CS 412

FEB 25TH – NEURAL NETWORKS

HTF – CHAPTER 11

Neural Networks

Networks of processing units (neurons) with connections (synapses) between them

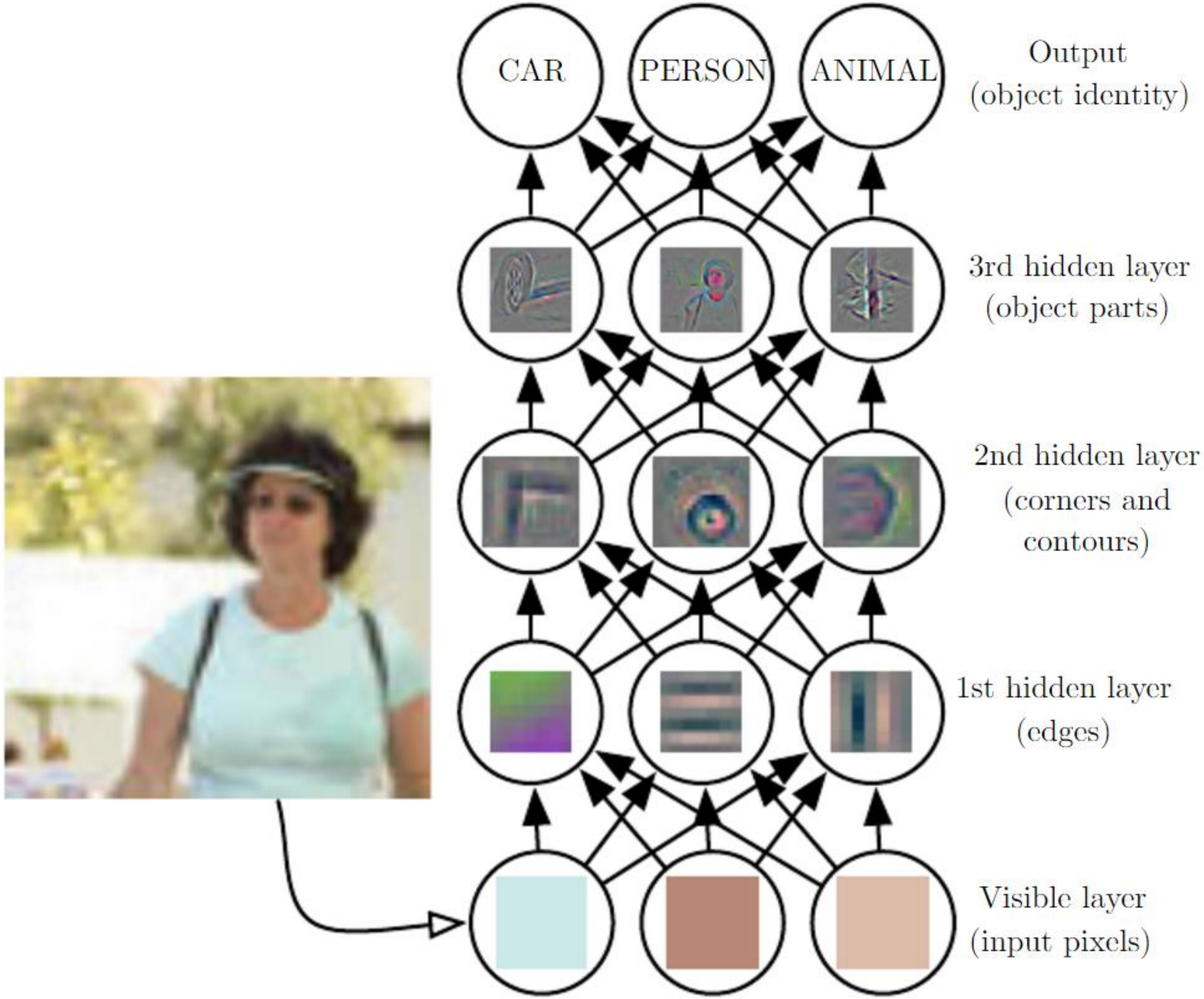
Large number of neurons: 10^{10}

Large connectitivity: 10^5

Parallel processing

Distributed computation/memory

Robust to noise, failures


CAR
PERSON
ANIMAL
Output
(object identity)
3rd hidden layer
(object parts)
2nd hidden layer
(corners and
contours)
1st hidden layer
(edges)
Visible layer
(input pixels)

Understanding the Brain

Levels of analysis for an information processing system such as sorting (Marr, 1982)

1. Computational theory: goal of computation and abstract definition of the task
2. Representation and algorithm: how to represent input and output, and how to transform from input to output
3. Hardware implementation

Reverse engineering: From hardware to theory

Parallel processing: SIMD vs MIMD

Neural net: SIMD with modifiable local memory

Learning: Update by training/experience

GPU CPU

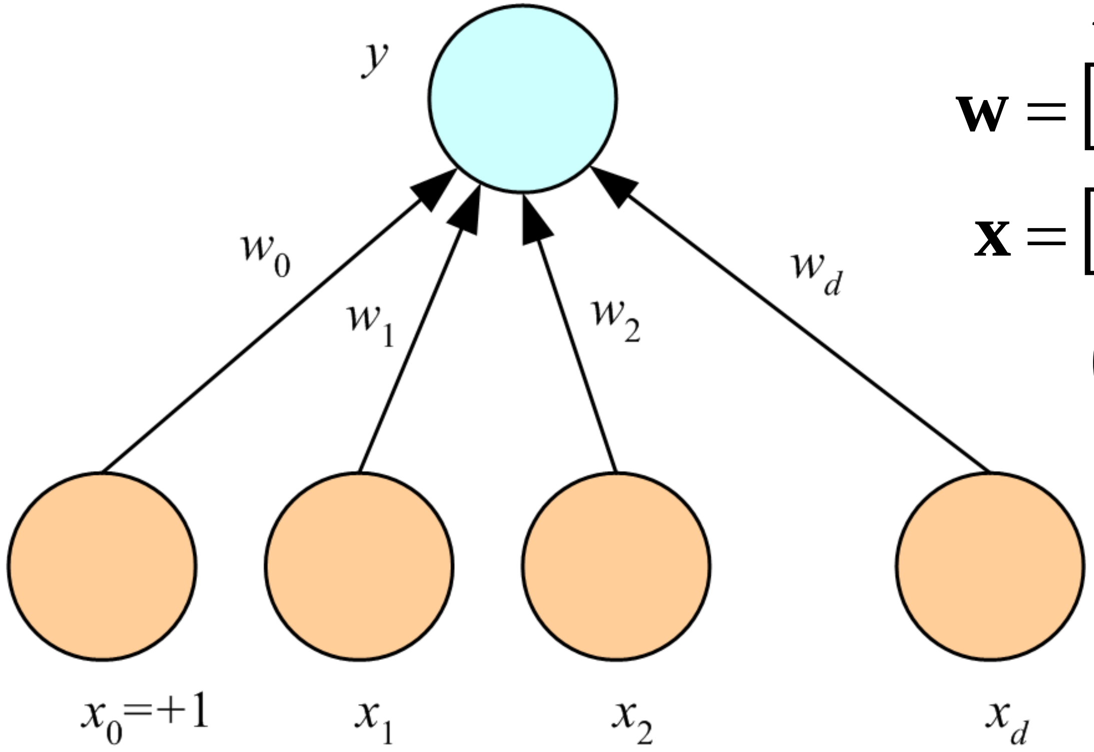
Perceptron

$$y = \sum_{j=1}^{d} w_j x_j + w_0 = \mathbf{w}^T \mathbf{x}$$

$$\mathbf{w} = [w_0, w_1, \ldots, w_d]^T$$

$$\mathbf{x} = [1, x_1, \ldots, x_d]^T$$

(Rosenblatt, 1962)



"adam"

Perceptron

What is the single-layer perceptron?

