

```
In [33]: import math
import pandas as pd
import numpy as np
import random
from statistics import mean, stdev, median, mode
from sklearn.model_selection import train_test_split, cross_val_score, cross_val_predict
from sklearn.decomposition import KernelPCA
import matplotlib.pyplot as plt
import matplotlib.pyplot as mp
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC
```

```
In [34]: data1 = np.loadtxt('data.csv')
#data1
```

```
In [35]: def minimum(x,y):
    min = np.argmin(y)

    return x[min]
# x=[1,2,3,4]
# y=[5,6,7,8]
# z = minimum(x,y)
# z
```

```
In [36]: def minimum3(x,y,z):
    min = np.argmin(z)

    return x[min], y[min], z[min]
```

```
In [37]: def decisionRegion(clf, X, Y):

# Lists to hold inpoints, predictions and assigned colors
xPred = []
yPred = []
cPred = []

# Use input points to get predictions here
for xP in range(-100,100):
    xP = xP/100.0
    for yP in range(-100,100):
        yP = yP/100.0
        xPred.append(xP)
        yPred.append(yP)
        if(clf.predict([[xP,yP]])=="1.0"):
            cPred.append("b")
        else:
            cPred.append("r")

## Visualize Results
#plot the points
mp.scatter(X,Y,s=3,c=colors)

#plot the regions
mp.scatter(xPred,yPred,s=3,c=cPred,alpha=.1)

#setup the axes
mp.xlim(-1,1)
mp.xlabel("Average Intensity")
mp.ylim(-1,1)
mp.ylabel("Intensity Variance")
```

```

In [38]: #shuffle the data and select training and test data
np.random.seed(100)
np.random.shuffle(data1)

features = []
digits = []

for row in data1:
    #import the data and select only the 1's and 5's
    if (row[0]==1 or row[0]==5):
        features.append(row[1:])
        digits.append(str(row[0]))

#Select the proportion of data to use for training.
#Notice that we have set aside 80% of the data for testing
numTrain = int(len(features)*.2)

trainFeatures = features[:numTrain]
testFeatures = features[numTrain:]
trainDigits = digits[:numTrain]
testDigits = digits[numTrain:]

```

```

In [39]: # Q)1)

```

```

In [40]: KPCA = KernelPCA(n_components = 2, kernel = 'poly', degree = 1)
data = KPCA.fit_transform(trainFeatures)

```

```

In [41]: train = pd.DataFrame(data = data)
train.head()

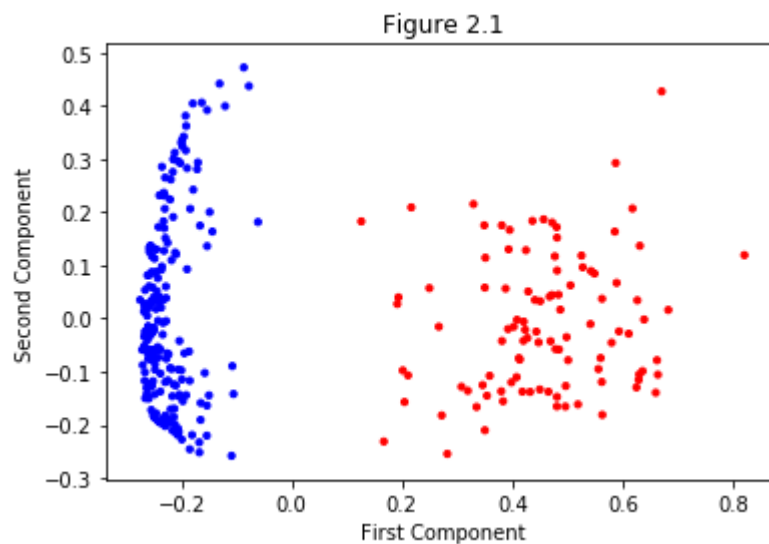
```

Out[41]:

	0	1
0	0.662562	-0.078380
1	-0.249190	-0.186351
2	0.387871	0.056345
3	0.636381	-0.098758
4	-0.171666	0.280755

```
In [42]: #Colors will be passed to the graphing library to color the points.  
#1's are blue: "b" and 5's are red: "r"  
colors = []  
for index in range(len(trainFeatures)):  
    if(trainDigits[index]=="1.0"):  
        colors.append("b")  
    else:  
        colors.append("r")  
  
#plot the data points  
  
plt.scatter(train[[0]],train[[1]], s=10,c=colors)  
plt.xlabel('First Component')  
plt.ylabel('Second Component')  
plt.title("Figure 2.1")
```

Out[42]: Text(0.5, 1.0, 'Figure 2.1')



```

In [43]: #Convert the 256D data (trainFeatures) to 2D data
#We need X and Y for plotting and simpleTrain for building the model.
#They contain the same points in a different arrangement

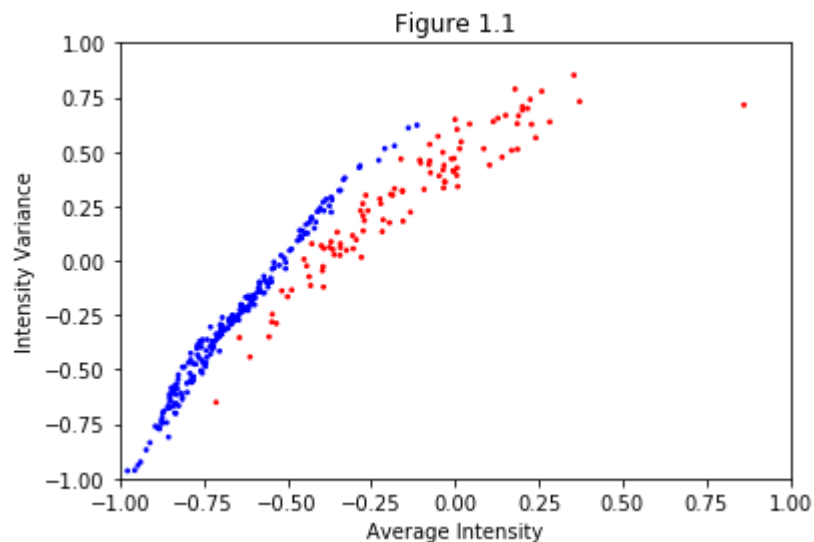
X = []
Y = []
simpleTrain = []

#Colors will be passed to the graphing library to color the points.
#1's are blue: "b" and 5's are red: "r"
colors = []
for index in range(len(trainFeatures)):
    #produce the 2D dataset for graphing/training and scale the data so it is
    #in the [-1,1] square
    xNew = 2*np.average(trainFeatures[index])+.75
    yNew = 3*np.var(trainFeatures[index])-1.5
    X.append(xNew)
    Y.append(yNew)
    simpleTrain.append([xNew,yNew])
    #trainDigits will still be the value we try to classify. Here it is the st
    #ring "1.0" or "5.0"
    if(trainDigits[index]=="1.0"):
        colors.append("b")
    else:
        colors.append("r")

#plot the data points
### https://matplotlib.org/api/\_as\_gen/matplotlib.pyplot.scatter.html
mp.scatter(X,Y,s=3,c=colors)

#specify the axes
mp.xlim(-1,1)
mp.xlabel("Average Intensity")
mp.ylim(-1,1)
mp.ylabel("Intensity Variance")
mp.title("Figure 1.1")
#display the current graph
mp.show()

```



```
In [44]: # 1)a) Yes. The components of KPCA seem to separate the data better than mean
          and variance components.
          # It is evident in the figure 2.1 and 1.1 as you can see that there are less
          number of points near the decision region with KPCA.
```

```
In [45]: KPCA = KernelPCA(n_components = 2, kernel = 'poly', degree = 3)
          data3 = KPCA.fit_transform(trainFeatures)
```

```
In [46]: train3 = pd.DataFrame(data = data3)
          train3.head()
```

Out[46]:

	0	1
0	1.704466	-0.165921
1	-0.762189	-0.610113
2	1.149909	0.109357
3	1.653838	-0.186883
4	-0.400848	0.914819

```

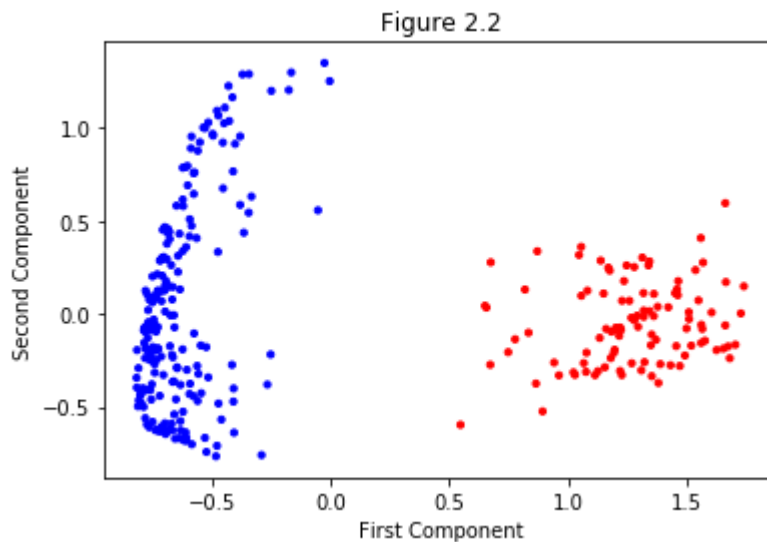
In [47]: #Colors will be passed to the graphing library to color the points.
#1's are blue: "b" and 5's are red: "r"
colors = []
for index in range(len(trainFeatures)):
    if(trainDigits[index]=="1.0"):
        colors.append("b")
    else:
        colors.append("r")

#plot the data points

plt.scatter(train3[[0]],train3[[1]], s=10,c=colors)
plt.xlabel('First Component')
plt.ylabel('Second Component')
plt.title("Figure 2.2")

```

Out[47]: Text(0.5, 1.0, 'Figure 2.2')



```

In [48]: # 1)b) Yes. This kpca (degree 3) seperates the data better than the above one
(degree 1) because there are less data points near the
# decision boundary when compared to the no of data points near the decision b
oundary for kpca of degree 1

```

```

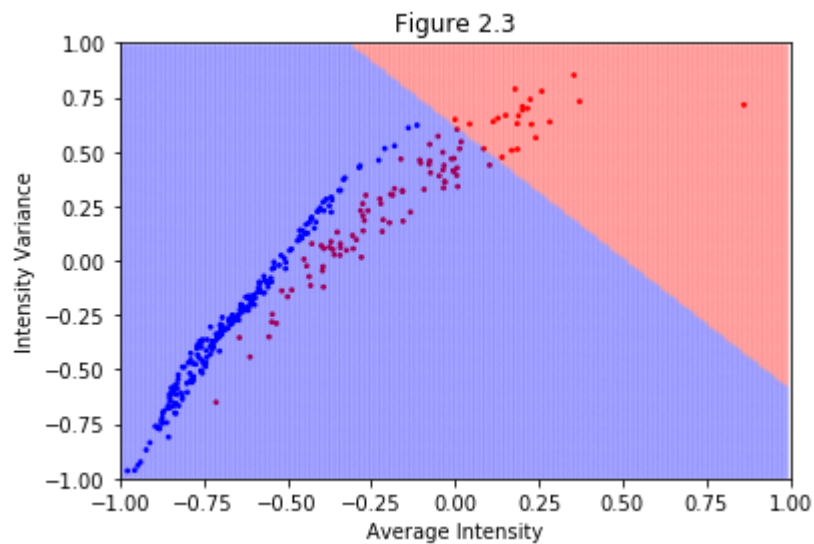
In [49]: # Q)2)

```

```
In [50]: LR = LogisticRegression(penalty = 'l2',C = 0.01)
clf = LR.fit(simpleTrain, trainDigits)

decisionRegion(clf, X, Y)
mp.title("Figure 2.3")
mp.show()
```

