1

Problem-3: (a): Determine the phase sequence of the set of voltages:

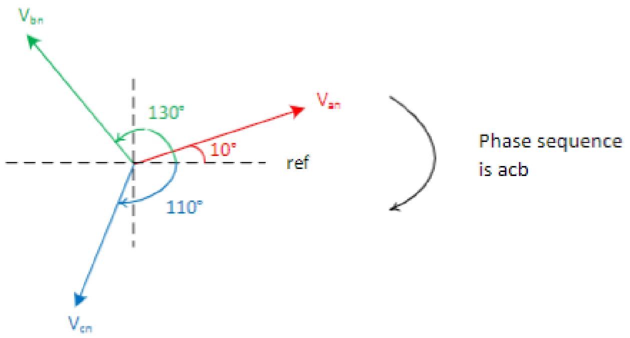
$$v_{an} = 200\sqrt{2}\sin(\omega t + 10°)\ [V]$$
$$v_{bn} = 200\sqrt{2}\sin(\omega - 230°)\ [V]$$
$$v_{cn} = 200\sqrt{2}\sin(\omega t - 110°)\ [V]$$

(b): Find the line voltages such as: V_{ab}, V_{bc} and V_{ca} for Y-connected source.

(c): Draw the phasor diagram of the phase and line voltages.

(a): The phasor form of the voltages are:

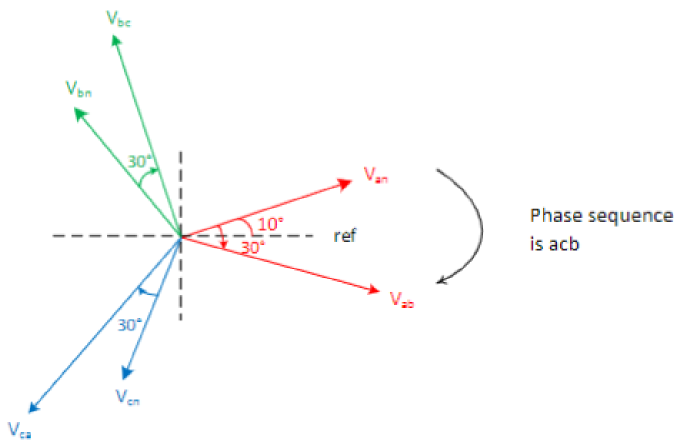
$$V_{an} = \frac{200\sqrt{2}}{\sqrt{2}}\angle 10°$$
$$= 200\angle 10°$$
$$V_{bn} = 200\angle - 230°$$
$$V_{cn} = 200\angle - 110°$$



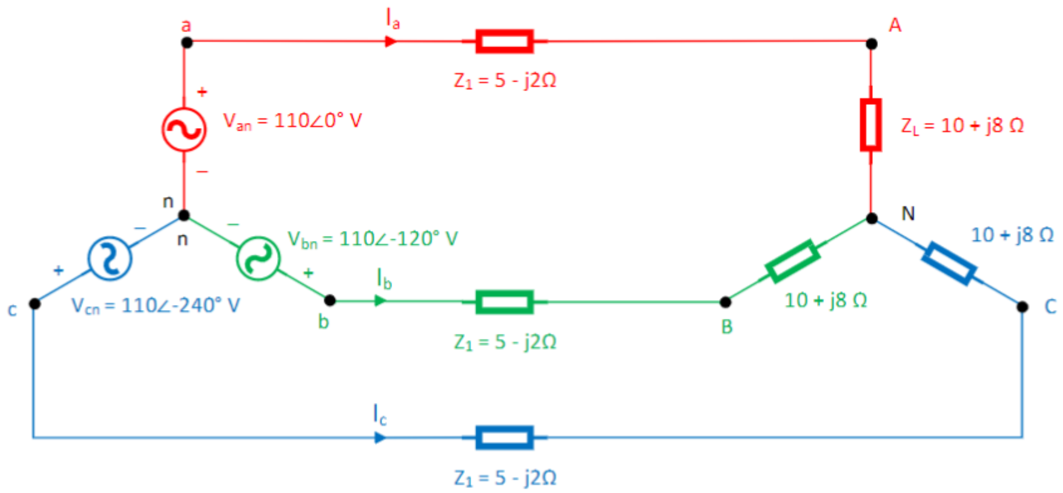
(b): Line voltages:

$$V_{ab} = V_{an} - V_{bn}$$
$$= 200\angle 10° - 200\angle\ 130°$$
$$= 200(0.985 + j0.174) - 200(-0.643 + j0.766)$$
$$= 200(1.628 - j0.592) \approx 346.46\angle - 20°\ V$$
$$V_{bc} = V_{bn} - V_{cn}$$
$$= V_{ab}\angle 120° \text{ For acb phase sequence}$$
$$= 346.46\angle 100°\ V$$
$$V_{ca} = V_{cn} - V_{an}$$
$$= V_{ab}\angle - 120°$$
$$\approx 346.46\angle - 140°\ V$$

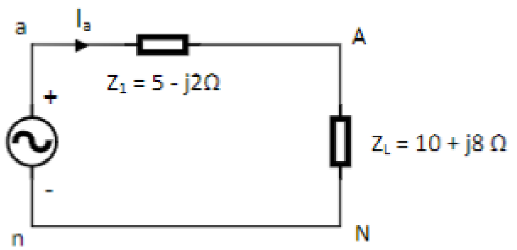
(c):



Problem-4: Calculate the currents in the three wire Y-Y balanced system.



a. The given 3-phase circuit is balanced circuit. Therefore I_a, I_b and I_c will be obtained taking 1-phase equivalent circuit.



$$Z_{eq} = Z_1 + Z_L = 15 + j6\ \Omega = 16.15\angle 21.8°\ \Omega$$

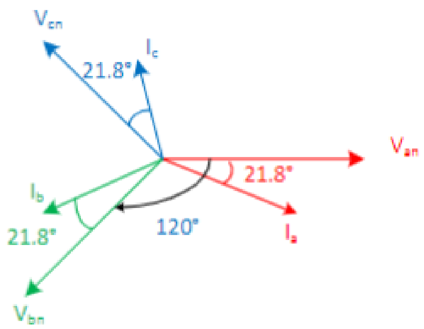
$$I_a = \frac{V_{an}}{Z_{eq}} = \frac{110\angle 0°}{16.15\angle 21.8°} = 6.81\angle -21.8°\ A$$

Since the source voltage is in positive sequence and both the source and load are balanced,

$$I_b = I_a\angle -120° = 6.81\angle -141.8°\ A$$

$$I_c = I_a\angle -240° = 6.81\angle -261.8° A = 6.81\angle 98.2°\ A$$

b. Phasor diagram of the currents and phase voltages.



3

Problem: A balanced three- phase three-wire system has a Y – connected load with V_{ab} = 400 V (RMS). Each phase contains three loads in parallel: -j100 Ω, 100 Ω and 50+j50 Ω. Find:

a) V_{cn}
b) I_a
c) The total active power drawn by the load.
d) The pf of the load.

The per-phase impedance of the loads is:

Z_1 = -j 100Ω

Z_2 = 100 Ω

Z_3 = 50 + j50 Ω

So, $Z_{eq} = Z_1 \,||\, Z_2 \,||\, Z_3$

= -j 100 Ω || 100 || (50+j50)

= 50+j0 Ω (Resistive)

a) $V_{an} = \frac{400}{\sqrt{3}} \angle 0^{\circ}$, $V_{cn} = \frac{400}{\sqrt{3}} \angle -240^{\circ}$ V

b) $I_a = \frac{V_{an}}{Z_{ph}} = \frac{\frac{400}{\sqrt{3}} \angle 0^{\circ}}{50} = \frac{8}{\sqrt{3}} \angle 0^{\circ}$ A

c) $S = 3\, V_{an} I_a^* = 3 \frac{400}{\sqrt{3}} \frac{8}{\sqrt{3}} = 3200$

P = 3200 W.

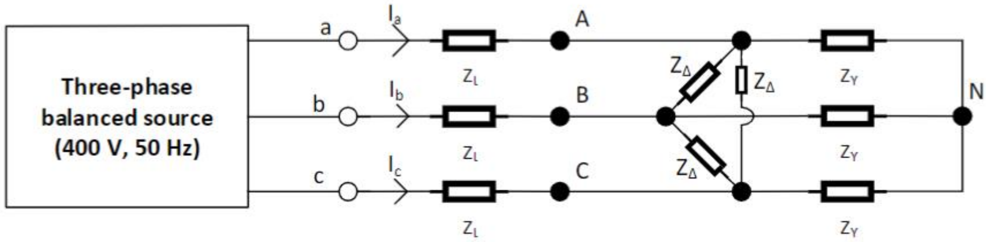
d) $\emptyset = 0^{\circ},\ pf = Cos\ 0^{\circ} = 1$

4

Problem-4: The circuit as shown below is excited by a balanced three phase source with a line voltage of 400 V. If $Z_L = 1+2j\ \Omega$, $Z_\Delta = 3-3j\ \Omega$ and $Z_Y = 1+j\ \Omega$:

a) Determine the magnitude of the line current of the combined loads.
b) Determine the total complex power drawn by the load.
c) Determine the pf at which the source is operating.
d) Draw the power triangle of the system.

A Y-connected capacitor bank is now connected across the source to compensate the reactive power drawn by the load such that the source will operate at power factor of 0.866 lagging. Find the per phase capacitance of the capacitor bank.



Solution:

The Δ-connected load will be transformed to Y-connected load.

So,

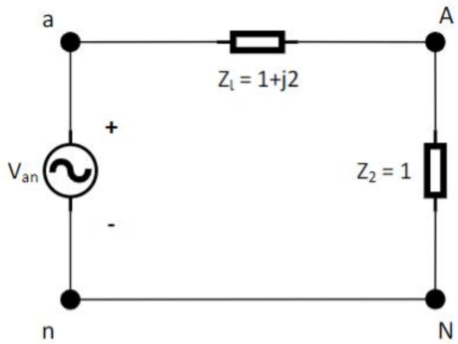
$$Z'_Y = \frac{Z_\Delta}{3} = 1 - j\Omega$$

Both Z_Y' and Z_Y are balanced load and parallel

So,

$$Z_2 = Z'_Y \parallel Z_Y = (1 - j) \parallel (1 + j) = 1\Omega\ (Resistive)$$

Since the system is balanced, per-phase analysis of the circuit is performed as shown below:



(a):

$$V_{an} = \frac{400}{\sqrt{3}}\angle 0^\circ$$

$$I_{an} = \frac{V_{an}}{Z_l + Z_2} = \frac{\frac{400}{\sqrt{3}}\angle 0^\circ}{2 + j2} = \frac{400}{\sqrt{3} \times 2\sqrt{2}\angle 45^\circ} = \frac{200}{\sqrt{6}}\angle -45^\circ = 81.649\angle -45^\circ\ A$$

(b):

$$S_{3\emptyset} = 3V_{an}I_a^*$$

$$S_{3\emptyset} = \frac{400}{\sqrt{3}}\frac{200}{\sqrt{6}}\angle 45^\circ \text{ VA}$$

$$S_{3\emptyset} = 40000\sqrt{2}\angle 45^\circ \text{ VA}$$

$$S_{3\emptyset} = 40\sqrt{2}\angle 45^\circ \text{ kVA}$$

$$S_{3\emptyset} = 56.56\angle 45^\circ \text{ kVA}$$

(c): The pf at which the source is operating:

$$Cos\emptyset = Cos\ 45^\circ = \frac{1}{\sqrt{2}} = 0.707\ (lag)$$

(d):

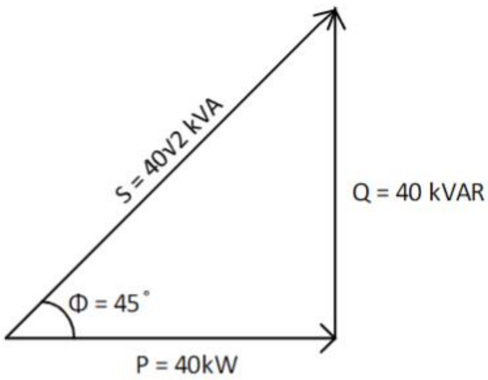
$$S_{3\emptyset} = 40\sqrt{2}\angle 45^\circ \text{ kVA}$$

$$S_{3\emptyset} = 40\sqrt{2}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) \text{ kVA}$$

$$S_{3\emptyset} = 40 + j40 \text{ kVA}$$

$$S_{3\emptyset} = P + jQ$$

P = 40 kW and Q = 40 kVAR



When the Y-connected capacitor bank is connected across the source, the source operating pf is 0.866 (lag).