- Just the linear discriminator
- No support vector constraints

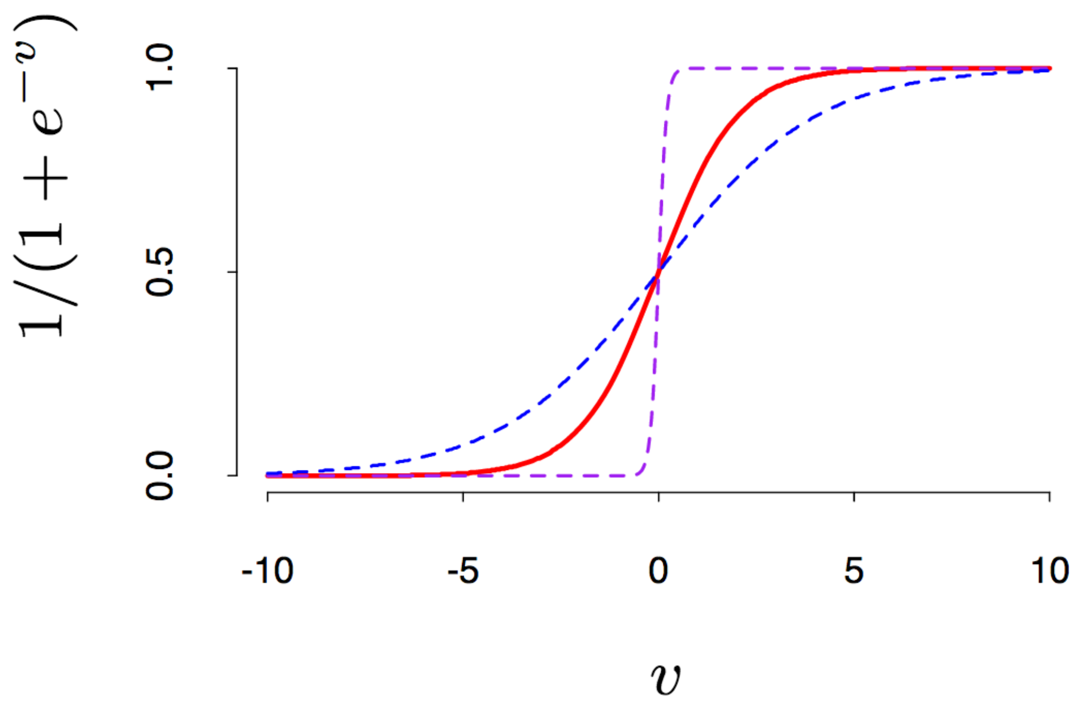
"sgd"

How do we train it?

- Stochastic gradient descent
  - Small changes based on the data – minimizing loss (what loss should we minimize?)
  - Update = learning factor*(DesiredOutput – Actual Output) * Input

$$\Delta w_{ij}^t = \eta\left(r_i^t - y_i^t\right)x_j^t$$

  - Descent is moderated by our learning factor (eta)



Perceptron

How do we make a regression model into a classification model?

- Activation function (here: sigmoid)
- Like logistic regression, there is no unique solution, so we also have to consider the rate at which the sigmoid transitions, this is the **activation rate**, s (here: ½,1,10)

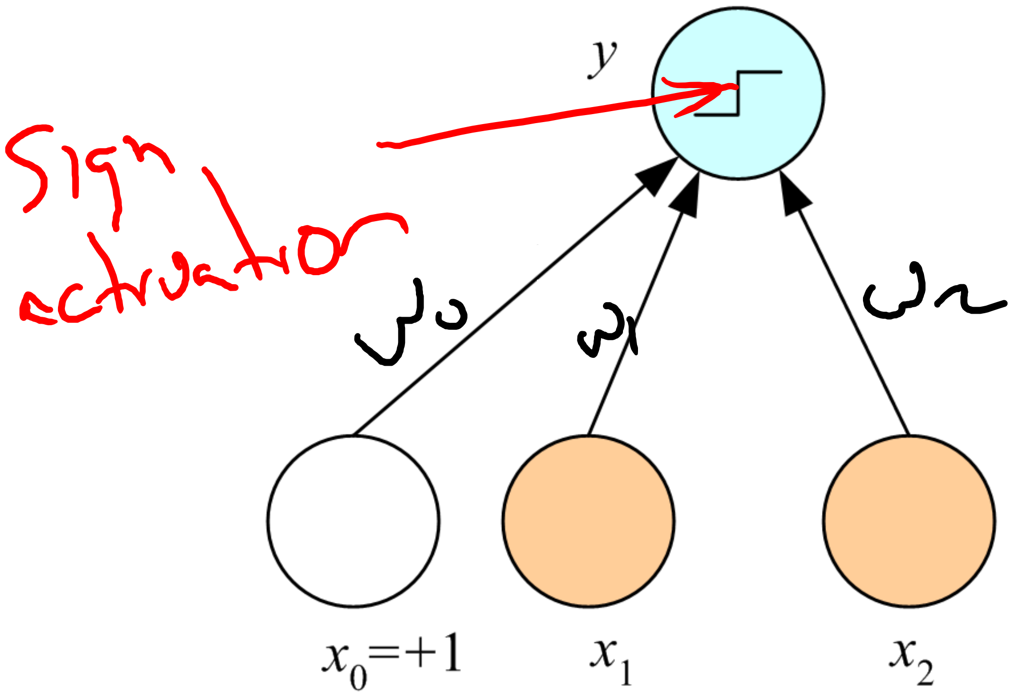
Why do we prefer this to the sign function?

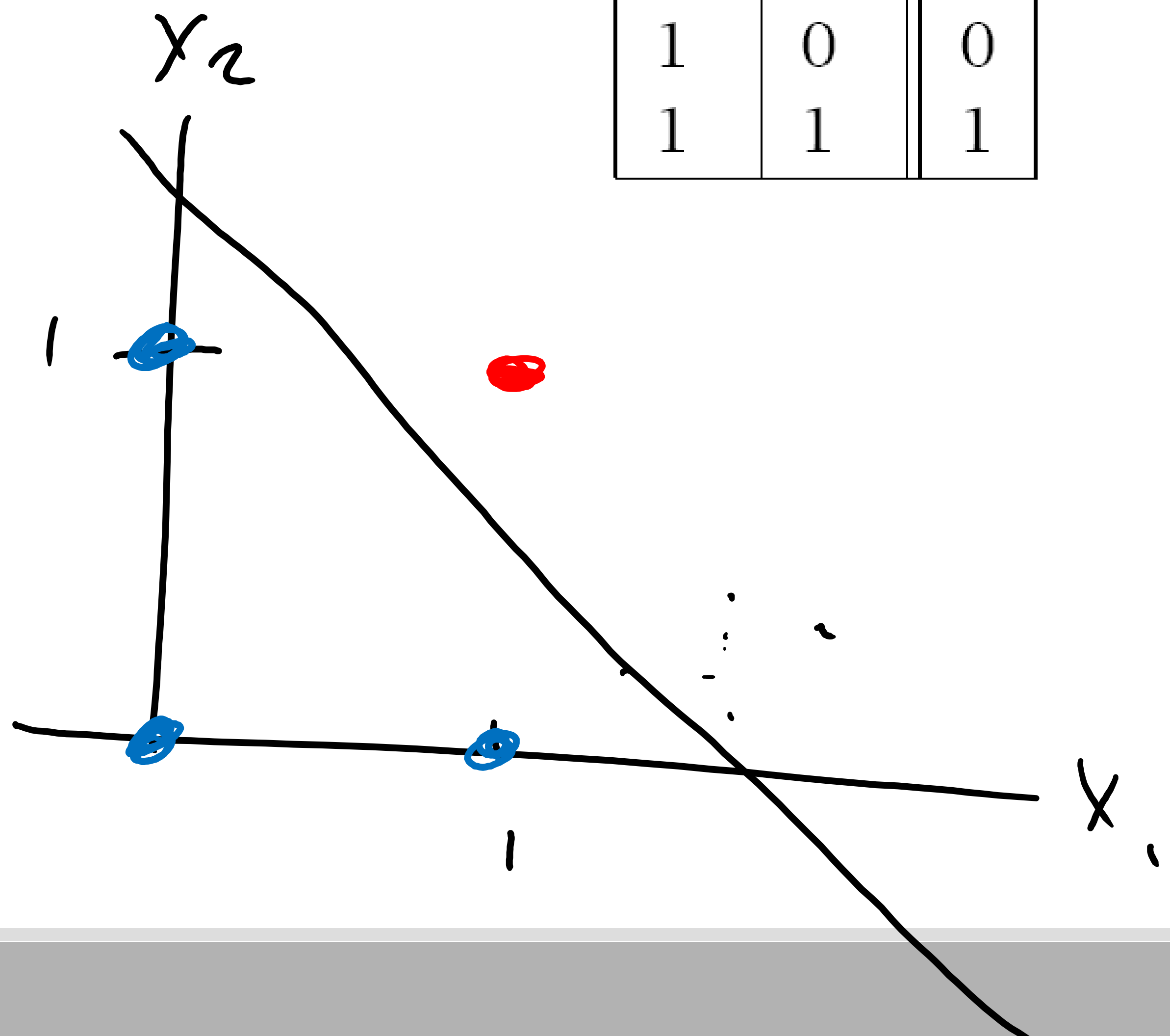
- They are differentiable and non-linear

Learning Boolean AND

x_1	x_2	r
0	0	0
0	1	0
1	0	0
1	1	1

What are the weights for this perceptron?
(purple line)





