HW1

1) Signum function:- $sgn(x) = \begin{cases} 1 & \text{for } x > 0 \\ 0 & \text{for } x = 0 \\ -1 & \text{for } x < 0 \end{cases}$

-1 - False

1 - true

$f(x) = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2$

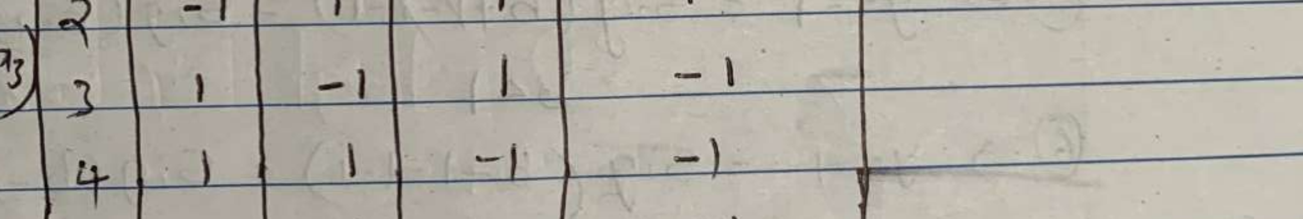
$\bar{x}_1 x_2 x_3$:- AND of $\bar{x}_1$, x_2 and x_3

~~y=sig~~ (~~b-x1+~~x_2+x_3)

~~ass~~

	x_1	x_2	x_3	Desired o/p
1	-1	-1	-1	-1
2	-1	1	1	1
3	1	-1	1	-1
4	1	1	-1	-1
5	-1	-1	1	-1
6	1	-1	-1	-1
7	-1	1	-1	-1
8	1	1	1	-1

