

Practice problems 8

1. For a balanced transportation problem of the form given below,

$$\begin{aligned}
 &\text{Minimize } \sum_i \sum_j c_{ij} x_{ij} \\
 &\text{subject to } \sum_i x_{ij} = b_j, \quad j = 1, 2, \dots, n, \\
 &\text{subject to } \sum_j x_{ij} = a_i, \quad i = 1, 2, \dots, m, \\
 &\mathbf{x} \geq \mathbf{0},
 \end{aligned}$$

with $\sum_i a_i = \sum_j b_j$, check the correctness of the following statements with proper justification.

- (a) If \mathbf{B} is a basis matrix of the above problem then the entries of \mathbf{B}^{-1} are either 1, -1 or 0.
- (b) If the transportation problem (P) given above is feasible then it has an optimal solution.
- (c) If $m = 4$ and $n = 4$, then there exists a θ -loop with exactly $m + n - 1$ cells.
- (d) In every row and column of the transportation array there must be at least one basic cell.
- (e) If all the a_i and b_j are positive integers and x^* is an optimal solution satisfying the condition that $\Delta = \{(i, j) : i, j, \text{ such that } x_{ij}^* \text{ is not an integer}\} \neq \phi$, then the cells corresponding to nonzero components of x^* contains a θ -loop.
- (f) If \mathfrak{B} is a collection of $m + n - 1$ basic cells and if the θ -loop in $\mathfrak{B} \cup \{(p, q)\}$ is considered, then

$$\sum \{c_{ij} : (i, j) \text{ gets the allocation } + \theta\} - \sum \{c_{ij} : (i, j) \text{ gets the allocation } - \theta\} = c_{pq} - z_{pq} \text{ (where } z_{ij} \text{ is as in simplex).}$$
- (g) If $m = 4$ and $n = 4$, then every column in the simplex table (leaving out the $B^{-1}b$) corresponding to any BFS will have atleast one zero entry.
- (h) If in the 1st row of the array the cost c_{1j} is changed to $c_{1j} + 5$, for all $j = 1, 2, \dots, n$, then the optimal solution for the new problem is always equal to the optimal solution of the old problem.
- (i) If all the a_i 's and b_j 's of the above transportation problem are integers then any basic feasible solution of the above problem will also take integer values.
- (j) If $\bar{a}_1, \bar{a}_2, \bar{a}_3$ (where $\bar{a}_i = B^{-1}a_i$, a_i is the i th column of A), are three columns in any iteration (table) of the simplex algorithm applied to this problem, then $\{\bar{a}_1, \bar{a}_2, \bar{a}_3\}$ is linearly independent and the sum of the elements of the vector $u = \bar{a}_1 + \bar{a}_2 + \bar{a}_3$, is equal to 3.
- (k) If \mathcal{B} is a collection of basic cells of a transportation array and if $(p, q) \in \mathcal{B}$, then there exists at least one more cell from \mathcal{B} in either the p -th row or q -th column of the array.

2. Consider the following transportation problem with c_{ij} 's, a_i 's and b_j 's as given below.

							a_i
	2	0	3	5	6	3	10
	1	3	4	0	9	7	24
	3	7	9	1	8	1	9
	4	8	7	3	6	8	36
b_j	24	4	1	19	23	8	

- Construct a basic feasible solution such that $\{(1, 1), (1, 3), (3, 4)\} \subseteq \mathcal{B}$.
- Construct a θ -loop which includes the cells $(1, 1), (1, 4), (3, 4)$ and $(3, 6)$.
- Is it true that given any collection of three cells in the above array there will be a BFS of the above problem containing those three cells?
- Solve the following transportation problem with, $\{(1, 2), (1, 5), (2, 1), (2, 4), (3, 2), (3, 3), (3, 6), (4, 4), (4, 5)\}$ as the initial set of basic cells and by taking
 - $u_3 = 0$ in every stage of solving (that is the third supply is removed).
 - $u_3 = 1$ initially (that is the third supply constraint is removed) and then in the next iteration (if any) by taking $v_3 = 0$ (that is the third destination constraint is removed).