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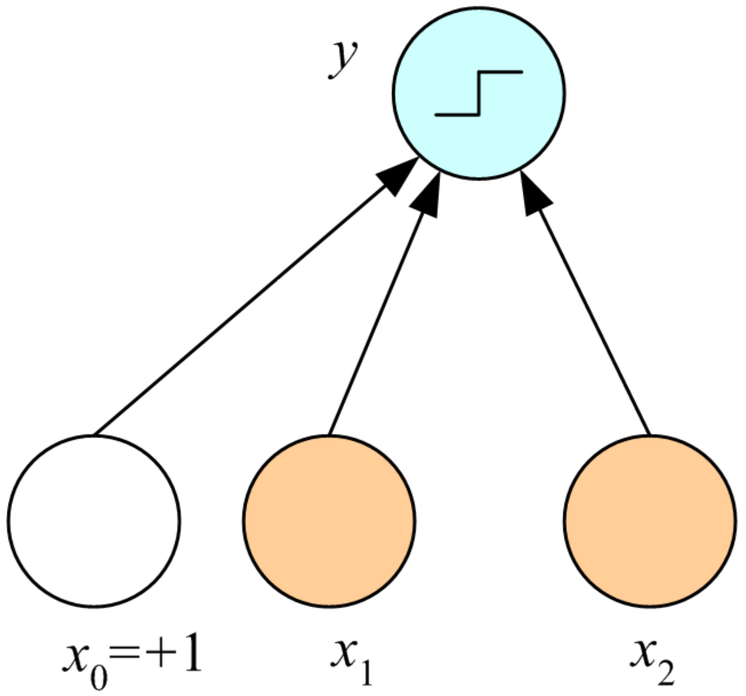

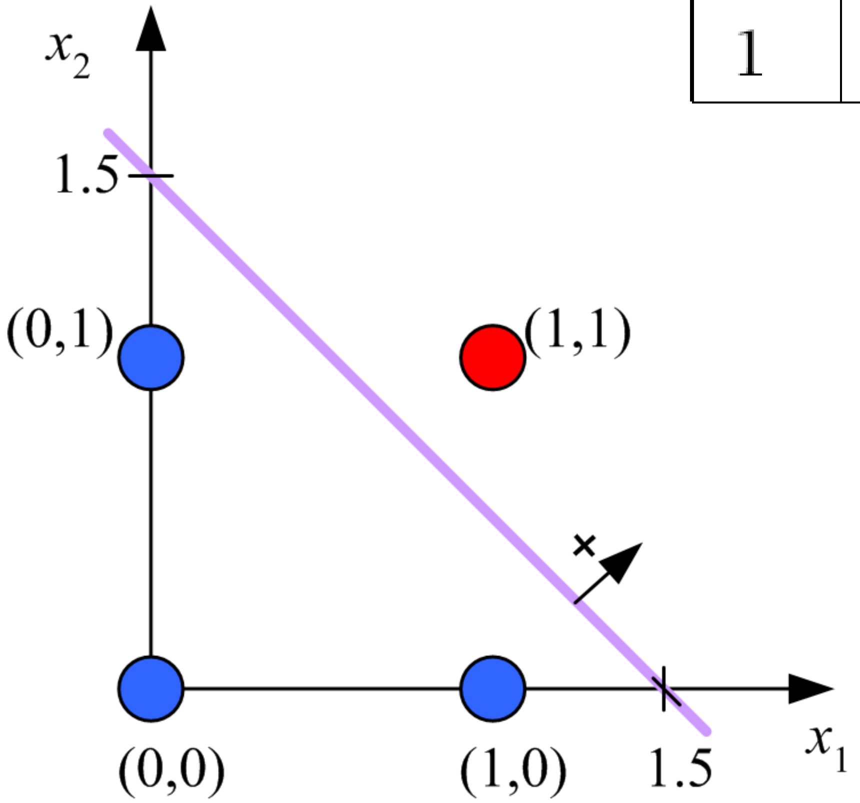

$w_0 + w_1 x_1 + w_2 x_2 = 0$

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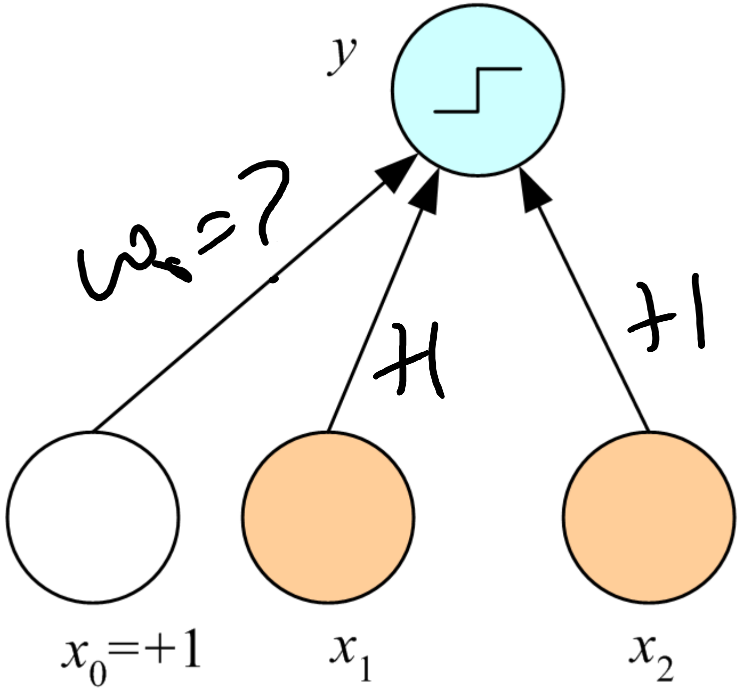