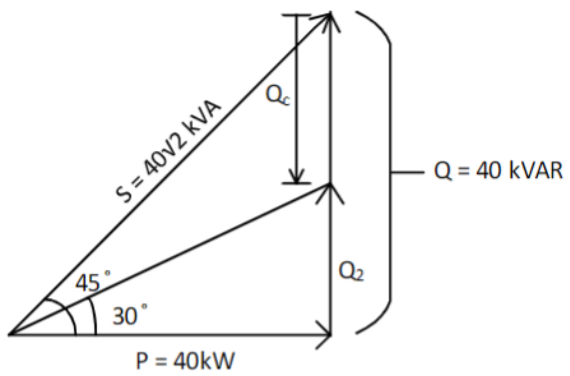
$$\emptyset = Cos^{-1}(0.866)$$

$$\emptyset = 30°$$

$$Q_2 = 40\, tan30°\ \text{kVAR}$$

Reactive power injected by the capacitor bank is:

$$Q_C = Q - Q_2$$



$$Q_C = 40 - 40\, tan30°\ \text{kVAR}$$

$$Q_C = 40\,(1 - \ tan30°)\ \text{kVAR}$$

$$Q_C = 40\left(1 - \frac{1}{\sqrt{3}}\right)\ \text{kVAR}$$

$$Q_C = 16.906\ \ \text{kVAR}$$

The total reactive power injected by the capacitor bank is:

$$Q_c = 3\frac{{V_{ph}}^2}{X_c}$$

$$X_c = 3\frac{{V_{ph}}^2}{Q_c}$$

$$X_c = 3\frac{\left(\frac{400}{\sqrt{3}}\right)^2}{16906}$$

$$\frac{1}{\omega C} = \frac{(400)^2}{16906}$$

$$C = \frac{16906}{(400)^2 \times 100\pi}$$

$$C = \frac{16906}{160000 \times 100\pi}$$

$$C = \frac{16906}{16\pi} \times 10^{-6}\ \mathrm{F}$$

$$C = 336.3\ \mu\mathrm{F}$$

5

Problem-5: The following three parallel-connected three-phase loads are fed by a balanced three-phase source.
Load 1: 250 kVA, 0.8 pf lagging
Load 2: 300 kVA, 0.95 pf lagging
Load 3: 400 kVA, unity pf.

If the line voltage is 13.8 kV, calculate the line current and the operating power factor of the source. Assume that the line impedance is zero.

$$S_1 = 250\ \angle Cos^{-1}0.8\text{ kVA}$$

$$S_1 = 250\ \times 0.8 + j0.6 \times 250\text{ kVA}$$

$$S_1 = 200 + j150\text{ kVA}$$

$$S_2 = 300\ \angle - Cos^{-1}0.95\text{ kVA}$$

$$S_2 = 285 - j93.67\text{ kVA}$$

$$S_3 = 450 + j0\text{ kVA}$$

So, total power drawn by the loads from the source is:

$$S = S_1 + S_2 + S_3$$

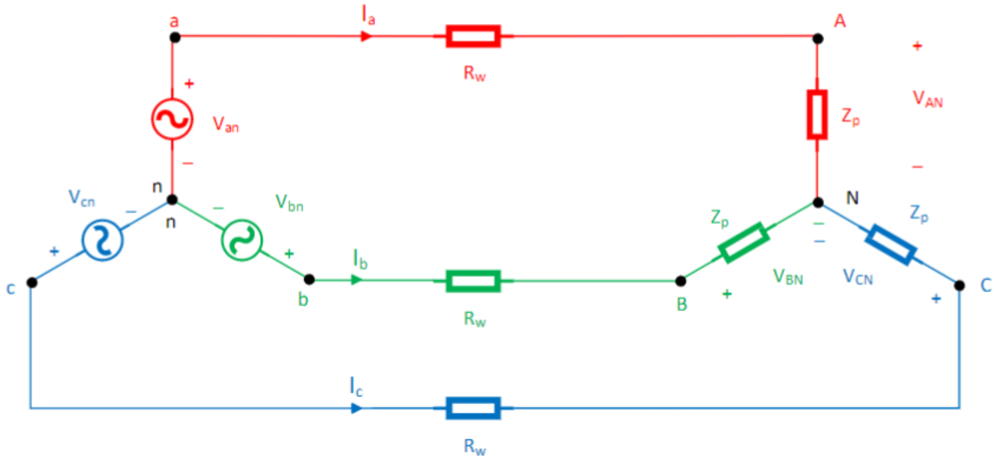
$$S = 935 + j56.33\text{ kVA}$$

$$S = 936.695\ \angle 3.448^{\circ}\text{ kVA}$$

6

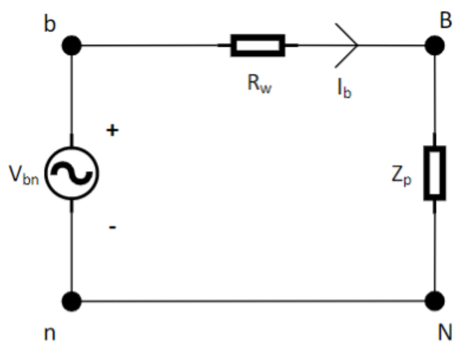
Problem-3: In the balanced three-phase system of figure, Z_p= 12+j5Ω and I_b= 20 ∠0° A with positive phase sequence. If the source is operating with power factor of 0.935 lagging. Find:

a) R_w
b) V_{bn}
c) V_{ab}
d) Complex power supplied by the source.



Given: Z_p = 12+j5Ω, I_b = 20 ∠0° A and pf angle Φ = $Cos^{-1}(0.935)$ = 20.77°

Since the system is balanced Y-Y connected system, the per-phase analysis can be performed as shown below:



a) The impedance Z_{bn} :

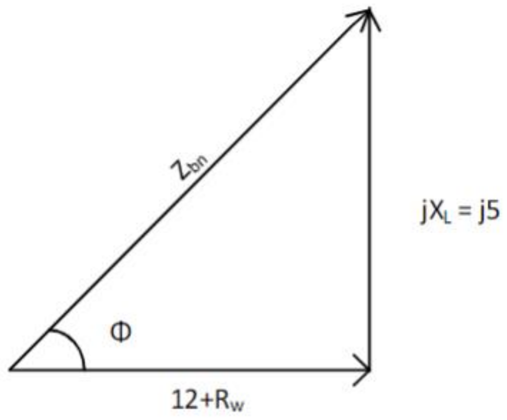
$$Z_{bn} = R_w + Z_p$$

$$Z_{bn} = (12 + R_w) + j5\ \Omega$$

$$\emptyset = tan^{-1}\frac{5}{12 + R_w} = 20.77°$$

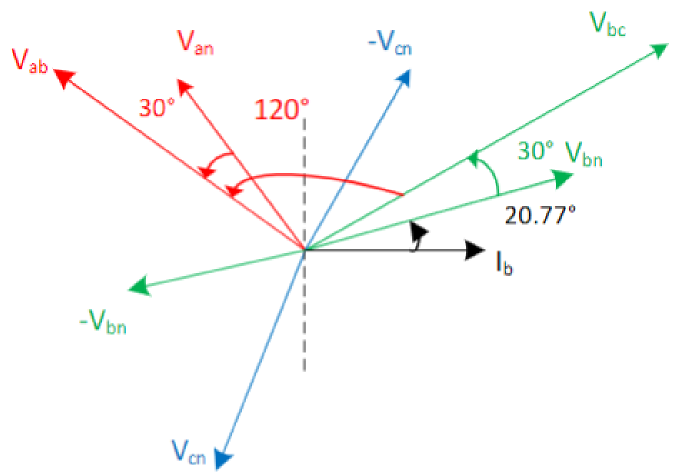
$$\frac{5}{12 + R_w} = 0.38$$

$$R_w = 1.18\ \Omega$$



b) V_{bn}

$$V_{bn} = I_b(Z_p + R_w)$$
$$\therefore\ Z_{bn} = Z_p + R_w = 13.18 + j5 = 14.097 \angle 20.77° \,\Omega$$
$$V_{bn} = 20 \angle 0° \times 14.097 \angle 20.77° \,V$$
$$V_{bn} = 282 \angle 20.77° \,V$$



c) V_{ab}

$$V_{bc} = \sqrt{3} \times 282 \angle (20.77° + 30°) \,V$$
$$V_{bc} = 488.44 \angle 50.77° \,V$$

V_{ab} will lead V_{bc} by an angle of 120˚.

So,

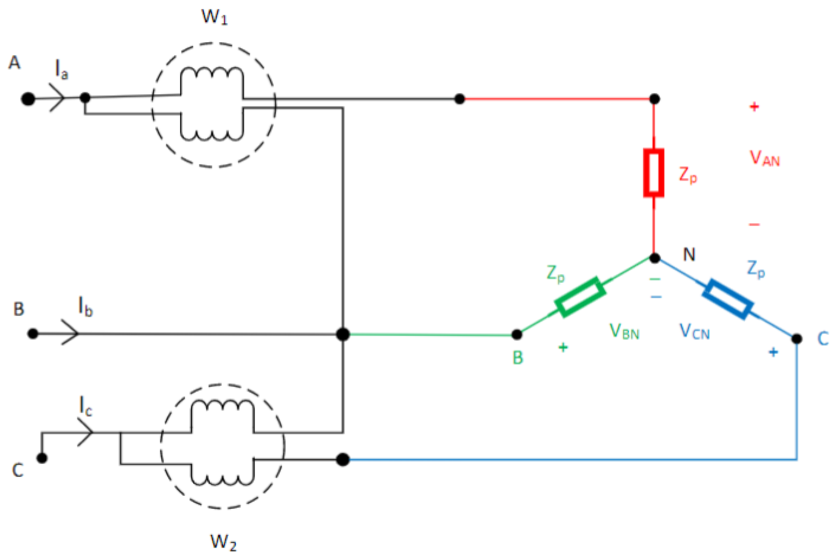
$$V_{ab} = V_{bc} \angle 120° \,V$$
$$V_{ab} = 488.44 \angle 170.77° \,V$$

d) Complex power supplied by the source:

$$S = \sqrt{3}V_L I_L \angle\Phi$$
$$S = \sqrt{3} \times 488.44 \times 20 \angle 20.77° \text{ VA}$$
$$S = 16920.06 \angle 20.77° \text{ VA}$$
$$S = 16.92 \angle 20.77° \text{ kVA}$$
$$S = 15.82 + j6 \text{ kVA}$$

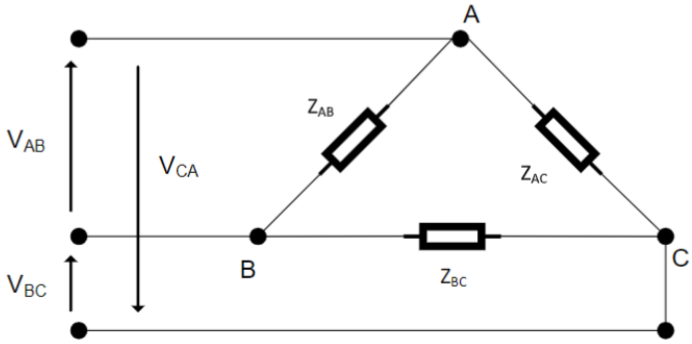
7

Problem-4: Calculate the readings of two wattmeter (W_1 and W_2) connected to measure the total power for a balanced Y-connected load shown in the figure, fed from a three-phase, 400V balanced supply with phase sequence as a-b-c. The load impedance per-phase is 20+j15Ω. Also, find the line and phase currents, power factor of load, total power (P), total reactive VA (Q) and total VA (S).



