Physics II: Electromagnetism

PH 102

Lecture-2

March-June 2022

The operator $\vec{\nabla}$

$$\vec{\nabla} \equiv \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$ which means of course $$\nabla_x \equiv \frac{\partial}{\partial x}, \quad \nabla_y \equiv \frac{\partial}{\partial y}, \quad \nabla_z \equiv \frac{\partial}{\partial z}$$

Important: $\vec{\nabla}$ is a vector operator, alone it does not have a meaning.

In PH101, you have already encountered the derivative as an operator (recall momentum operator $-i\hbar\partial/\partial x$ in quantum mechanics).

$\vec{\nabla}T$ means that $\vec{\nabla}$ is a vector operator that acts upon a scalar field T to give a vector field. (<u>$\vec{\nabla}T$ does not mean $\vec{\nabla}$ is a vector that multiplies a scalar T</u>).

Since $\vec{\nabla}$ is an operator, $T\vec{\nabla} \neq \vec{\nabla}T$ (unlike ordinary algebra)

Still an operator, "hungry for something to differentiate"

The operator $\vec{\nabla}$ has already acted to give a Vector field

Differential Calculus of the vector field

What else can we do with the del or grad operator? $\longrightarrow$ Try combining it with a vector field

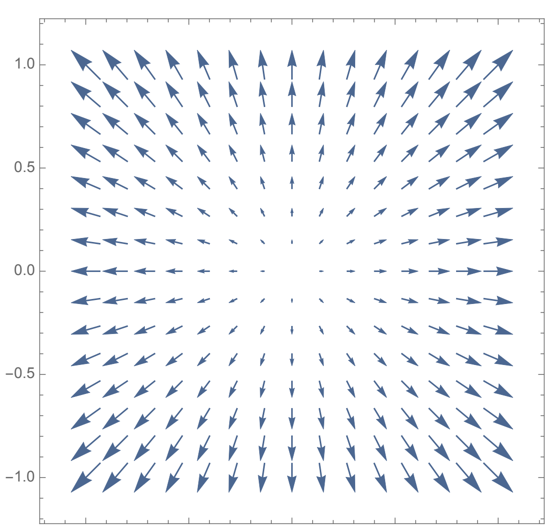
$$(\text{a vector}).\vec{\nabla} \qquad\qquad \vec{\nabla}.(\text{a vector})$$

(Still a hungry operator) (is a Scalar)

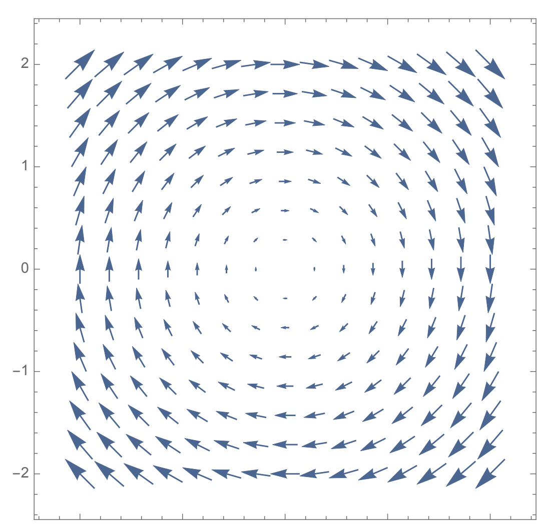
The Divergence:

$$\begin{aligned}\vec{\nabla}.\vec{v} &= \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right).(v_x\hat{x} + v_y\hat{y} + v_z\hat{z}) \\ &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\end{aligned}$$

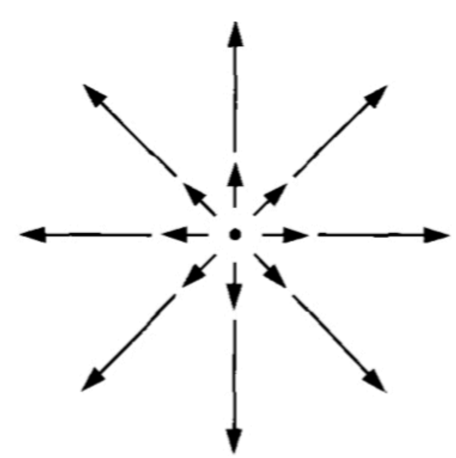
Divergence is a measure of how much the vector is spread out (diverges) from the point in question.