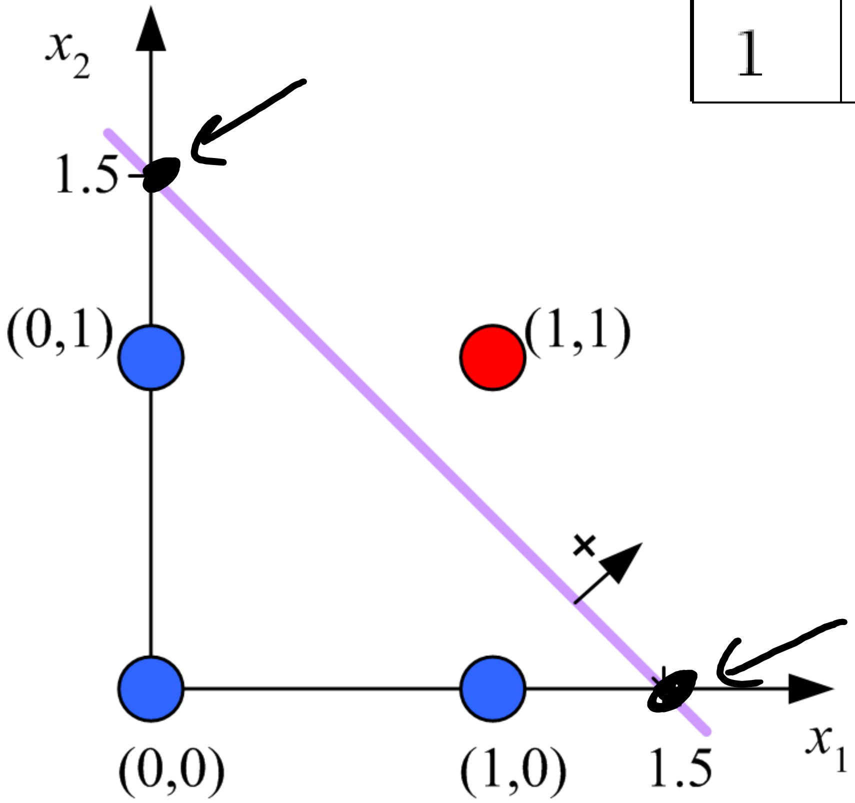

$W_0 + 1.5 = 0$

Learning Boolean AND

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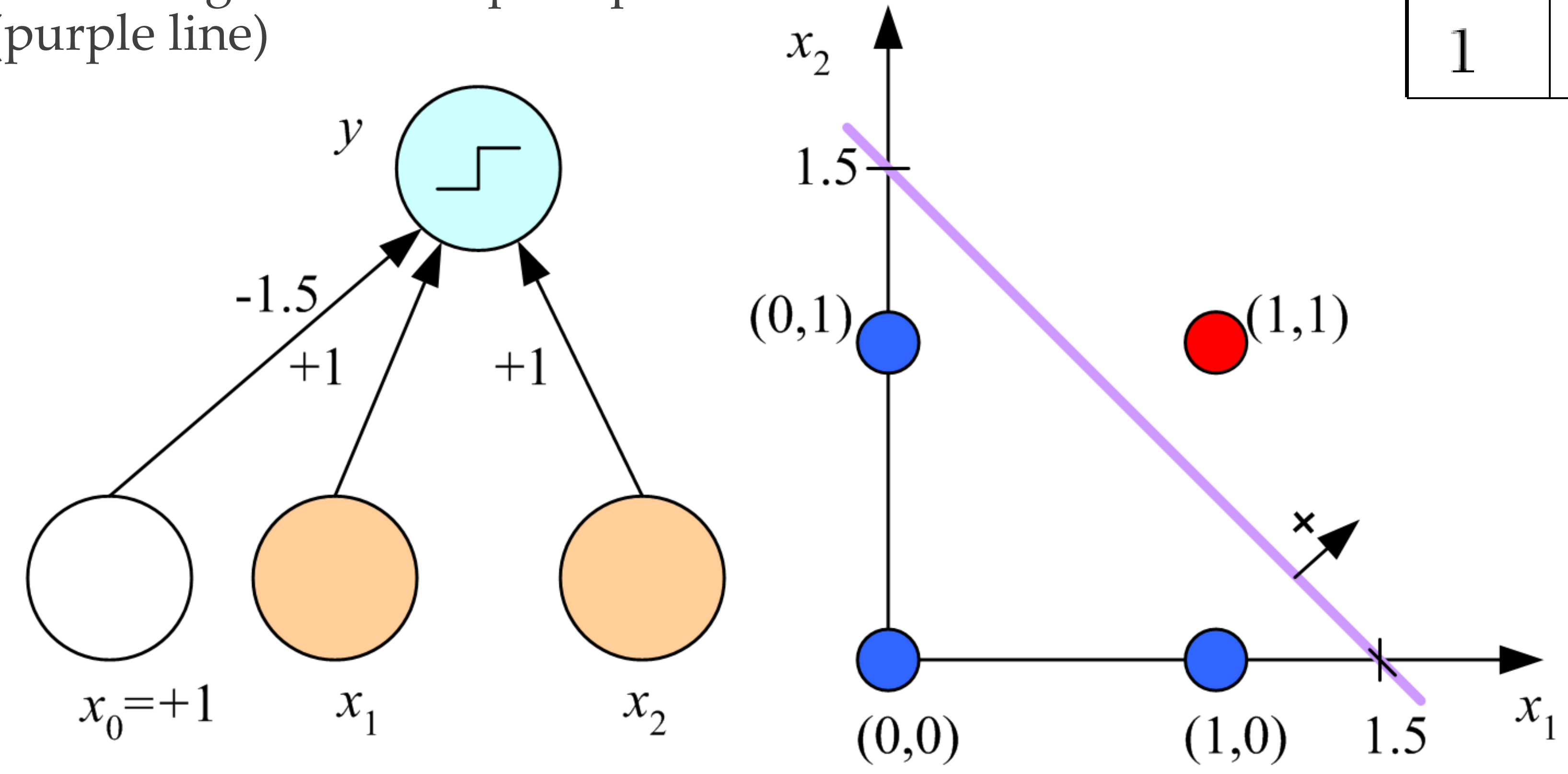
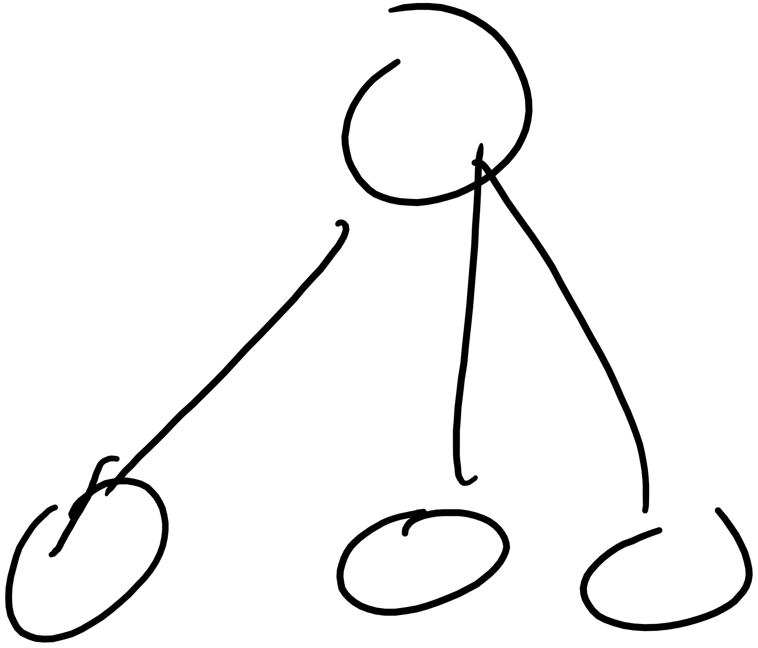
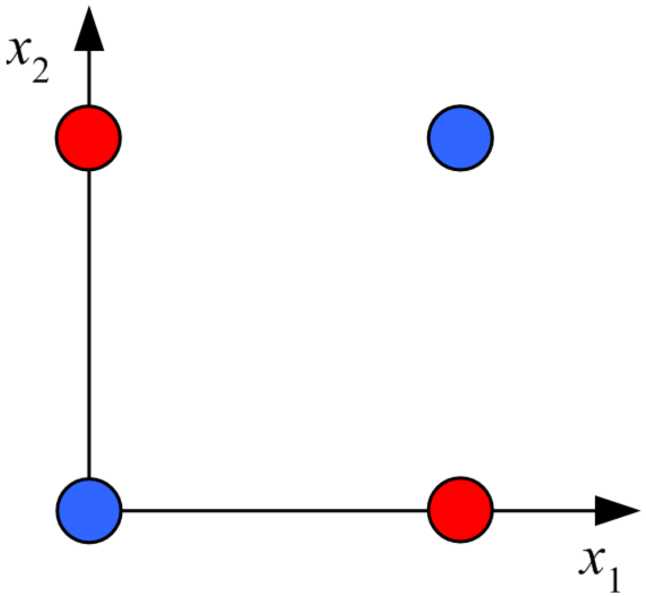


XOR

x_1	x_2	r
0	0	0
0	1	1
1	0	1
1	1	0



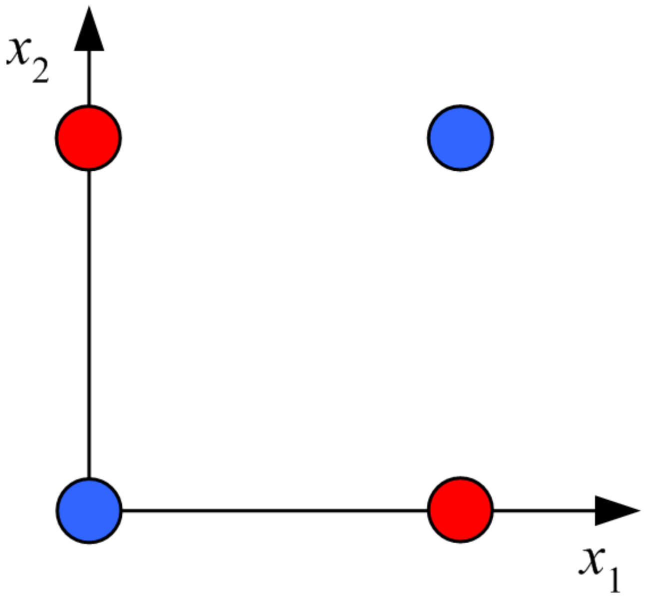


XOR

x_1	x_2	r
0	0	0
0	1	1
1	0	1
1	1	0

No w_0, w_1, w_2 satisfy:

$$
\begin{aligned}
&&& w_0 \le 0 \\
&& w_2 + {} & w_0 > 0 \\
w_1 + {} &&& w_0 > 0 \\
w_1 + {} & & w_2 + {} & w_0 \le 0
\end{aligned}
$$



Perceptron

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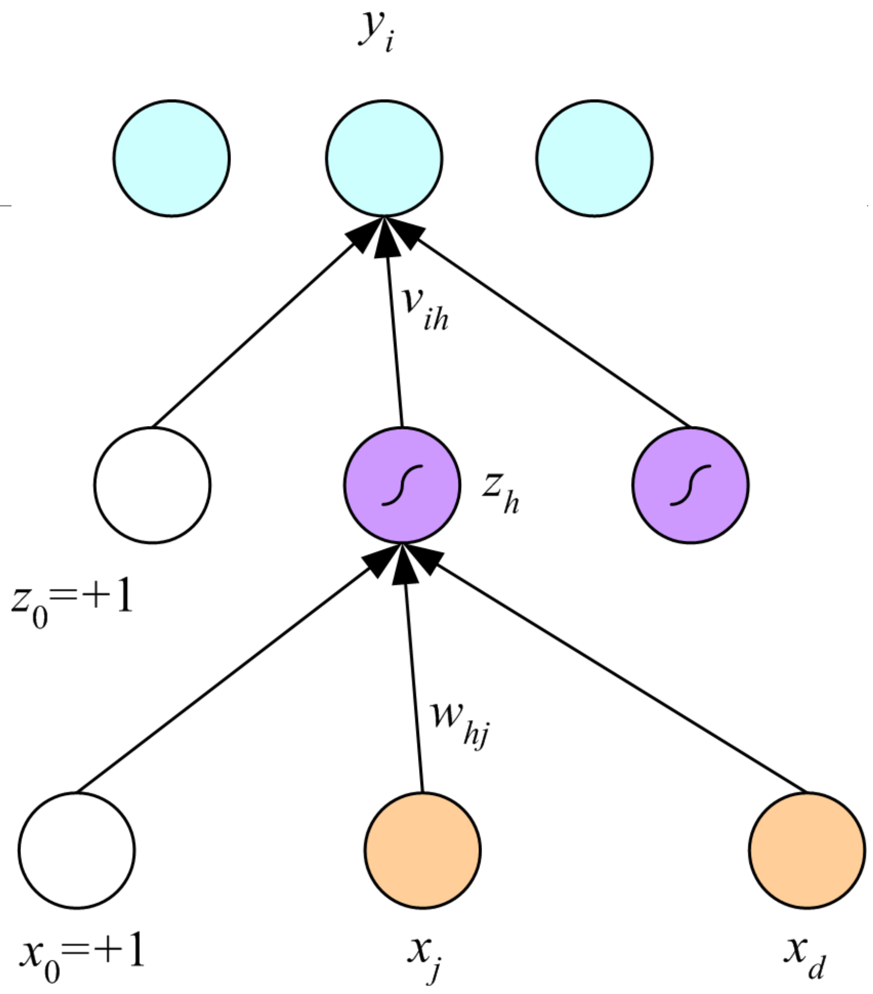
Let's add multiple layers to the perceptron