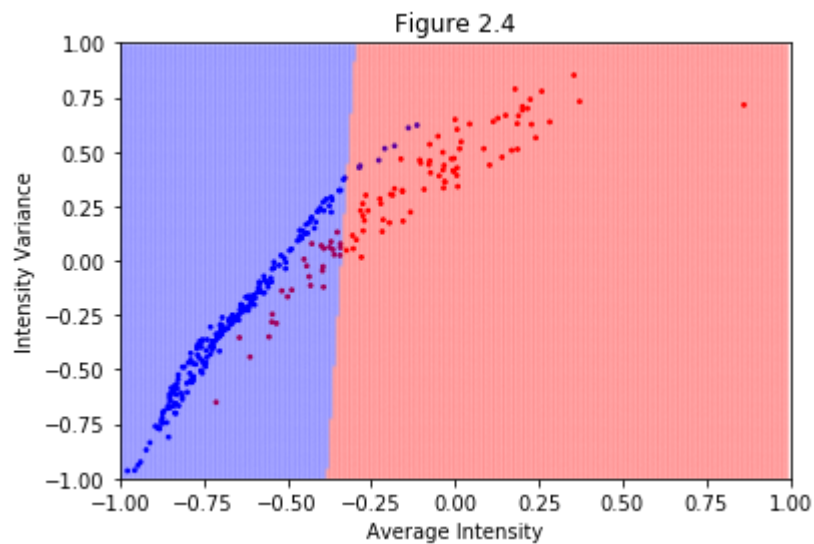
C:\Users\kalya\Anaconda3\lib\site-packages\sklearn\linear_model\logistic.py:432: FutureWarning: Default solver will be changed to 'lbfgs' in 0.22. Specify a solver to silence this warning.
FutureWarning)




```
In [51]: LR = LogisticRegression(penalty = 'l2',C = 2.0)
clf = LR.fit(simpleTrain, trainDigits)

decisionRegion(clf, X, Y)
mp.title("Figure 2.4")
mp.show()
```

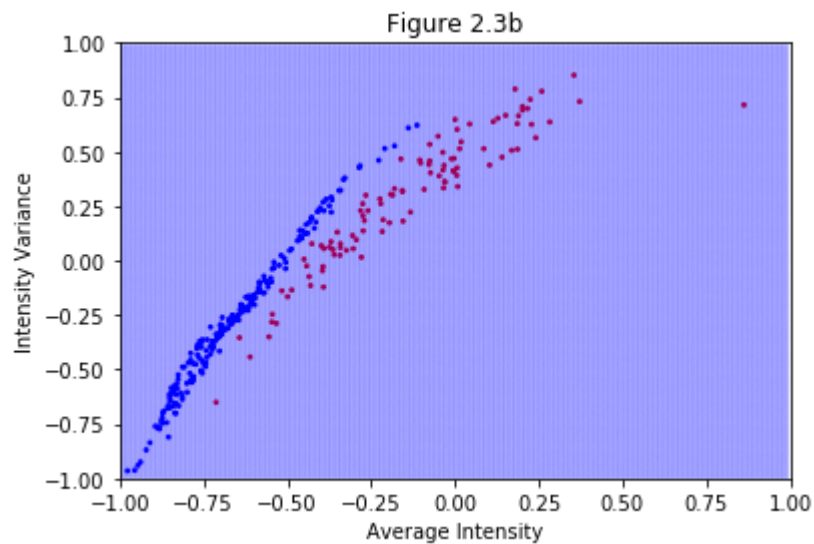
C:\Users\kalya\Anaconda3\lib\site-packages\sklearn\linear_model\logistic.py:432: FutureWarning: Default solver will be changed to 'lbfgs' in 0.22. Specify a solver to silence this warning.
FutureWarning)



```
In [52]: LR = LogisticRegression(penalty = 'l1',C = 0.01)
clf = LR.fit(simpleTrain, trainDigits)

decisionRegion(clf, X, Y)
mp.title("Figure 2.3b")
mp.show()
```

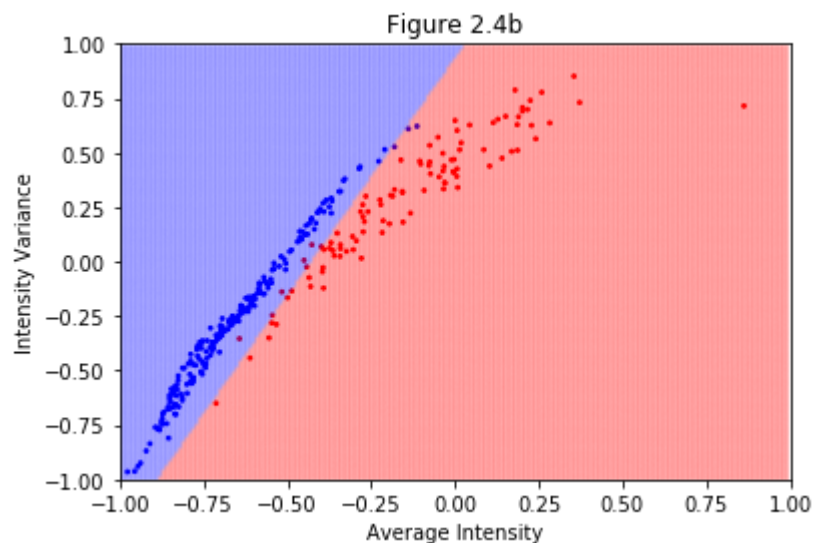
C:\Users\kalya\Anaconda3\lib\site-packages\sklearn\linear_model\logistic.py:432: FutureWarning: Default solver will be changed to 'lbfgs' in 0.22. Specify a solver to silence this warning.
FutureWarning)



```
In [53]: LR = LogisticRegression(penalty = 'l1',C = 2.0)
clf = LR.fit(simpleTrain, trainDigits)

decisionRegion(clf, X, Y)
mp.title("Figure 2.4b")
mp.show()
```

C:\Users\kalya\Anaconda3\lib\site-packages\sklearn\linear_model\logistic.py:432: FutureWarning: Default solver will be changed to 'lbfgs' in 0.22. Specify a solver to silence this warning.
FutureWarning)



```
In [54]: #Q) 2) Graduate student question:

# From the figures 2.3 and 2.3b, for C = 0.01, both l1 and l2 underfit the data but l1 completely underfit the data
# For C = 2.0, the regularization techniques behaved conversely, l1 has less number of training errors compared to l2
# It looks like the regularization parameter C has more role to play than regularization techniques in making a model overfit or underfit.
```

```
In [55]: # Q) 3)
```

```

In [56]: # USING 2D dimensional data
x = []
y = []
z = []
p = []
for i in range(1,25):
    c = np.random.uniform(0.01,100)
    model2 = SVC(C= c, gamma= 'scale')
    #model2.predict(testFeatures)
    cvs = cross_val_score(model2, simpleTrain, trainDigits, cv = 10, scoring=
'accuracy')
    err = 1-cvs
    evsm = 1-(cvs.mean())
    p.append(err)
    x.append(c)
    y.append(evsm)
    z.append([x,evsm])

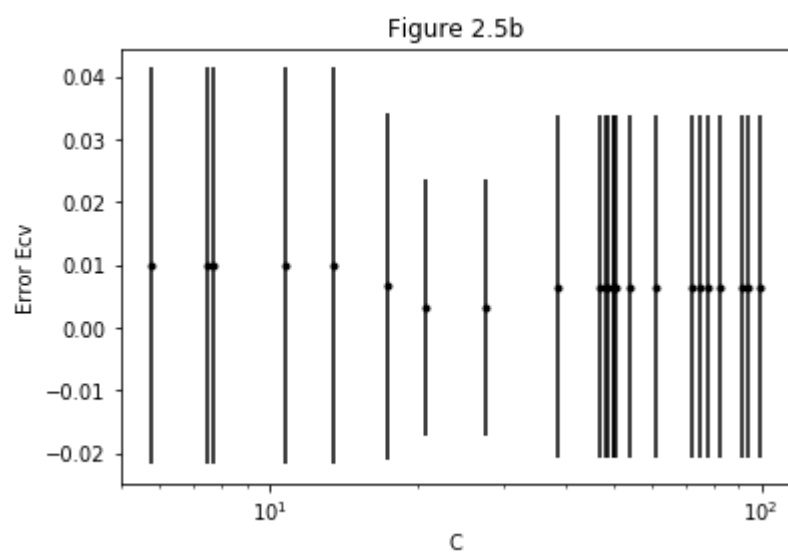
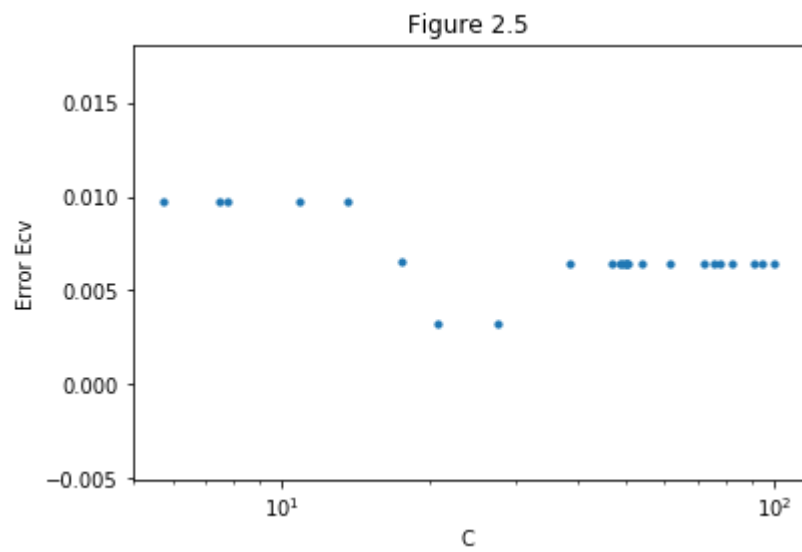
# print(len(x))
# print(len(y))
# print(count)
mp.scatter(x,y, s=10)
mp.xlabel("C")
mp.xscale('log')
mp.ylabel("Error Ecv")
mp.title("Figure 2.5")
mp.show()

m=[]
std =[]
#print(y)
for i in range(len(p)):
    m.append(mean(p[i]))
    std.append(2*stdev(p[i]))

mp.errorbar(x, m, yerr=std, fmt='.k');

mp.xlabel("C")
mp.xscale('log')
mp.ylabel("Error Ecv")
mp.title("Figure 2.5b")
mp.show()