① ⇒ ~~y = -1 = sig(b~~



$y = sig(b - x_1 + x_2 + x_3)$

① $\Rightarrow y = -1 = sig(b + 1 - 1 - 1) = sig(b - 1)$

$b - 1 < 0$

$b < 1$

② $\Rightarrow y = 1 = sig(b + 1 + 1 + 1) = sig(b + 3)$

$b + 3 > 0$

$b > -3$ — Ⓐ

③ $\Rightarrow y = -1 = sig(b - 1 - 1 + 1) = sig(b - 1)$

$b - 1 < 0$

$b < 1$

④ $\Rightarrow y = -1 = sig(b - \cancel{1} + \cancel{1} - 1) = sig(b - 1)$

$b < 1$

⑤ $\Rightarrow y = -1 = sig(b + \cancel{1} - \cancel{1} - 1) = sig(b - 1)$

$b < 1$

⑥ $\Rightarrow y = -1 = sig(b - 1 - 1 - 1) = sig(b - 3)$

$b < 3$

⑦ $\Rightarrow y = -1 = sig(b + 1 + \cancel{1} - \cancel{1}) = sig(b + 1)$

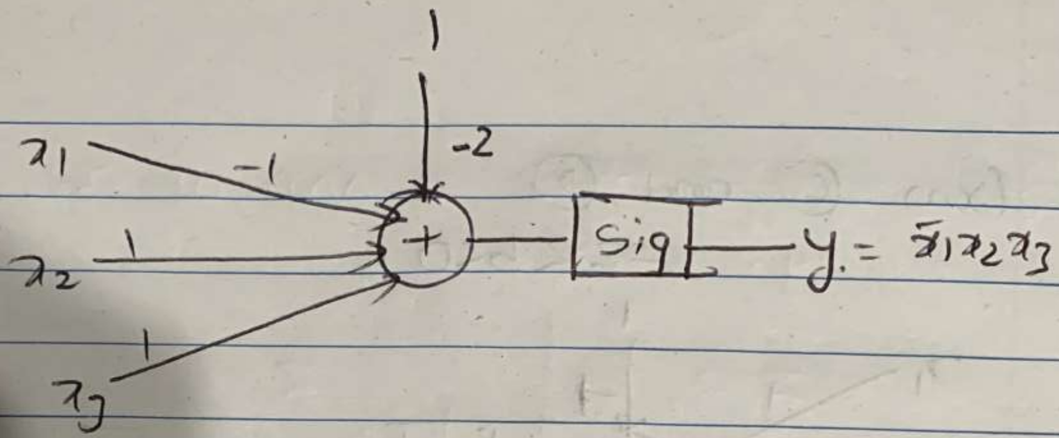
$b + 1 < 0$

$b < -1$ — Ⓑ

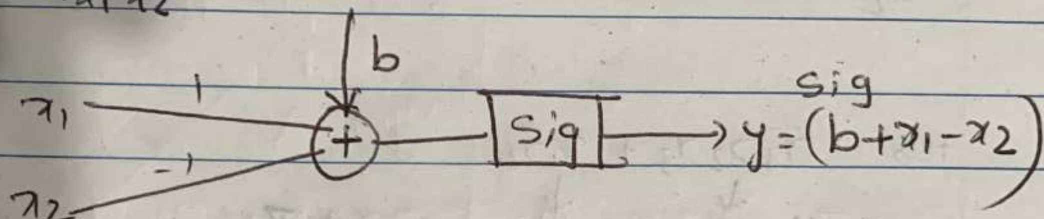
⑧ $\Rightarrow y = -1 = sig(b - \cancel{1} + \cancel{1} + 1) = sig(b + 1)$

$b < -1$

from Ⓐ and Ⓑ :- $-3 < b < -1$



Now $x_1 \bar{x}_2$



S	x_1	x_2	Desired o/p $(x_1 \bar{x}_2)$
1	-1	-1	-1
2	1	-1	1
3	-1	1	-1
4	1	1	-1

① $\Rightarrow$ $y = -1 = \text{sig}(b - 1 + 1) = \text{sig}(b)$

$b < 0$ — Ⓒ

② $\Rightarrow$ $y = 1 = \text{sig}(b + 1 + 1) = \text{sig}(b + 2)$

$b > -2$ — Ⓓ

③ $\Rightarrow$ $y = -1 = \text{sig}(b - 1 - 1) = \text{sig}(b - 2)$

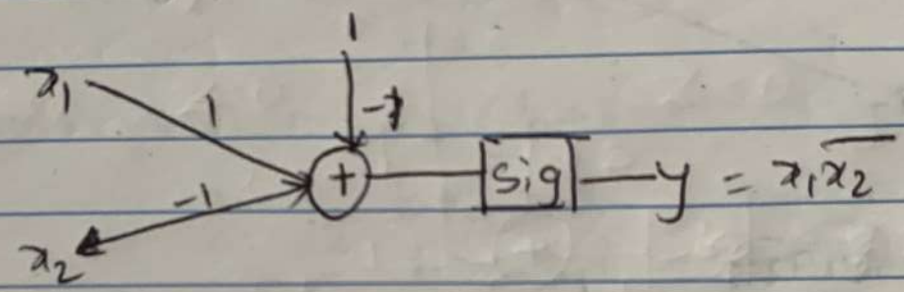
$b < 2$

④ $\Rightarrow$ $y = -1 = \text{sig}(b + 1 - 1) = \text{sig}(b)$

$b < 0$

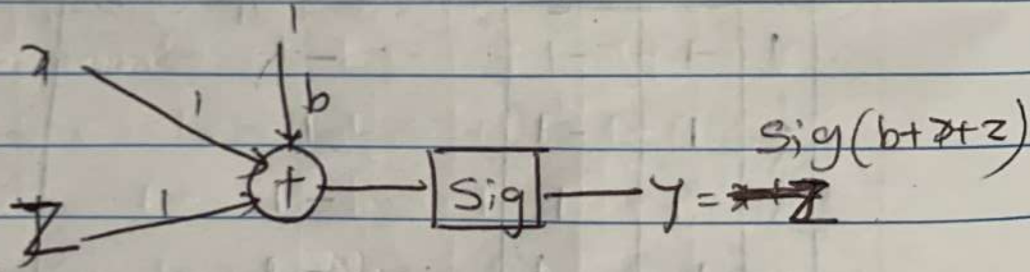
from (C) and (D)

