Econ 512 HW 6

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The codes and comments have been specified in the MATLAB file hw6_512_v2.m.

1 Question 1: Formulation of the Dynamic Programming Problem

Based on the notations given in the question, the firm's Bellman equation can be written as follows:

$$V(K,p) = \max_{K' \in \{0,1....K\}} p(K-K') - 0.2(K-K')^{1.5} + \delta * E_{p'|p} V(K',p'); \quad (1)$$

Here, $p' = 0.5 + 0.5p + u$ and u is normally distributed with mean 0 and std deviation 0.1.

- State Variables: K and p. Here, K is the stock of lumber at the beginning of the period and p is the current price of lumber.
- Policy Variable: K' - the amount of lumber that they choose to leave for the next period. Hence, the amount of harvest this period is (K - K').
- State Space: $K \in \{0, 1, ..., 100\}$ and $p \in (-\inf, \inf)$.
- Set of feasible actions: $K' \in \{0, ..., K\}$.
- As per the formulation in the Bellman equation, the state variable that represents the stock of lumber next period is the policy variable this period (K').
- The price of lumber follows an AR(1) process, as given in the question. We approximate the space of p by a finite number of discrete points by the Tauchen representation; the grid points and the corresponding transition probabilities are derived in Question 2.

2 Question 2: Grid Points & Transition Probabilities

Based on the AR(1) process given, $p' = 0.5 + 0.5p + u \implies p' = 0.5p + (0.5 + u)$ $\implies p' = 0.5p + u'$, where u' is normally distributed with mean 0.5 and std deviation 0.1.

Hence, the parameters of the Tauchen procedure are: $N = 21$, $\mu = 0.5$, $\rho = 0.5$ and $\sigma = 0.1$.

3 Question 3: Algorithm for Value Function Iteration

1. Fix some K, where $K \in \{0, 1, ..., 100\}$. The set of feasible actions then becomes $K' \in \{0, 1, ..., K\}$.

2. Given K, I then calculate the profit matrix. The profit matrix has 21 rows (no. of grid points for price) and $(K + 1)$ columns (no. of feasible actions given K). Each cell (i,j) in the profit matrix corresponds to profit from the 'i'th point in the price grid and 'j'th element of the feasible action space.

3. Start with an initial guess of the value function. In the code, I have taken the initial guess to be a matrix of zeros with the appropriate dimensions.

4. I then calculate the matrix r which corresponds to the RHS of the Bellman equation [current profit (calculated in step 3) + discounted expected future profits given V (based on the transition matrix for prices derived in Question 2), for each element in the price grid and for each element in the feasible action space]. More specifically,

$$r = profit + delta * prob * V; \tag{2}$$

5. I then calculate the maximum of profits across all feasible elements in the action space, for each element in the price grid.

6. Compare the vector obtained in step 5 to the initial guess of the value function in step 3. If their difference is greater than 1e-6, then update V with that obtained in step 5 and repeat steps 4-5. If their difference is less than 1e-6, then the process is said to have converged.

7. Once the process converges, I calculate the value function and the associated policy function for different values of p, given K.

8. Repeat steps 1-6 for different values of K to calculate the value function and policy function associated with each value of K.

Figure 1 below displays the results of the VFI. Different lines in the first graph corresponds to different current prices of lumber. Figure 2 shows the relationship between the value of the firm and initial stock of lumber for $p \in \{0.9, 1, 1.1\}$.

4 Question 4: Algorithm

The algorithm used here is similar to that used for VFI in Question 3, except for the following modifications.

1. In Step 1, I fix K, where K belongs to the set of all possible values of the policy function derived in Q3. As before, the set of feasible actions then becomes $K' \in \{0, 1, ..., K\}$.

2. Given K, I then calculate the profit matrix. The profit matrix has 21 rows (no. of grid points for price) and $(K+1)$ columns (no. of feasible actions given K). The profit matrix represents the discounted expected value of profits in period (t+1), where expectation is calculated from the perspective of period t. Hence, the profit matrix corresponds to profits from all possible prices in the current period and from all possible values of the control variable in period (t+1).

3. Remaining steps are same as in Question 3.

Figure 3 displays the relationship between price of lumber in period t and the control variable in period (t+1), for different amounts of lumber left in stock after period t. Different lines in the graph represent different amounts of lumber left in stock at the beginning of period t+1.

Figure 1: Relationship between optimal value and the initial stock of lumber. Different lines in the graph correspond to different current prices of lumber.

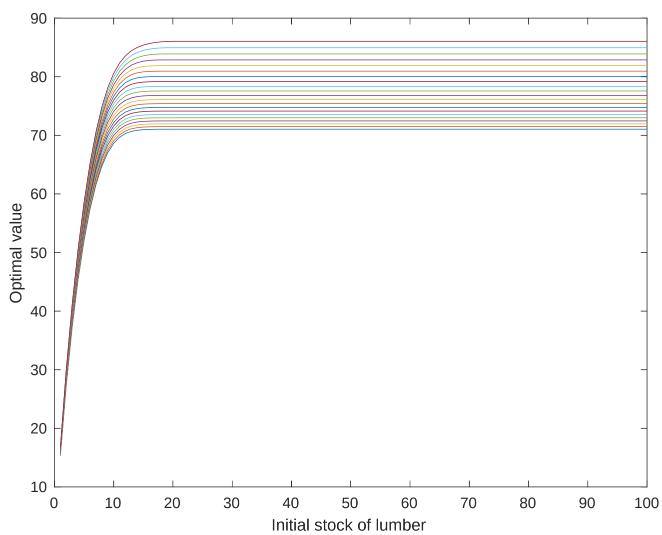

Figure 2: Relationship between optimal value and the initial stock of lumber for p = 0.9,1 and 1.1

