	3	**0.05434**	0.07096	0.06901	0.08740	0.08494	0.08793	0.07083	0.06681	0.07152
	5	0.21098	0.20813	0.23067	0.24814	0.25227	0.26722	0.25228	0.22598	**0.18380**
	8	NaN	0.39476	0.42790	0.52843	0.58467	0.48796	0.42769	0.41669	0.37803
	10	NaN	0.44100	0.49854	0.61452	0.65891	0.55862	0.47784	0.47400	0.42550
WFG6^{-1}	3	**0.05555**	0.07064	0.06940	0.08761	0.08656	0.09107	0.07121	0.06708	0.07144
	5	0.22326	0.20879	0.23472	0.23969	0.25092	0.27386	0.25563	0.22895	**0.18108**
	8	NaN	0.39207	0.44079	0.51870	0.58752	0.49671	0.42929	0.41820	0.38577
	10	NaN	0.43261	0.50390	0.61078	0.66057	0.56964	0.47855	0.47419	0.42065
WFG7^{-1}	3	**0.05528**	0.07491	0.06984	0.09135	0.08764	0.08919	0.07122	0.06709	0.07665
	5	0.24217	0.21990	0.24024	0.26035	0.25841	0.26725	0.25555	0.22880	**0.19349**
	8	NaN	0.39812	0.44856	0.57769	0.61427	0.48629	0.42953	0.41873	0.36740
	10	NaN	0.43765	0.50225	0.65658	0.68282	0.55764	0.48052	0.47575	0.39785
WFG8^{-1}	3	**0.05541**	0.07182	0.07039	0.08438	0.08642	0.09138	0.07125	0.06708	0.07267
	5	0.19352	0.21132	0.23438	0.22655	0.24985	0.27496	0.25585	0.22920	**0.18775**
	8	NaN	0.39399	0.44081	0.49022	0.57774	0.49801	0.42906	0.41764	0.37975
	10	NaN	0.43266	0.50071	0.57353	0.65643	0.57134	0.47790	0.47233	0.40987
WFG9^{-1}	3	**0.05449**	0.06858	0.06769	0.08732	0.08791	0.08518	0.07062	0.06719	0.07407
	5	**0.18347**	0.20468	0.23095	0.23795	0.25551	0.25081	0.24740	0.22190	0.19209
	8	NaN	0.39243	0.43482	0.51895	0.59280	0.46407	0.42810	0.41925	0.37479
	10	NaN	0.44781	0.49704	0.60650	0.66438	0.52325	0.47860	0.47591	0.40130