- At each level we have a **regression** model defined by the activation function and **always** a constant w_0

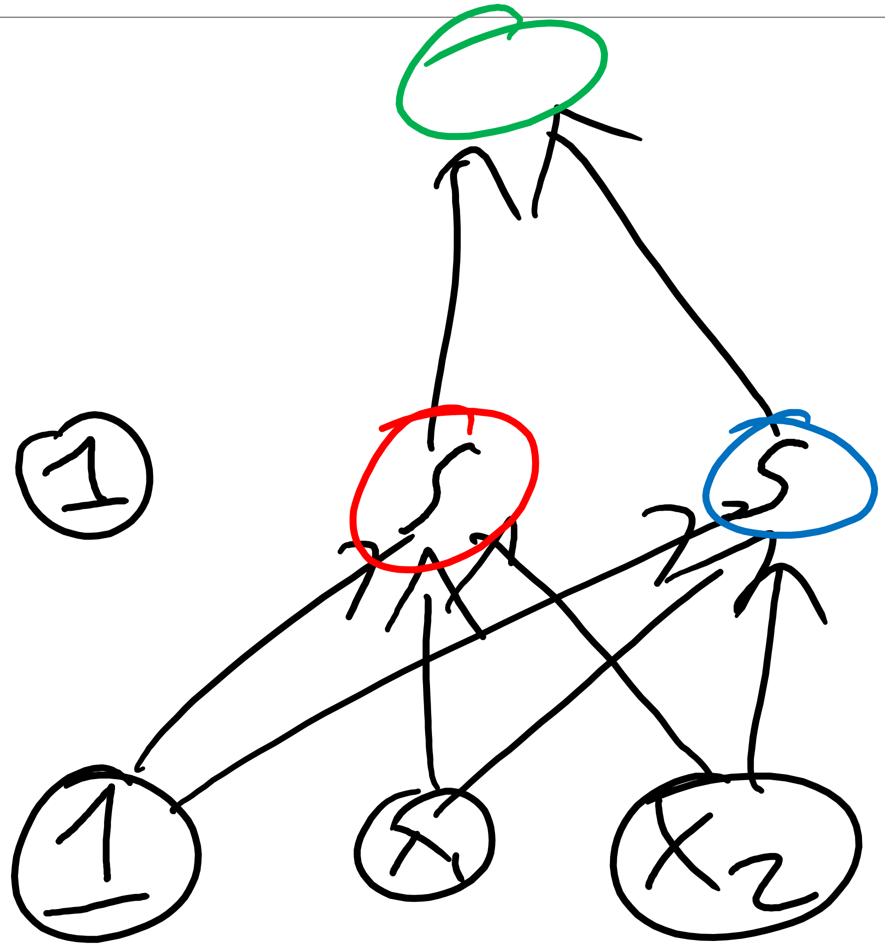


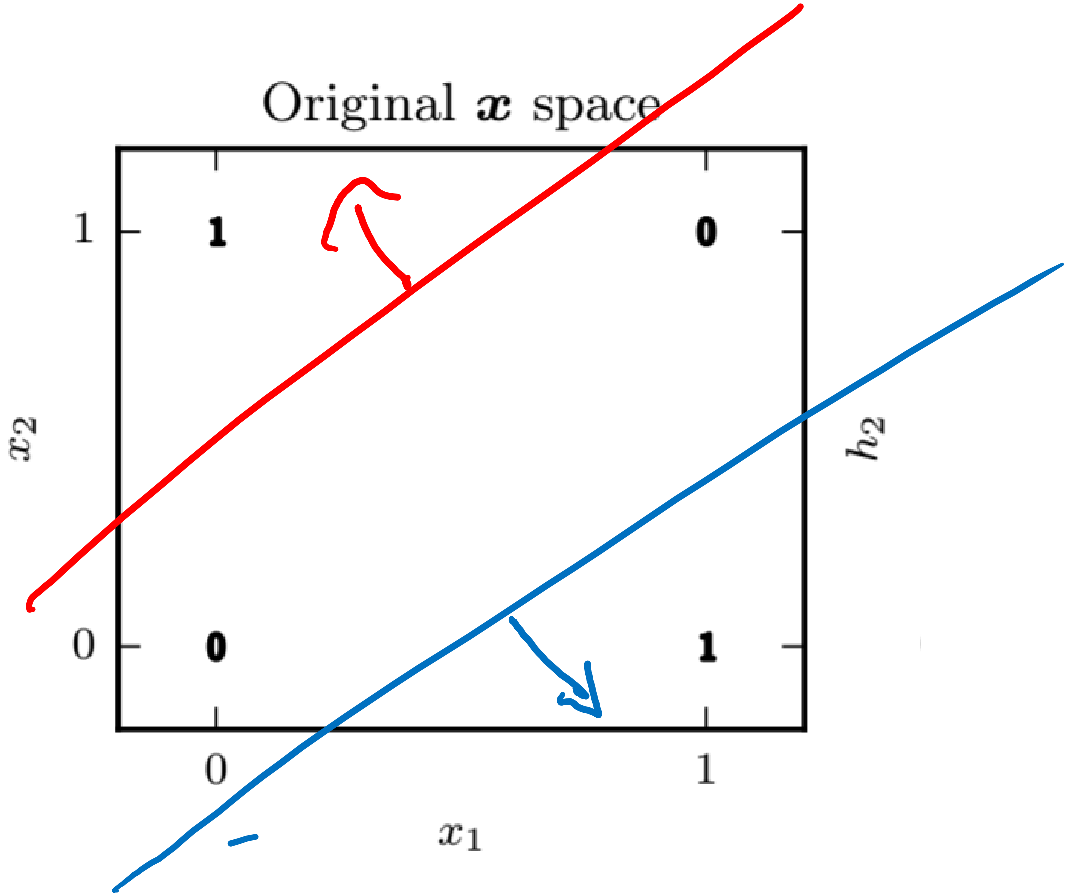
Perceptron

How do we use this to solve the XOR problem?