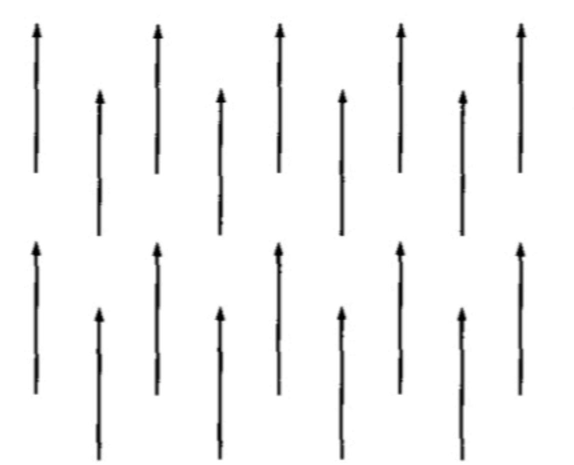


non-zero



zero divergence:


Positive


zero divergence

Differential Calculus of the vector field

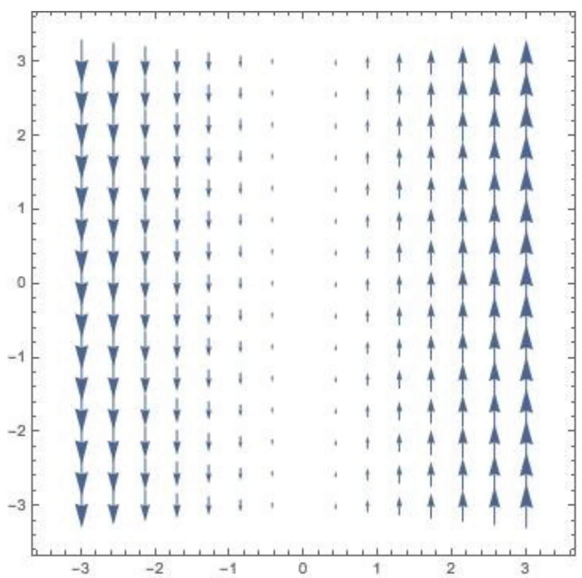
The Curl: Another operation with the gradient operator

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

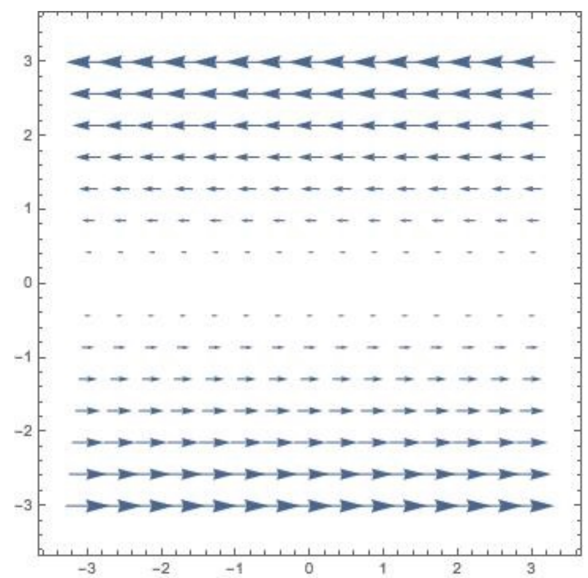
Curl means how much a vector swirls around the point in question. If vector field is a velocity field of a fluid then non-zero curl indicates rotational flow.

Or, if you like:

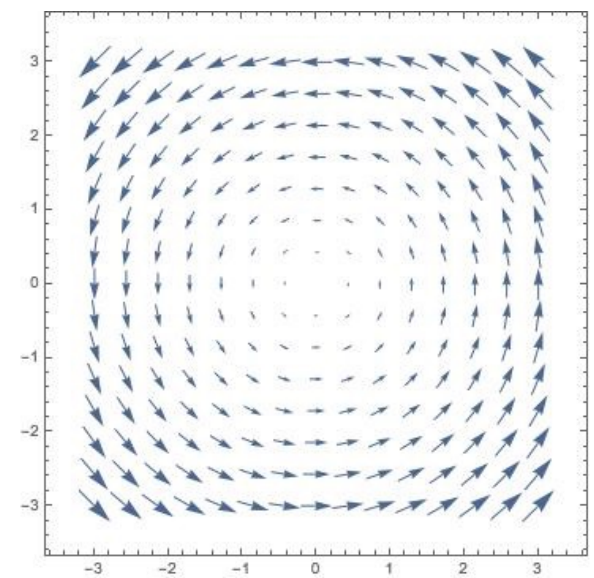
$$(\vec{\nabla} \times \vec{v})_i = \sum_{j,k} \varepsilon_{ijk} \nabla_j v_k$$