```



```
In [57]: # from Figure 2.5, the error is low for x = 3
c_opt = minimum(x,y)
c_opt
```

```
Out[57]: 27.404470110232598
```

```

In [58]: # USING 256D dimensional data
x256 = []
y256 = []
p256 = []

for i in range(1,25):
    c256 = np.random.uniform(0.01,100)
    model2 = SVC(C= c256, gamma= 'scale')
    #model2.predict(testFeatures)
    cvs256 = cross_val_score(model2, trainFeatures, trainDigits, cv = 10, scoring='accuracy')
    err256 = 1-cvs256
    evsm256 = 1-(cvs256.mean())
    p256.append(err256)
    x256.append(c256)
    y256.append(evsm256)

# print(len(x))
# print(len(y))
# print(count)
mp.scatter(x256,y256, s=10)
mp.xlabel("C")
mp.xscale('log')
mp.ylabel("Error Ecv")
mp.title("Figure 2.6")
mp.show()

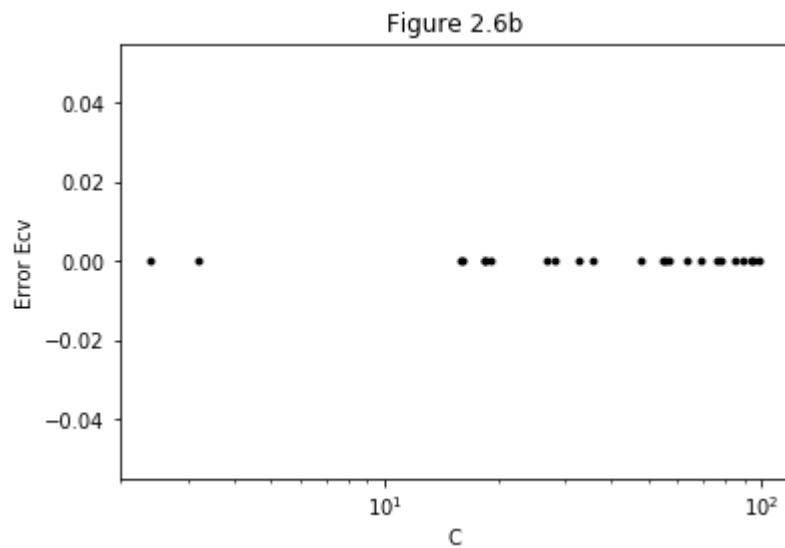
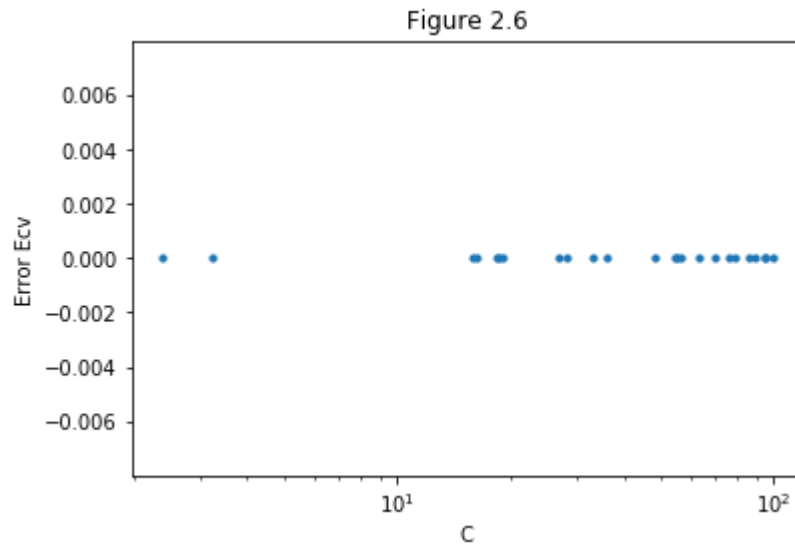
m256=[]
std256 = []
#print(y)
for i in range(len(p256)):
    m256.append(mean(p256[i]))
    std256.append(2*stdev(p256[i]))

mp.errorbar(x256, m256, yerr=std256, fmt='.k');

mp.xlabel("C")
mp.xscale('log')
mp.ylabel("Error Ecv")
mp.title("Figure 2.6b")
mp.show()

print(np.sort(x256))

```

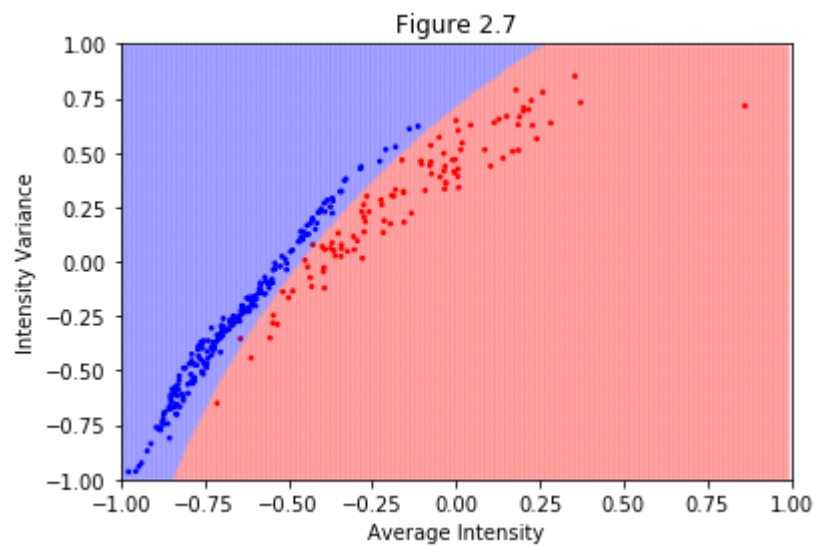


```
[ 2.38892019  3.22532004 15.85696226 16.20335476 18.43898396 18.48234987
19.08740535 26.97663363 28.27696536 32.96013892 35.85365496 48.01854316
54.69987977 55.4738074  56.65342317 63.14267019 69.49737684 75.77161746
78.54534258 85.53950334 89.36686788 94.14115424 94.5088163  98.84508942]
```

```
In [59]: # Since the 256 dimensional data is linearly seperable, we got zero training e
rrors for all C values.
# But if you want me to pick a value for C, I would pick 44 because a small va
lue of C would underfit the test data set and
# a large value of C would overfit the test data
```

```
In [60]: svm = SVC(C = c_opt, gamma= 'auto')
clf = svm.fit(simpleTrain, trainDigits)

decisionRegion(clf, X, Y)
mp.title("Figure 2.7")
mp.show()
```




```

In [74]: # Gamma vs Degree
# It is taking a lot of time for the following cell to run for degree of 10 and
# 20 with Gamma = 'scale'.
# So, I used Gamma = 1 for those two degree

x = []
y = []
z = []
p = []
c_opt4_1 = []
degree1 = [2,5]
degree2 = [10,20]
r1= []
for i in degree1:
    x1 = []
    y1 = []
    z1 = []
    p1 = []
    for j in range(1,25):
        c1 = np.random.uniform(0.01,100)
        svm = SVC(kernel = 'poly', C = c1, degree = i, gamma= 'scale')
        cvs = cross_val_score(svm, simpleTrain, trainDigits, cv = 10, scoring=
'accuracy', n_jobs = -1)
        err = 1-cvs
        evsm = 1-(cvs.mean())
        p.append(err)
        x.append(c1)
        y.append(evsm)
        z.append(i)
        p1.append(err)
        x1.append(c1)
        y1.append(evsm)
        z1.append(i)
    coptimal = minimum(x1,y1)
    c_opt4_1.append(coptimal)
    r1.append(y[np.argmin(y1)])
    mp.scatter(x1,y1, s=10)
    mp.xlabel("C")
    mp.xscale('log')
    mp.ylabel("Error Ecv")
    mp.title("Degree: " + str(i))
    mp.show()
#     svm = SVC(kernel = 'poly', C = coptimal, degree = i, gamma= 'scale')
#     svm.fit(simpleTrain, trainDigits)
#     decisionRegion(svm, X, Y)
#     mp.title("Degree: " + str(i))
#     mp.show()

c_opt4_2 = []
r2 = []
for i in degree2:
    x2 = []
    y2 = []
    z2 = []
    p2 = []
    for j in range(1,25):

```

```

c2 = np.random.uniform(0.01,100)
svm = SVC(kernel = 'poly', C = c2, degree = i, gamma= 1)
cvs = cross_val_score(svm, simpleTrain, trainDigits, cv = 10, scoring=
'accuracy', n_jobs = -1)
err = 1-cvs
evsm = 1-(cvs.mean())
p2.append(err)
x2.append(c2)
y2.append(evsm)
z2.append(i)
p1.append(err)
x1.append(c2)
y1.append(evsm)
z1.append(i)
coptimal = minimum(x2,y2)
c_opt4_2.append(coptimal)
r2.append(y[np.argmin(y2)])
mp.scatter(x2,y2, s=10)
mp.xlabel("C")
mp.xscale('log')
mp.ylabel("Error Ecv")
mp.title("Degree: " + str(i))
mp.show()
#     svm = SVC(kernel = 'poly', C = coptimal, degree = i, gamma= 1)
#     svm.fit(simpleTrain, trainDigits)
#     decisionRegion(svm, X, Y)
#     mp.title("Degree: " + str(i))
#     mp.show()

c_opt = c_opt4_1 + c_opt4_2
r = r1 + r2
degree = degree1 + degree2

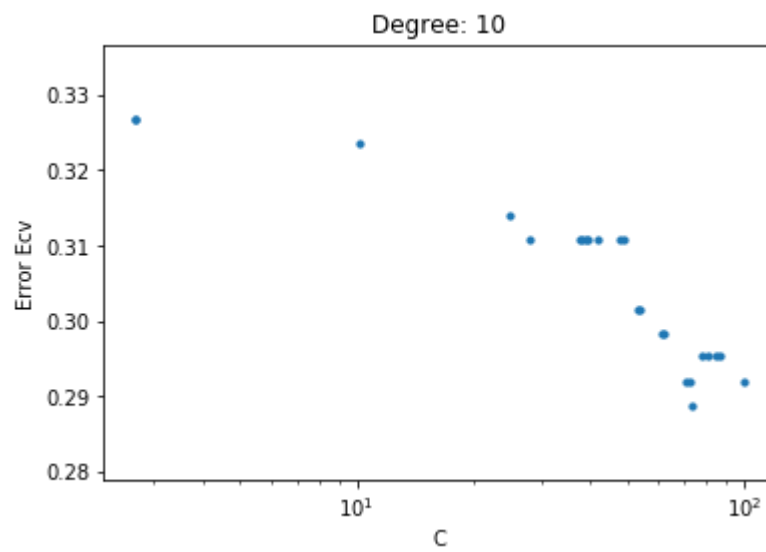
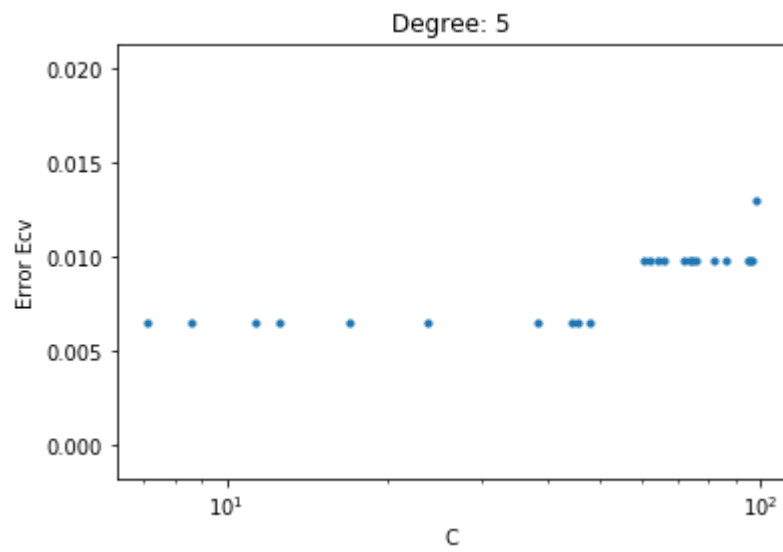
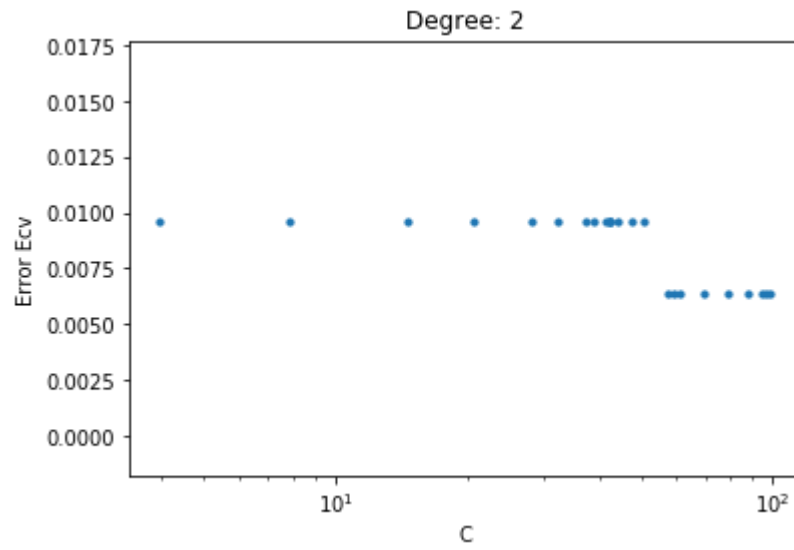
print('\n')
print("Optimal C Values for Degrees " + str(degree) + " are: " + str(c_opt))

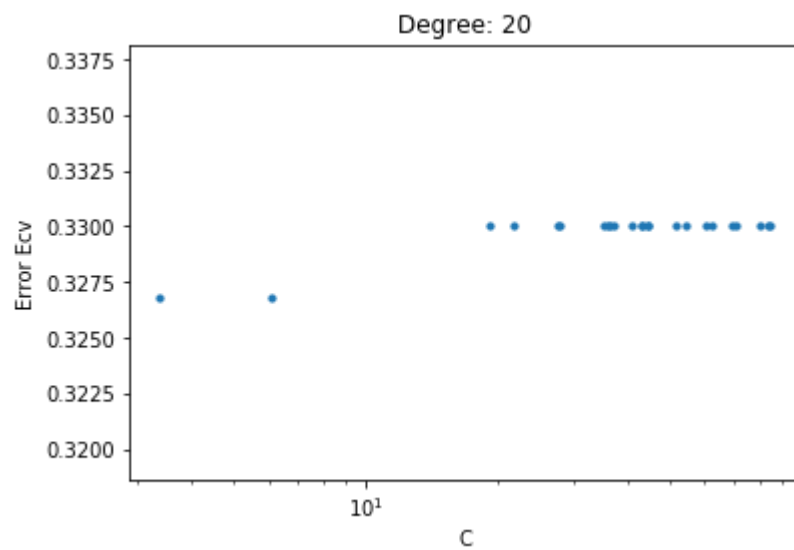
d_opt, c_optf, Min_Ecv = minimum3(z,x,y)
print('\n')
print("Min Ecv Value is: " + str(Min_Ecv))
print("Optimal degree is: " + str(d_opt))
print("Optimal C Value is: " + str(c_optf))

plt.scatter(degree,r)
plt.xlabel('Degree')
plt.ylabel('Min Ecv')
plt.title('Gamma vs Ecv')
plt.show()

plt.scatter(degree,c_opt)
plt.xlabel('Degree')
plt.ylabel('C value of Min Ecv')
plt.title('Degree vs C')
plt.show()

