# 1-Probability Theory:

## Probability

- Probability implies 'likelihood' or 'chance'. When an event is certain to happen then the probability of occurrence of that event is 1 and when it is certain that the event cannot happen then the probability of that event is 0.

- Hence the value of probability ranges from 0 to 1.

Formula

**P(A)=(Number of favourable cases / Total number of equally likely cases)=m/n**

Thus to calculate the probability we need information on number of favorable cases and total number of equally likely cases. This can he explained using following example.

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**Example**
**Problem Statement:**

**A coin is tossed. What is the probability of getting a head?**

**Solution:**

**Total number of equally likely outcomes (n) = 2 (i.e. head or tail)**

**Number of outcomes favorable to head (m) = 1**

**P(head)=1/2 = 0.5**

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## What Is a Joint Probability?

- Joint probability is a statistical measure that calculates the likelihood of two events occurring together and at the same point in time.
- Joint probability is the probability of event Y occurring at the same time that event X occurs.

## The Formula for Joint Probability Is

Notation for joint probability can take a few different forms. The following formula represents the probability of events intersection:

$$P\ \left(X \bigcap Y\right)$$

**where:**

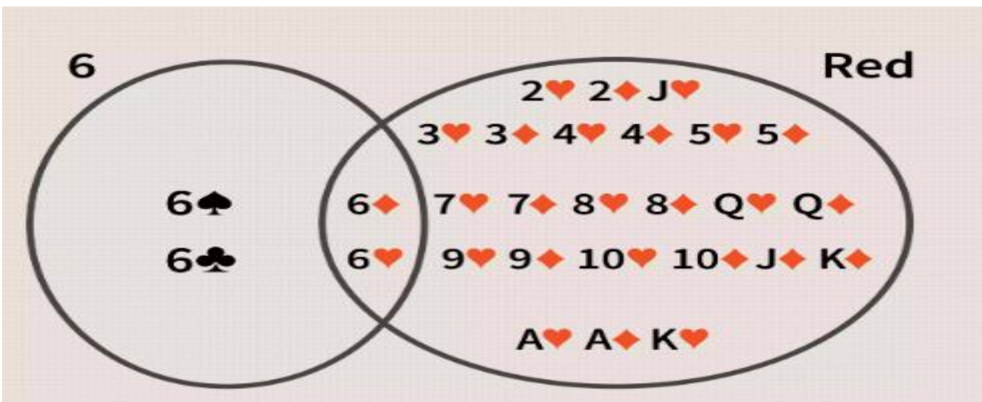
$X, Y = \text{Two different events that intersect}$

$P(X \text{ and } Y), P(XY) = \text{The joint probability of X and Y}$

Joint probability is a measure of two events happening at the same time, and can only be applied to situations where more than one observation can occur at the same time. For example, from a deck of 52 cards, the joint probability of picking up a card that is both red and 6 is P(6 ∩ red) = 2/52 = 1/26, since a deck of cards has two red sixes—the six of hearts and the six of diamonds. Because the events "6" and "red" are independent in this example, you can also use the following formula to calculate the joint probability:

$$P(6 \cap red) = P(6) \times P(red) = 4/52 \times 26/52 = 1/26$$

The symbol "∩" in a joint probability is referred to as an intersection. The




**Conditional Probability**

- The ***conditional probability*** of an event *B* is the probability that the event will occur given the knowledge that an event *A* has already occurred.
- This probability is written *P(B|A)*, notation for the *probability of B given A*.
- In the case where events *A* and *B* are *independent* (where event *A* has no effect on the probability of event *B*), the conditional probability of event *B* given event *A* is simply the probability of event *B*, that is *P(B)*.

**If events *A* and *B* are not independent, then the probability of the *intersection of A and B* (the probability that both events occur) is defined by**

$$P(A \text{ and } B) = P(A)P(B|A).$$

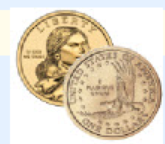
From this definition, the conditional probability $P(B|A)$ is easily obtained by dividing by $P(A)$:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

## Independent Events

Events can be "Independent", meaning each event is **not affected** by any other events.

Example: Tossing a coin.



Each toss of a coin is a perfect isolated thing.

What it did in the past will not affect the current toss.

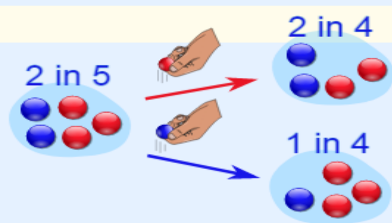
The chance is simply 1-in-2, or 50%, just like ANY toss of the coin.

So each toss is an **Independent Event**.

## Dependent Events

But events can also be "dependent" ... which means they **can be affected by previous events** ...

Example: Marbles in a Bag



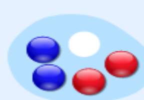

2 blue and 3 red marbles are in a bag.

What are the chances of getting a blue marble?