$-2 < b < 0$



For $\bar{x}_1 x_2 x_3 + x_1\bar{x}_2$

$\underbrace{\bar{x}_1 x_2 x_3}_{x} + \underbrace{x_1\bar{x}_2}_{z} = x + z$



S	x	z	O/P
1	-1	-1	-1
2	-1	1	1
3	1	-1	1
4	1	1	1

(1) $\Rightarrow y = -1 = \text{sig}(b-1-1) = \text{sig}(b-2)$

$b < 2$ — (i)

(2) $\Rightarrow y = 1 = \text{sig}(b-1+1) = \text{sig}(b)$

$b > 0$

③ $\Rightarrow y = 1 = sig(b+1-1) = sig(b)$

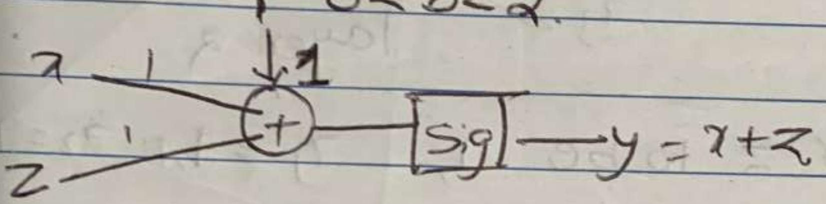
$b > 0$ — (ii)

④ $\Rightarrow y = 1 = sig(b+1+1) = sig(b-2)$

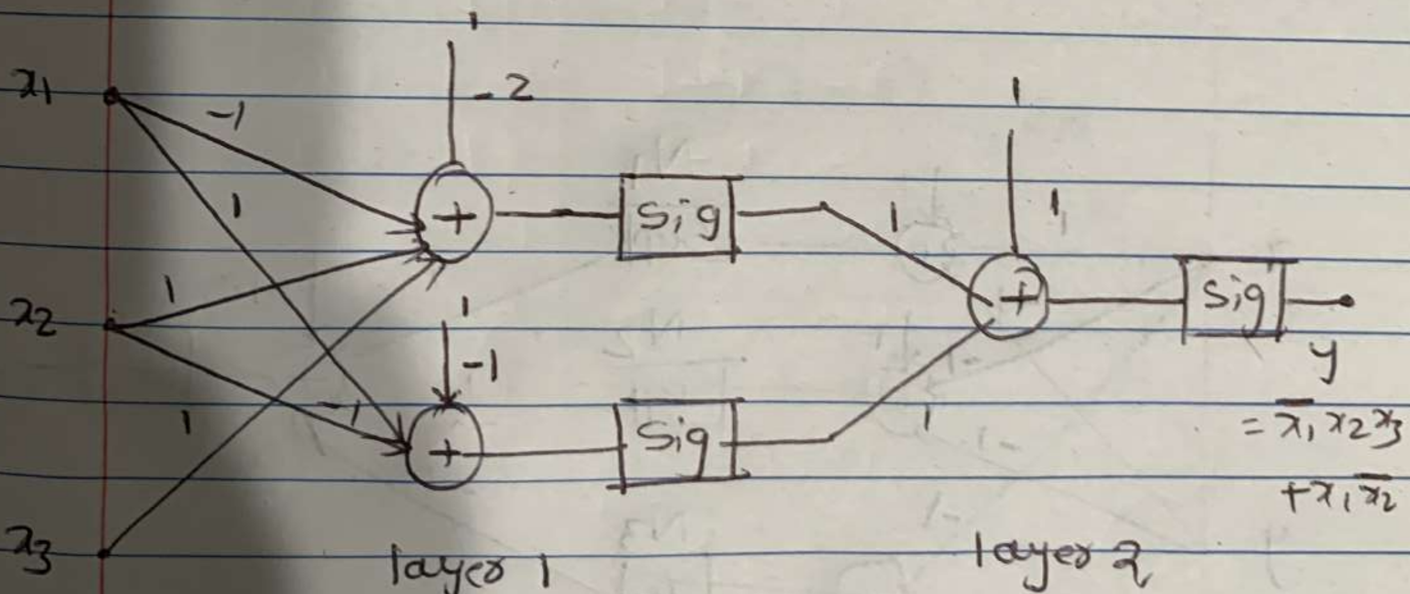
$b > 2$

from (i) and (ii)