8

Problem: A three phase supply with an effective line voltage 240V has an unbalanced Δ-connected load as shown in the figure.



The impedance of the loads are:

$$Z_{AB} = 25\angle 90°\ \Omega$$
$$Z_{BC} = 15\angle 30°\ \Omega$$
$$Z_{CA} = 20\angle 0°\ \Omega$$

Obtain the total complex power absorbed by the load.

Solution:

The rms value of the phase currents:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{240}{25} = 9.6\text{ A}$$
$$I_{BC} = \frac{V_{BC}}{Z_{BC}} = \frac{240}{15} = 16\text{ A}$$
$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{240}{20} = 12\text{ A}$$

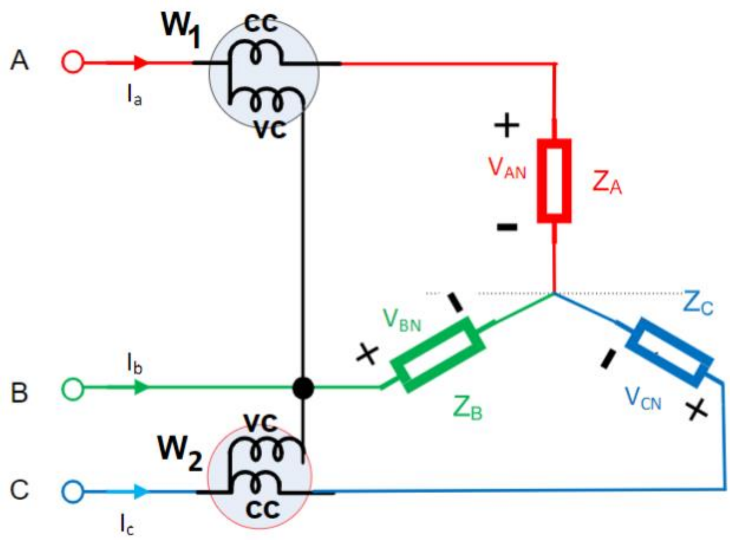
Hence, the complex powers in the three phases are:

$$S_{AB} = I_{AB}^2 Z_{AB} = (9.6)^2 \times 25\angle 90° = 2304\angle 90° = 0 + \text{j}2304\text{ VA}$$
$$S_{BC} = I_{BC}^2 Z_{BC} = (16)^2 \times 15\angle 30° = 3840\angle 30° = 3325 + \text{j}1920\text{ VA}$$
$$S_{CA} = I_{CA}^2 Z_{CA} = (12)^2 \times 20\angle 0° = 2880\angle 0° = 2880 + \text{j}0\text{ VA}$$

The total complex power absorbed by the load is:

$$S_{3\Phi} = S_{AB} + S_{BC} + S_{CA} = 6205 + \text{j}4224\text{ VA}$$

Problem-3: Calculate the readings of two wattmeter (W_1 and W_2) connected to measure the total power for a balanced Y-connected load shown in the figure, fed from a three-phase, 400V balanced supply with phase sequence as a-b-c. The load impedance per-phase is 20+j15Ω. Also, find the line and phase currents, power factor of load, total power (P), total reactive VA (Q) and total VA (S).

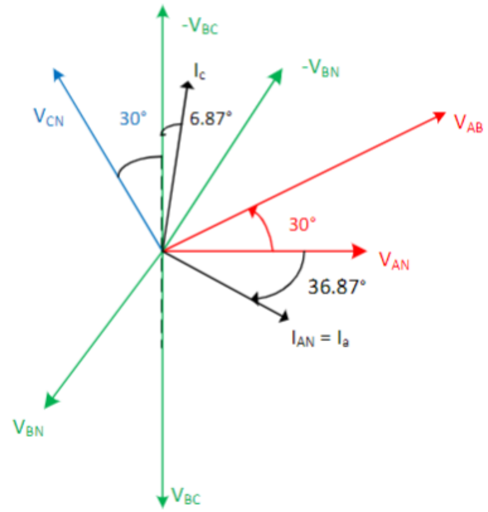


Solution: Given: V_L = 400V, Z_p = 20+j5Ω = 25 ∠ + 36.87° Ω
As the star connected load is balanced, the phase voltages are:

$$V_{AN} = \frac{400}{\sqrt{3}}\angle 0° = 231\angle 0°\text{ V}$$
$$V_{BN} = 231\angle -120°\text{ V}$$
$$V_{CN} = 231\angle +120°\text{ V}$$

So, line voltages are:

$$V_{AB} = 400\angle +30°\text{ V}$$
$$V_{BC} = 400\angle -90°\text{ V}$$
$$V_{CA} = 400\angle +150°\text{ V}$$



So, current in phase A:

$$I_a = \frac{V_{AN}}{Z_p} = \frac{231\angle 0°}{25\angle + 36.87°} = 9.24\angle - 36.87°\text{ A}$$

So,

$$I_b = 9.24\angle - 156.87°\text{ A}$$
$$I_c = 9.24\ \angle 83.13°\text{ A}$$

Line currents = Phase currents.

Pf of the load = Cos (36.87˚) = 0.8(lagging)

$$Reading\ of\ W_1 = V_{AB} I_a Cos\angle(V_{AB} and\ I_a)$$
$$Reading\ of\ W_1 = 400 \times 9.24 \times Cos\angle(66.87°)\text{ W}$$
$$P_1 = 1.45\text{ kW}$$

$$Reading\ of\ W_2 = V_{CB} I_c Cos\angle(V_{CB} and\ I_a)$$
$$Reading\ of\ W_1 = 400 \times 9.24 \times Cos\angle(6.87°)\text{ W}$$
$$P_2 = 3.67\text{ kW}$$

Total average power absorbed by the load

$$P_T = P_1 + P_2 = 1.45 + 3.67\text{ kW} = 5.12\text{ kW}$$

$$Total\ reactive\ VA = Q_T = \sqrt{3} V_L\, I_L Sin\emptyset$$
$$Q_T = \sqrt{3} \times 400 \times 9.24 \times Sin 36.86° = 3.842\text{ kVAR}$$

$$Total\ VA = S = \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 9.24 = 6.403\text{ kVA}$$

$$The\ complex\ power\ S_T = P_T + jQ_T = \sqrt{3} V_T I_T \angle\emptyset$$

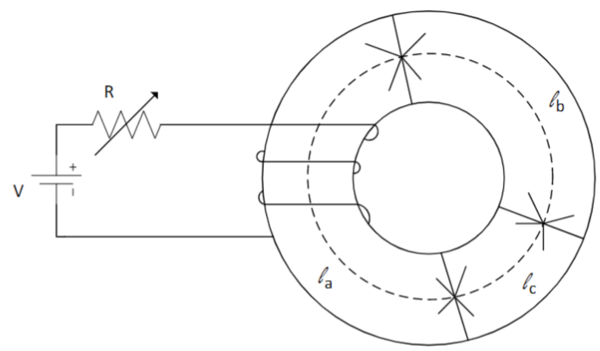
$$S_T = \sqrt{3} \times 400 \times 9.24\angle 36.86°$$

$$S_T = 6.403\ \angle 36.87°\text{ kVA}$$

$$S_T = 5.121 + j3.842\text{ kVA}$$

Problem-4: A toroid is composed of three ferromagnetic materials and is equipped with a coil having 100 turns as depicted in figure. Material 'a' is a nickel-iron alloy having a mean arc length ℓ_a of 0.3m. Material 'b' is a medium silicon steel and has a mean arc length ℓ_b of 0.2 m. Material 'c' is of cast steel having mean arc length ℓ_c of 0.1 m. Each material has a cross-sectional area of 0.001m^2.

a) Find the magnetomotive force needed to establish a magnetic flux Φ of 0.6 mWb.
b) What current must be made to flow through the coil?
c) Compute the relative permeability and reluctance of each ferromagnetic material.



Solution:

a) To obtain the total mmf of the coil, we need to apply Ampere's circulital law:
Thus,

$$F = F_a + F_b + F_c = H_a\ell_a + H_b\ell_b + H_c\ell_c$$

Given:

$$\emptyset = 0.6 \text{ mWb} = 6 \times 10^{-4} \text{ Wb}$$

Flux Density: $B_a = B_b = B_c = \frac{\emptyset}{A_c}$

So,

$$B_a = B_b = B_c = \frac{6 \times 10^{-4}}{1 \times 10^{-3}} = 0.6 \text{ T}$$

Hence looking into the B-H curve for three metals;

H_a, H_b and H_c for B = 0.6T will be obtained.

For nickel-iron alloy, H_a =10 AT/m for B = 0.6T.

For medium-silicon steel, H_b = 77 AT/m for B = 0.6T.

For cast steel, H_c = 270 AT/m for B = 0.6T.

Total required mmf is:

$$F = F_a + F_b + F_c = H_a\ell_a + H_b\ell_b + H_c\ell_c$$

$$F = 10 \times 0.3 + 77 \times 0.2 + 27 \times 0.1 = 3 + 15.4 + 27 = 45.4 \text{ AT}$$

b) The current 'i' to establish the flux of 0.6 mWb in the core is:

$$i = \frac{mmf}{N} = \frac{45.4}{100} = 0.454 \text{ A}$$

c) Relative permeability of nickel-iron alloy:

$$\mu_a = \frac{B_a}{H_a} = \frac{0.6}{10} = 0.06\ H/m$$

So,

$$\mu_{ra} = \frac{0.06}{\mu_0} = \frac{0.06}{4\pi \times 10^{-7}} \approx 47746$$

Reluctance of nickel-iron alloy:

$$S_a = \frac{l_a}{\mu_a A_c} = \frac{0.3}{0.06 \times 0.001} \approx 5000\ \frac{\text{AT}}{\text{Wb}}$$

Relative permeability of medium silicon steel:

$$\mu_b = \frac{B_b}{H_b} = \frac{0.6}{77}\ H/m$$

So,

$$\mu_{rb} = \frac{0.6}{77 \times \mu_0} = \frac{0.6}{77 \times 4\pi \times 10^{-7}} \approx 6200.8$$

Reluctance of medium silicon steel:

$$S_b = \frac{l_b}{\mu_b A_c} = \frac{0.2}{\frac{0.6}{77} \times 0.001} \approx 25667\ \frac{\text{AT}}{\text{Wb}}$$

Relative permeability of cast steel:

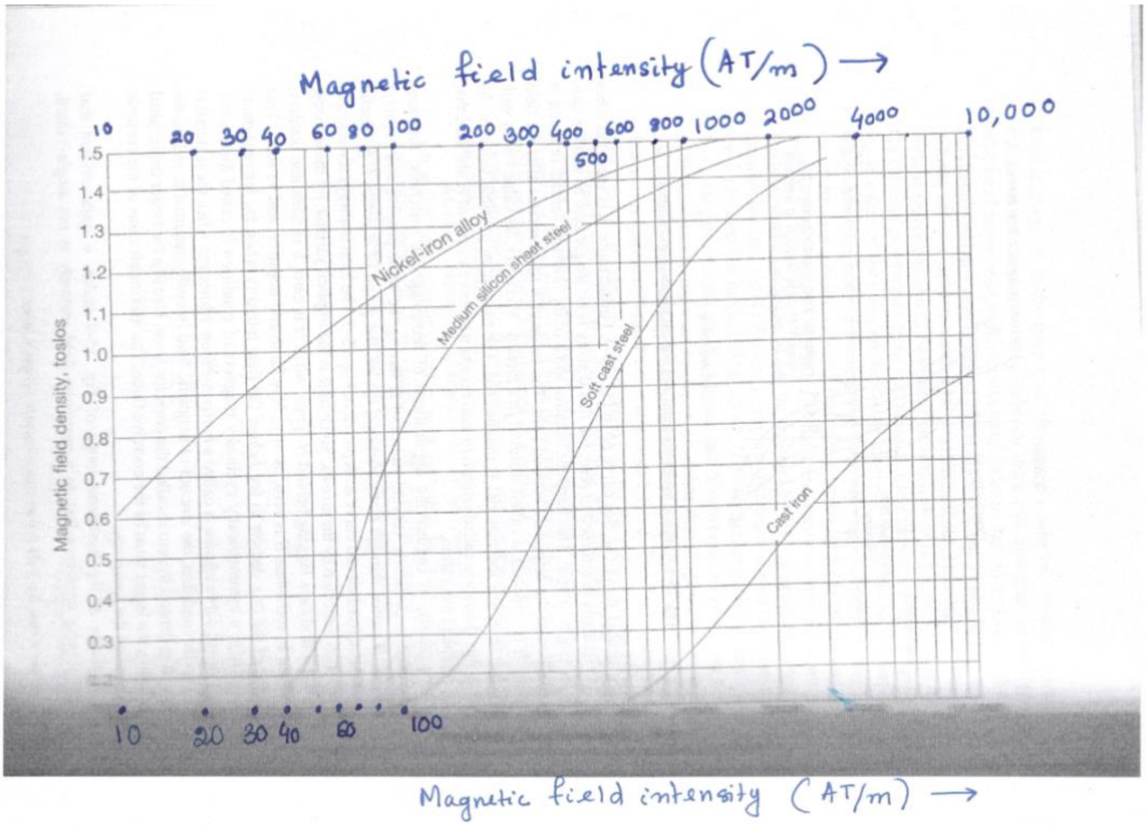
$$\mu_c = \frac{B_c}{H_c} = \frac{0.6}{270}\ H/m$$

So,

$$\mu_{rc} = \frac{0.6}{270 \times \mu_0} = \frac{0.6}{270 \times 4\pi \times 10^{-7}} \approx 1768$$

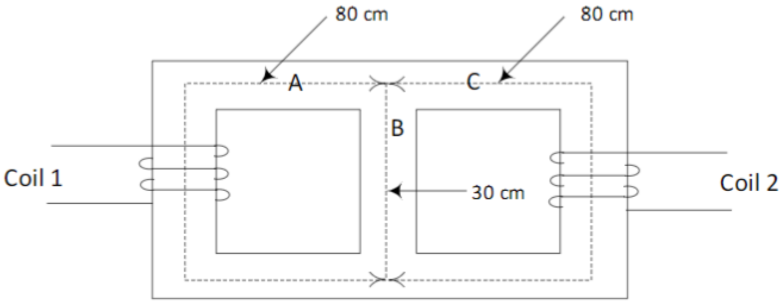
Reluctance of cast steel:

$$S_c = \frac{l_c}{\mu_c A_c} = \frac{0.1}{\frac{0.6}{270} \times 0.001} \approx 45000\ \frac{\text{AT}}{\text{Wb}}$$

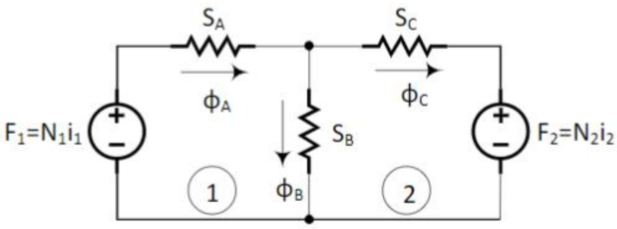


11

Problem-5: The cross-section of the core is 25 cm^2. At a particular situation, the flux in branches A and B is 3500 μwb and that in branch C is zero. Find the required ATs in coil 1 and coil 2. ($\mu_r = 1000$)



Solution-5: The magnetic equivalent circuit:



$$\text{Since } \emptyset_C = 0, \emptyset_A = \emptyset_B$$

$$\text{For loop-1: } F_1 - \emptyset_A S_A - \emptyset_B S_B = 0$$

$$F_1 = \emptyset_A S_A + \emptyset_B S_B$$

$$= \emptyset_A (S_A + S_B)$$

$$S_A = \frac{l_A}{\mu_0 \mu_r A_C} = \frac{80 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 25 \times 10^{-4}} = 2.5465 \times 10^5 \frac{AT}{wb}$$

$$S_B = \frac{l_B}{\mu_0 \mu_r A_C} = \frac{30 \times 10^{-2}}{4\pi \times 10^{-7} \times 1000 \times 25 \times 10^{-4}} = 9.5494 \times 10^4 \frac{AT}{wb} = 0.955 \times 10^5 \frac{AT}{wb}$$

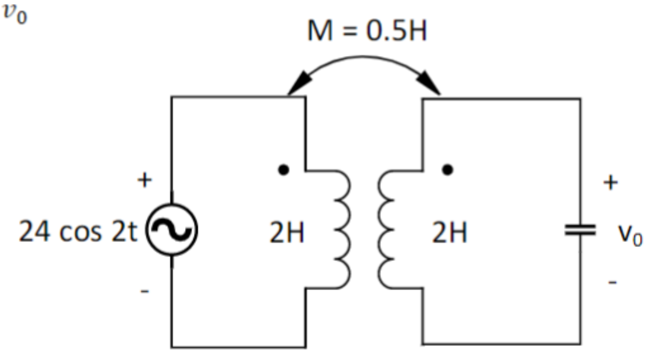
$$F_1 = \emptyset_A (S_A + S_B)$$

$$= 3500 \times 10^{-6} (2.5465 + 0.955) \times 10^5$$

$$= 1225.5\ AT$$

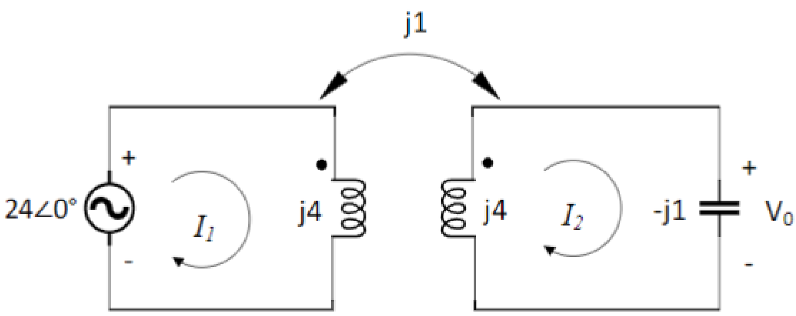
$$F_2 = \emptyset_B S_B = 3500 \times 10^{-6} \times 0.955 \times 10^5 = 334.25\ AT$$

Problem-6: Find v_0



Solution-6: Frequency model: $\omega = 2\, rad/s$

NOTE: All the calculations are based on maximum value of the excitation voltage.



Applying Faraday's law of electromagnetic induction to mesh-1:

$$24 - (j4I_1 - jI_2) = 0$$

$$24 - j4I_1 + jI_2 = 0 \text{ ------------------ (1)}$$

Similarly for mesh-2:

$$-(j4I_2 - jI_1) - (-j1)I_2 = 0$$

$$-j3I_2 + jI_1 = 0$$

$$I_1 = 3I_2 \text{ ----------------(2)}$$

Substituting (2) in (1):

$$24 - j4I_1 + jI_2 = 0$$

$$24 - j4 \times 3I_2 + jI_2 = 0$$

$$I_2 = \frac{24}{j11} = -j\frac{24}{11}A$$

$$V_0 = (-j1)I_2$$

$$= -j1 \times -j\frac{24}{11}$$

$$= -\frac{24}{11}V$$

So, $V_0(t) = -\frac{24}{11}\cos(2t)\,V$

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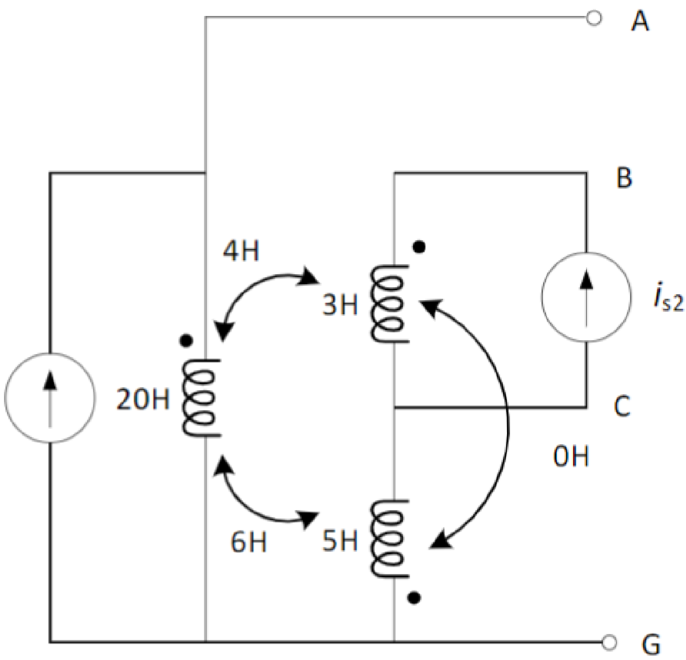
Problem: Let $i_{s_1}(t) = 4t\ A$ and $i_{s_2}(t) = 10t\ A$ in the circuit.



Find: (a) v_{AG} ; (b) v_{CG} ; (c) v_{BG}

(a): For v_{AG}

$$v_{AG} - 20\frac{di_{s_1}}{dt} - 4\frac{di_{s_2}}{dt} = 0$$

$$v_{AG} - 20 \times 4 - 4 \times 10 = 0$$

$$v_{AG} = 120V$$

(b): For v_{CG}

$$v_{CG} = -6 \times \frac{di_{s_1}}{dt}$$

$$= -6 \times 4 = -24V$$

(c):

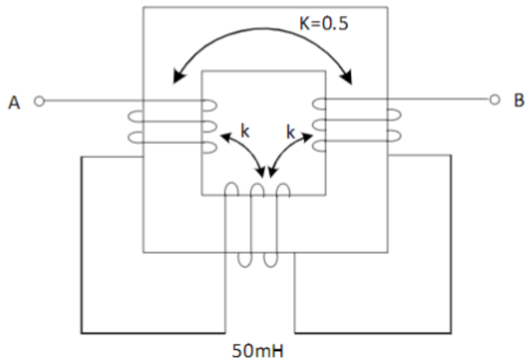
$$v_{BG} - 3\frac{di_{s_2}}{dt} - 4\frac{di_{s_1}}{dt} = -6\frac{di_{s_1}}{dt}$$

$$v_{BG} = 3\frac{di_{s_2}}{dt} + 4\frac{di_{s_1}}{dt} - 6\frac{di_{s_1}}{dt}$$

$$= 3 \times 10 + 4 \times 4 - 6 \times 4$$

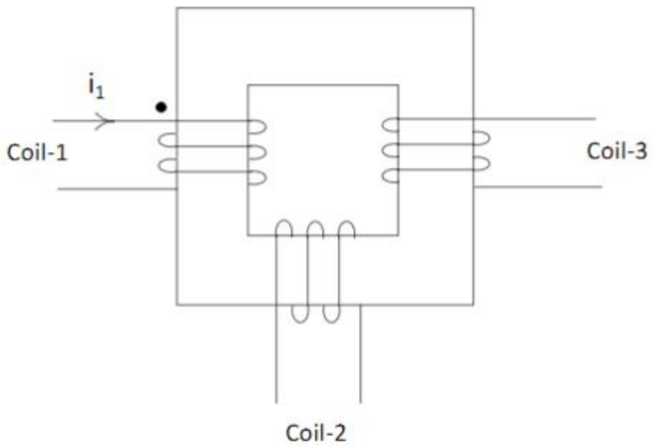
$$v_{BG} = 30 + 16 - 24 = 22V$$

Problem-1: Obtain the equivalent inductance across AB of the magnetically coupled circuit. If this circuit is excited by a source $v(t) = 200\sin(100t)V$. Find the energy stored in the circuit at $t = 5ms$.