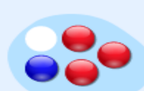
The chance is **2 in 5**

**But after taking one out** the chances change!

So the next time:



if we got a **red** marble before, then the chance of a blue marble next is **2 in 4**



if we got a **blue** marble before, then the chance of a blue marble next is **1 in 4**

This is because we are **removing** marbles from the bag.

So the next event **depends on** what happened in the previous event, and is called **dependent**.

**Bayes's theorem**

- In statistics and probability theory, the Bayes' theorem (also known as the Bayes' rule) is a mathematical formula used to determine the conditional probability of events.
- Essentially, the Bayes' theorem describes the probability of an event based on prior knowledge of the conditions that might be relevant to the event.

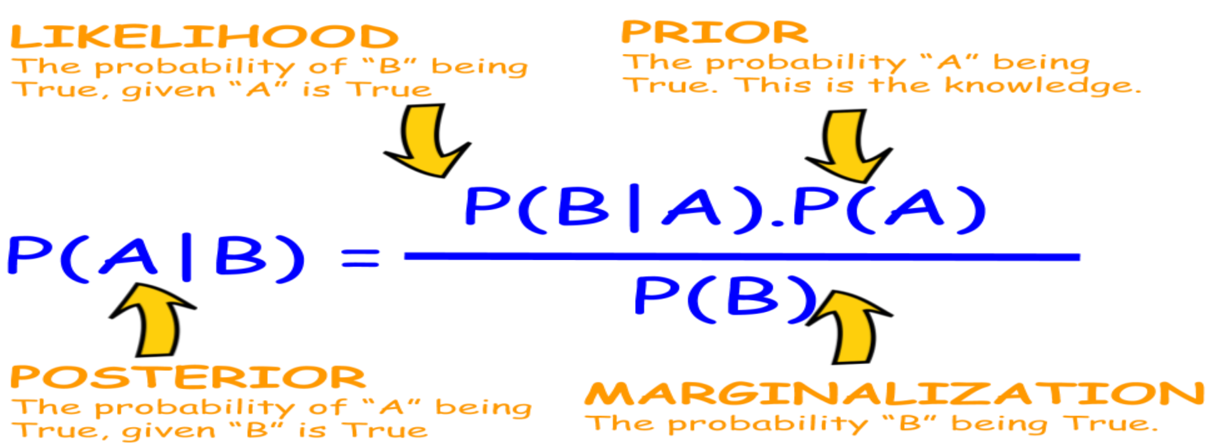
$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

$A, B$ = events

$P(A|B)$ = probability of A given B is true

$P(B|A)$ = probability of B given A is true

$P(A), P(B)$ = the independent probabilities of A and B




Example:
Imagine 100 people at a party, and you tally how many wear pink or not, and if a man or not, and get these numbers:

| | Pink | notPink |
|---|---|---|
| Man | 5 | 35 |
| notMan | 20 | 40 |

Bayes' Theorem is based off just those 4 numbers!

| | Pink | notPink | |
|---|---|---|---|
| Man | 5 | 35 | 40 |
| notMan | 20 | 40 | 60 |
| | 25 | 75 | 100 |

And calculate some probabilities:

- the probability of being a man is $P(Man) = \frac{40}{100} = 0.4$
- the probability of wearing pink is $P(Pink) = \frac{25}{100} = 0.25$
- the probability that a man wears pink is $P(Pink|Man) = \frac{5}{40} = 0.125$
- the probability that a person wearing pink is a man **P(Man|Pink) = ...**

But all your data is **ripped up**! Only 3 values survive:

- **P(Man) = 0.4,**
- **P(Pink) = 0.25 and**
- **P(Pink|Man) = 0.125**

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Imagine a pink-wearing guest leaves money behind ... was it a man? We can answer this question using Bayes' Theorem:

$$P(Man|Pink) = \frac{P(Man)\ P(Pink|Man)}{P(Pink)}$$

$$P(Man|Pink) = \frac{0.4 \times 0.125}{0.25} = 0.2$$

**Cumulative Probability**

- A cumulative probability refers to the probability that the value of a random variable falls within a specified range.
- Frequently, cumulative probabilities refer to the probability that a random variable is less than or equal to a specified value.
- Consider a coin flip experiment. If we flip a coin two times, we might ask: What is the probability that the coin flips would result in one or fewer heads? The answer would be a cumulative probability.
- It would be the probability that the coin flip results in zero heads plus the probability that the coin flip results in one head. Thus, the cumulative probability would equal:

$$P(X \leq 1) = P(X = 0) + P(X = 1) = 0.25 + 0.50 = 0.75$$

The table below shows both the probabilities and the cumulative probabilities associated with this experiment.

| Number of heads | Probability | Cumulative Probability |
|---|---|---|
| 0 | 0.25 | 0.25 |
| 1 | 0.50 | 0.75 |
| 2 | 0.25 | 1.00 |