$0 < b < 2$



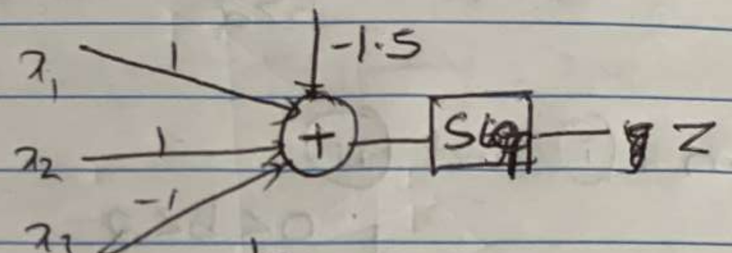
Now, final network is.



2) The second layer is AND of x_1, x_2 and $\bar{x}_3$

i.e $y = x_1 x_2 \bar{x}_3$

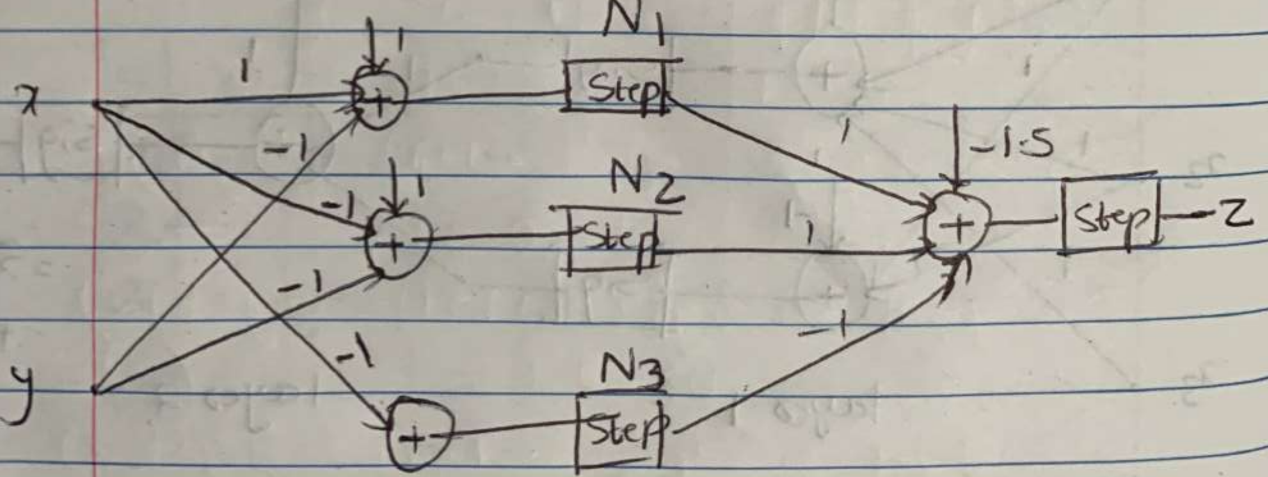
So,



layer 2

For Z to be 1, $x_1 = 1$, $x_2 = 1$ and $x_3 = 0$

<u>Given network</u>



layer 1

layer 2

In layer 1, a) output of $N_1 = 1$