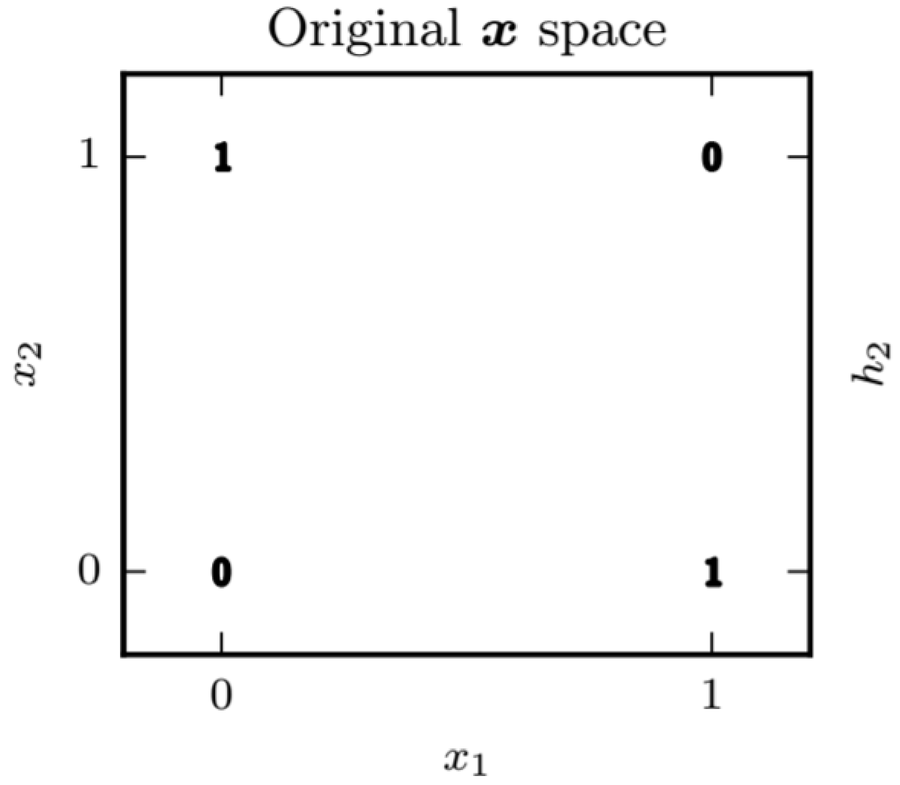
$X_0 = 1$

XOR





XOR



Perceptron

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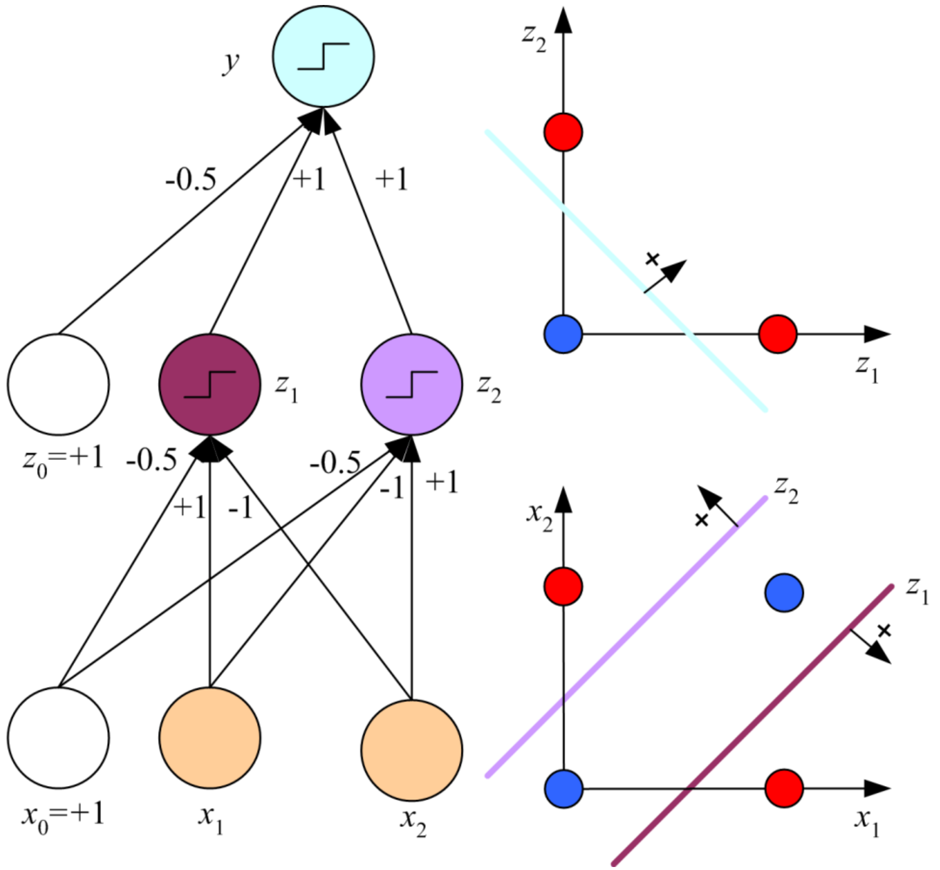
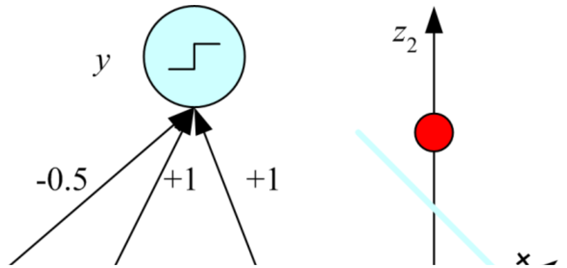
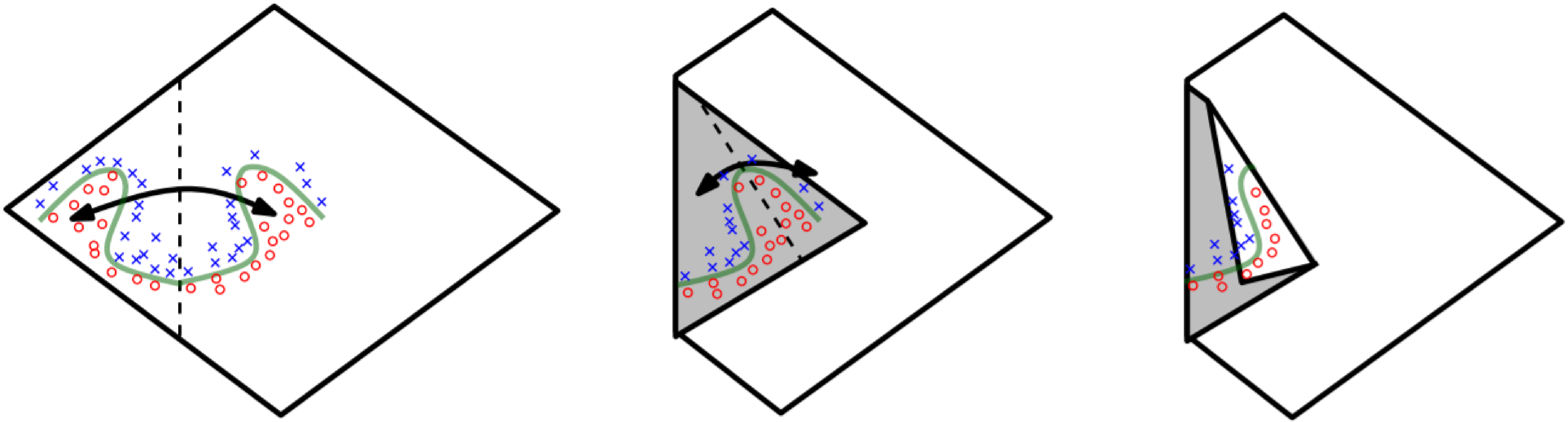


Perceptron

How do we use this to solve the XOR problem?

- What is happening here?





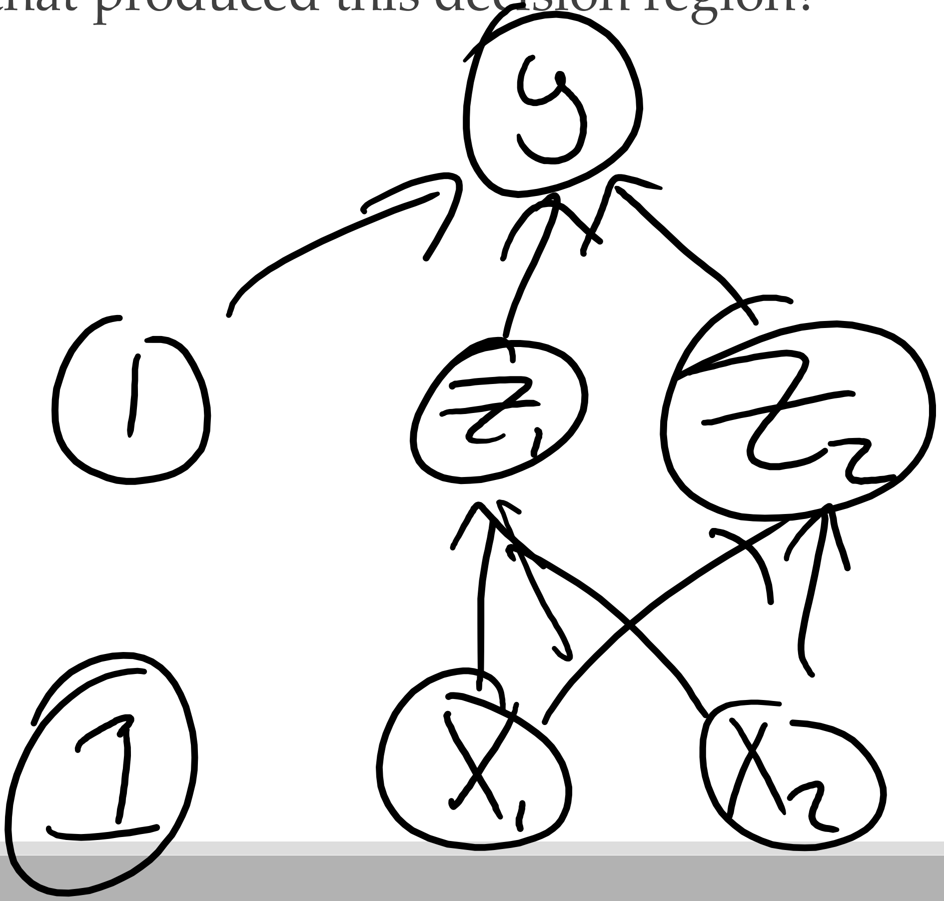
How to train?

