

Homework K-3

1) ~~$f(x,y)$~~
 $f(x,y) = -\log(1-x-y) - \log x - \log y$

a) Gradient $g = \begin{bmatrix} \frac{\partial f}{\partial w_1} \\ \frac{\partial f}{\partial w_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$

$$\frac{\partial f}{\partial x} = -\frac{1}{1-x-y}(-1) - \frac{1}{x} = \frac{1}{1-x-y} - \frac{1}{x}$$

$$\frac{\partial f}{\partial y} = -\frac{1}{1-x-y}(-1) - \frac{1}{y} = \frac{1}{1-x-y} - \frac{1}{y}$$

$$g = \begin{bmatrix} \frac{1}{1-x-y} - \frac{1}{x} \\ \frac{1}{1-x-y} - \frac{1}{y} \end{bmatrix} = \frac{1}{1-x-y} \begin{bmatrix} \frac{x-1+x+y}{x} \\ \frac{y-1+x+y}{y} \end{bmatrix}$$

$$= \frac{1}{1-x-y} \begin{bmatrix} \frac{2x+y-1}{x} \\ \frac{x+2y-1}{y} \end{bmatrix}$$

Hessian:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x} \left[(1-x-y)^{-1} - x^{-1} \right]$$

$$= -1(1-x-y)^{-2}(-1) + x^{-2}$$

$$= \frac{1}{(1-x-y)^2} + \frac{1}{x^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y} \left[(1-x-y)^{-1} - y^{-1} \right]$$

$$= \frac{1}{(1-x-y)^2} + \frac{1}{y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y} \left[(1-x-y)^{-1} - x^{-1} \right] = \frac{1}{(1-x-y)^2}$$

$$H = \begin{bmatrix} \frac{1}{(1-x-y)^2} + \frac{1}{x^2} & \frac{1}{(1-x-y)^2} \\ \frac{1}{(1-x-y)^2} & \frac{1}{(1-x-y)^2} + \frac{1}{y^2} \end{bmatrix}$$

$$H = \frac{1}{(1-x-y)^2} \begin{bmatrix} \frac{x^2 + (1-x-y)^2}{x^2} & 1 \\ 1 & \frac{y^2 + (1-x-y)^2}{y^2} \end{bmatrix}$$