```



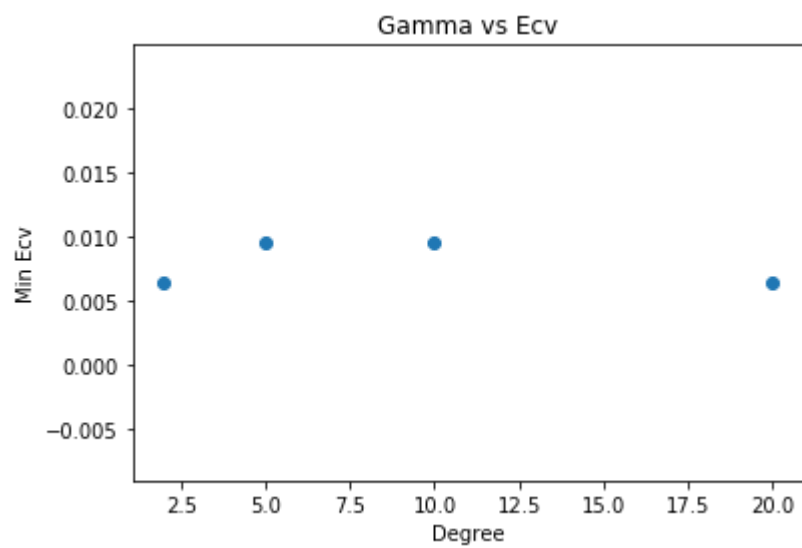


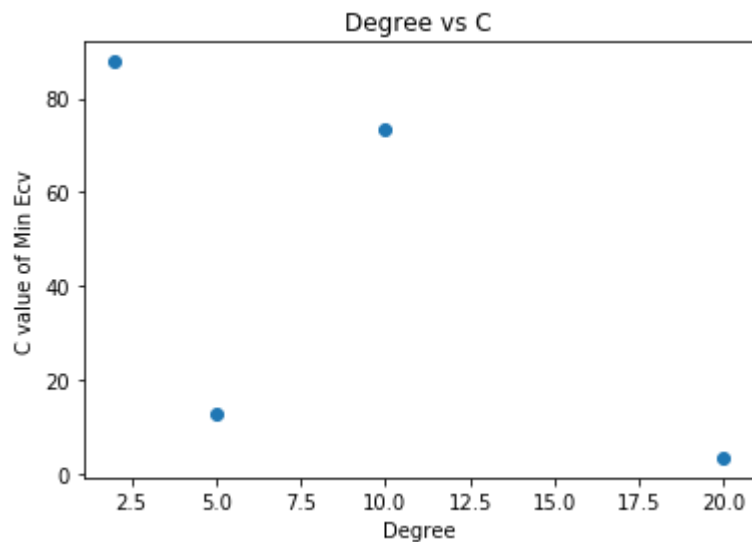
Optimal C Values for Degrees [2, 5, 10, 20] are: [87.83078117113321, 12.607149031715023, 73.48612969804714, 3.3801631824841496]

Min Ecv Value is: 0.006350806451612789

Optimal degree is: 2

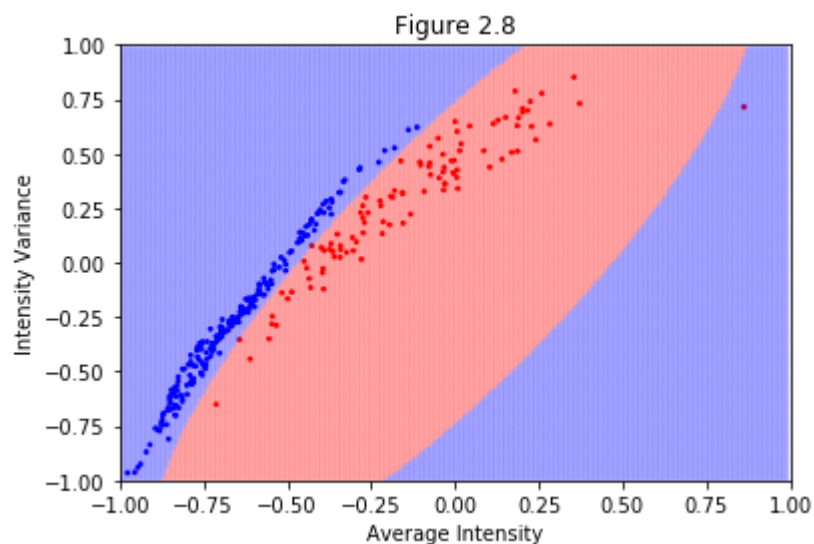
Optimal C Value is: 87.83078117113321





In [75]: *# Q) 3)*
With smaller degree, there is higher chance of underfitting so, to compensate that we need a greater C value which gave me lesser Ecv.
And in case of higher degree which causes overfitting, we need smaller C value to get less cross validation error Ecv
It is evident in the above observations.

In [76]: `svm = SVC(C = c_optf, kernel = 'poly', degree = d_opt, gamma= 'scale')`
`clf = svm.fit(simpleTrain, trainDigits)`
`decisionRegion(clf, X, Y)`
`mp.title("Figure 2.8")`
`mp.show()`

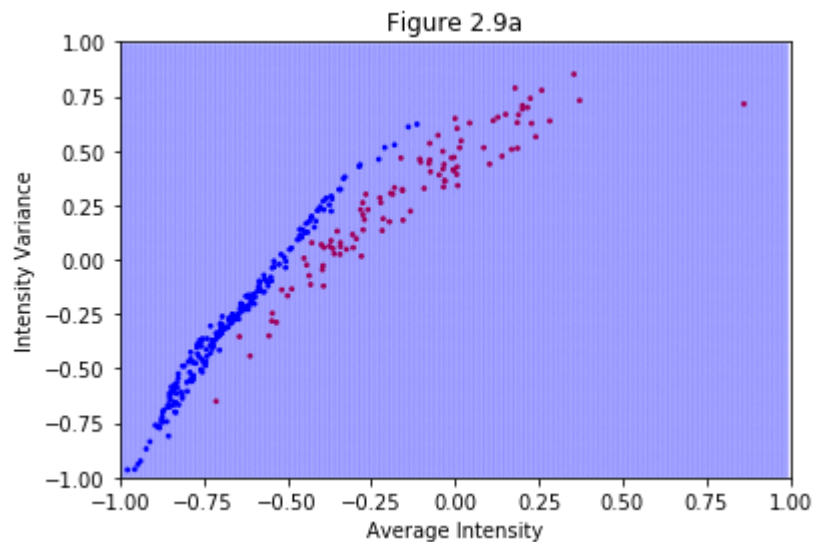


In [77]: *# Q)3) From the above plot, it is evident that this SVM model is best in terms of separating the training data.*

In [78]: *# Graduate Student Question*

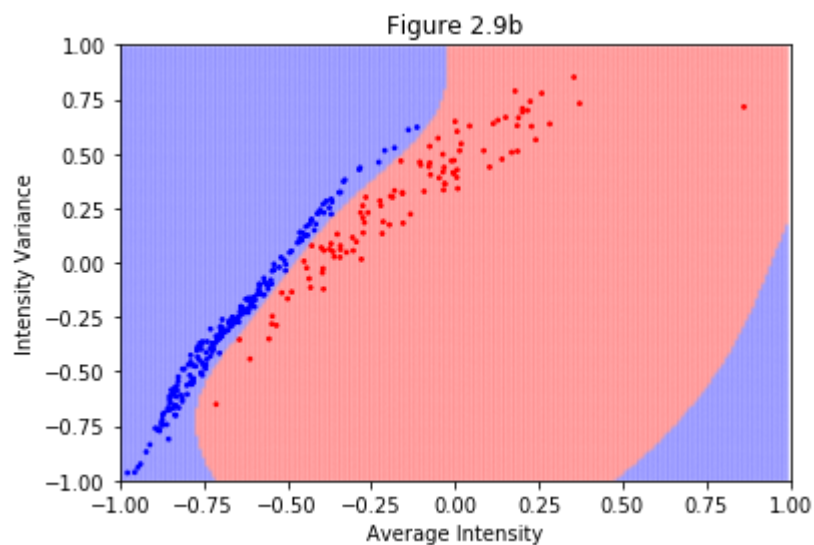
```
In [79]: svm = SVC(C = 0.000001, gamma = 'scale')
clf = svm.fit(simpleTrain, trainDigits)

decisionRegion(clf, X, Y)
mp.title("Figure 2.9a")
mp.show()
```



```
In [80]: svm = SVC(C = 10000, gamma = 'scale')
clf = svm.fit(simpleTrain, trainDigits)

decisionRegion(clf, X, Y)
mp.title("Figure 2.9b")
mp.show()
```



```
In [81]: # Q)3) Graduate Student Question
# When C is low, the hinge loss would be high and svm model produces high marg
in and the model underfits. It is evident in the figure 2.9a
# When C is high, the hinge loss would be low and SVM model tries to overfit t
he data. Check the figure 2.9b
```