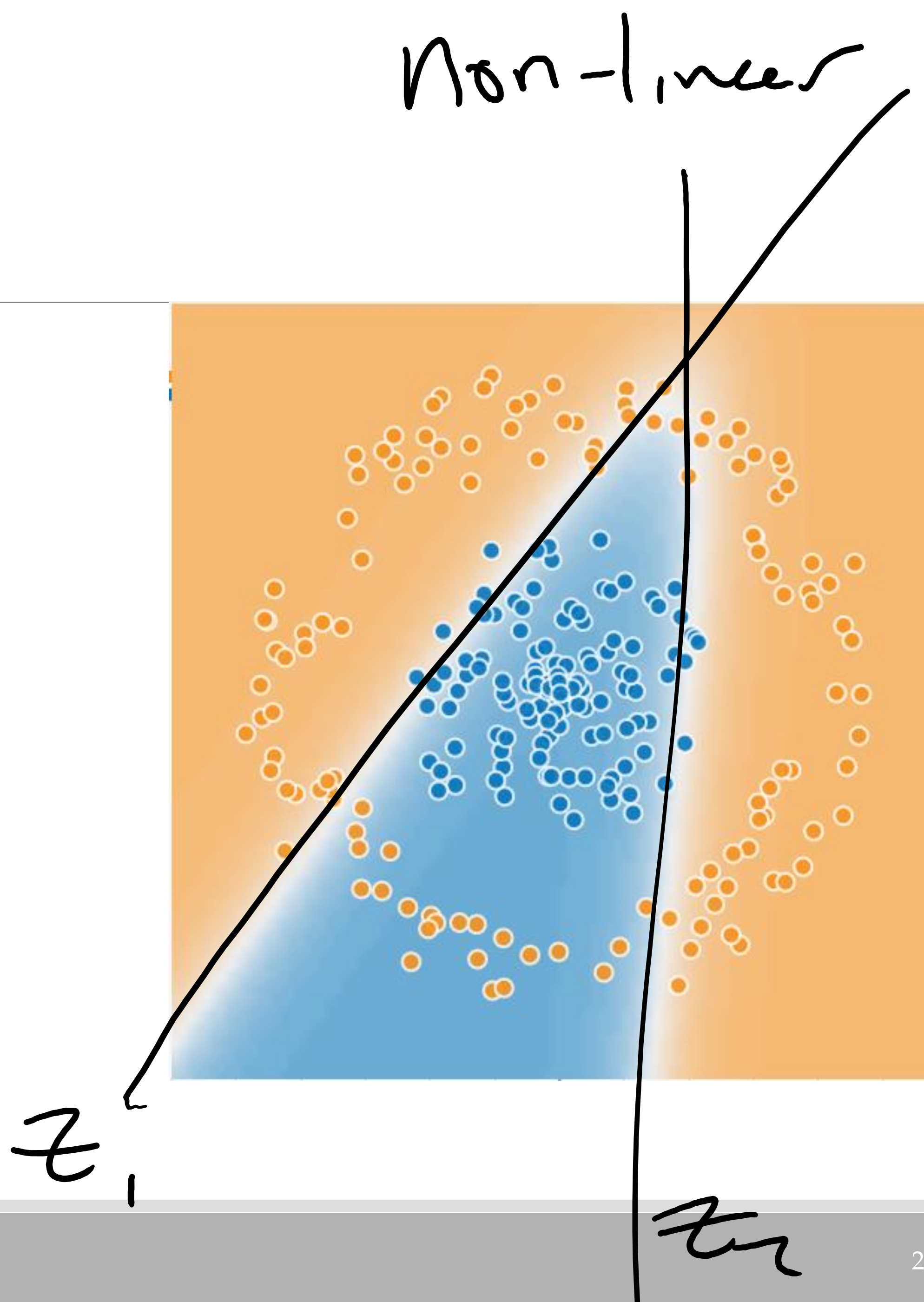
We (hopefully) have some idea of how a particular set of weights causes the neural network to make a decision

- We have some vector of inputs X_d that are all fed as parameters to some number of nodes
- Each of these nodes outputs a sigmoid function to the next hidden layer
- This process eventually leads to the final layer, which makes the final prediction

How to train?

What is the structure of the neural network that produced this decision region?





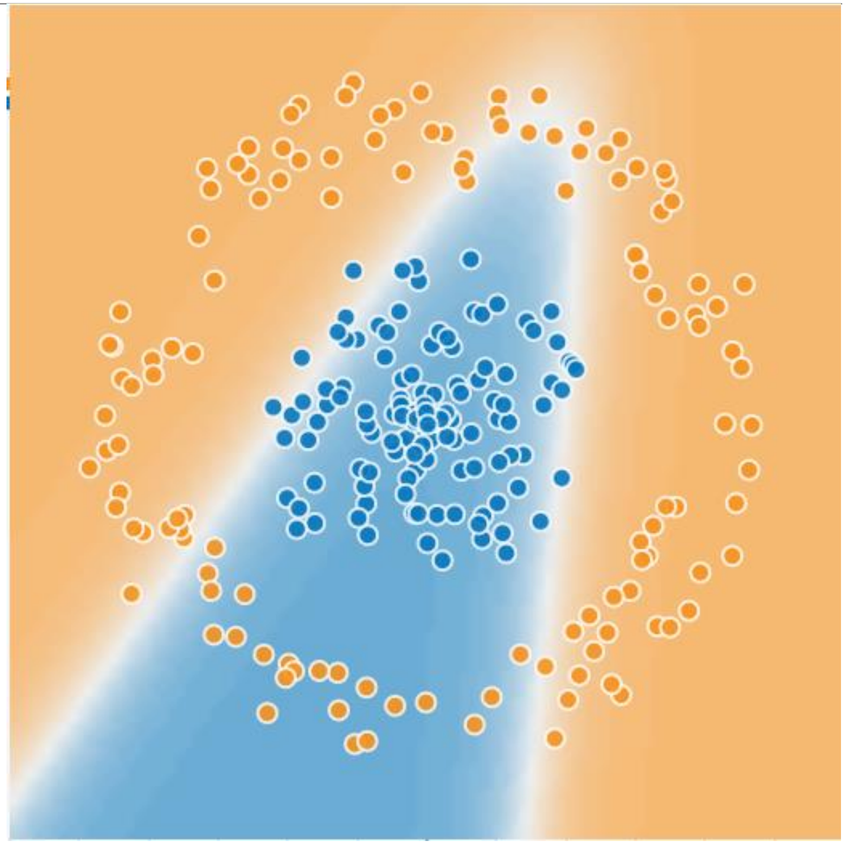
How to train?

0.5

What is the structure of the neural network that produced this decision region?

- Blue is positive, orange is negative
- What do the white regions represent?

regions of low certainty



How to train?