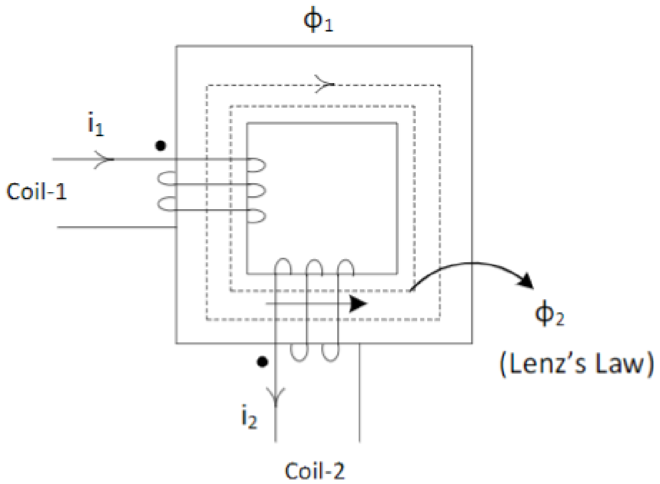


Solution-1: (a)- Equivalent inductance of the circuit across AB:

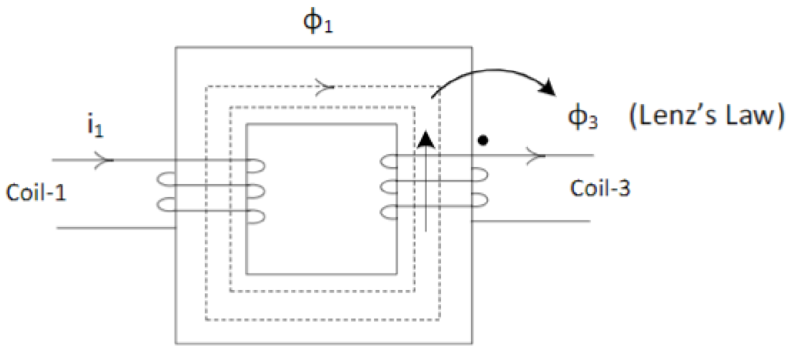
To obtain the dotted terminals of the coils, modified circuit is as follows,



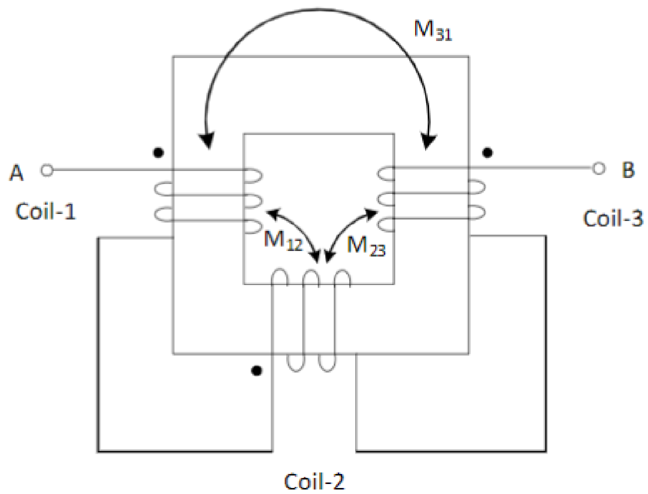
To obtain the dotted terminals of the coil-2, the circuit is:



To obtain the dotted terminals of the coil-3:



Hence, the dotted terminals of the coils are:



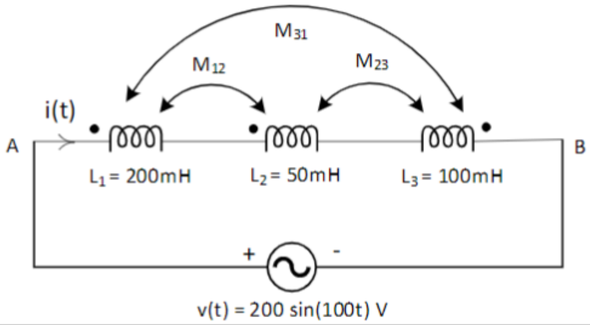
Mutual inductance between the coils:

$$M_{12} = k\sqrt{L_1 L_2} = 0.5\sqrt{200 \times 50} = 50mH$$

$$M_{23} = 0.5\sqrt{50 \times 100} = 35.355\ mH$$

$$M_{31} = 0.5\sqrt{200 \times 100} = 70.71\ mH$$

The equivalent circuit diagram:



$$v(t) = \left(L_1\frac{di}{dt} + M_{12}\frac{di}{dt} - M_{31}\frac{di}{dt}\right) - \left(L_2\frac{di}{dt} + M_{12}\frac{di}{dt} - M_{23}\frac{di}{dt}\right) - \left(L_3\frac{di}{dt} - M_{31}\frac{di}{dt} - M_{23}\frac{di}{dt}\right) = 0$$

So, $v = [(L_1 + M_{12} - M_{31}) + (L_2 + M_{12} - M_{23}) + (L_3 - M_{31} - M_{23})]\frac{di}{dt}$

$$L_{eq} = (L_1 + M_{12} - M_{31} + L_2 + M_{12} - M_{23} + L_3 - M_{31} - M_{23})\ mH$$

$$= (L_1 + L_2 + L_3 + 2M_{12} - 2M_{23} - 2M_{31})$$

$$(200 + 50 + 100 + 2 \times 50 - 2 \times 35.35 - 2 \times 70.71)\ mH$$

$$(350 + 100 - 70.7 - 141.42) = (450 - 212.12) mH$$

$$L_{eq} = 237.88\ mH$$

(b): The inductance reactance of the circuit:

$$jX_L = j\omega L_{eq} = j\ 100 \times 237.88 \times 10^{-3} \approx j23.8\ \Omega$$

$$i(t) = \frac{200}{23.8}\sin(100t - 90°)\ A$$

At $t = 5ms$, $\omega t: 5 \times 10^{-3} \times 100$

$\longrightarrow 0.5\ rad = 28.65°$

$$i(t = 5ms) = \frac{200}{23.8}\sin(28.65° - 90°)$$

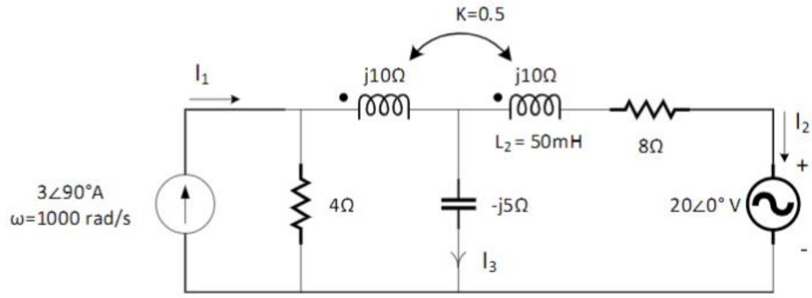
$$\approx 8.403 \times -0.87756 \approx -7.374\ A$$

So, the energy stored in the coupled circuit at $t = 5\ ms$:

$$W = \frac{1}{2}L_{eq}\{i(t = 5\ ms)\}^2$$

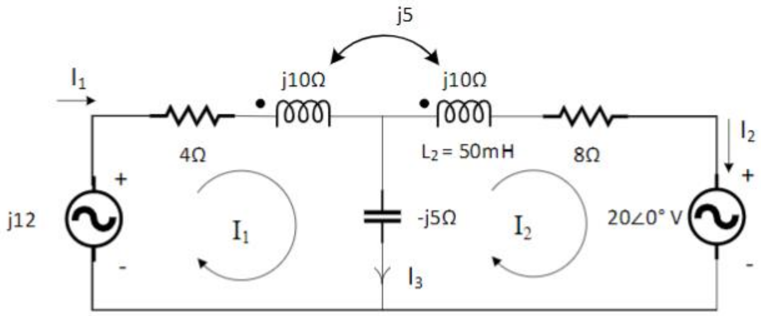
$$= \frac{1}{2}237.88 \times 10^{-3}(-7.374)^2 \approx 6467.5 \times 10^{-3} J \approx 6.5\ J$$

Problem-2: Determine the currents I_1, I_3 and I_3 in the circuit as shown in the figure. Find the energy stored in the coupled coils at $t = 2ms$.



Solution-2: The mutual inductance of the coils, $M = k\sqrt{L_1L_2} = 0.5\sqrt{j10 \times j10} = j5\ \Omega$

Transform the current source to voltage source:



The voltage equation for mesh-1:

$$j12 - 4I_1 - j10I_1 - j5I_2 - (-j5)(I_1 - I_2) = 0$$

$$j12 - (4 + j10 - j5)I_1 - j10I_2 = 0$$

$$(4 + j5)I_1 + j10I_2 \text{---------------------(1)}$$

The voltage equation for mesh-2:

$$20\angle 0^\circ - (-j5)(I_2 - I_1) - j10I_2 - j5I_1 - 8I_2 = 0$$

$$20 + (j5 - j10 - 8)I_2 - j10I_1 = 0$$

$$j10I_1 + (8 + j5)I_2 = 20\text{-----------------}(2)$$

$$\begin{bmatrix} 4+j5 & j10 \\ j10 & 8+j5 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} j12 \\ 20 \end{bmatrix}$$

$$\Delta = 107 + j60,\ \Delta_1 = -60 - j296,\ \Delta_2 = 40 - j100$$

$$I_1 = \frac{\Delta_1}{\Delta} = 2.462\angle 72.18^\circ\ A$$

$$I_2 = \frac{\Delta_2}{\Delta} = 878\angle -97.48^\circ\ mA$$

$$I_3 = I_1 - I_2 = 3.329\angle 74.89^\circ\ A$$

$$i_1(t) = 2.462\sqrt{2}\cos(\omega t + 72.18^\circ)\ A$$

$$i_2(t) = 878\sqrt{2} \times 10^{-3}\cos(\omega t - 97.48^\circ)\ A$$

At $t = 2\ ms$, $\omega t = 1000 \times 2 \times 10^{-3} = 2\ rad = 114.6^\circ$

So at $t = 2\ ms$:

$$i_1(t = 2 \times 10^{-3}) = -3.46\ A$$

$$i_2(t = 2 \times 10^{-3}) = 1.187\ A$$

The total energy stored in the coupled inductor:

$$W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

Given: $j\omega L_1 = j10$

$$L_1 = \frac{10}{1000} = 10\ mH = L_2$$

$$M = \frac{5}{1000} = 5\ mH$$

So,

$$W = \frac{1}{2} \times 10 \times 10^{-3}(-3.460)^2 + \frac{1}{2} \times 10 \times 10^{-3} \times (1.187)^2 + 5 \times 10^{-3}(-3.460)(1.187)$$

$$W = 59.858 \times 10^{-3} + 7.045 \times 10^{-3} - 20.654 \times 10^{-3}$$

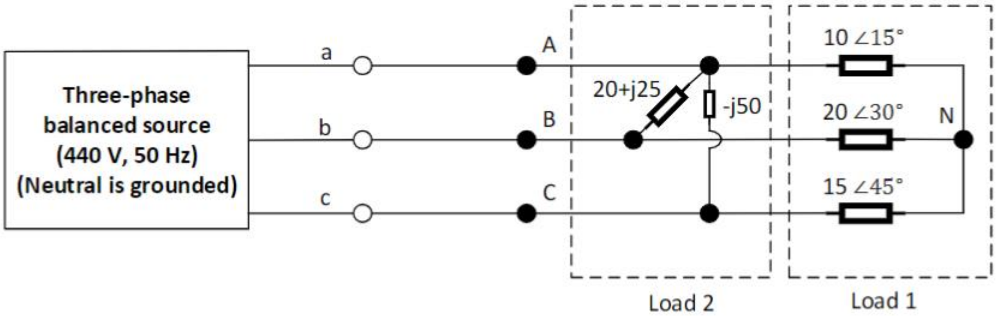
Q1. The circuit as shown below is excited by a balanced three-phase source with a line voltage of 440V. **[6 Marks]**

a) Determine the complex power absorbed by load 1, load 2 and the power factor at which source is operating.

b) Calculate the source line currents and draw the phasor diagram considering V_{an} as the reference phasor.