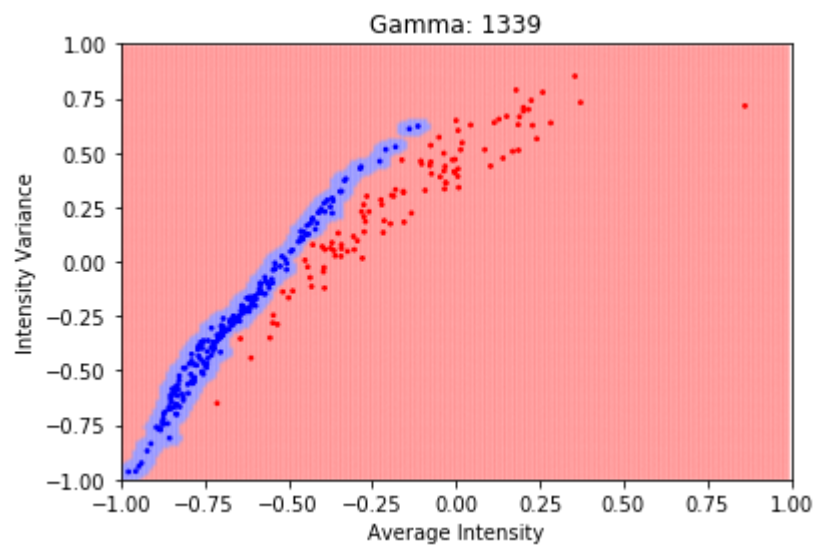
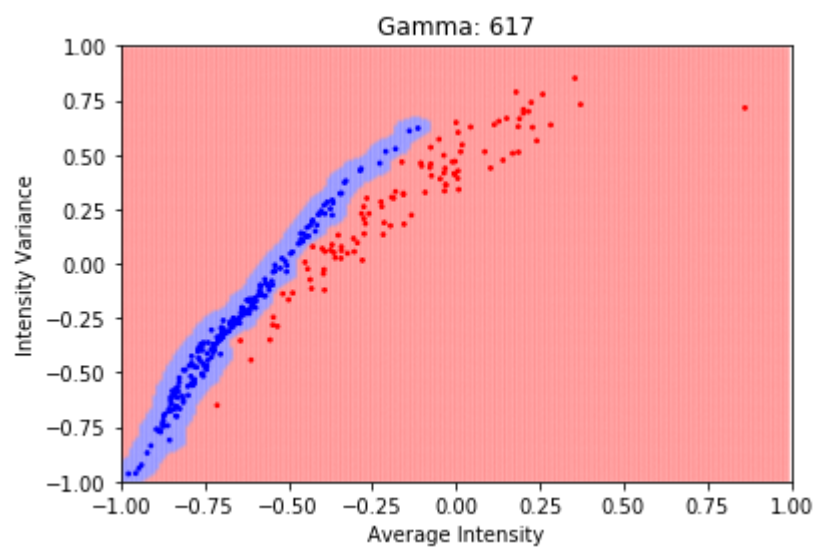
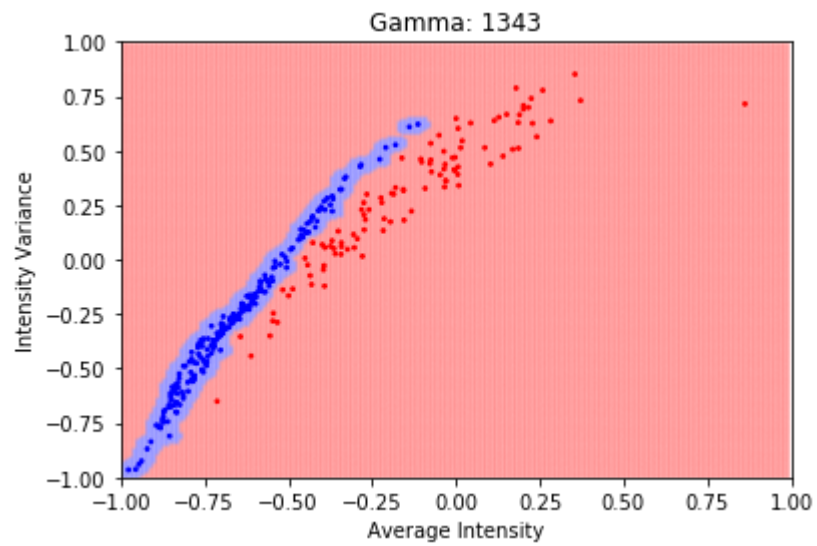
In [82]: *# Extra Credit*

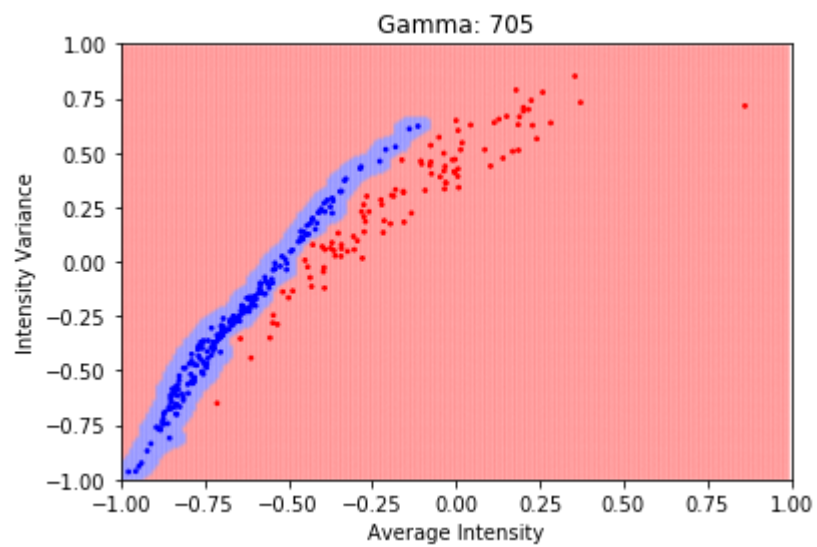
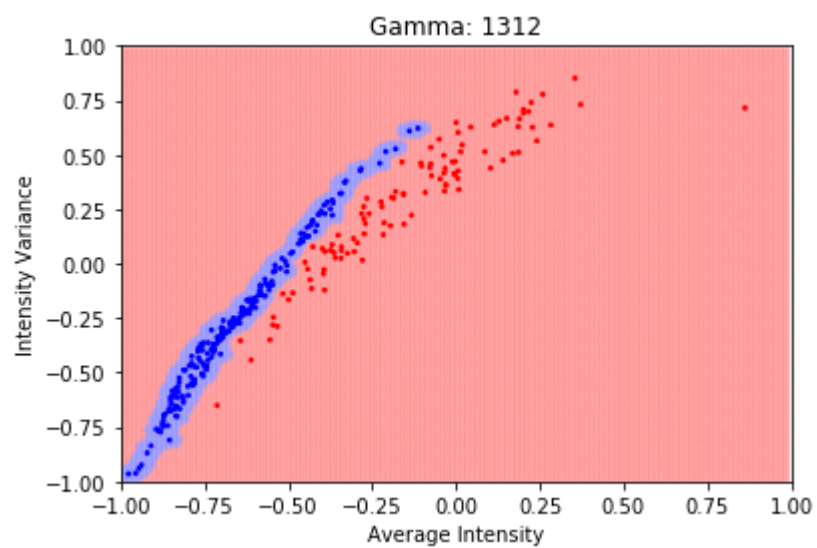
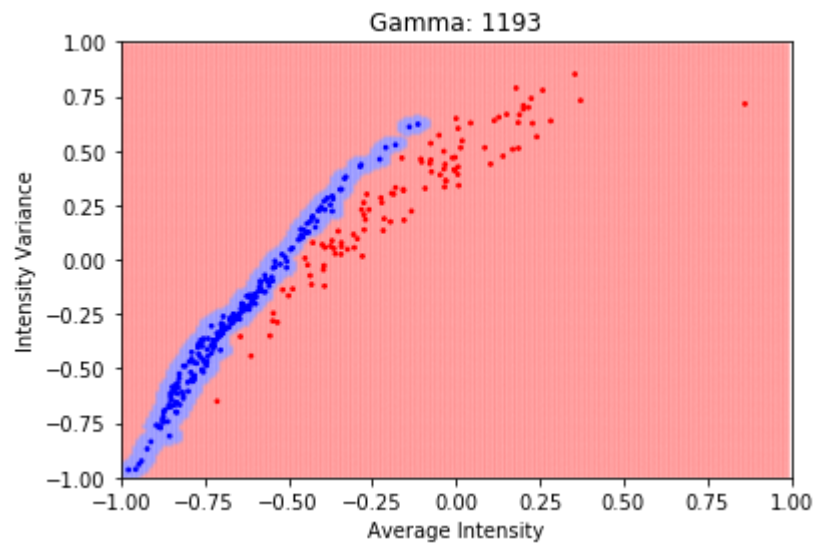
In []:

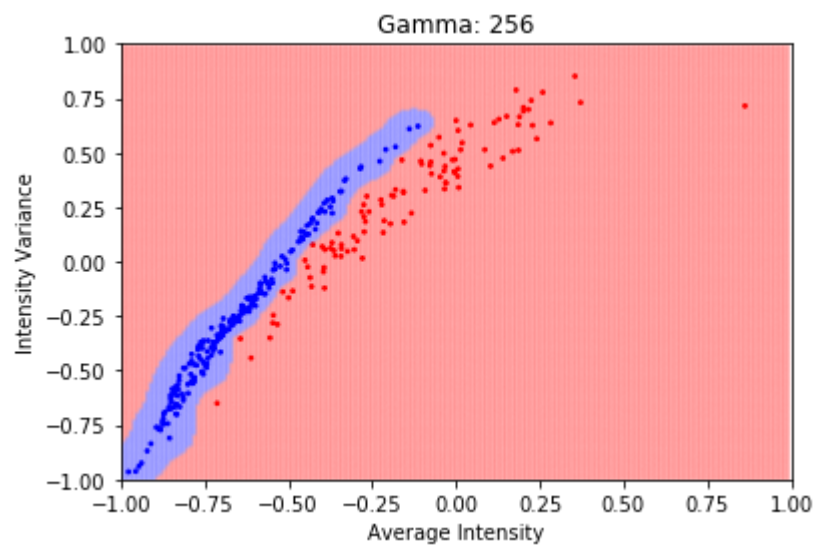
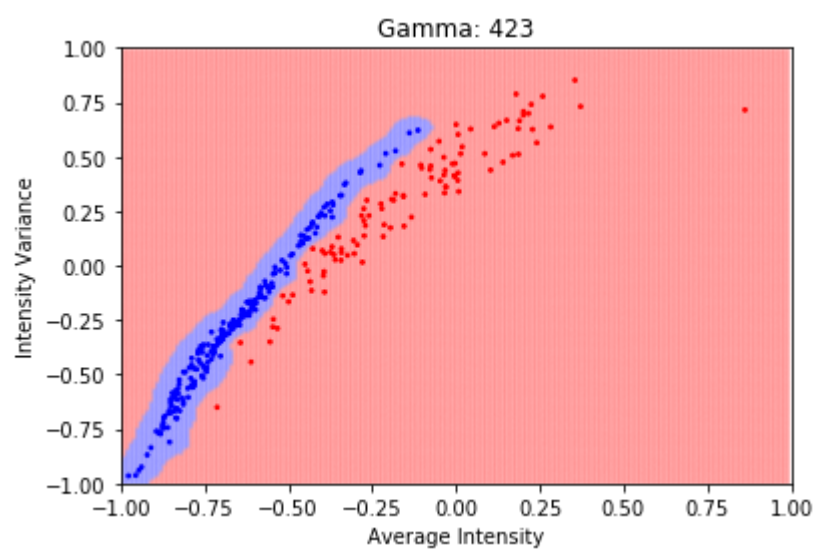
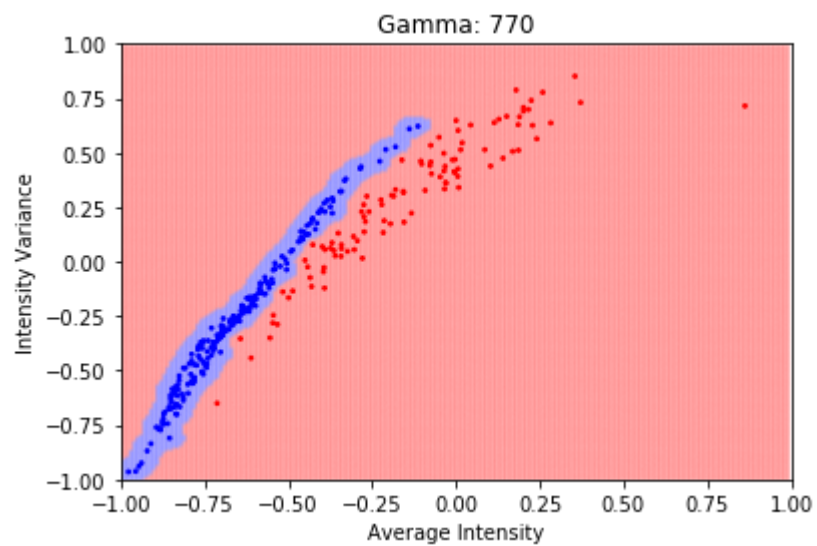
```
In [83]: # Varying only C. Keeping C and Gamma constant
x = []
y = []
z = []
p = []
degree = [2,5,10,20]

for i in range(1,10):
    gamma = np.random.randint(1,1500)
    svm = SVC(kernel = 'rbf', gamma= gamma)
    cvs = cross_val_score(svm, simpleTrain, trainDigits, cv = 10, scoring='accuracy')
    err = 1-cvs
    evsm = 1-(cvs.mean())
    p.append(err)
    x.append(gamma)
    y.append(evsm)
    z.append([x,evsm])
    svm.fit(simpleTrain, trainDigits)
    decisionRegion(svm,X, Y)
    mp.title("Gamma: " + str(gamma))
    mp.show()

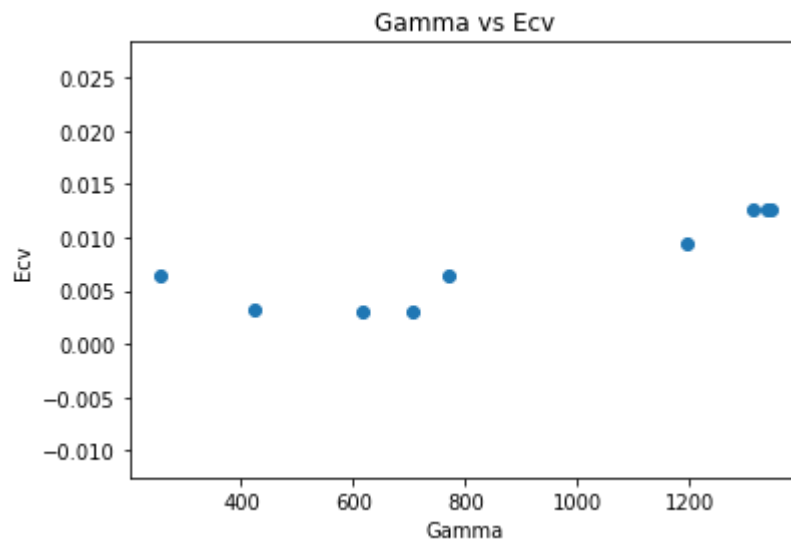
z = minimum(x,y)
print("Optimal Value of Gamma = " + str(z))
mp.scatter(x,y)
mp.xlabel('Gamma')
mp.ylabel('Ecv')
mp.title("Gamma vs Ecv")
mp.show()
```





Optimal Value of Gamma = 617



In [84]: *# From the above plots, we could say that the models with smaller values of gamma underfit the data.*
And the models with greater values of gamma formed smaller islands i.e. over fit the data.

```

In [92]: # Varying both C and Gamma
x1 = []
y1 = []
z1 = []
p1 = []
c_opt4 = []
g = []
r = []

for i in range(1,5):
    x = []
    y = []
    z = []
    p = []
    gamma = np.random.randint(1,1500)
    g.append(gamma)
    for j in range(1,25):
        c = np.random.uniform(0.01,100)
        svm = SVC(kernel = 'rbf', C = c, gamma= gamma)
        cvs = cross_val_score(svm, simpleTrain, trainDigits, cv = 10, scoring=
'accuracy', n_jobs = -1)
        err = 1-cvs
        evsm = 1-(cvs.mean())
        p.append(err)
        x.append(c)
        y.append(evsm)
        z.append(gamma)
        p1.append(err)
        x1.append(c)
        y1.append(evsm)
        z1.append(gamma)
    coptimal = minimum(x,y)
    c_opt4.append(coptimal)
    r.append(y[np.argmin(y)])
    mp.scatter(x,y, s=10)
    mp.xlabel("C")
    mp.ylabel("Error Ecv")
    mp.title("Gamma: " + str(gamma))
    mp.show()
    svm = SVC(kernel = 'rbf', C = coptimal, gamma= gamma)
    svm.fit(simpleTrain, trainDigits)
    decisionRegion(svm, X, Y)
    mp.title("Gamma: " + str(gamma))
    mp.show()

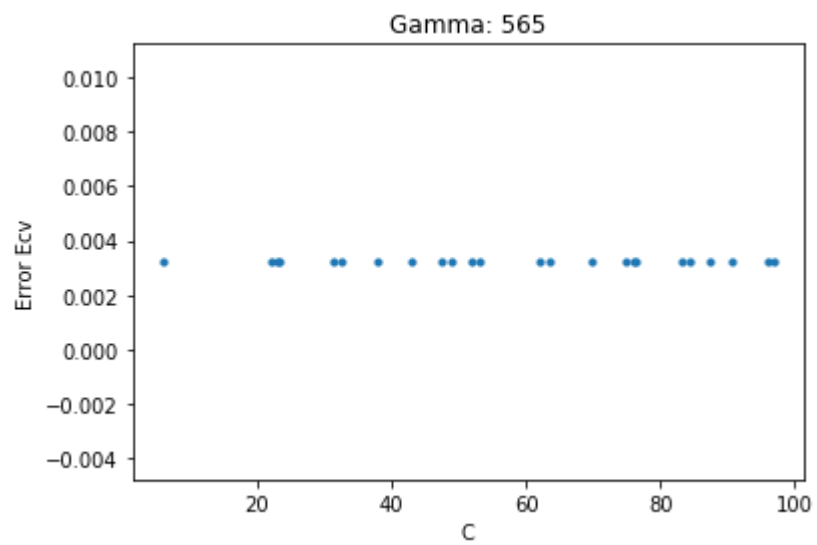
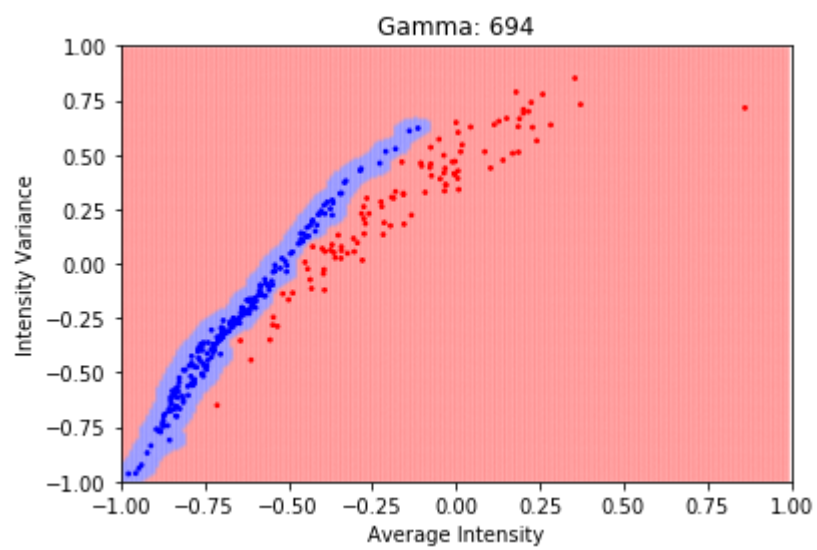
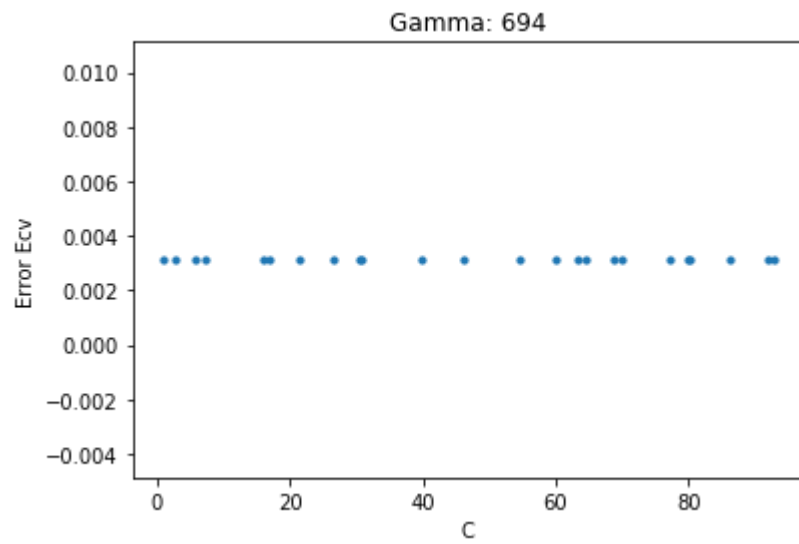
print('\n')
print("Optimal C Values for Gamma " + str(g) + " are: " + str(c_opt4))

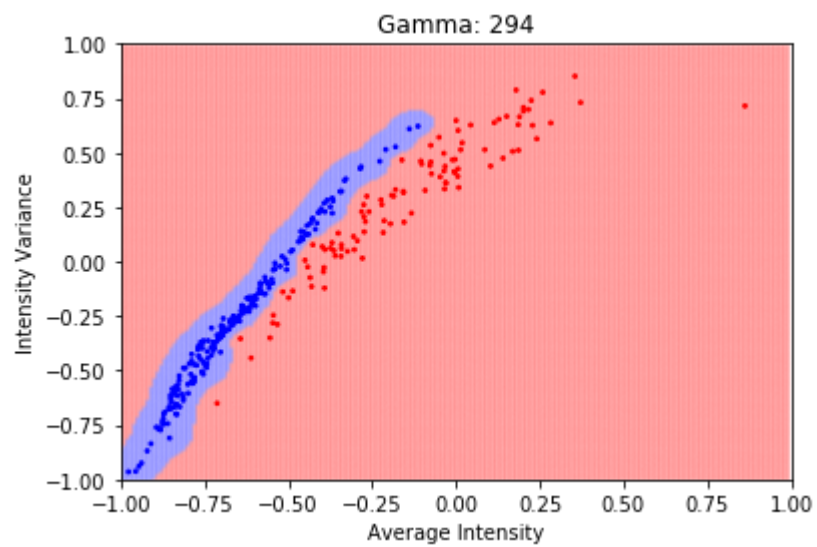
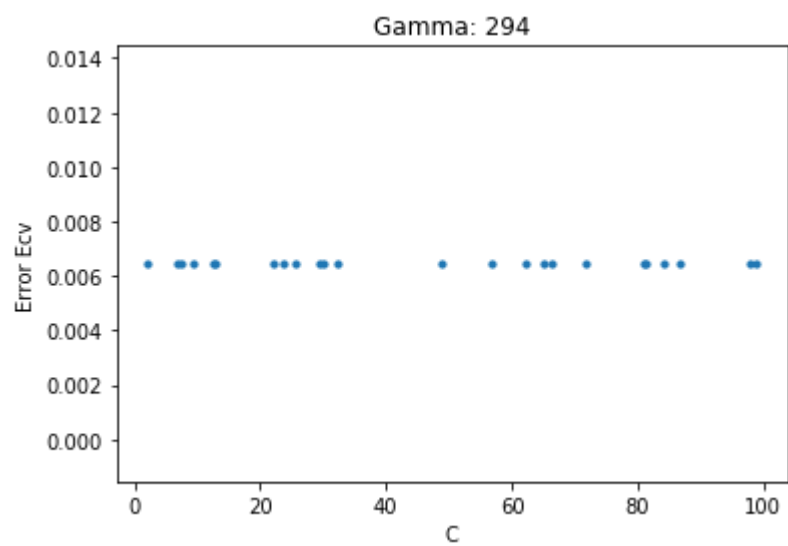
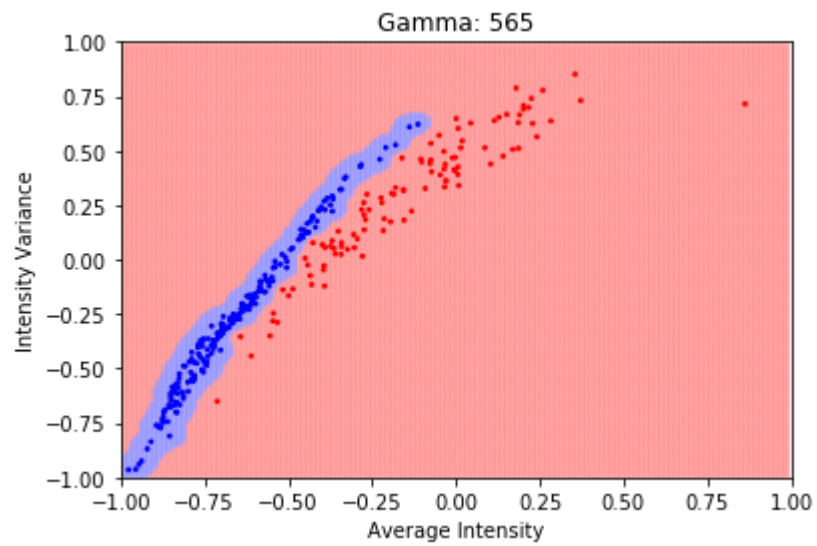
g_opt, c_optf, Min_Ecv = minimum3(z1,x1,y1)
print('\n')
print("Min Ecv Value is: " + str(Min_Ecv))
print("Optimal Gamma is: " + str(g_opt))
print("Optimal C Value is: " + str(c_optf))

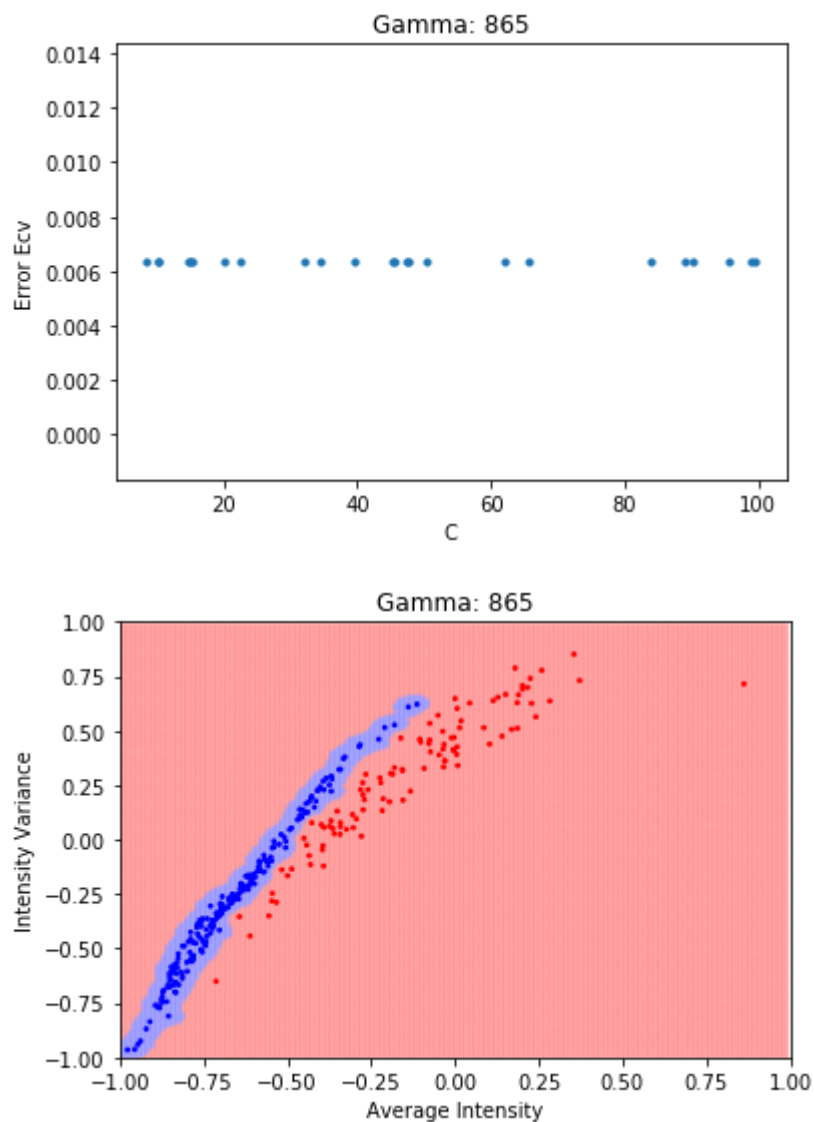
plt.scatter(g,r)
plt.xlabel('Gamma')

