Homework 1: k-Nearest Neighbors

CS412

Released: January 22nd

Due: January 29th, 11:30pm on Gradescope

1 Getting started

Import your data into the language of your choice. In this zip file, there are two starter code files, a Jupyter notebook and a simple Python file. Also included is the `data.csv` file which contains all the handwritten digit data. We will only be using 1's and 5's for this assignment, but I have provided the entire dataset for those of you who are interested in experimenting with categorical classification beyond binary. The first column in the `data.csv` is the value of the digit it represents. In the starter code, I have selected just the 1's and 5's and then separated them into a training and testing set.

Notice that each row is 256 pixels saved from left to right, top to bottom forming a 16x16 square. Each pixel has an intensity which ranges from -1 to 1, where 1 is the darkest.
I have also pregenerated two simple features to represent each point: mean intensity and intensity variance. The starter code plots these points

2 Drawing your first graph

Once the data is imported, you will want to select a way to graphically represent the data in two-dimensions. For this, we use mean intensity and intensity variance. This gives us a 2-dimensional feature space so we can plot decision regions. Additionally, since the features are not perfect seperators, we make the machine learning of the problem more interesting and visual to study.

Once this is done, we create a graph of your data points normalized to $[-1, 1]$, so that all of your points fall between -1 and 1 on the x and y axes. You do **not** need to standardize to mean and standard deviation. Mark the 1's as red and the 5's as blue. **Label this Figure 1.1 in your report**. See that there are axis labels and use the `matplotlib` package documentation to label the figure.

3 1-Nearest Neighbor

In this section, color the regions of the graph by which is the closest in your two-dimensional space. Use Euclidean distance. Use the same normalization and axes that you had in Figure 1.1. **Label this Figure 1.2 in your report**. This is already done for you in the starter code.
Answer the following questions in your report:

a) Do you believe that this model suffers from underfitting or overfitting?
   Why or why not?

b) Do you expect accuracy to increase, decrease or stay about the same as a result of our transformation to the 2-dimensional space. Why?

Use 10-fold cross validation to determine the cross validation error for the set under the following conditions and present their error for the following 1-Nearest-Neighbor problems. The 2-dimension problem should be only based on your two dimensions from part 2 and the 256 dimension problem should be on the points in the 256-dimensional image space. **(1c) Comment on any differences you see in the results and what may have resulted in them**. You may need to find or modify your packages to get these values.

a) Euclidean distance - 2 dimensions

b) (EC) Manhattan distance - 2 dimensions

c) (EC) Chebyshev distance - 2 dimensions

d) Euclidean distance - 256 dimensions

e) (EC) Manhattan distance - 256 dimensions

f) (EC) Chebyshev distance - 256 dimensions

4 k-Nearest Neighbor

Consider all of the odd k-Neighbor models between 1-49. Produce a graph of the 10-fold cross validation results for each of the 25 candidates and show their result. The x axis should be k and the y axis should be the cross-validation error (E_{cv}). Do this for both the 256-dimension space. **Label this Figure 1.3** and report the value of k which you think yields the best result. (2a) Explain your answer.

(2b) Use the optimal value of k from Figure 1.3 and graph the 2-dimensional decision region for this k-Nearest neighbor model and **label this Figure 1.4**. Does this model suffer from overfitting or underfitting? Explain your answer.

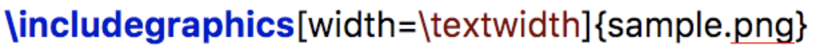
(2c) Graduate student question: (EC for undergrad) Provide the estimated error of the 25 models at the 95% confidence level by utilizing the variance of the cross validation fold errors. Select your model as that with the lowest 95% upper bound. Does this make a model more likely to overfit or underfit the data? Explain

5 Extra Credit

Find two new datasets, one which has a low optimum value for k and one which has a high optimum value for k. Explain what in the data may have led to this result and support your answer with figures as necessary. Additionally, give theE_{cv} for these models. To search for potential datasets, search the UCI datasets. Most are available as `.csv`

Making your report

For the code and the report, we highly recommend use of Jupyter notebook. I find it pretty simple to get running on either Windows or UNIX machines and it provides a very nice way to mix code and report. Included in the zip is some helper documentation to get you started. When you finally submit, there will be two submissions on gradescope. One for your pdf report and another for your zip file containing all your code.
I have included this LaTeX code here to help with compiling your report if you choose to. You can include images by placing them in the source file and adding:



```
\includegraphics[width=\textwidth]{sample.png}
```

Also, **HERE** is a resource for coding in R. **HERE** is a resource for starting code in Python. Starter code will be posted on Thursday, Jan 24.