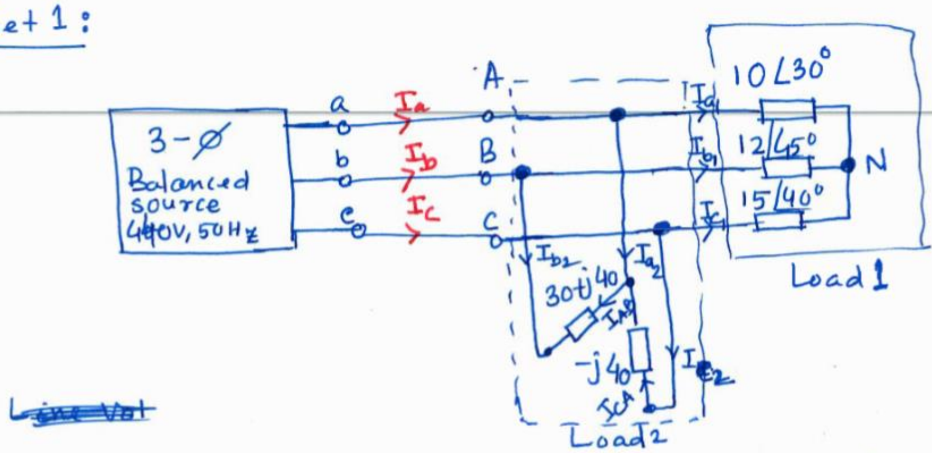
c) A Δ-connected capacitor bank is now connected across the source to compensate the reactive power drawn from the source such that the source will operate at unit power factor. Find the per-phase capacitance of the capacitor bank.

Neglect the impedance of the line and consider the positive phase sequence for the analysis.



Example Solved

Set 1:



(a) Complex power absorbed by the Load 1 and 2 :

$$V_{an} = \frac{440}{\sqrt{3}} \angle 0^\circ$$

$$V_{bn} = \frac{440}{\sqrt{3}} \angle -120^\circ$$

$$V_{cn} = \frac{440}{\sqrt{3}} \angle +120^\circ$$

Load 1 :

$Z_A = 10\angle 30^\circ \ \Omega$

$Z_B = 12\angle 45^\circ \ \Omega$

$Z_C = 15\angle 40^\circ \ \Omega$

Load 2 :

$Z_{AB} = 30 + j40 \ \Omega$

$Z_{CA} = -j40$

Voltage at load neutral :

$$V_N = \frac{\frac{V_{an}}{Z_A} + \frac{V_{bn}}{Z_B} + \frac{V_{cn}}{Z_C}}{\frac{1}{Z_A} + \frac{1}{Z_B} + \frac{1}{Z_C}}$$

$$= \frac{\frac{440}{\sqrt{3}}\left[\frac{1\angle 0^\circ}{10\angle 30^\circ} + \frac{1\angle -120^\circ}{12\angle 45^\circ} + \frac{1\angle 120^\circ}{15\angle 40^\circ}\right]}{\left(\frac{1}{10\angle 30^\circ} + \frac{1}{12\angle 45^\circ} + \frac{1}{15\angle 40^\circ}\right)}$$

$$V_N = 19.3\angle 19.65^\circ \ V$$

Current in load 1 :

$$I_{a1} = \frac{V_{an} - V_N}{Z_A} = \frac{\frac{440}{\sqrt{3}}\angle 0^\circ - 19.3\angle 19.65^\circ}{10\angle 30^\circ} = 23.6\angle -31.57^\circ A$$

$$I_{b1} = \frac{V_{bn} - V_N}{Z_B} = \frac{\frac{440}{\sqrt{3}}\angle -120^\circ - 19.3\angle 19.65^\circ}{\ldots} = 22.4\angle -167.65^\circ A$$

$$I_{c_1} = \frac{V_{cn} - V_N}{Z_c}$$

$$= \frac{\frac{440}{\sqrt{3}}\angle 120^\circ - 19.3\angle 19.65^\circ}{15\angle 40^\circ}$$

$$= 17.21\angle 84.22^\circ \quad A$$

Complex power absorbed by load1:

$$S_1 = I_{a_1}^2 Z_A + I_{b_1}^2 Z_B + I_{c_1}^2 Z_c$$

$$= (23.6)^2 \times 10\angle 30^\circ + (22.4)^2 \times 12\angle 45^\circ + (17.21)^2 (15\angle 40^\circ)$$

$$= (48233 + j\,2784.8) + (4259.9 + j\,4259.9) + (3403.2 + j\,2855)$$

$$= 12486.4 + j\,9899.7 \qquad (\cancel{10313 + j\,8188 \text{ VA}})$$

1

$$S_1 = 15934.7\angle 38.41^\circ \text{ VA} \qquad (\cancel{13168.2\angle 38.45^\circ \text{ VA}})$$

Complex power absorbed by load2:

$$I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{440\angle 30^\circ}{30+j40} = \frac{440\angle 30^\circ}{50\angle 53.13^\circ} = 8.8\angle -23.13^\circ \text{ A}$$

$$I_{CA} = \frac{V_{CA}}{Z_{CA}} = \frac{440\angle -210^\circ}{-j40} = \frac{440\angle -210^\circ}{40\angle -90^\circ} = 11\angle -120^\circ \text{ A}$$

$$S_2 = I_{AB}^2 Z_{AB} + I_{CA}^2 Z_{CA}$$

$$= (8.8)^2 (30+j40) + (11)^2 (-j40)$$

$$= 2323.2 + j\,3097.6 - j\,4840 = 2323.2 - j\,1743.1 \text{ (VA)}$$

1

pf at which source is operating:

$$S = S_1 + S_2$$

$$= 12486.4 + j9899.7 + 2323.2 - j1743$$

$$= 148096 + j\,8156.7 \text{ VA} \quad (\quad)$$

$$= 16907.28 \angle 28.84^\circ \text{ VA} \quad (\quad)$$

$$pf = \cos(28.84^\circ) \text{ (Lag)}$$

$$= 0.876 \text{ (Lag)} \quad 1 \quad (\cancel{0.8756 \text{ (lag)}})$$

(b) Calculate the line currents (I_a, I_b, I_c) drawn from the source and the phasor diagram:

Load-1:

$$\left.\begin{aligned} I_{a_1} &= 23.6 \angle -31.57^\circ \text{ A} \\ I_{b_1} &= 22.4 \angle 167.65^\circ \text{ A} \\ I_{c_1} &= 17.21 \angle 84.22^\circ \text{ A} \end{aligned}\right\} \text{load-1}$$

Load 2:

$$I_{a_2} = I_{AB} - I_{CA}$$

$$= 8.8 \angle -23.13^\circ - 11 \angle -120^\circ$$

$$= \cancel{14.907 \angle}$$

$$= (8.09 - j3.46) - (-5.5 - j9.53)$$

$$= 8.09 - j3.46 + 5.5 + j9.53$$

$$= 13.59 + j6.07$$

$$= 14.88 \angle 24.07^\circ \text{ A}$$

$$I_{b_2} = -I_{AB} = -8.8 \angle 23.13^\circ \text{ A}$$

$$I_{c_2} = I_{CA} = 11 \angle -120^\circ \text{ A}$$

Line current from source:

$I_a = I_{a_1} + I_{a_2}$

$= 23.6\angle{-31.57^\circ} + 14.88\angle{24.07^\circ}$

$= 20.11 - j12.36 + (13.59 + j6.07)$

$= \cancel{43.70} - j6.29$ 33.70 $= 33.7 - j6.29$

$= \cancel{45.15}\ 34.28\angle{-10.57^\circ}$ A

I_a

$=$

0.5

$I_b = I_{b_1} + I_{b_2}$

$= 22.4\angle{-167.65^\circ} + (-8.8)\angle{-23.13^\circ}$

$= 30\angle{-177.38^\circ}$ A

0.5

I

$I_c = I_{c_1} + I_{c_2}$

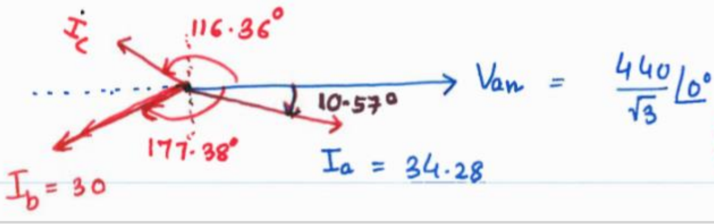
$= 17.21\angle{84.27^\circ} + 11\angle{-120^\circ}$

$= 8.476\angle{116.36^\circ}$ A

0.5

$\cancel{I_c = 7.7\angle{116.1^\circ}\ A}$

$\cancel{= -3.39 + j6.9}$



0.5

(c) The per-phase capacitance of the capacitor bank for the source operating at unity pf:

$$Q_{C(3\phi)} = 8156.7 \text{ VAR}$$

$$= 3\frac{V_{ph}^2}{X_c}$$

So $X_c = 3\dfrac{V_{ph}^2}{Q_{C(3\phi)}}$

$$= 3\frac{\left(\frac{440}{\cancel{3}}\right)^2}{8156.7} = \frac{3\times(440)^2}{8156.7}$$

$$= 23.735\times 3$$

$$\Rightarrow \frac{1}{\omega C} = 23.\cancel{735}\times 3$$

$$\Rightarrow C = \frac{1}{3\times 23.735\times 314} = (0.000134)/3$$
$$= (134\ \mu F)/3$$

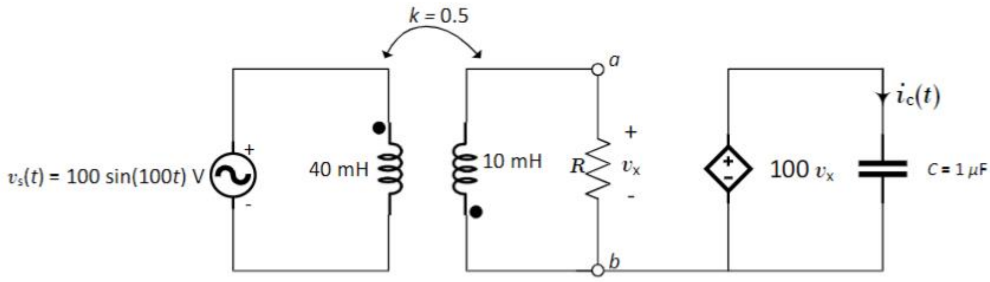
$$C = \frac{134}{3}\mu F$$
$$= 44.7\ \mu F$$

1

17

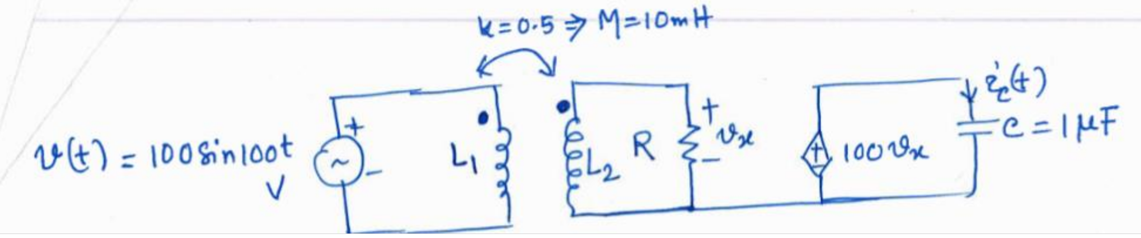
Q2. In the circuit given below, a resistor 'R' is connected across the terminal 'ab'. Find the capacitor current $i_c(t)$ and the energy stored in the magnetically coupled circuit at $t = 5$ ms if (a) $R = \infty$ and (b) $R = 1\,\Omega$.

[6 Marks]



Example Solved

...t1: $L_1 = 10\text{ mH}$, $L_2 = 40\text{ mH}$, $k = 0.5 \Rightarrow M = 10\text{ mH}$



(a) $R = \infty$:

$$v(t) - L_1\frac{di_1}{dt} = 0 \Rightarrow i_1 = \int \frac{v(t)}{L_1} = \int \frac{100 \sin 100t}{L_1}$$

$$\Rightarrow i_1(t) = \frac{100}{L_1}\left(-\frac{\cos 100t}{100}\right)$$

$$i_1(t) = -\frac{\cos 100t}{L_1} \quad \cancel{\cos 100t}$$

$$v_x = M\frac{di_1}{dt}$$

$$= M\frac{d}{dt}\left(-\frac{\cos 100t}{L_1}\right)$$

$$\Rightarrow v_x = \frac{100M}{L_1} \sin 100t$$

$$i_c(t) = C\frac{dv_c}{dt} = C\times 100\,\frac{dv_x}{dt} = C\times 100\,\frac{d}{dt}\left(\frac{100M}{L_1}\sin 10\right.$$

$$\text{So}\quad \boxed{i_c(t) = \frac{M}{L_1}\cos 100t = \frac{10}{10}\cos 100t = \cos 100t \text{ A}} \quad (1$$

Energy stored at $t = 5\,ms$: $\omega t = 0.5\,rad = 28.65^\circ$

$$w = \frac{1}{2}L_1 i_1^2 = \frac{1}{2}L_1\left(-\frac{\cos 100t}{L_1}\right)^2$$

$$= \frac{\cos^2(100t)}{2L_1} = \frac{\cos^2(28.65^\circ)}{2\times 10\times 10^{-3}} = 3$$

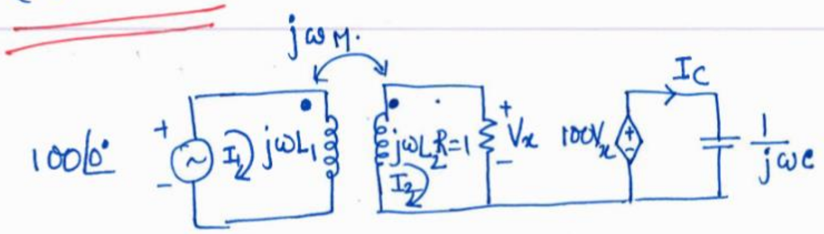
Or

$$i_1(t = 5ms) = -\left.\frac{\cos(100t)}{10\times 10^{-3}}\right|_{t=5ms} = -100\cos(28.65^\circ)$$

$$= -87.76 \text{ A}$$

$$w\Big|_{t=5ms} = \frac{1}{2}\times 10\times 10^{-3}\times(-87.76)^2 = \boxed{38.5\,J} \quad (1)$$

(b) R = 1 :



Loop 1:

$$100 - (j\omega L_1 I_1 - j\omega M I_2) = 0$$

$$\Rightarrow 100 = j\omega L_1 I_1 - j\omega M I_2 \quad \text{——} \quad (1)$$

Loop 1:

$$-I_2 - (j\omega L_2 I_2 - j\omega M I_1) = 0$$

$$\Rightarrow I_2 = j\frac{\omega M}{1 + j\omega L_2} \Rightarrow I_1 = \frac{1 + j\omega L_2}{j\omega M}I_2 \quad \text{—} (2$$

Substituting (2) in (1):

$$100 = \left[\frac{L_1}{M}\left(1+j\omega L_2\right) - j\omega M\right] I_2$$

$$\Rightarrow \quad I_2 = \frac{100}{\frac{L_1}{M}\left(1+j\omega L_2\right) - j\omega M} = \frac{100}{\frac{L_1}{M} + j\left(\frac{\omega L_1 L_2}{M} - \omega M\right)}$$

$$= \frac{100}{\frac{10}{10} + j\left(\frac{100\times 10\times 40\times 10^{-6}}{10\times 10^{-3}} - 100\times 10\times 10^{-3}\right)}$$

$$= \frac{100}{3.16} \angle -71.57^\circ$$

$$i_2(t) = \frac{100}{3.16} \sin\left(100t - 71.57^\circ\right) \text{ A} \quad 1$$

$$I_1 = \frac{1+j\omega L_2}{j\omega M} I_2$$

$$= \frac{1+j\,100\times 40\times 10^{-3}}{j\,100\times 10\times 10^{-3}} \times \frac{100}{3.16} \angle -71.57^\circ$$

$$= 130.4 \angle 274.4^\circ \text{ A}$$

$$i_1(t) = 130.4 \sin\left(100t + 274.4^\circ\right) \text{ A} \quad 1$$

$$v_x(t) = i_2(t) = \frac{100}{3.16} \sin\left(100t - 71.57^\circ\right) \text{ A}$$

$$i_c(t) = C\frac{dv_c}{dt} = 100C\frac{dv_x}{dt} = 100C\frac{di_2}{dt}$$

$$= 100C \times \frac{100}{3.16} \times 100 \cos\left(100t - 71.57^\circ\right)$$

$$\boxed{i_c(t) = \frac{1}{3.16}\cos\left(100t - 71.57^\circ\right)}$$ 1

Energy stored at $t = 5ms$:

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 - M i_1 i_2$$

$$t = 5ms \Rightarrow \omega t = 28.65^\circ$$

$$i_1(t = 5ms) = 130.4\sin\left(28.65 + 274.4^\circ\right)$$

$$= -109.3\ A$$

$$i_2(t = 5ms) = \frac{100}{3.16}\sin\left(28.65 - 71.57^\circ\right)$$

$$= -21.55\ A$$

$$w\Big|_{t=5ms} = \frac{1}{2}\,10\times10^{-3}\left(-109.3\right)^2 + \frac{1}{2}\times40\times10^{-3}\left(-21.55\right)^2$$

$$- 10\times10^{-3}\times\left(-109.3\right)\left(-21.55\right)$$

$$\boxed{w\Big|_{t=5ms} = 45.53\,J}$$ 1

18

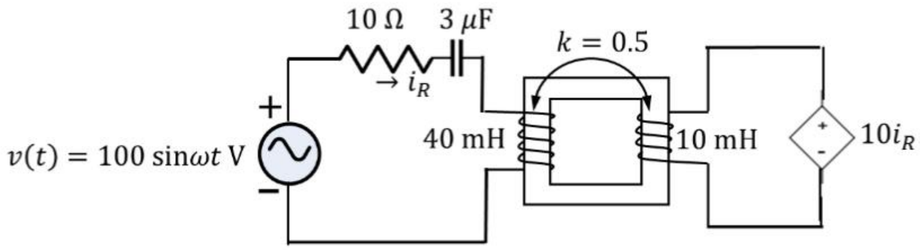
Q1. Determine

(a) The value of the resonant frequency (f_0) in Hz and

(b) The magnitude of the impedance seen from the source at half power frequencies for the network shown below.

(c) The type of filter, if the output will be taken across the 10 W resistor. Give justification with amplitude-frequency plot.

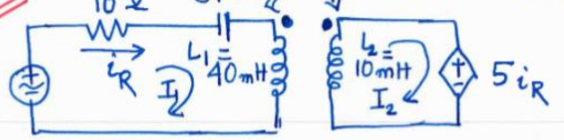
[5]



Example Solved

Set6: Q1 $k = 0.5 \Rightarrow M = 10\,mH$

$V = 100\angle 0°$



0.5

Loop 1:

$$V - 10I_1 - \frac{1}{j\omega C}I_1 - \left(j\omega L_1 I_1 - j\omega M I_2\right) = 0$$

$$\Rightarrow V - \left(10 + \frac{1}{j\omega C} + j\omega L_1\right)I_1 + j\omega M I_2 = 0 \quad \text{—— (1)}$$

0.5

Loop 2:

$$-5i_R - \left(j\omega L_2 I_2 - j\omega M I_1\right) = 0$$

$$\Rightarrow -5I_1 - j\omega L_2 I_2 + j\omega M I_1 = 0$$

$$\Rightarrow \left(j\omega M - 5\right)I_1 = j\omega L_2 I_2$$

$$\Rightarrow I_2 = \frac{j\omega M - 5}{j\omega L_2} I_1 \quad \text{—— (2)}$$

0.5

Substituting (2) in (1):

$$V - \left(10 + \frac{1}{j\omega C} + j\omega L_1\right) I_1 + j\omega M \frac{j\omega M - 5}{j\omega L_2} I_1 = 0$$

$$\Rightarrow V - I_1\left[10 + \frac{1}{j\omega C} + j\omega L_1 - j\omega M \frac{(j\omega M - 5)}{j\omega L_2}\right] = 0$$

$$\Rightarrow V = I_1\left[10 - j\frac{1}{\omega C} + j\omega L_1 - \frac{j\omega M(j\omega M - 5)}{j\omega L_2}\right] \quad \text{—— (3)}$$

$$Z = 10 - j\frac{1}{\omega C} + j\omega L_1 - \frac{j\omega M(j\omega M - 5)}{j\omega L_2}$$

$$= 10 - j\frac{1}{\omega C} + j\omega L_1 - j\omega\frac{M^2}{L_2} + \frac{5M}{L_2}$$

$$= \left(10 + \frac{5M}{L_2}\right) - j\frac{1}{\omega C} + j\omega L_1 - j\omega\frac{M^2}{L_2}$$

(a) $\text{Im}\{Z\} = 0$

$$-\frac{1}{\omega_0 C} + \omega_0 L_1 - \omega_0\frac{M^2}{L_2} = 0$$

$$\Rightarrow \omega_0^2 L_1 C - \frac{\omega_0^2 M^2 C}{L_2} = 1 \quad 1$$

$$\omega_0^2 \left(L_1 C - \frac{M^2 C}{L_2} \right) = 1$$

$$\Rightarrow \omega_0^2 = \frac{1}{C\left(L_1 - \frac{M^2}{L_2}\right)}$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{C\left(L_1 - \frac{M^2}{L_2}\right)}}$$

$$= \frac{1}{\left[3\times10^{-6}\left(40\times10^{-3} - \frac{10\times10^{-3}\times10\times10^{-3}}{10\times10^{-3}}\right)\right]^{\frac{1}{2}}}$$

$$\omega_0 = \frac{10^4}{3} \text{ rad/s}$$

$$f_0 = 530.52 \text{ Hz}$$

1

(b) Impedance amplitude at HPBW:

$$Z_{HPBW} = \sqrt{2}R$$
$$= 10\sqrt{2}$$
$$\approx 14.1\,\Omega$$

0.5

(c) From (3)

$$I_1 = \frac{V}{\left(10 + \frac{5M}{L_2}\right) - j\frac{1}{\omega C} + j\omega L_1 - j\omega\frac{M^2}{L_2}}$$

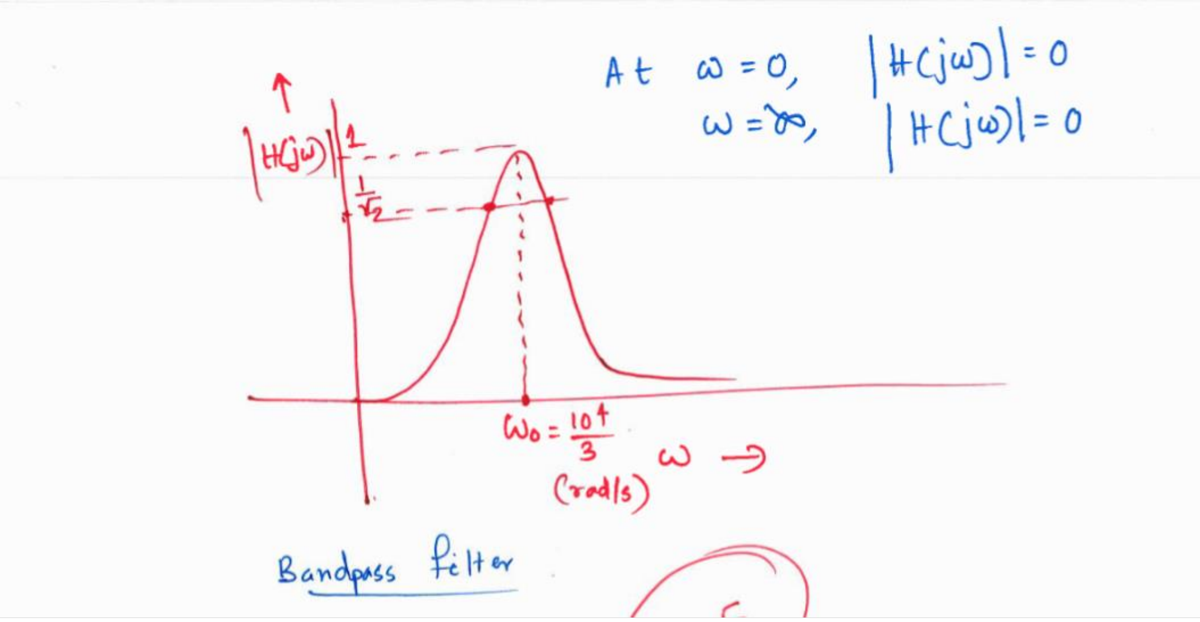
0.5

$$= \frac{V}{\left(10 + \frac{5M}{L_2}\right) + j\left\{\omega\left(L_1 - \frac{M^2}{L_2}\right) - \frac{1}{\omega C}\right\}}$$

$$V_0 = I_1 \times 10 = \frac{10V}{\left(10 + \frac{5M}{L_2}\right) + j\left\{\omega\left(L_1 - \frac{M^2}{L_2}\right) - \frac{1}{\omega C}\right\}}$$

$$H(j\omega) = \frac{V_0}{V} = \frac{10}{\left(10 + \frac{5M}{L_2}\right) + j\left\{\omega\left(L_1 - \frac{M^2}{L_2}\right) - \frac{1}{\omega C}\right\}}$$

$\omega \rightarrow$



19

1. In the balanced three-phase system of Fig. 1, the load impedance $\boldsymbol{Z_P = 8+j5\ \Omega}$. Assume positive (+) phase sequence and $\boldsymbol{W_1 > W_2}$. If the source is operating with a power factor of **0.98** and $\boldsymbol{W_1 = 15}$ ***kW***, find the values of (a) $\boldsymbol{R_W}$, (b) $\boldsymbol{W_2}$, (c) total real power absorbed by the load and (d) the reactive power supplied to the load. **[2+1+1+1]**

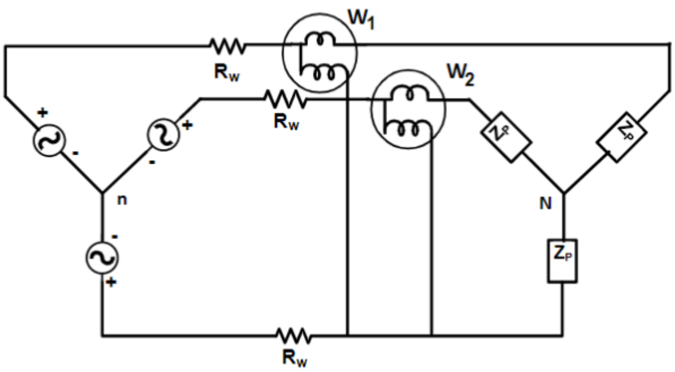


Fig. 1

Solution: (a) R_W = 16.631 Ω (b) W_2 = 7.044 kW (c) Total real power = 22.044 kW

(d) Total reactive power = 13.780 kVAR

20

6. (a) Short circuit test was conducted on a 50 kVA, 2400/240 V, 50 Hz transformer with the low voltage side short circuited and the instruments placed on the high voltage side. The readings of the instruments were 96 V, 20.8 A, and 1080 W.

   (i) If the transformer delivers a load of 50 kVA at 0.8 pf (lead) on the low voltage side at the rated terminal voltage of 240 V, what should be the input voltage at the high voltage side? You may use the approximate equivalent circuit of the transformer. [4]

   (ii) If the efficiency of the transformer while delivering the load in part (i) is known to be 95%, determine the efficiency of the transformer when it delivers a load of 25 kW at a pf of 0.5 (lag). [4]

   (b) A 4-pole 440 V 50 Hz Y connected 3-phase induction motor has an efficiency of 0.80 when its output is 60 HP. Assume that at this load the stator and rotor copper losses are equal to the total core loss. The mechanical (rotational) losses are one third of the core loss. Calculate the motor speed. [4]

X

21

1. The terminal voltage of the balanced 3-phase delta connected load shown in Figure 1 is maintained constant at 100 V and the total power drawn by the load is 6 kW at 0.83 pf (lag).

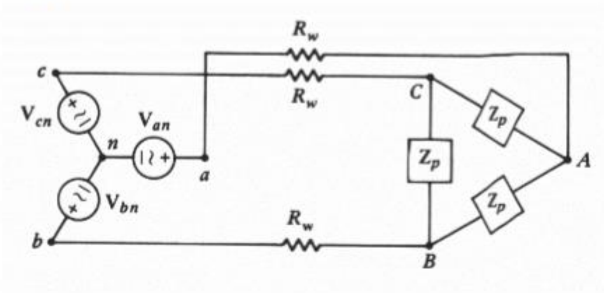


Figure 1

   a. Calculate the impedance Z_p and the current drawn by the load.
   b. If the resistance of the lines, R_w = 0.8 Ω/phase,
      i. draw the per-phase equivalent circuit of the system, and
      ii. calculate the terminal voltage of the source, total power loss in the lines, and the total complex power supplied by the source.

[3+5]

Q1. *a*. Power drawn per phase $= 2\text{kW at } 0.83 \text{ pf (lag)} = (2000/0.83)\angle\cos^{-1} 0.83$

$= 2409.64\angle 33.9°$

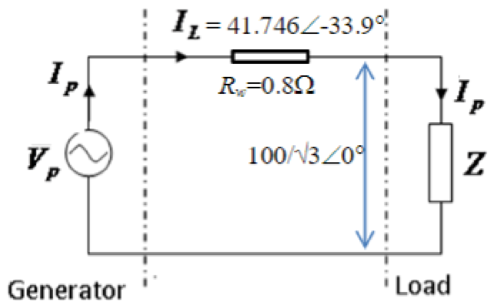
$= 2000.03 + j\ 1343.96$ VA

Therefore, $Z_p = \dfrac{100^2}{(2409.64\angle 33.9°)^*} = 4.15\angle 33.9°\ \Omega$ [1.5]

Impedance / star phase $= Z_p / 3 = 1.383\angle 33.9°\ \Omega = 1.148 + j0.771$

Current drawn by the load $= I_L = I_{ph} = \dfrac{(100/\sqrt{3})\angle 0°}{1.383\angle 33.9°} = 41.746\angle -33.9°\ A$ [1.5]

b. The per phase equivalent circuit is as shown.



Then, the source voltage per phase may be calculated as:

$V_{sp} = V_a\angle 0° + I_a \times R_w$

$= (100/\sqrt{3})\angle 0° + 41.746\angle -33.9° \times 0.8 = 85.45 - j18.62$

$= 87.455\angle -12.29°$

Terminal voltage of the source $V_t = \sqrt{3} \times 87.455 = 151.5$ V [2]

Power lost in $R_w = 41.746^2 \times 0.8 = 1394.18$ W

Total power loss in the line $= 3 \times 1394.18 = 4182.54$ W [1]

Therefore, total power output

$= 3 \times (2000.03 + j\ 1343.96 + 1394.18)$ VA

$= 10182.6 + j\ 4031.9$

$= 10951.8\angle 21.60°$ VA [2]

The complex power may also be calculated as

$= 3 \times 87.455\angle -12.29° \times (41.746\angle -33.9°)^*$

22

1. The details of an iron-cored inductor are shown in Figure 1 (a) where the dimensions are in cm. The core is 2 cm deep. The linearized B-H curve of the core is shown in Figure 1 (b).

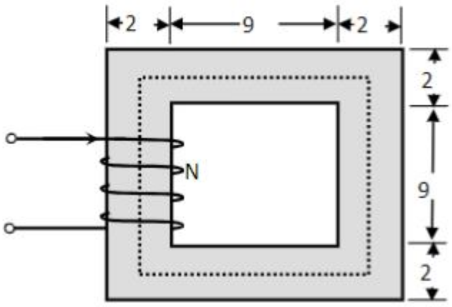


Figure 1(a)

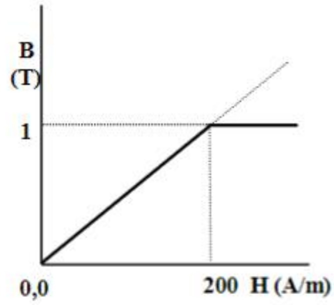
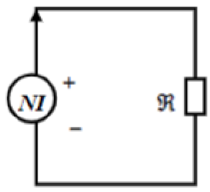


Figure 1(b)

a. Draw the magnetic equivalent circuit for the system. [2]
b. Calculate the inductance of the coil if the number of turn N=22. [1]
c. What is the maximum current I_{max} that the inductor can take before it saturates? [2]

Sol: 1a. The magnetic equivalent circuit as shown below



where, $\mu = \frac{B}{H} = \frac{1}{200}$

Mean length l = 4×11 cm = 0.44 m

Cross section A = 2×2×10^{-4} m^2 = 4×10^{-4} m^2

Reluctance $\Re = \frac{l}{\mu A} = \frac{0.44}{(1/200)\times 4\times 10^{-4}} = 0.22\times 10^{6}\ \mathrm{H}^{-1}$ [2]

1b. Inductance $L = \frac{N^2}{\Re} = \frac{22^2}{0.22\times 10^6} = 2.2\times 10^{-3}\ \mathrm{H}$ [1]

1c. Saturation occurs at B = 1 T,

Therefore $\varphi_{max} = 1\times 4\times 10^{-4} = \frac{Ni_{max}}{\Re} = \frac{22\times i_{max}}{0.22\times 10^6}$ and, then

$$i_{max} = \frac{Ni_{max}}{\Re} = \frac{4\times 10^{-4}\times 0.22\times 10^6}{22} = 4\ \mathrm{A}$$ [2]