```

```
plt.ylabel('Min Ecv')  
plt.title('Gamma vs Ecv')  
plt.show()  
  
plt.scatter(g,c_opt4)  
plt.xlabel('Gamma')  
plt.ylabel('C value of Min Ecv')  
plt.title('Gamma vs C')  
plt.show()
```





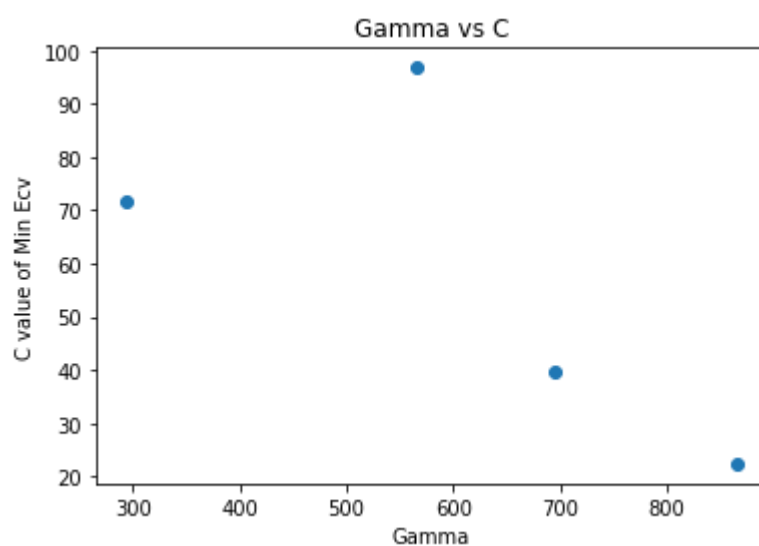
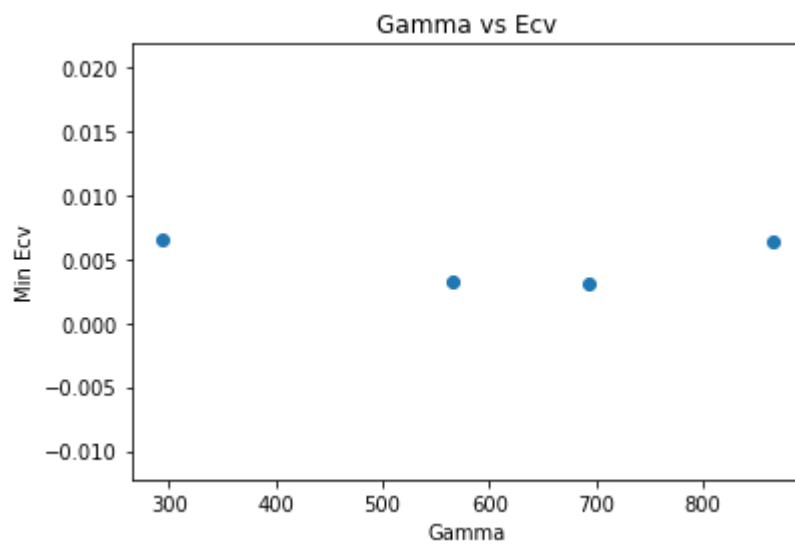


Optimal C Values for Gamma [694, 565, 294, 865] are: [39.87402383420889, 96.79593140378634, 71.77385236560717, 22.442276346482515]

Min Ecv Value is: 0.0031250000000000444

Optimal Gamma is: 694

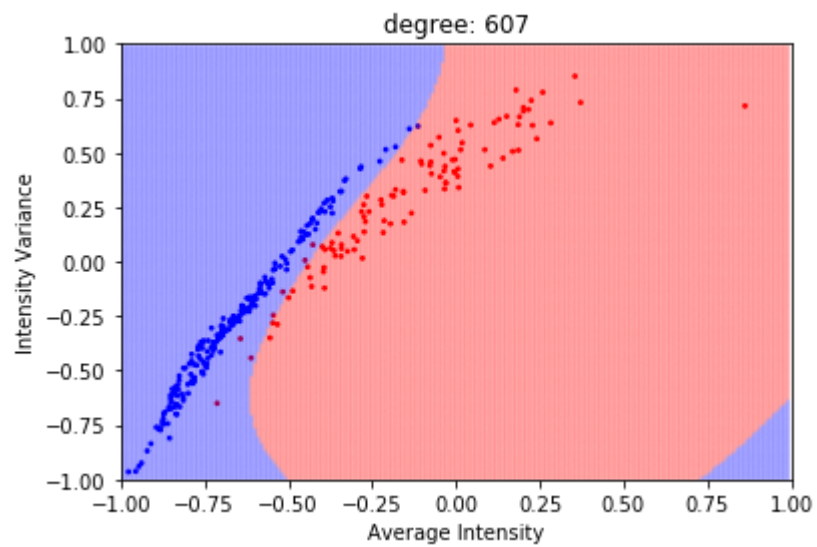
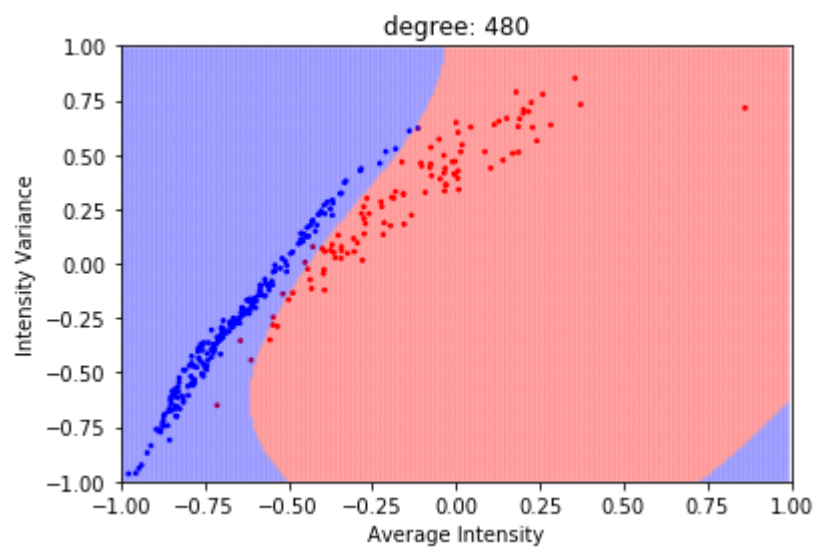
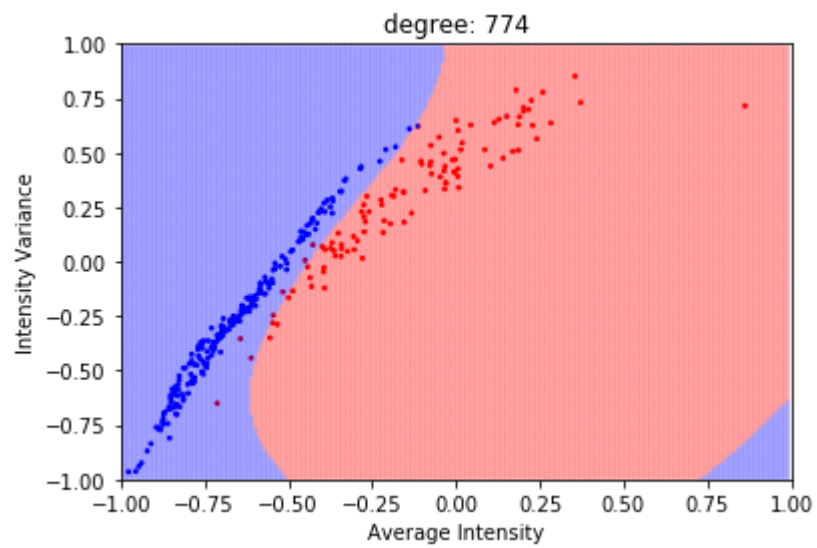
Optimal C Value is: 39.87402383420889