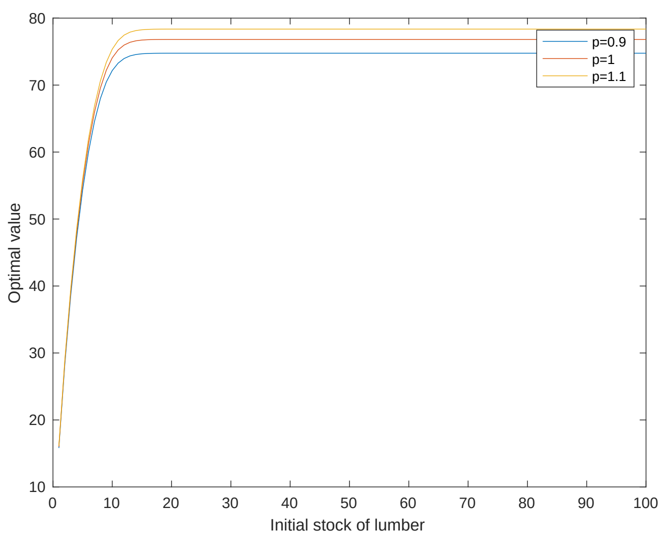

Figure 3: Relationship between price of lumber in period t and the control variable in period (t+1), for different amounts of lumber left in stock after period t. Different lines in the graph capture this relationship for different value of the state variable in period t+1.

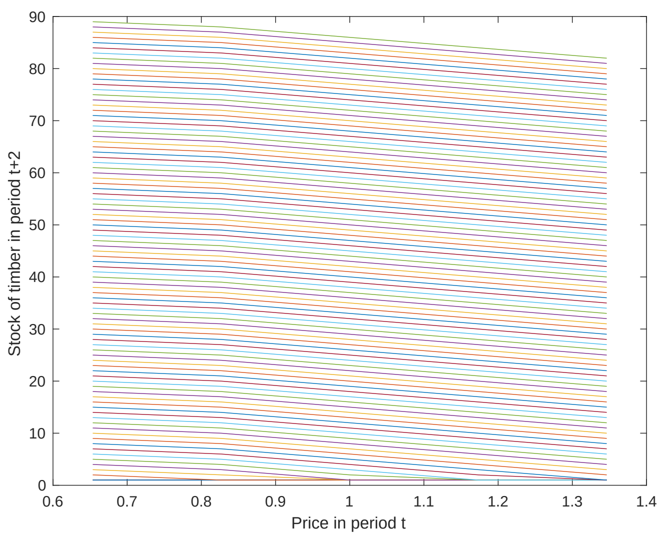

Figure 4: Q6: Relationship between optimal value and the initial stock of lumber. Different lines in the graph correspond to different current prices of lumber.

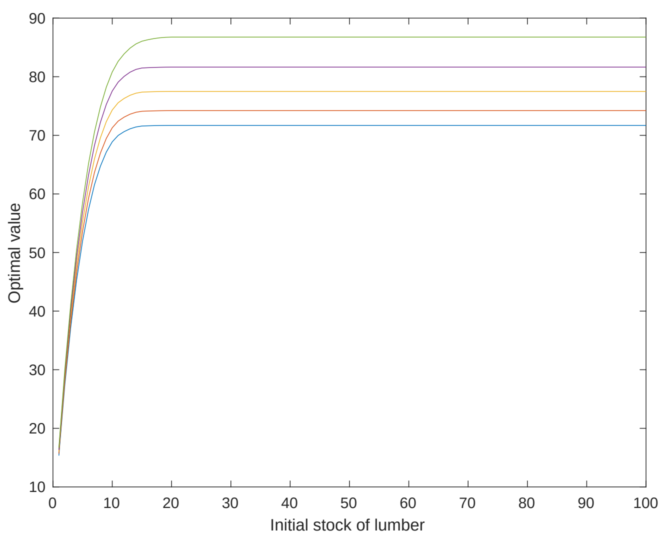

Figure 5: Q6: Relationship between optimal value and the initial stock of lumber for p = 0.9,1 and 1.1

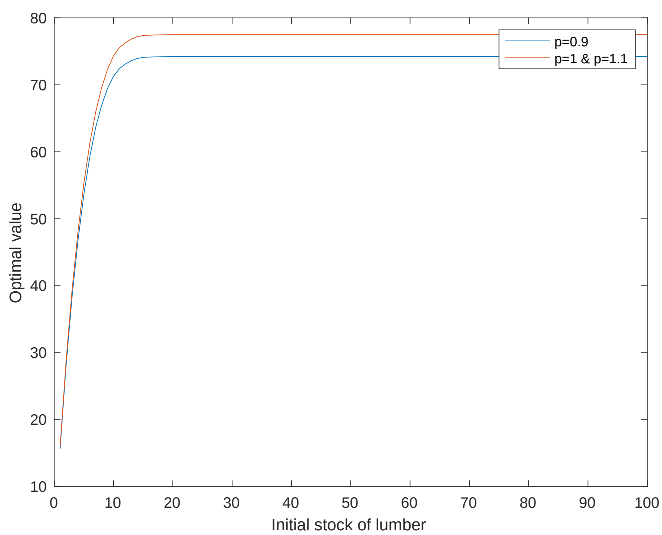

Figure 6: Q6: Relationship between price of lumber in period t and the control variable in period (t+1), for different amounts of lumber left in stock after period t. Different lines in the graph correspond to the relationship for different value of the state variable in period t+1.