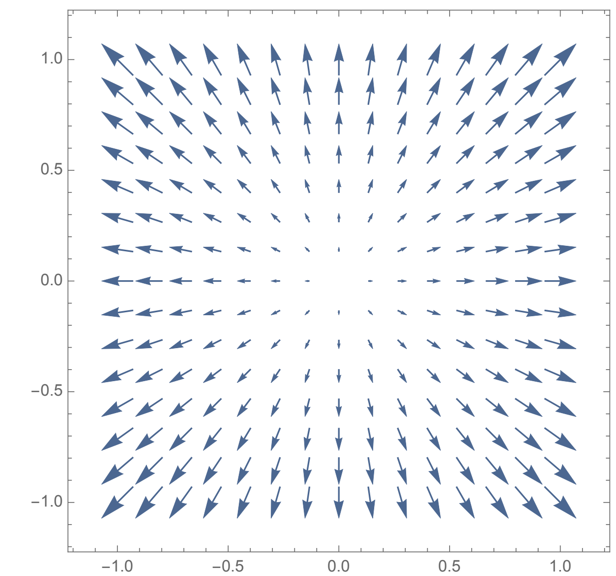
$$\vec{V}_1 = x\hat{y}$$
$$\vec{\nabla} \times \vec{V}_1 = \hat{z}$$



$$\vec{V}_2 = -y\hat{x}$$
$$\vec{\nabla} \times \vec{V}_2 = \hat{z}$$



$$\vec{V}_3 = -y\ \hat{x} + x\ \hat{y}$$
$$\vec{\nabla} \times \vec{V}_3 = 2\hat{z}$$

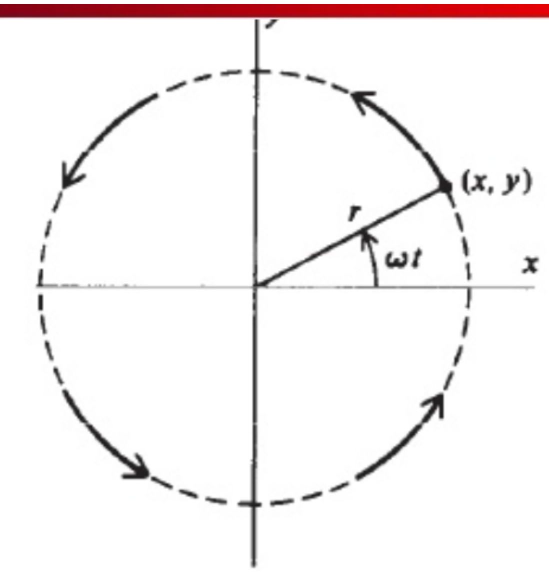


$$\vec{V}_4 = x\hat{x} + y\hat{y}$$
$$\vec{\nabla} \times \vec{V}_4 = 0$$

Vector fields with zero curl is called "irrotational" for obvious reasons!

How does the curl represent swirling of a vector field

Consider water is flowing on a circular path shown in figure. A small volume of water at point (x,y) at time t has coordinates

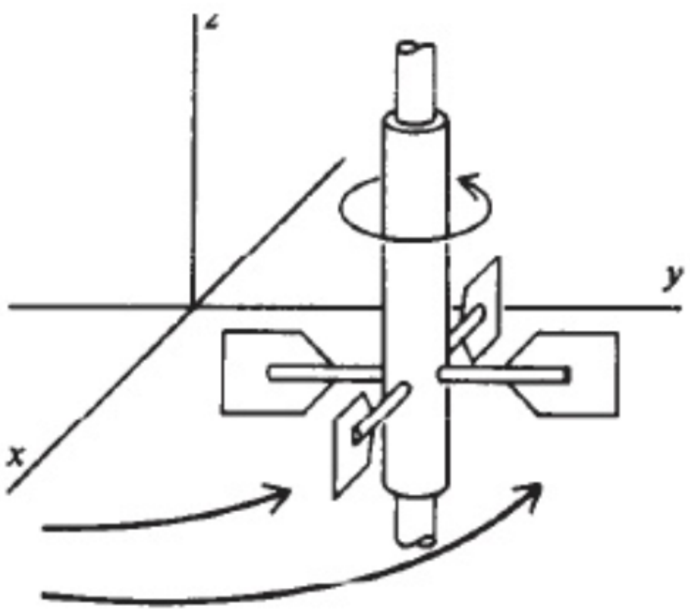


$$x = r\cos\omega t, y = r\sin\omega t$$

Velocity vector field at point (x,y)