i.e $y_1 = sig(1 + x - y) = 1$

$1 + x - y \geq 0 \Rightarrow \underline{x - y \geq -1}$ — l_A

$\cancel{y \leq x+1}$

b) output of $N_2 = 1$

i.e $y_2 = sig(1 - x - y) = 1$

$1 - x - y \geq 0$

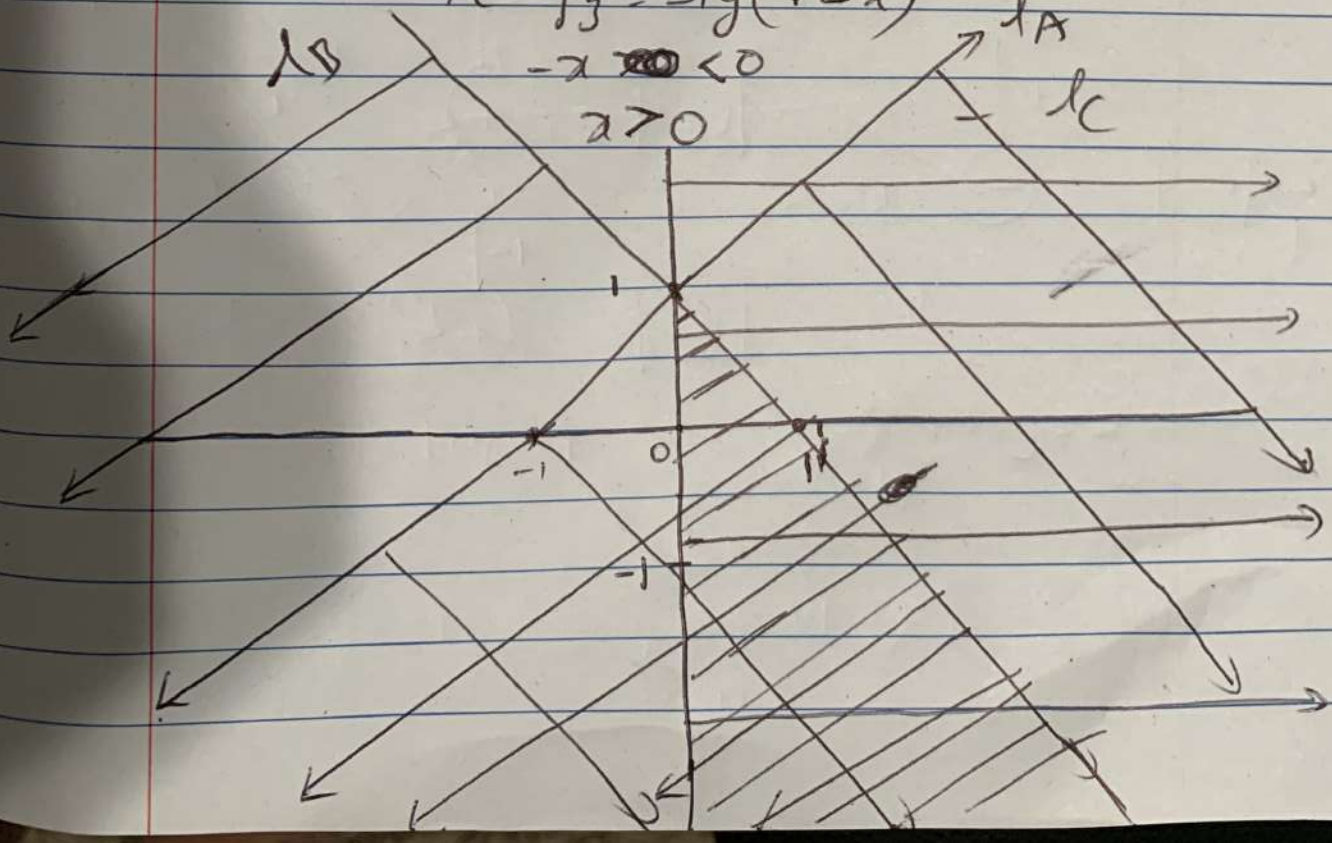
$x + y \leq 1$ — l_B

c) output of $N_3 = 0$

i.e $y_3 = sig(-x)$

$-x < 0$

$x > 0$



boundary region