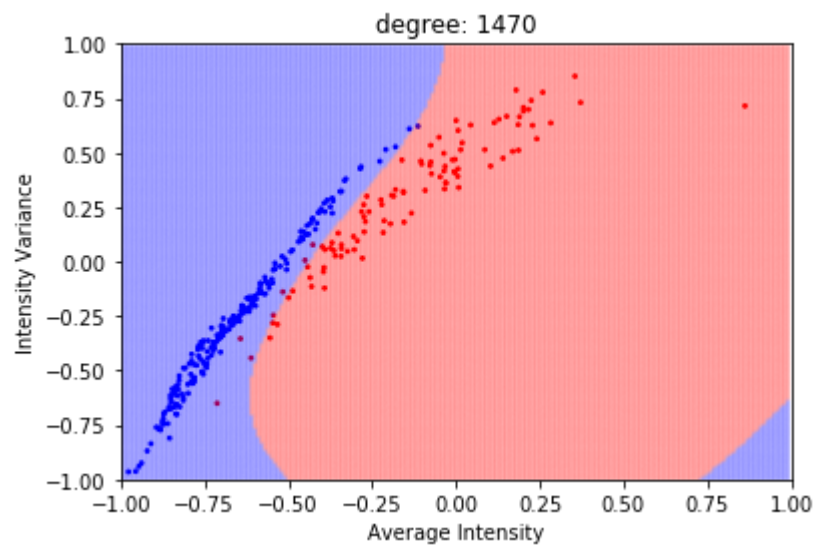
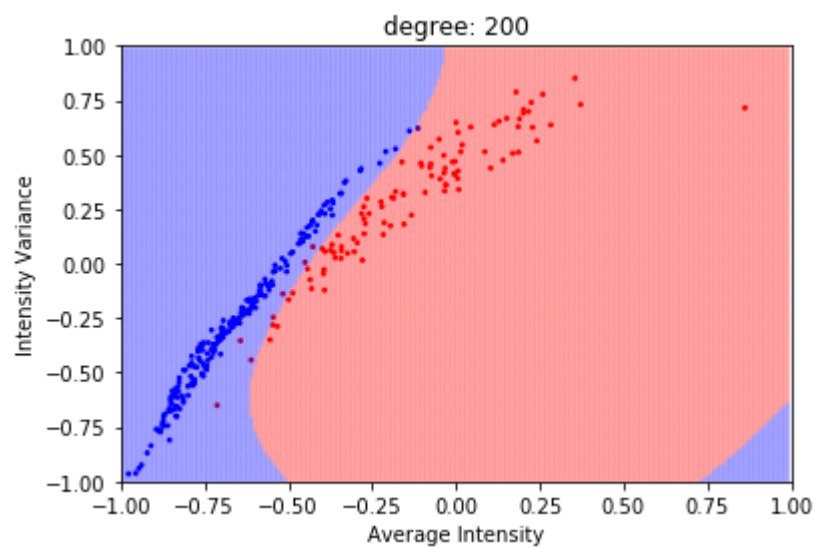
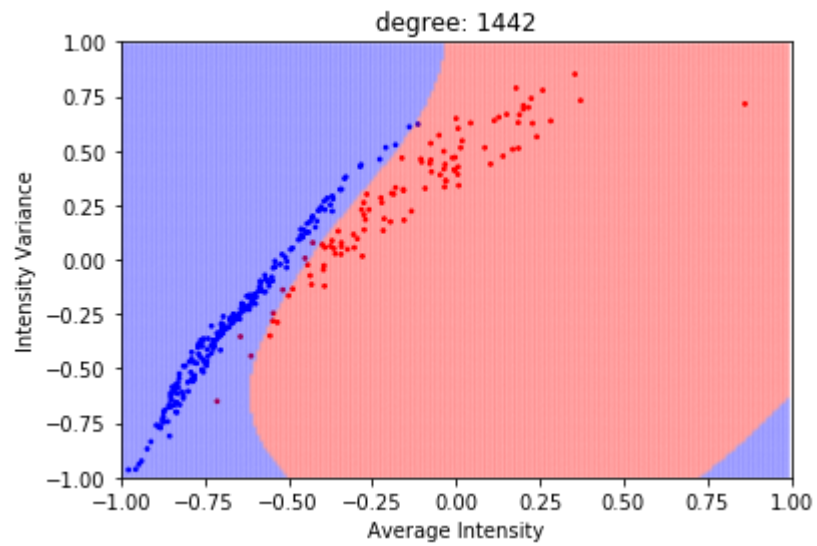


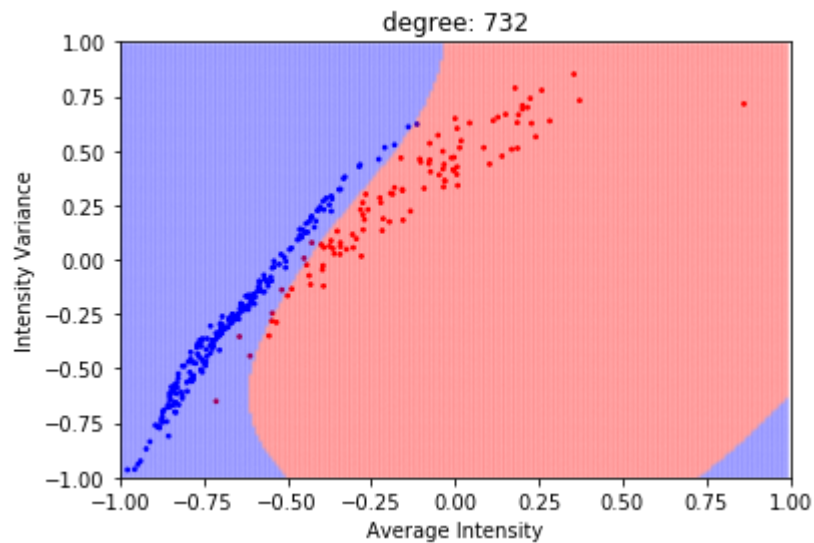
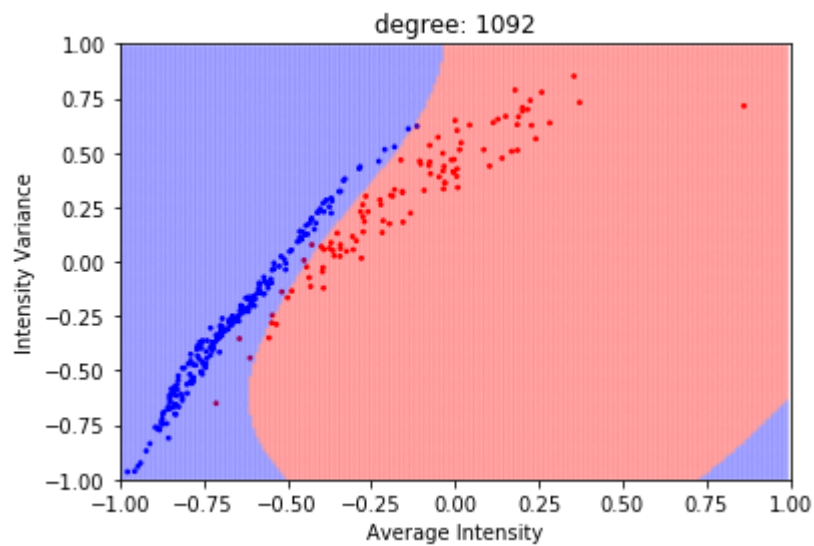
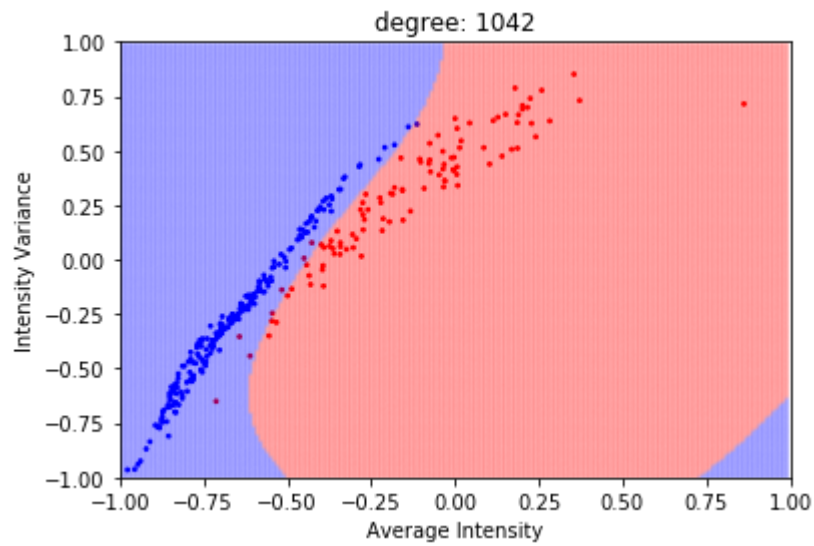
In [93]: *# As models with higher values of Gamma tend to overfit the data, we need smaller value of C to get minimum cross validation error.
Similarly, we need higher value of C for models with smaller value of Gamma as these models tend to underfit the data
It makes sense from the above Gamma vs C plot*

```
In [90]: # Varying only degree. Keeping C and Gamma constant
x = []
y = []
z = []
p = []
degree = [2,5,10,20]

for i in range(1,10):
    degree = np.random.randint(1,1500)
    svm = SVC(kernel = 'rbf', gamma= 'scale', degree = degree)
    cvs = cross_val_score(svm, simpleTrain, trainDigits, cv = 10, scoring='accuracy')
    err = 1-cvs
    evsm = 1-(cvs.mean())
    p.append(err)
    x.append(gamma)
    y.append(evsm)
    z.append([x,evsm])
    svm.fit(simpleTrain, trainDigits)
    decisionRegion(svm,X, Y)
    mp.title("degree: " + str(degree))
    mp.show()
```







In [91]: *# As you can see in above plots, the parameter degree has no effect on the model performance.
This makes sense as degree parameter is only valid for the poly kernel*

In []:

In []: