———Supplementary Online Material———

Multiple Surrogate Assisted Many-objective Optimization for Computationally Expensive Engineering Design

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ABSTRACT

This document contains supplementary results for the above study which could not be included in the main body of the paper due to the page limitations.

1 Performance metrics

Inverted generation distance (IGD) and hypervolume (HV) are the most common metrics used for an quantitative assessment in the multiobjective optimization domain. Computation of IGD requires a reference POF, while computation of HV requires a reference point. The aspect of uniformity of solutions in the reference set can always be questioned. As for hypervolume, the choice of the reference point used for HV computation is crucial and can adversely affect the interpretations about relative performance [1,2]. For HV computation, there are recent reports that suggest use of a slightly larger than true nadir vector as a reference point [1,2]. In our experiments, we have used the reference point to be 1.1 multiplied by the theoretical nadir vector for problems. Given a set of solutions and their corresponding objective vectors, we first discard all solutions that are dominated by the reference point. The objective vectors of the remaining solutions are normalized using the ideal and the modified nadir vector and HV is computed using a reference point which is 1.1^M in the normalized space (M is the number of objectives). All HV computations reported in this paper are based on exact method (WFG algorithm) [3]. HV and IGD computations in this paper are based on solutions obtained from the archive i.e. the set of all fully evaluated solutions so far.

2 IGD statistics

2.1 IGD statistics obtained for DTLZ1-DTLZ7 problems

Table 1.　IGD statistics (best, mean, and worst) of DTLZ1-7

Problems	M	$SaMaO_{ASF-ED}$			$SaMaO_{ASF-ASF}$			$SaMaO_{ED-ED}$			K-RVEA			RVEA			ParEGO		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
DTLZ1	3	38.22	57.259	69.507	32.697	54.899	65.747	36.202	54.564	72.441	42.698	83.709	126.799	42.650	82.870	115.100	13.420	**52.470**	112.700
	4	31.778	44.958	55.442	25.524	44.001	63.014	24.196	**40.624**	52.051	38.728	61.613	95.279	39.650	59.180	97.710	18.630	45.450	87.760
	6	7.3	**19.64**	30.654	11.663	23.927	39.610	5.878	19.927	33.964	10.009	20.885	32.256	12.240	22.940	36.850	NaN	NaN	NaN
	8	0.711	**5.331**	16.886	2.641	5.868	14.605	1.234	5.743	11.654	2.526	8.142	14.041	1.250	7.406	15.660	NaN	NaN	NaN
	10	0.192	0.331	0.465	0.220	0.349	0.549	0.177	**0.286**	0.564	0.234	0.395	0.703	0.193	0.339	1.105	NaN	NaN	NaN
DTLZ2	3	0.066	0.079	0.101	0.063	**0.070**	0.086	0.063	0.074	0.089	0.086	0.109	0.167	0.227	0.288	0.335	0.151	0.191	0.243
	4	0.169	0.193	0.228	0.149	**0.162**	0.180	0.15	0.176	0.2	0.198	0.221	0.261	0.280	0.332	0.383	0.289	0.337	0.408
	6	0.303	0.327	0.357	0.301	**0.324**	0.356	0.301	0.325	0.352	0.375	0.397	0.448	0.375	0.404	0.440	NaN	NaN	NaN
	8	0.39	**0.411**	0.432	0.386	0.417	0.470	0.385	0.413	0.46	0.435	0.476	0.508	0.466	0.541	0.704	NaN	NaN	NaN
	10	0.45	0.469	0.492	0.463	0.483	0.508	0.447	**0.466**	0.488	0.488	0.537	0.562	0.539	0.608	0.733	NaN	NaN	NaN
DTLZ3	3	135.358	174.489	200.073	94.774	171.437	212.268	105.747	174.519	224.037	125.327	225.062	329.058	133.700	256.100	347.900	81.150	**145.500**	261.600
	4	93.751	145.538	168.362	74.369	141.142	197.430	81.118	143.169	175.271	94.966	198.689	336.823	89.950	198.600	306.300	66.930	**138.100**	209.400
	6	37.938	77.834	110.366	31.092	**70.577**	101.804	42.032	77.769	111.722	21.191	78.399	148.230	43.540	95.970	157.700	NaN	NaN	NaN
	8	4.06	22.1	52.855	7.771	20.093	51.782	6.344	21.063	45.191	7.922	**19.320**	36.220	8.569	25.270	42.170	NaN	NaN	NaN
	10	0.789	**1.085**	1.68	0.752	1.209	1.800	0.636	1.188	1.723	0.832	1.271	2.180	0.761	1.228	1.836	NaN	NaN	NaN
DTLZ4	3	0.119	0.21	0.459	0.112	0.196	0.345	0.117	**0.195**	0.318	0.132	0.309	0.594	0.205	0.399	0.959	0.387	0.646	0.947
	4	0.22	0.32	0.539	0.221	**0.306**	0.434	0.218	0.325	0.585	0.267	0.379	0.497	0.320	0.514	0.737	0.505	0.725	0.960
	6	0.398	0.488	0.612	0.393	0.489	0.583	0.374	**0.465**	0.567	0.404	0.511	0.618	0.503	0.615	0.800	NaN	NaN	NaN
	8	0.489	0.569	0.673	0.518	0.588	0.661	0.5	**0.555**	0.649	0.495	0.568	0.665	0.554	0.628	0.731	NaN	NaN	NaN
	10	0.574	0.622	0.673	0.562	**0.609**	0.698	0.546	0.61	0.699	0.535	0.587	0.638	0.599	0.667	0.761	NaN	NaN	NaN
DTLZ5	3	0.01	**0.013**	0.016	0.010	0.014	0.019	0.011	0.017	0.026	0.044	0.080	0.139	0.201	0.247	0.316	0.039	0.055	0.072
	4	0.017	0.026	0.04	0.021	0.029	0.044	0.015	**0.025**	0.037	0.037	0.062	0.099	0.149	0.294	0.393	0.090	0.288	0.428
	6	0.021	**0.03**	0.042	0.021	0.034	0.052	0.02	0.03	0.042	0.029	0.045	0.063	0.159	0.280	0.431	NaN	NaN	NaN
	8	0.015	0.02	0.033	0.016	0.021	0.029	0.013	**0.017**	0.025	0.015	0.023	0.030	0.104	0.260	0.748	NaN	NaN	NaN
	10	0.008	**0.01**	0.013	0.008	0.011	0.014	0.008	0.01	0.013	0.007	0.010	0.014	0.224	0.488	0.746	NaN	NaN	NaN
DTLZ6	3	0.055	0.98	2.117	0.115	1.413	4.373	0.064	**0.79**	2.204	2.582	3.202	4.028	3.651	4.960	5.613	5.030	6.378	6.867
	4	0.11	0.911	2.669	0.106	2.365	4.503	0.03	**0.713**	2.163	1.578	2.506	3.192	3.027	4.044	5.208	5.652	5.916	6.034
	6	0.036	0.464	1.862	0.164	2.256	3.110	0.062	**0.431**	1.351	0.541	1.463	1.997	1.025	2.524	3.600	NaN	NaN	NaN
	8	0.052	**0.111**	0.331	0.058	1.126	1.739	0.038	0.164	0.814	0.332	0.581	0.853	0.247	1.004	1.870	NaN	NaN	NaN
	10	0.019	0.046	0.348	0.019	0.196	0.484	0.014	**0.041**	0.089	0.033	0.062	0.112	0.140	0.297	0.751	NaN	NaN	NaN
DTLZ7	3	0.072	0.218	0.849	0.067	**0.159**	0.811	0.074	0.226	0.798	0.103	0.164	0.350	0.400	0.515	0.637	0.621	0.829	1.201
	4	0.183	0.258	0.59	0.158	**0.210**	0.359	0.182	0.3	0.799	0.235	0.405	0.680	0.532	0.691	0.926	0.719	0.892	1.149
	6	0.428	0.524	1.005	0.482	**0.523**	0.605	0.455	0.532	0.716	0.622	0.722	0.982	0.889	1.088	1.808	NaN	NaN	NaN
	8	0.636	**0.771**	0.861	0.719	0.845	0.953	0.736	0.856	1.031	0.965	1.079	1.347	1.162	1.359	1.634	NaN	NaN	NaN
	10	0.985	**1.053**	1.112	0.935	1.173	1.335	1.109	1.221	1.356	1.089	1.185	1.333	1.343	1.900	3.327	NaN	NaN	NaN