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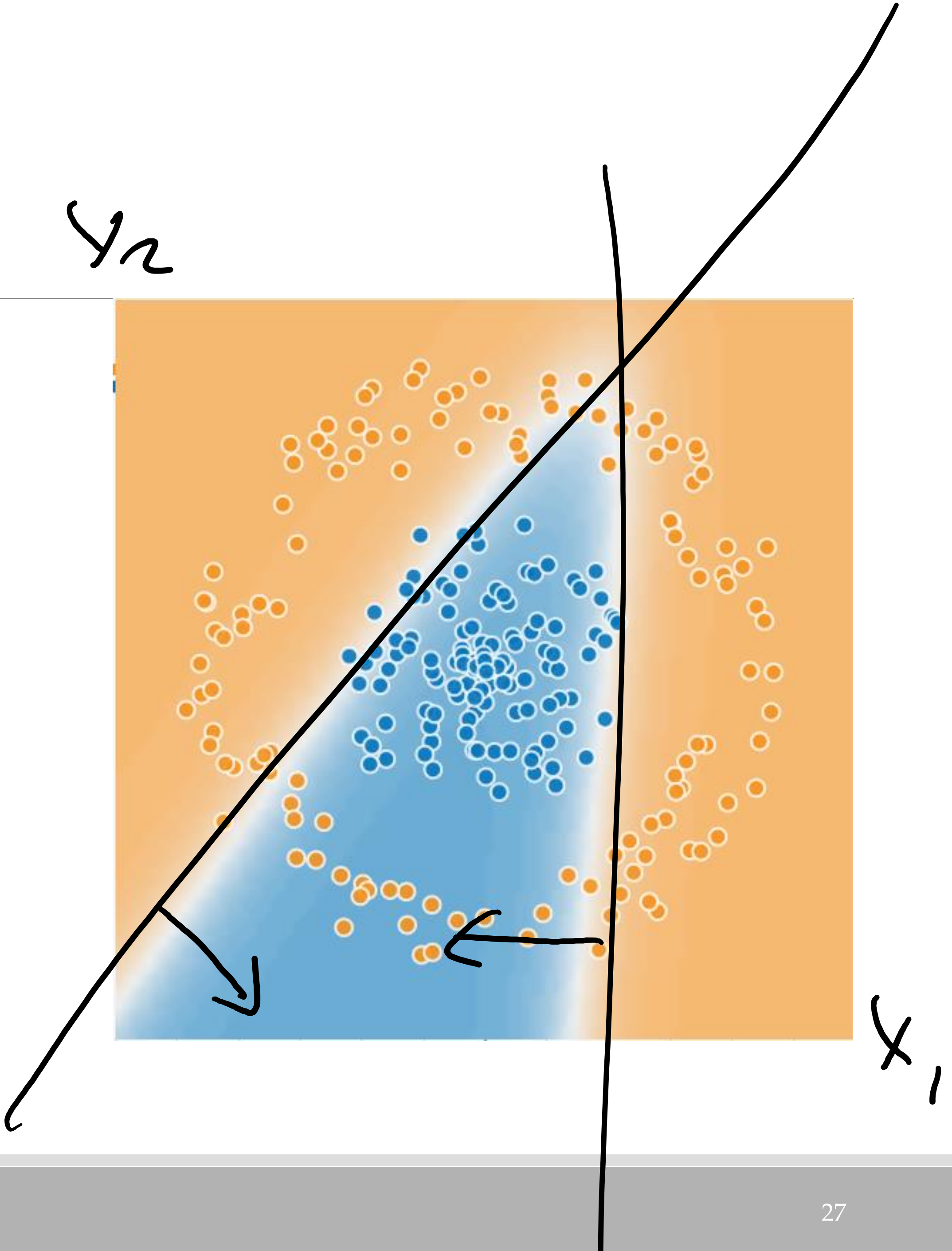
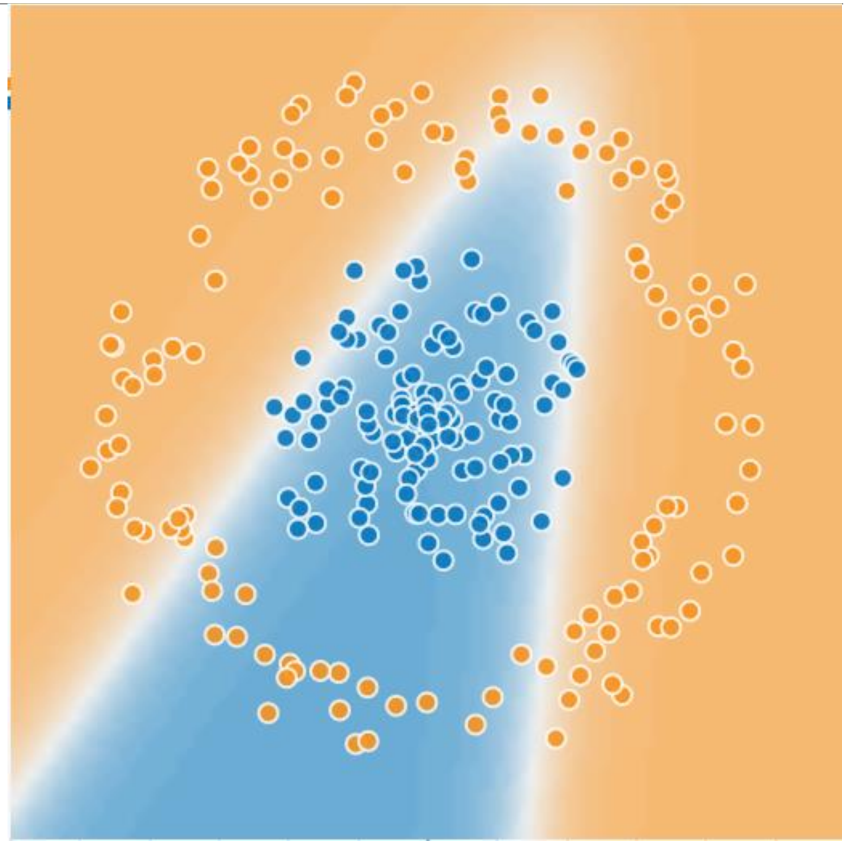
Two lines from two middle “hidden nodes” with sigmoid behavior

What might the weights look like for each of these nodes?



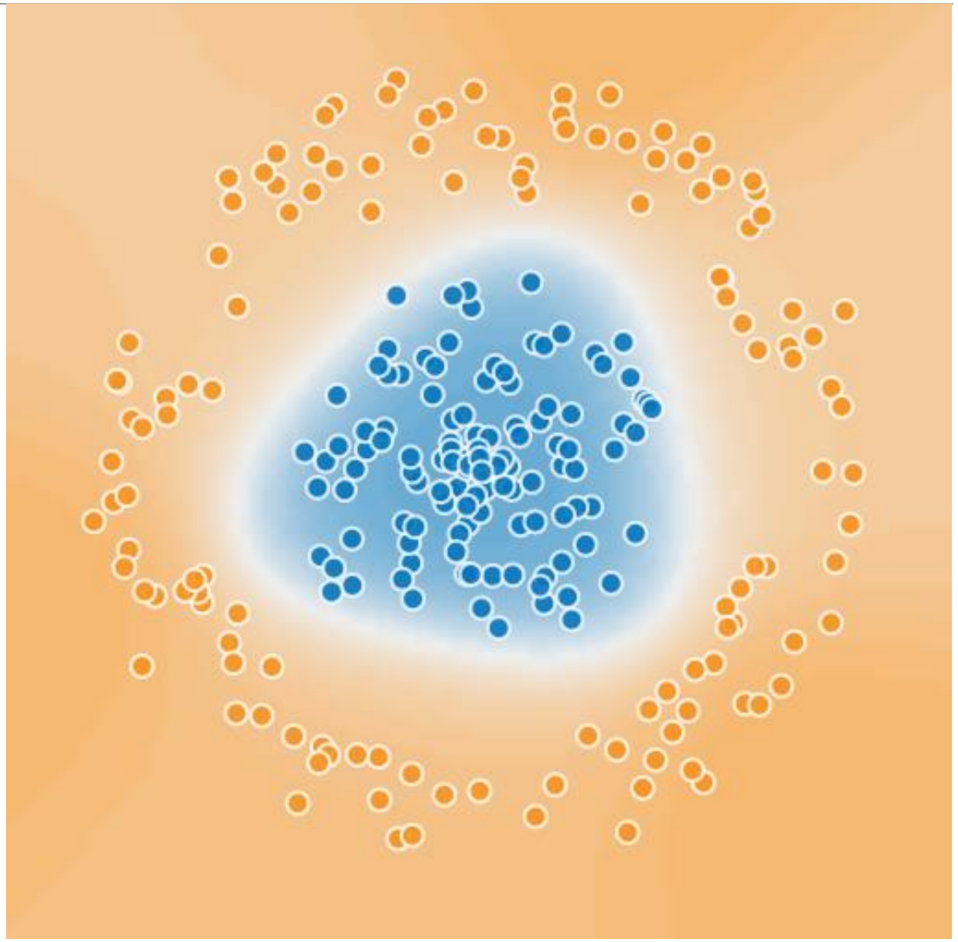
How to train?

Which would help more? Increasing the number of nodes in our hidden layer or increasing the number of hidden layers?



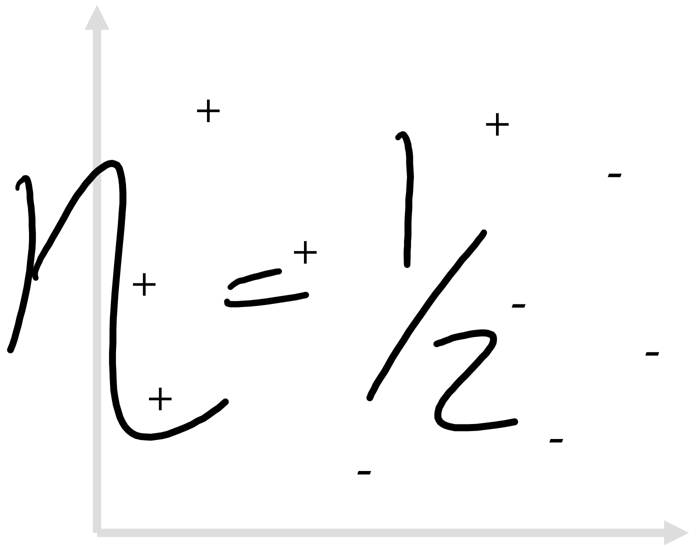
How to train?

With three internal nodes, we can now generate three linear models to separate the data.



Algorithm 8.4: Perceptron algorithm

```
Input: linearly separable data set x_i ∈ ℝ^D, y_i ∈ {−1, +1} for i = 1 : N;
Initialize θ_0;
k ← 0;
repeat
    k ← k + 1;
    i ← k mod N;
    if ŷ_i ≠ y_i then
        θ_{k+1} ← θ_k + y_i x_i
    else
        no-op
until converged;
```



$\eta = 1/2$

$\eta(\hat{y} - y)x$

What’s wrong with this approach?

It’s an easy way to separate linear data. But what’s wrong?

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How do we get around this?

- Random weights
- Feedback between nodes