

ECE/CS 559 - Spring 2020 - Homework #3

Due: 03/02/2020, 11pm.

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1. (50 pts) In this computer experiment, we will implement the gradient descent method and Newton's method. Let $f(x, y) = -\log(1-x-y) - \log x - \log y$ with domain $\mathcal{D} = \{(x, y) : x+y < 1, x > 0, y > 0\}$.
 - (a) Find the gradient and the Hessian of f on paper.
 - (b) Begin with an initial point in $w_0 \in \mathcal{D}$ with $\eta = 1$ and estimate the global minimum of f using the Gradient descent method, which will provide you with points w_1, w_2, \dots . Report your initial point w_0 and η of your choice. Draw a graph that shows the trajectory followed by the points at each iteration. Also, plot the energies $f(w_0), f(w_1), \dots$, achieved by the points at each iteration. Note: During the iterations, your point may "jump" out of \mathcal{D} where f is undefined. If that happens, change your initial starting point and/or η .
 - (c) Repeat part (b) using Newton's method.
 - (d) Compare the speed of convergence of gradient descent and Newton's method, i.e. how fast does each method approach the estimated global minimum?
2. (50 pts) Perform the following steps:
 - (a) Let $x_i = i, i = 1, \dots, 50$.
 - (b) Let $y_i = i + u_i, i = 1, \dots, 50$, where each u_i should be chosen to be an arbitrary real number between -1 and 1 .
 - (c) Find the linear least squares fit to $(x_i, y_i), i = 1, \dots, 50$ analytically using the matrix pseudoinverse. Note that the linear least squares fit is the line $y = w_0 + w_1x$, where w_0 and w_1 should be chosen to minimize $\sum_{i=1}^{50} (y_i - (w_0 + w_1x_i))^2$.
 - (d) Plot the points $(x_i, y_i), i = 1, \dots, 50$ together with their linear least squares fit.
 - (e) Find (on paper) the gradient of $\sum_{i=1}^{50} (y_i - (w_0 + w_1x_i))^2$ (derivatives with respect to w_0 and w_1).
 - (f) (Re)find the linear least squares fit numerically using the gradient descent algorithm. Compare with (c).