2.2 IGD statistics obtained for WFG1-9 problems

Table 2. IGD statistics (best, mean, and worst) of WFG1-9

Problems	M	SaMaO$_{ASF-ED}$			SaMaO$_{ASF-ASF}$			SaMaO$_{ED-ED}$			K-RVEA			RVEA			ParEGO		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
WFG1	3	1.608	1.815	2.257	1.607	1.832	2.091	1.686	1.883	2.389	1.656	**1.794**	2.139	1.775	2.108	2.394	1.697	1.798	2.318
	4	1.811	2.090	2.318	1.826	2.049	2.544	1.803	2.061	2.332	1.876	2.105	2.478	1.934	**2.018**	2.225	1.982	2.137	2.517
	6	2.245	2.409	2.650	2.257	**2.382**	2.555	2.285	2.435	2.621	2.316	2.474	2.738	2.683	2.822	3.298	NaN	NaN	NaN
	8	2.542	**2.705**	3.052	2.471	2.708	2.958	2.552	2.763	2.999	2.632	2.819	3.194	2.896	3.074	3.755	NaN	NaN	NaN
	10	2.893	3.084	3.295	2.859	**3.036**	3.303	2.814	3.070	3.349	2.972	3.143	3.456	3.275	3.529	4.162	NaN	NaN	NaN
WFG2	3	0.348	0.456	0.575	0.388	0.438	0.497	0.369	0.450	0.508	0.273	**0.335**	0.418	0.595	0.771	0.903	0.652	0.777	0.872
	4	0.478	0.598	0.728	0.403	**0.508**	0.646	0.532	0.619	0.732	0.356	0.548	0.605	0.601	0.831	1.031	0.914	1.098	1.407
	6	0.634	0.741	0.889	0.555	0.705	0.853	0.629	0.780	0.997	0.501	**0.564**	0.697	0.919	1.191	1.897	NaN	NaN	NaN
	8	0.752	0.872	1.086	0.731	0.848	1.063	0.806	0.911	1.505	0.694	**0.759**	0.894	1.040	1.727	2.940	NaN	NaN	NaN
	10	0.956	**1.082**	1.261	0.987	1.169	1.397	0.936	1.110	1.369	0.902	0.988	1.178	1.595	2.393	3.892	NaN	NaN	NaN
WFG3	3	0.283	0.367	0.436	0.268	0.358	0.432	0.237	**0.344**	0.443	0.309	0.380	0.458	0.557	0.681	1.065	0.426	0.477	0.568
	4	0.307	0.453	0.552	0.313	0.413	0.519	0.384	0.496	0.615	0.149	**0.213**	0.296	0.440	0.621	0.828	0.368	0.447	0.512
	6	0.745	0.899	1.071	0.605	0.749	0.943	0.732	0.900	1.091	0.317	**0.513**	0.691	1.094	1.486	2.572	NaN	NaN	NaN
	8	0.566	0.762	0.952	0.493	0.640	0.808	0.693	0.816	1.028	0.435	**0.578**	0.804	1.218	1.793	2.793	NaN	NaN	NaN
	10	0.686	0.865	1.048	0.554	0.736	0.854	0.775	0.948	1.110	0.538	**0.693**	0.935	1.665	3.465	6.425	NaN	NaN	NaN
WFG4	3	0.380	0.420	0.453	0.384	**0.411**	0.475	0.377	0.418	0.457	0.390	0.436	0.488	0.533	0.582	0.657	0.509	0.569	0.658
	4	0.692	0.753	0.882	0.686	**0.737**	0.833	0.736	0.809	0.917	0.701	0.890	1.180	1.066	1.243	1.527	1.175	1.494	2.039
	6	1.782	1.864	1.953	1.748	1.847	1.941	1.800	1.891	2.008	1.676	**1.736**	1.844	2.289	2.632	3.302	NaN	NaN	NaN
	8	2.997	**3.169**	3.493	3.088	3.352	3.876	3.061	3.232	3.447	2.954	3.181	3.971	4.348	5.275	8.286	NaN	NaN	NaN
	10	4.615	5.290	7.246	4.746	6.224	8.981	4.713	**5.050**	5.603	4.733	5.305	6.208	6.487	7.741	9.607	NaN	NaN	NaN
WFG5	3	0.309	**0.397**	0.529	0.324	0.402	0.467	0.319	0.400	0.497	0.327	0.419	0.625	0.589	0.699	0.794	0.586	0.684	0.729
	4	0.742	0.843	0.939	0.643	**0.744**	0.870	0.745	0.895	1.004	0.704	0.815	1.007	1.016	1.171	1.351	1.120	1.301	1.493
	6	1.791	1.897	2.041	1.750	1.837	1.934	1.781	1.950	2.098	1.719	**1.813**	1.912	2.211	2.357	2.655	NaN	NaN	NaN
	8	2.930	**3.119**	3.425	2.991	3.134	3.547	3.078	3.206	3.355	2.997	3.150	3.833	4.145	4.496	5.106	NaN	NaN	NaN
	10	4.550	4.884	5.523	4.768	5.343	6.097	4.631	**4.834**	5.105	4.525	4.841	5.568	5.895	6.459	7.401	NaN	NaN	NaN
WFG6	3	0.548	0.637	0.746	0.518	**0.631**	0.715	0.553	0.635	0.720	0.540	0.695	0.789	0.758	0.841	0.892	0.631	0.790	0.942
	4	0.869	0.958	1.060	0.725	**0.877**	1.016	0.909	0.986	1.073	0.923	1.028	1.156	1.110	1.253	1.410	1.236	1.330	1.418
	6	2.016	2.097	2.195	1.904	**1.991**	2.093	2.059	2.134	2.251	2.095	2.211	2.372	2.318	2.460	2.802	NaN	NaN	NaN
	8	3.197	3.330	3.464	3.145	**3.276**	3.395	3.432	3.536	3.684	3.594	3.782	3.983	4.107	4.576	5.488	NaN	NaN	NaN
	10	4.672	4.892	5.099	4.691	**4.847**	4.977	4.903	5.201	5.544	5.289	5.563	5.750	5.920	6.414	7.163	NaN	NaN	NaN
WFG7	3	0.513	0.552	0.575	0.489	0.550	0.599	0.503	**0.548**	0.578	0.522	0.560	0.600	0.598	0.667	0.729	0.563	0.645	0.716
	4	0.783	0.848	0.946	0.708	**0.831**	0.887	0.790	0.874	0.967	0.761	0.973	1.249	1.222	1.429	1.910	1.377	1.558	1.792
	6	1.844	1.926	2.018	1.820	1.950	2.024	1.847	1.963	2.039	1.772	**1.864**	1.984	2.381	2.644	3.432	NaN	NaN	NaN
	8	3.143	**3.240**	3.372	3.264	3.405	3.567	3.182	3.292	3.435	3.234	3.382	3.731	4.519	5.045	5.916	NaN	NaN	NaN
	10	4.854	5.467	7.427	5.518	6.115	7.488	4.821	**5.207**	6.559	4.950	5.405	6.694	6.649	7.433	9.046	NaN	NaN	NaN
WFG8	3	0.605	0.660	0.717	0.571	**0.644**	0.724	0.611	0.670	0.783	0.639	0.705	0.795	0.802	0.893	0.991	0.745	0.843	0.899
	4	1.040	1.154	1.277	1.026	**1.121**	1.259	1.084	1.213	1.300	1.280	1.427	1.592	1.561	1.684	1.876	1.619	1.786	1.954
	6	2.108	2.174	2.275	1.968	**2.148**	2.246	2.119	2.220	2.283	2.165	2.299	2.428	2.584	2.818	3.167	NaN	NaN	NaN
	8	3.272	**3.400**	3.511	3.335	3.445	3.563	3.420	3.539	3.670	3.480	3.592	3.837	4.708	5.149	6.108	NaN	NaN	NaN
	10	4.765	**4.907**	5.005	4.830	5.042	5.283	4.987	5.144	5.277	5.073	5.248	5.494	6.269	7.056	7.998	NaN	NaN	NaN
WFG9	3	0.503	**0.620**	0.732	0.537	0.642	0.834	0.527	0.629	0.775	0.470	0.656	0.792	0.701	0.822	0.953	0.490	0.640	0.869
	4	0.797	**0.955**	1.247	0.820	0.990	1.328	0.819	0.979	1.314	0.828	1.099	1.443	1.210	1.349	1.506	0.985	1.232	1.448
	6	1.831	2.044	2.510	1.862	2.101	2.580	1.850	**2.003**	2.287	1.785	2.099	3.081	2.321	2.568	3.352	NaN	NaN	NaN
	8	3.092	3.622	4.647	3.195	3.791	4.503	3.103	**3.496**	4.211	3.110	3.873	5.126	4.259	4.853	5.672	NaN	NaN	NaN
	10	5.357	6.279	7.215	5.528	6.623	7.492	4.954	**5.861**	6.636	5.021	6.159	8.068	6.164	7.209	9.154	NaN	NaN	NaN

2.3 IGD statistics obtained for DTLZ1^{-1}-DTLZ4^{-1} problems

Table 3.   IGD statistics (best, mean, and worst) of DTLZ1^{-1}-DTLZ4^{-1}

Problems	M	SaMaO$_{ASF-ED}$			SaMaO$_{ASF-ASF}$			SaMaO$_{ED-ED}$			K-RVEA		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
DTLZ1^{-1}	3	42.866	65.394	78.893	46.246	**64.981**	83.149	59.130	72.359	95.132	74.502	84.601	99.595
	4	74.634	**85.607**	100.255	73.706	88.919	99.946	84.433	97.583	114.420	86.283	100.255	111.087
	6	136.782	151.043	166.223	135.354	149.997	166.124	139.842	155.629	172.516	121.038	**133.477**	150.650
	8	151.716	174.816	189.181	161.357	**174.706**	185.369	169.316	184.184	193.758	156.159	177.837	185.420
	10	176.544	190.863	202.639	180.408	**190.357**	203.428	191.928	198.020	211.895	184.394	194.013	208.754
DTLZ2^{-1}	3	0.377	0.555	0.771	0.371	0.534	0.761	0.473	0.632	0.846	0.439	**0.532**	0.646
	4	0.701	0.914	1.090	0.700	0.891	1.006	0.810	0.992	1.146	0.658	**0.791**	0.915
	6	1.477	1.606	1.698	1.515	1.596	1.681	1.526	1.612	1.709	1.266	**1.357**	1.445
	8	1.802	1.950	2.057	1.862	1.941	2.019	1.868	1.961	2.085	1.756	**1.834**	1.909
	10	2.025	**2.184**	2.292	2.103	2.190	2.301	2.130	2.226	2.323	2.031	2.205	2.298
DTLZ3^{-1}	3	449.944	536.144	613.007	415.195	**522.256**	591.138	446.320	544.276	644.275	598.910	666.319	730.355
	4	598.892	**683.621**	766.584	623.193	694.789	757.791	557.947	698.914	773.050	737.554	785.650	827.294
	6	964.069	1036.972	1088.493	990.216	1034.419	1070.411	960.639	1038.746	1078.974	935.810	**1006.935**	1086.322
	8	1154.329	1232.447	1283.507	1174.687	**1225.227**	1290.521	1166.996	1247.673	1301.139	1140.382	1232.060	1289.777
	10	1308.860	1395.855	1453.558	1314.338	**1390.115**	1452.036	1367.405	1419.355	1479.143	1329.477	1419.592	1468.607
DTLZ4^{-1}	3	0.391	**0.627**	0.901	0.416	0.646	0.927	0.465	0.673	0.976	0.509	0.888	1.203
	4	0.704	0.967	1.223	0.732	**0.900**	1.365	0.825	1.066	1.378	1.001	1.228	1.529
	6	1.333	1.624	1.870	1.332	**1.598**	1.836	1.318	1.687	1.892	1.430	1.774	1.974
	8	1.737	1.999	2.112	1.712	**1.996**	2.146	1.914	2.083	2.182	1.981	2.129	2.244
	10	2.146	2.280	2.372	2.064	**2.244**	2.355	2.113	2.265	2.340	2.142	2.274	2.358

2.4 IGD statistics obtained for WFG1^{-1}-WFG9^{-1} problems

Table 4. IGD statistics (best, mean, and worst) of WFG1^{-1}-WFG9^{-1}

	M	SaMaO$_{ASF-ED}$			SaMaO$_{ASF-ASF}$			SaMaO$_{ED-ED}$			K-RVEA		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
WFG1^{-1}	3	1.189	**1.711**	2.045	1.295	1.760	2.025	1.023	1.725	2.163	1.090	1.731	2.026
	4	1.296	**1.829**	2.218	1.178	1.853	2.266	1.524	1.884	2.243	1.372	1.835	2.231
	6	1.456	**1.810**	2.093	1.560	1.814	2.176	1.536	1.880	2.254	1.611	1.942	2.273
	8	1.502	**1.916**	2.364	1.676	1.965	2.289	1.546	1.945	2.244	1.820	2.032	2.267
	10	1.775	**2.061**	2.386	1.633	2.087	2.459	1.929	2.093	2.444	1.891	2.111	2.473
WFG2^{-1}	3	0.380	0.422	0.465	0.378	0.420	0.454	0.355	0.420	0.473	0.363	**0.418**	0.467
	4	0.709	0.748	0.798	0.702	0.740	0.798	0.706	0.748	0.797	0.665	**0.733**	0.791
	6	1.146	1.391	1.670	1.145	1.381	1.670	1.143	1.390	1.672	1.154	**1.328**	1.669
	8	1.461	1.940	2.583	1.439	**1.905**	2.633	1.468	2.051	2.635	1.469	2.064	2.635
	10	1.896	2.678	3.663	1.910	**2.585**	3.667	1.957	2.804	3.667	1.967	2.847	3.667
WFG3^{-1}	3	1.463	1.497	1.602	1.382	1.486	1.580	1.270	**1.324**	1.417	1.379	1.430	1.475
	4	2.670	2.721	2.758	2.571	2.702	2.750	2.455	**2.549**	2.621	2.503	2.621	2.692
	6	5.537	5.667	5.786	5.581	5.663	5.748	5.530	**5.611**	5.692	5.551	5.617	5.698
	8	9.304	9.431	9.570	9.277	9.422	9.508	9.317	9.405	9.505	9.291	**9.388**	9.488
	10	13.718	13.893	13.976	13.683	**13.866**	13.952	13.764	13.888	13.985	13.752	13.867	13.989
WFG4^{-1}	3	0.812	0.874	0.917	0.755	**0.870**	0.917	0.823	0.873	0.916	0.861	0.899	0.946
	4	1.214	1.281	1.373	1.212	**1.273**	1.367	1.204	1.290	1.408	1.244	1.310	1.421
	6	2.647	2.934	3.371	2.590	**2.918**	3.314	2.572	2.979	3.374	2.652	3.007	3.263
	8	4.644	5.394	6.172	4.727	**5.349**	6.172	4.759	5.596	6.297	5.140	5.676	6.147
	10	7.438	8.518	9.571	7.554	**8.433**	9.343	7.957	8.731	9.637	8.283	8.927	9.612
WFG5^{-1}	3	0.585	0.624	0.647	0.600	**0.621**	0.650	0.584	0.624	0.655	0.610	0.630	0.647
	4	0.881	**0.904**	0.943	0.882	0.905	0.930	0.883	0.910	0.942	0.885	0.908	0.943
	6	1.948	2.022	2.182	1.939	2.021	2.174	1.938	2.027	2.171	1.935	**2.014**	2.163
	8	3.670	3.872	4.290	3.623	**3.862**	4.123	3.684	3.914	4.313	3.691	3.912	4.305
	10	5.818	6.323	6.876	5.790	**6.288**	6.856	5.909	6.390	6.914	5.900	6.403	6.928
WFG6^{-1}	3	0.462	0.471	0.485	0.439	**0.466**	0.483	0.450	0.466	0.481	0.461	0.473	0.489
	4	0.761	0.780	0.807	0.753	**0.775**	0.797	0.754	0.778	0.804	0.754	0.777	0.806
	6	1.889	2.005	2.174	1.879	**1.987**	2.167	1.885	2.006	2.160	1.917	2.000	2.171
	8	3.645	3.942	4.300	3.646	**3.924**	4.139	3.707	4.018	4.381	3.737	4.019	4.372
	10	5.978	6.473	6.794	5.936	**6.419**	6.766	6.050	6.597	7.120	6.057	6.607	7.140
WFG7^{-1}	3	0.592	0.683	0.729	0.553	0.676	0.733	0.555	**0.669**	0.723	0.683	0.714	0.735
	4	1.014	1.048	1.086	0.979	**1.039**	1.084	0.987	1.040	1.092	1.019	1.060	1.116
	6	2.159	2.332	2.514	2.205	**2.310**	2.472	2.207	2.366	2.553	2.250	2.376	2.589
	8	3.886	4.288	4.661	3.809	**4.245**	4.656	4.058	4.467	4.963	4.063	4.490	5.007
	10	6.195	6.844	7.500	6.195	**6.810**	7.500	6.553	7.156	7.902	6.509	7.232	7.863
WFG8^{-1}	3	0.329	0.358	0.377	0.327	**0.346**	0.388	0.330	0.354	0.379	0.362	0.389	0.420
	4	0.701	0.731	0.770	0.694	**0.726**	0.769	0.701	0.729	0.769	0.700	0.733	0.770
	6	1.876	2.000	2.153	1.859	**1.968**	2.146	1.855	2.004	2.137	1.908	2.006	2.134
	8	3.703	4.006	4.391	3.661	**3.934**	4.175	3.735	4.065	4.420	3.765	4.060	4.412
	10	6.067	6.554	6.980	6.002	**6.496**	6.962	6.066	6.677	7.188	6.102	6.695	7.224
WFG9^{-1}	3	0.331	0.347	0.362	0.330	0.347	0.366	0.331	0.350	0.366	0.333	**0.346**	0.365
	4	0.684	0.722	0.759	0.687	**0.719**	0.756	0.688	0.725	0.760	0.703	0.729	0.767
	6	1.936	2.096	2.222	1.914	**2.083**	2.187	1.962	2.102	2.223	1.980	2.104	2.250
	8	3.782	**4.131**	4.582	3.817	4.151	4.552	3.891	4.184	4.584	3.828	4.214	4.612
	10	6.103	6.724	7.386	6.077	**6.685**	7.292	5.941	6.805	7.432	6.068	6.885	7.543

3 HV statistics

3.1 HV statistics obtained for DTLZ1-DTLZ7 problems

Table 5. HV statistics (best, mean, and worst) of DTLZ1-7

Problems	M	SaMaO-ASF+ED			SaMaO-ASF+ASF			SaMaO-ED+ED			K-RVEA		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
DTLZ1	3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	10	2.294	1.257	0.288	2.222	1.266	0.144	2.240	**1.553**	0.026	1.922	0.844	0.000
DTLZ2	3	0.722	0.702	0.666	0.718	0.699	0.657	0.719	**0.704**	0.674	0.648	0.608	0.504
	4	0.966	0.919	0.842	0.934	0.876	0.807	0.968	**0.931**	0.880	0.857	0.808	0.679
	6	1.390	**1.307**	1.172	1.302	1.246	1.118	1.357	1.294	1.203	1.215	1.163	1.108
	8	1.837	**1.744**	1.615	1.770	1.657	1.556	1.804	1.724	1.553	1.713	1.611	1.503
	10	2.336	**2.256**	2.168	2.307	2.208	2.090	2.329	2.243	2.093	2.338	2.134	2.031
DTLZ3	3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	10	1.346	**0.564**	0.000	1.423	0.493	0.000	1.289	0.410	0.000	0.850	0.254	0.000
DTLZ4	3	0.611	0.446	0.011	0.623	0.440	0.166	0.607	**0.451**	0.115	0.551	0.223	0.000
	4	0.822	0.577	0.096	0.863	0.582	0.229	0.813	**0.590**	0.348	0.627	0.324	0.050
	6	1.168	0.850	0.429	1.237	0.876	0.588	1.184	**0.920**	0.567	1.119	0.725	0.318
	8	1.599	1.309	0.961	1.548	1.301	1.108	1.615	**1.405**	0.802	1.668	1.337	0.749
	10	2.161	1.993	1.665	2.243	2.019	1.680	2.230	2.024	1.687	2.344	**2.180**	2.001
DTLZ5	3	0.259	**0.256**	0.252	0.258	0.254	0.242	0.255	0.250	0.243	0.211	0.163	0.102
	4	0.208	**0.199**	0.173	0.209	0.195	0.170	0.208	0.198	0.189	0.186	0.159	0.115
	6	0.194	0.180	0.159	0.191	0.176	0.152	0.197	**0.186**	0.171	0.194	0.177	0.144
	8	0.220	0.212	0.164	0.220	0.212	0.179	0.222	0.213	0.187	0.222	**0.215**	0.198
	10	0.259	**0.255**	0.240	0.258	0.254	0.231	0.260	0.254	0.215	0.260	0.245	0.214
DTLZ6	3	0.234	0.045	0.000	0.190	0.046	0.000	0.219	**0.063**	0.000	0.000	0.000	0.000
	4	0.187	0.042	0.000	0.179	0.018	0.000	0.199	**0.045**	0.000	0.000	0.000	0.000
	6	0.199	0.069	0.000	0.173	0.007	0.000	0.191	**0.070**	0.000	0.000	0.000	0.000
	8	0.218	**0.163**	0.000	0.210	0.023	0.000	0.215	0.115	0.000	0.004	0.000	0.000
	10	0.255	**0.239**	0.000	0.254	0.138	0.000	0.256	0.225	0.050	0.258	0.225	0.061
DTLZ7	3	0.424	0.388	0.263	0.425	**0.398**	0.271	0.425	0.384	0.267	0.396	0.375	0.337
	4	0.462	0.431	0.369	0.460	**0.438**	0.401	0.457	0.422	0.336	0.414	0.383	0.346
	6	0.485	**0.432**	0.367	0.464	0.424	0.346	0.477	0.410	0.347	0.389	0.367	0.324
	8	0.515	0.428	0.313	0.509	**0.446**	0.384	0.441	0.341	0.128	0.385	0.350	0.308
	10	0.456	0.353	0.205	0.524	**0.480**	0.443	0.359	0.133	0.017	0.417	0.361	0.301

3.2 HV statistics obtained for WFG1-9 problems

Table 6.   HV statistics (best, mean, and worst) of WFG1-9

Problems	M	SaMaO-ASF+ED			SaMaO-ASF+ASF			SaMaO-ED+ED			K-RVEA		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
WFG1	3	0.355	**0.227**	0.000	0.303	0.196	0.041	0.326	0.173	0.000	0.291	0.216	0.138
	4	0.468	0.377	0.208	0.459	0.352	0.018	0.460	**0.379**	0.184	0.433	0.368	0.283
	6	0.472	0.381	0.209	0.477	0.364	0.208	0.474	0.376	0.204	0.479	**0.391**	0.280
	8	0.568	0.458	0.269	0.548	0.437	0.271	0.557	**0.463**	0.340	0.544	0.437	0.259
	10	0.596	0.496	0.301	0.662	0.480	0.243	0.639	0.519	0.389	0.619	**0.553**	0.370
WFG2	3	1.052	0.969	0.915	1.041	0.977	0.919	1.036	0.969	0.912	1.057	**1.012**	0.942
	4	1.317	1.176	0.950	1.312	**1.207**	0.986	1.253	1.145	0.974	1.316	1.116	1.051
	6	1.672	1.566	1.392	1.697	1.608	1.482	1.660	1.554	1.287	1.659	**1.619**	1.580
	8	2.111	2.063	1.994	2.122	2.055	1.879	2.105	1.995	1.392	2.110	**2.073**	2.016
	10	2.565	2.454	2.274	2.562	2.456	2.336	2.553	2.417	1.956	2.564	**2.528**	2.483
WFG3	3	0.732	0.688	0.657	0.733	0.700	0.665	0.751	0.691	0.655	0.748	**0.713**	0.663
	4	0.837	0.780	0.712	0.848	0.801	0.749	0.803	0.756	0.711	0.908	**0.889**	0.851
	6	0.985	0.934	0.855	1.048	1.007	0.946	1.016	0.935	0.872	1.149	**1.090**	1.025
	8	1.404	1.334	1.240	1.439	1.363	1.282	1.401	1.321	1.226	1.443	**1.404**	1.320
	10	1.710	1.656	1.591	1.733	1.665	1.611	1.701	1.623	1.544	1.790	**1.717**	1.629
WFG4	3	0.532	0.501	0.464	0.544	**0.506**	0.453	0.532	0.503	0.455	0.520	0.489	0.447
	4	0.784	0.699	0.631	0.754	**0.700**	0.622	0.740	0.674	0.616	0.762	0.681	0.612
	6	1.159	1.056	0.937	1.093	1.021	0.932	1.122	1.021	0.888	1.207	**1.136**	1.033
	8	1.643	1.467	1.196	1.611	1.421	1.288	1.626	1.456	1.262	1.689	**1.568**	1.364
	10	2.074	1.715	1.547	2.029	1.620	1.429	1.980	1.669	1.498	1.949	**1.830**	1.700
WFG5	3	0.615	**0.546**	0.421	0.592	0.533	0.478	0.607	0.542	0.460	0.573	0.523	0.355
	4	0.723	0.654	0.581	0.816	**0.696**	0.559	0.715	0.625	0.556	0.779	0.684	0.607
	6	1.164	1.036	0.912	1.200	**1.040**	0.925	1.166	0.992	0.918	1.116	1.017	0.894
	8	1.565	1.462	1.390	1.560	**1.464**	1.279	1.513	1.377	1.205	1.531	1.421	1.298
	10	1.819	1.690	1.549	1.819	1.696	1.485	1.690	1.551	1.405	1.901	**1.704**	1.470
WFG6	3	0.447	**0.359**	0.277	0.440	0.352	0.293	0.400	0.351	0.274	0.428	0.348	0.280
	4	0.676	0.541	0.451	0.760	**0.568**	0.443	0.667	0.535	0.401	0.634	0.557	0.467
	6	0.940	0.740	0.624	1.078	**0.855**	0.680	0.921	0.743	0.614	0.953	0.803	0.635
	8	1.664	1.516	1.170	1.669	**1.525**	1.380	1.577	1.345	0.861	1.533	1.340	1.194
	10	1.987	1.754	1.234	2.011	**1.783**	1.469	1.881	1.585	1.114	1.843	1.675	1.513
WFG7	3	0.466	0.413	0.373	0.460	0.410	0.360	0.487	**0.418**	0.387	0.428	0.397	0.362
	4	0.719	**0.649**	0.571	0.744	0.635	0.579	0.752	0.642	0.555	0.744	0.643	0.580
	6	1.158	0.940	0.746	1.081	0.942	0.847	1.101	0.950	0.776	1.160	**1.028**	0.933
	8	1.716	1.475	1.226	1.569	1.463	1.337	1.585	1.385	1.154	1.640	**1.515**	1.348
	10	1.927	**1.767**	1.528	1.852	1.711	1.590	1.780	1.605	1.375	2.009	1.744	1.487
WFG8	3	0.369	0.340	0.303	0.396	0.345	0.300	0.356	0.329	0.284	0.409	**0.369**	0.333
	4	0.475	0.403	0.332	0.527	0.433	0.358	0.472	0.369	0.291	0.501	**0.441**	0.376
	6	0.702	0.597	0.539	0.827	**0.679**	0.584	0.705	0.596	0.499	0.686	0.604	0.530
	8	1.253	1.053	0.836	1.383	**1.138**	0.975	1.312	1.031	0.829	1.253	1.064	0.909
	10	1.683	1.367	1.135	1.718	**1.438**	1.192	1.541	1.296	1.022	1.480	1.260	1.055
WFG9	3	0.475	**0.361**	0.287	0.449	0.346	0.264	0.443	0.359	0.275	0.487	0.345	0.250
	4	0.723	**0.587**	0.465	0.709	0.577	0.410	0.717	0.581	0.462	0.700	0.570	0.443
	6	1.091	0.849	0.618	1.068	0.865	0.669	1.022	0.868	0.607	1.046	**0.922**	0.716
	8	1.560	**1.390**	1.188	1.503	1.386	1.244	1.528	1.381	1.224	1.639	1.359	1.131
	10	1.835	**1.652**	1.466	1.794	1.589	1.326	1.774	1.644	1.434	1.880	1.620	1.296

3.3 HV statistics obtained for DTLZ1^{-1}-DTLZ4^{-1} problems

Table 7. HV statistics (best, mean, and worst) of DTLZ1^{-1}-DTLZ4^{-1}

| Problems	M	SaMaO-ASF+ED			SaMaO-ASF+ASF			SaMaO-ED+ED			K-RVEA		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
DTLZ1^{-1}	3	0.226	**0.183**	0.159	0.215	0.182	0.147	0.191	0.174	0.141	0.164	0.150	0.132
	4	0.047	**0.042**	0.037	0.049	0.040	0.033	0.044	0.037	0.030	0.038	0.033	0.030
	6	1.070E-03	7.911E-04	5.654E-04	1.045E-03	**7.983E-04**	5.570E-04	8.316E-04	6.083E-04	4.277E-04	9.584E-04	7.435E-04	5.507E-04
	8	2.011E-05	**1.304E-05**	9.186E-06	1.666E-05	1.301E-05	9.096E-06	1.135E-05	7.146E-06	5.280E-06	1.252E-05	7.868E-06	5.626E-06
	10	2.185E-07	1.472E-07	8.670E-08	2.191E-07	**1.623E-07**	8.590E-08	1.085E-07	7.820E-08	4.930E-08	1.078E-07	8.599E-08	5.020E-08
DTLZ2^{-1}	3	0.659	**0.624**	0.571	0.668	0.623	0.559	0.652	0.615	0.559	0.625	0.604	0.558
	4	0.341	0.307	0.262	0.352	0.281	0.252	0.331	0.293	0.240	0.351	**0.328**	0.303
	6	0.040	0.031	0.021	0.036	0.029	0.020	0.041	0.030	0.021	0.063	**0.057**	0.052
	8	3.088E-03	1.652E-03	6.177E-04	0.003	0.001	0.001	3.151E-03	1.897E-03	9.443E-04	5.612E-03	**4.010E-03**	2.243E-03
	10	1.514E-04	5.109E-05	2.090E-05	0.000	0.000	0.000	8.057E-05	2.865E-05	4.526E-06	2.625E-04	**7.385E-05**	8.563E-06
DTLZ3^{-1}	3	0.455	0.396	0.333	0.467	**0.400**	0.339	0.477	0.387	0.327	0.332	0.296	0.266
	4	0.210	**0.158**	0.124	0.203	0.154	0.127	0.193	0.158	0.118	0.147	0.121	0.102
	6	0.021	**0.015**	0.010	0.027	0.014	0.010	0.023	0.015	0.010	0.017	0.013	0.009
	8	1.600E-03	**8.929E-04**	5.133E-04	1.983E-03	8.898E-04	4.214E-04	1.596E-03	7.387E-04	4.136E-04	1.191E-03	5.789E-04	2.905E-04
	10	6.979E-05	**3.706E-05**	1.102E-05	0.000	0.000	0.000	4.477E-05	1.682E-05	3.452E-06	3.199E-05	1.233E-05	4.708E-06
DTLZ4^{-1}	3	0.565	**0.435**	0.330	0.549	0.421	0.308	0.522	0.418	0.299	0.540	0.343	0.232
	4	0.246	0.156	0.075	0.219	**0.172**	0.094	0.207	0.129	0.035	0.140	0.087	0.034
	6	0.015	0.005	0.001	1.886E-02	**7.335E-03**	1.516E-03	2.339E-02	5.204E-03	9.794E-04	4.253E-03	1.217E-03	3.320E-04
	8	2.510E-04	6.143E-05	1.657E-05	1.780E-04	**6.363E-05**	1.135E-05	1.251E-04	4.382E-05	9.654E-06	3.232E-05	9.970E-06	3.923E-06
	10	1.368E-06	2.911E-07	6.060E-08	0.000	0.000	0.000	4.841E-06	**5.568E-07**	1.019E-07	2.498E-07	1.020E-07	4.010E-08

4 HV statistics obtained for WFG1^{-1}-WFG9^{-1} problems

Table 8. HV statistics (best, mean, and worst) of WFG1^{-1}-WFG9^{-1}

Problems	M	SaMaO-ASF+ED			SaMaO-ASF+ASF			SaMaO-ED+ED			K-RVEA		
		Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst	Best	Mean	Worst
WFG1^{-1}	3	0.044	0.033	0.023	0.068	0.035	0.025	0.066	0.035	0.023	0.053	**0.038**	0.026
	4	4.973E-03	3.517E-03	2.326E-03	7.168E-03	3.444E-03	2.346E-03	0.005	0.003	0.002	6.229E-03	**4.265E-03**	2.523E-03
	6	6.969E-05	4.158E-05	2.840E-05	1.152E-04	**4.693E-05**	2.675E-05	7.502E-05	4.288E-05	2.527E-05	7.060E-05	4.443E-05	3.201E-05
	8	8.044E-07	4.465E-07	2.730E-07	8.205E-07	4.439E-07	2.868E-07	9.937E-07	**5.060E-07**	3.370E-07	5.875E-07	4.402E-07	2.735E-07
	10	6.800E-09	4.184E-09	3.000E-09	7.100E-09	4.180E-09	3.000E-09	9.100E-09	**5.192E-09**	2.900E-09	6.200E-09	4.632E-09	2.800E-09
WFG2^{-1}	3	0.106	0.084	0.063	0.103	0.081	0.066	0.115	**0.086**	0.065	0.096	0.076	0.066
	4	3.092E-03	**6.706E-04**	0.000E+00	3.035E-03	6.118E-04	0.000	0.002	0.000	0.000	1.753E-03	4.299E-04	0.000
	6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
WFG3^{-1}	3	0.107	0.102	0.097	0.113	0.104	0.097	0.113	**0.109**	0.103	0.113	0.108	0.103
	4	0.017	**0.015**	0.013	0.018	0.015	0.013	0.016	0.014	0.011	0.017	0.015	0.012
	6	1.002E-04	3.418E-05	0.000	1.163E-04	**3.972E-05**	0.000	0.000	0.000	0.000	8.883E-05	2.743E-05	0.000
	8	5.226E-07	**2.090E-08**	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
WFG4^{-1}	3	0.333	0.294	0.274	0.349	**0.295**	0.275	0.323	0.294	0.280	0.299	0.285	0.274
	4	0.111	**0.095**	0.087	0.106	0.094	0.077	0.107	0.093	0.079	0.102	0.088	0.071
	6	3.463E-03	**1.621E-03**	0.000E+00	3.722E-03	1.327E-03	0.000E+00	3.438E-03	1.156E-03	0.000	3.267E-03	1.106E-03	0.000
	8	1.405E-04	**1.706E-05**	0.000E+00	1.161E-04	6.266E-06	0.000E+00	3.976E-05	2.215E-06	0.000	8.330E-06	5.799E-07	0.000
	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
WFG5^{-1}	3	0.424	0.409	0.394	0.427	**0.411**	0.394	0.422	0.408	0.395	0.415	0.405	0.396
	4	0.190	**0.179**	0.172	0.191	0.178	0.165	0.182	0.174	0.166	0.185	0.176	0.169
	6	0.015	**0.013**	0.009	0.015	0.012	0.009	0.015	0.012	0.009	0.014	0.012	0.010
	8	9.167E-04	**5.124E-04**	1.400E-04	8.968E-04	4.636E-04	7.753E-05	5.474E-04	2.664E-04	3.043E-05	4.371E-04	2.514E-04	3.043E-05
	10	3.546E-05	**9.060E-06**	3.498E-07	2.449E-05	8.480E-06	0.000E+00	9.760E-06	2.496E-06	0.000	9.760E-06	2.434E-06	0.000
WFG6^{-1}	3	0.495	0.486	0.477	0.517	**0.490**	0.481	0.498	0.489	0.482	0.500	0.489	0.481
	4	0.232	0.226	0.217	0.235	0.226	0.218	0.235	0.227	0.212	0.242	**0.228**	0.215
	6	0.023	**0.020**	0.016	0.022	0.020	0.016	0.022	0.019	0.015	0.023	0.019	0.016
	8	1.355E-03	**8.550E-04**	4.116E-04	1.470E-03	8.297E-04	3.650E-04	8.879E-04	5.049E-04	1.163E-04	7.671E-04	4.939E-04	1.163E-04
	10	4.013E-05	1.319E-05	1.182E-06	5.199E-05	**2.074E-05**	1.182E-06	2.447E-05	6.041E-06	0.000	2.447E-05	6.123E-06	0.000
WFG7^{-1}	3	0.400	0.366	0.355	0.433	0.372	0.349	0.442	**0.376**	0.350	0.376	0.361	0.355
	4	0.147	**0.140**	0.128	0.172	0.140	0.124	0.149	0.139	0.121	0.150	0.137	0.117
	6	0.012	**0.007**	0.004	0.009	0.007	0.003	0.008	0.006	0.003	8.579E-03	5.981E-03	2.862E-03
	8	4.778E-04	2.066E-04	0.000	5.189E-04	**2.271E-04**	0.000	2.677E-04	7.983E-05	0.000	2.677E-04	7.785E-05	0.000
	10	1.104E-05	**1.609E-06**	0.000	9.613E-06	1.503E-06	0.000	2.518E-06	1.245E-07	0.000	2.518E-06	1.300E-07	0.000
WFG8^{-1}	3	0.603	0.577	0.555	0.611	**0.595**	0.559	0.605	0.582	0.563	0.581	0.560	0.539
	4	0.284	0.274	0.262	0.307	0.277	0.263	0.291	0.275	0.263	0.294	**0.279**	0.261
	6	0.038	**0.033**	0.028	0.038	0.033	0.028	0.036	0.031	0.026	0.037	0.031	0.024
	8	0.004	**0.002**	0.001	0.003	0.002	0.001	0.002	0.002	0.001	1.867E-03	1.407E-03	8.694E-04
	10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	7.795E-05	**3.676E-05**	1.613E-05
WFG9^{-1}	3	0.586	**0.571**	0.563	0.582	0.570	0.562	0.580	0.569	0.558	0.585	0.571	0.560
	4	0.287	0.272	0.256	0.293	**0.273**	0.254	0.294	0.272	0.257	0.287	0.270	0.249
	6	0.032	0.025	0.021	0.030	**0.026**	0.021	0.032	0.025	0.020	0.028	0.024	0.020
	8	2.232E-03	1.188E-03	4.300E-04	2.259E-03	**1.242E-03**	4.678E-04	1.713E-03	9.214E-04	2.833E-04	1.268E-03	8.107E-04	2.895E-04
	10	9.420E-05	2.823E-05	4.879E-06	9.008E-05	**2.834E-05**	1.201E-06	8.089E-05	1.793E-05	1.215E-06	5.747E-05	1.667E-05	1.201E-06

5 Results obtained for the engineering design optimization problem

We present the results after 500, 1000, 2000 and 4000 function evaluations. It is interesting to observe from the matrix plots in Fig. 1-Fig. 4 that SaMaO$_{ASF-ED}$ delivered a diverse set of non-dominated designs with all possible engine types at every checkpoint, whereas MaO could only deliver non-dominated designs involving one type of engine at 500 evaluations, two types of engines at 1000 evaluations and all types of engines at 2000 evaluations and onwards.
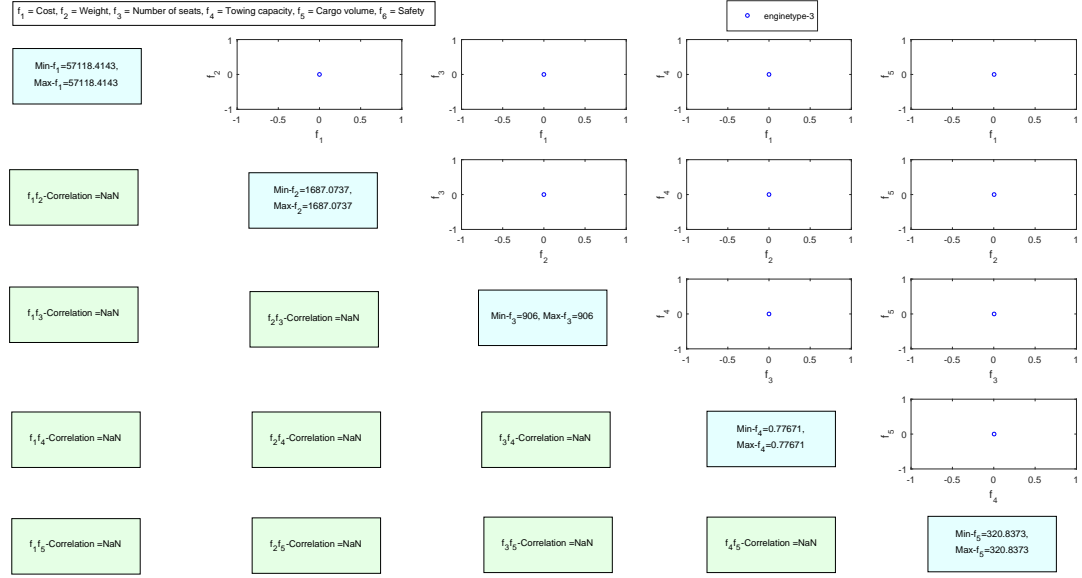
(a)
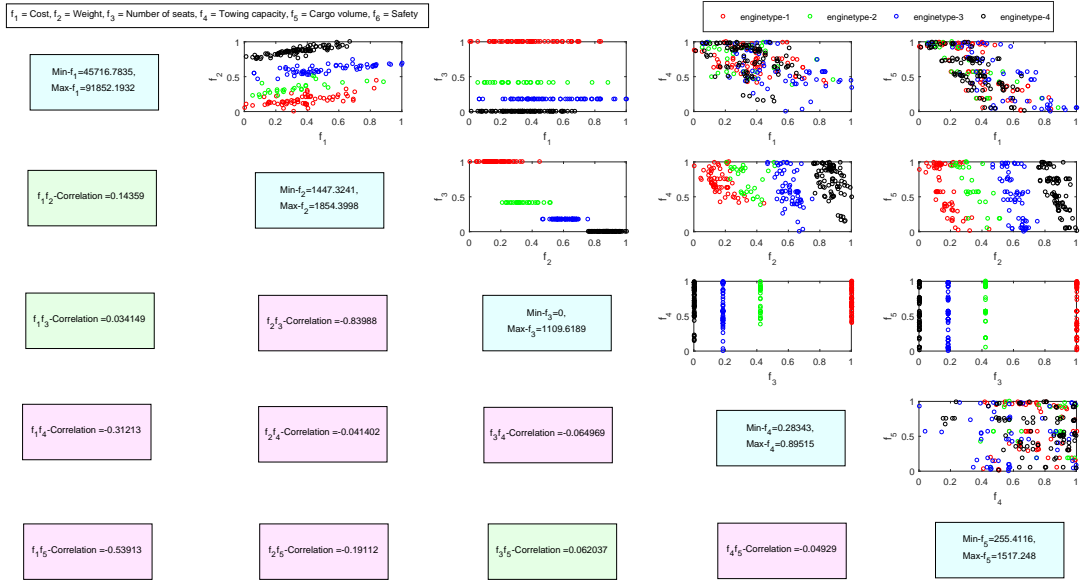
(b)

Fig. 1. Results obtained after 500 function evaluations: (a) MaO, (b) SaMaO$_{ASF-ED}$
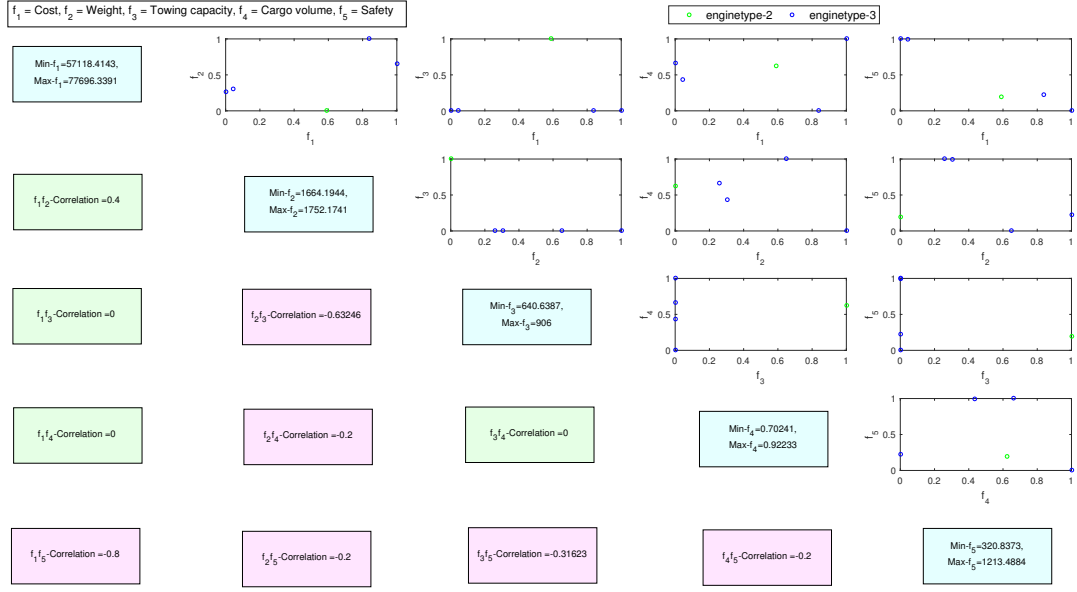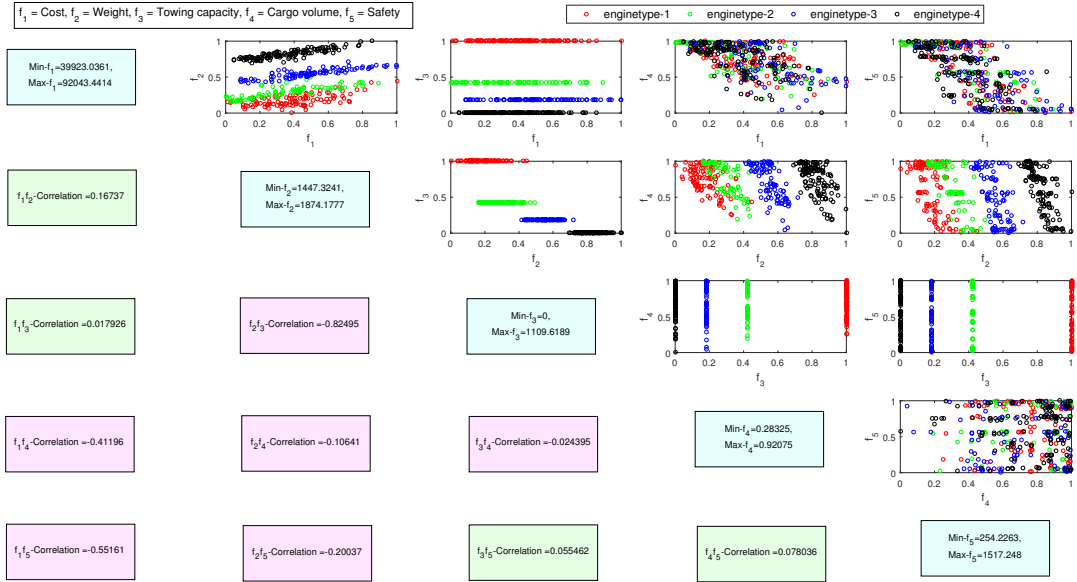
Fig. 2.   Results obtained after 1000 function evaluations: (a) MaO, (b) SaMaO$_{\text{ASF-ED}}$
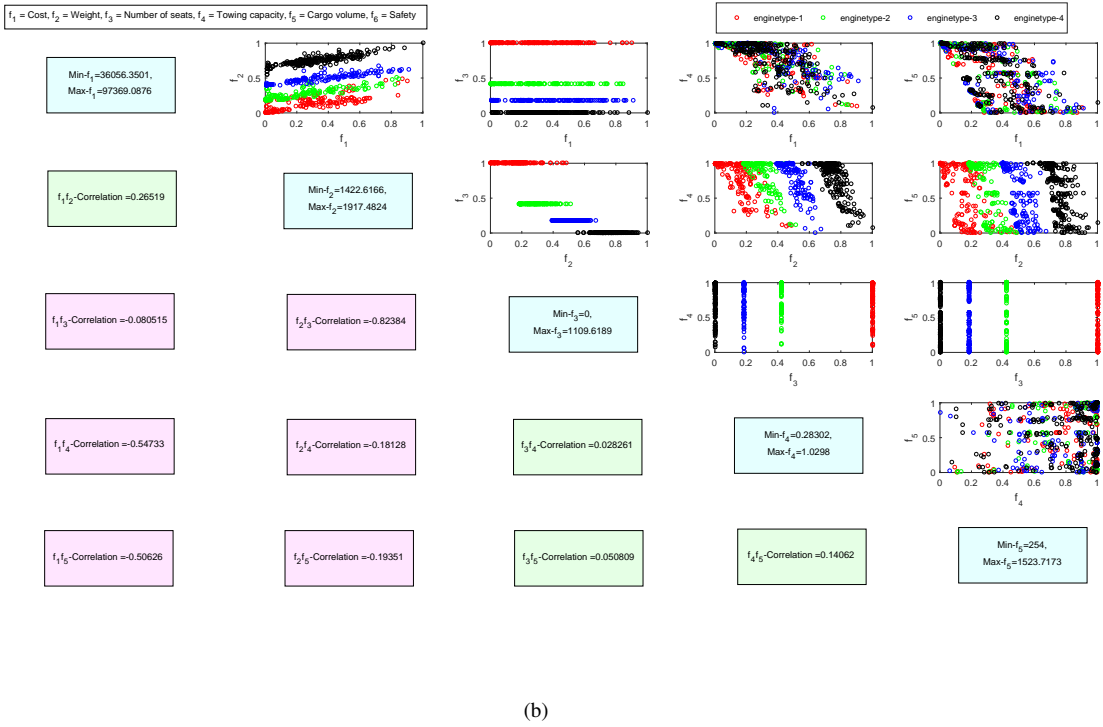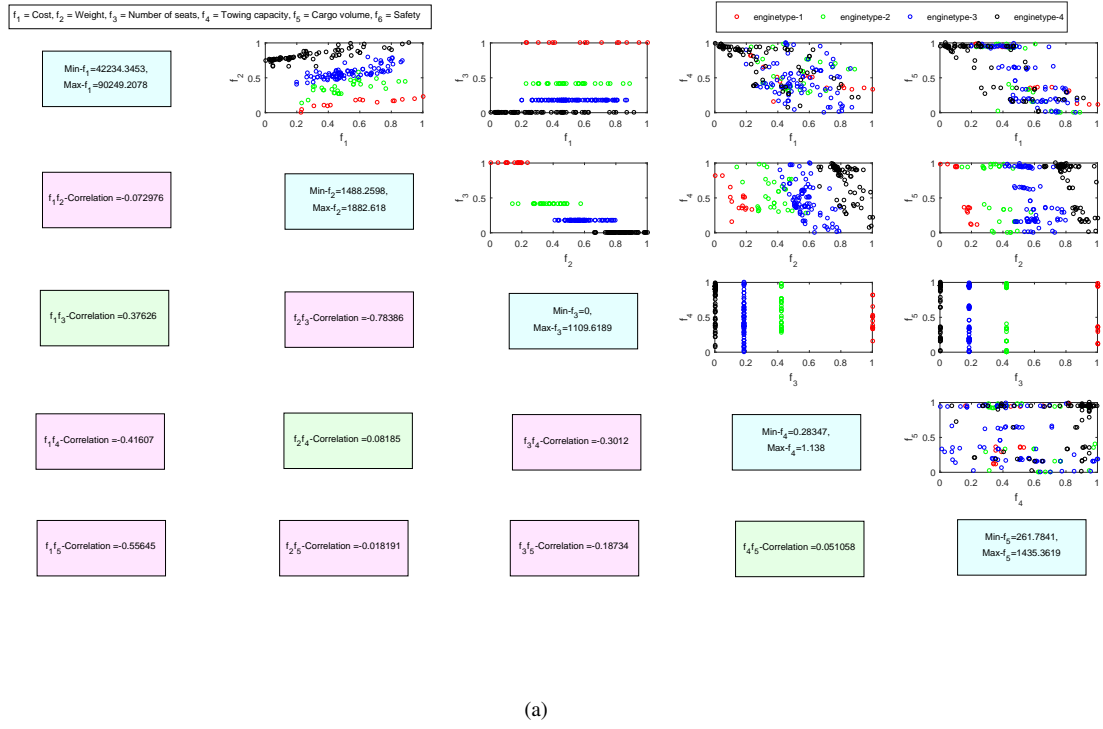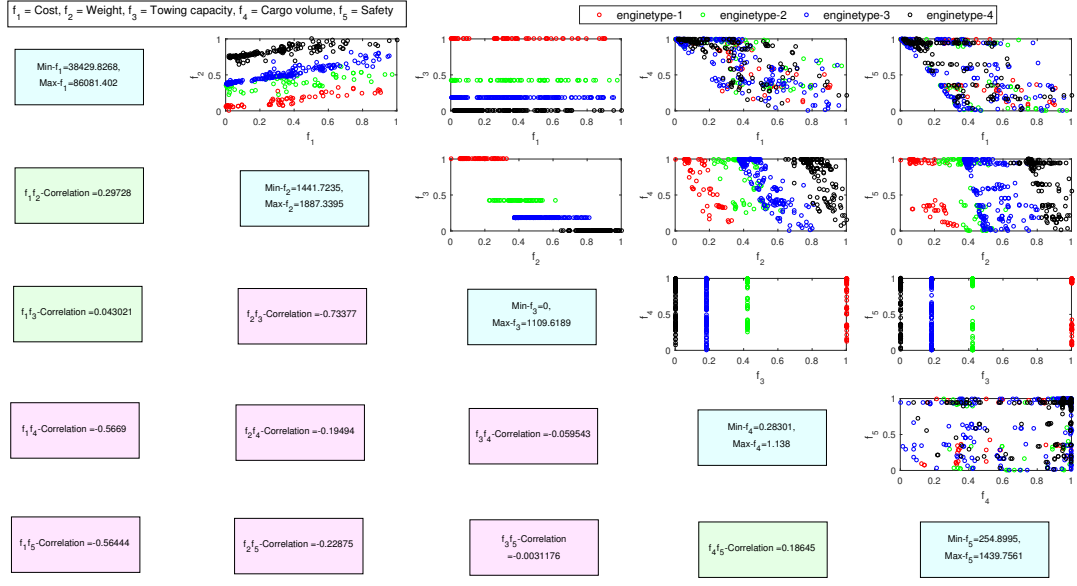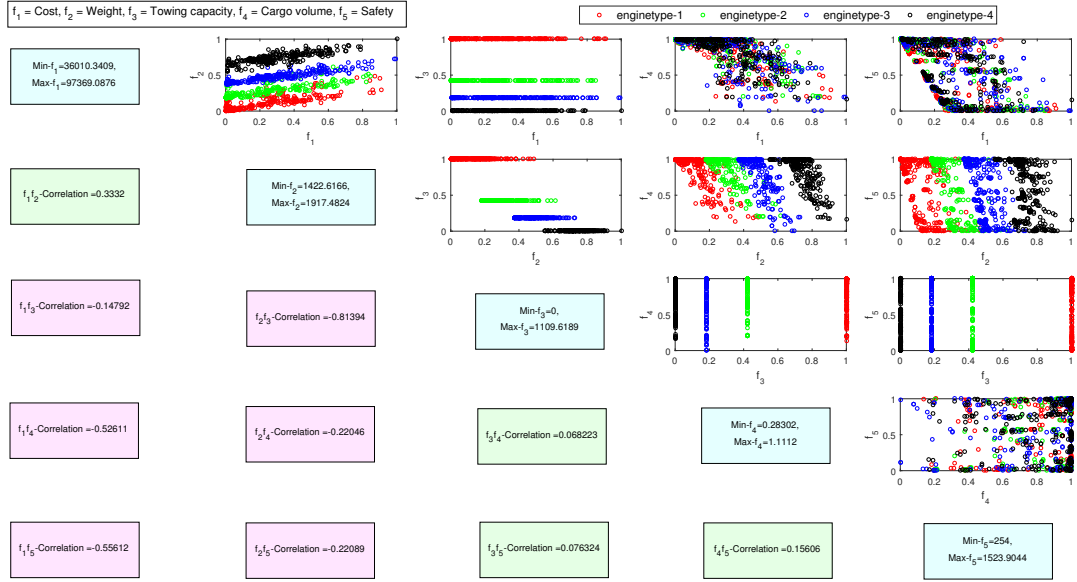
Fig. 3. Results obtained after 2000 function evaluations: (a) MaO, (b) SaMaO$_{\text{ASF-ED}}$

Fig. 4.   Results obtained after 4000 function evaluations: (a) MaO, (b) SaMaO$_{\text{ASF-ED}}$

We also compare the HV delievered by $SaMaO_{ASF-ED}$ and MaO at the start, 200, 500, 1000, 2000, 3000 and 4000 evaluation budget which is shown in Fig. 5:
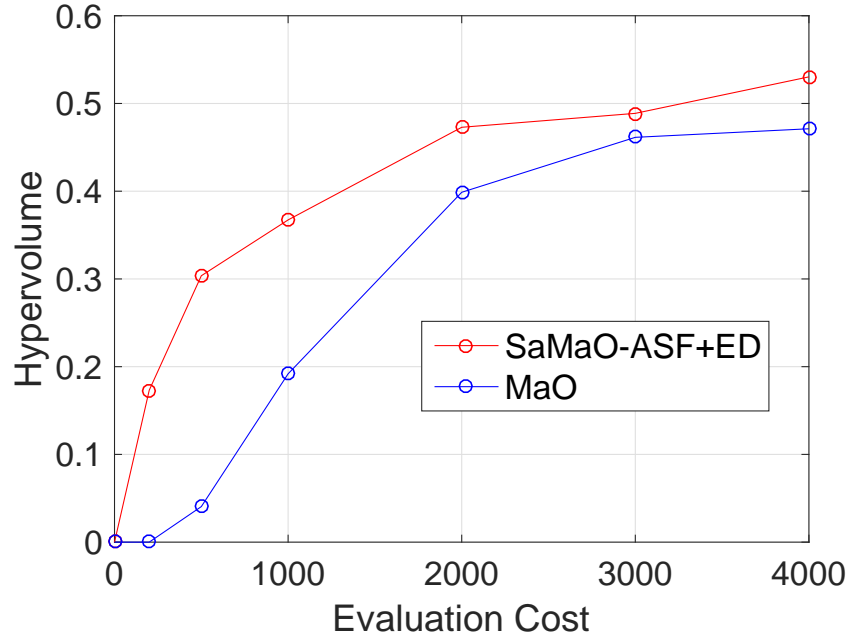

Fig. 5.   Comparison of MaO and $SaMaO_{ASF-ED}$ in terms of HV values delivered at different evaluations

In this comparison, the reference point for HV computation is taken as 1.1 times the nadir point computed using the combined set of solutions obtained at the end of 4000 evaluations using both $SaMaO_{ASF-ED}$ and MaO. The obtained ideal and nadir point at the end of 4000 evaluation budget is:

Z^I: 1.0E+04×(3.6010, 0.1423, -0.1110, -0.0001, -0.1524) and Z^N: 1.0E+04×(9.7369, 0.1917, 0, -0.00003, -0.0254).

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