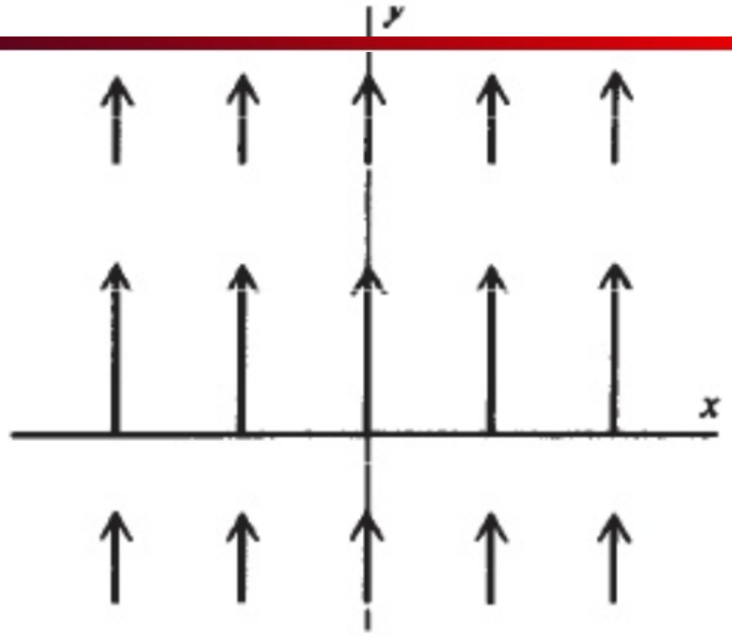
$$\begin{aligned} \vec{v} &= \hat{x}(dx/dt) + \hat{y}(dy/dt) = r\omega[-\hat{x}\sin\omega t + \hat{y}\cos\omega t] \\ &= \omega(-\hat{x}y + \hat{y}x) \end{aligned}$$

The curl of the velocity vector field: $\vec{\nabla} \times \vec{v} = 2\hat{z}\omega$ **(Non-zero)**



Curl of a vector field can be ideated by placing a paddle wheel in the flow. If the wheel rotates, the curl is non-zero. The wheel will rotate with its axis pointing in the direction of the curl.

More interesting example

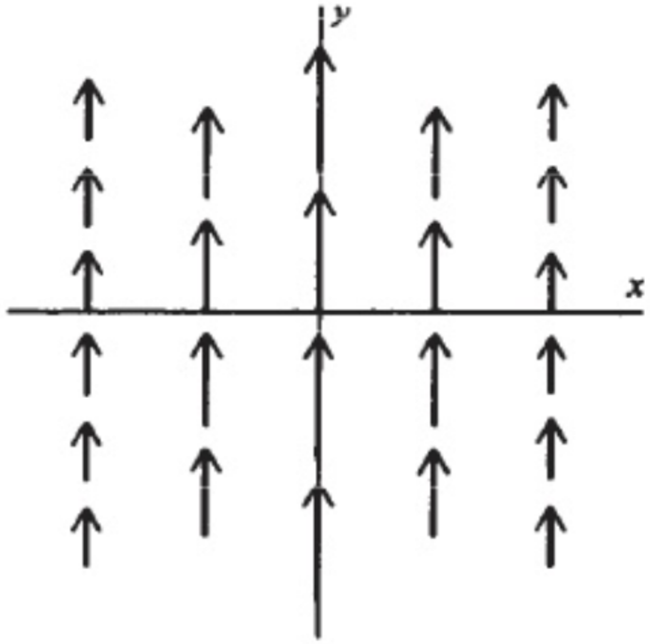


Imagine a vector field $\vec{v} = \hat{y} v_0 e^{-y^2/\lambda^2}$

Field lines point along *y-axis* and the magnitude varies with *y*

As expected the curl will be zero: $\vec{\nabla} \times \vec{v} = 0$

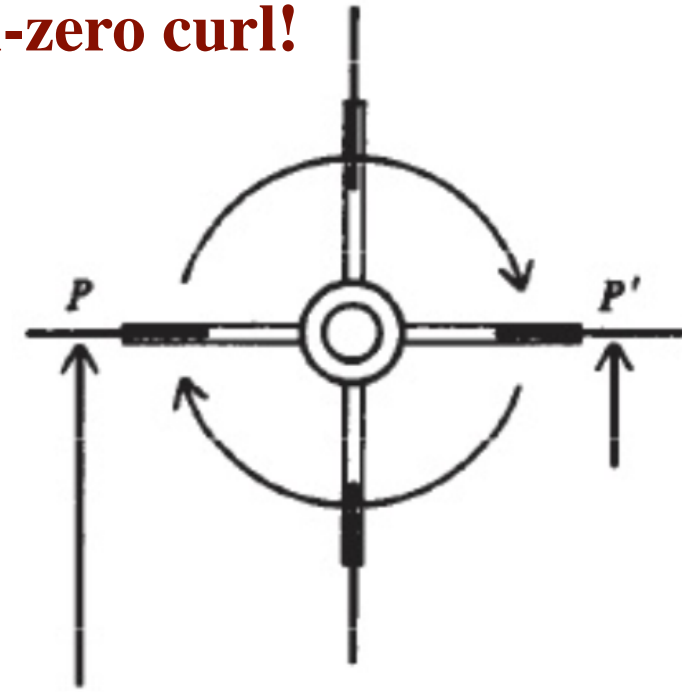
However, consider $\vec{v} = \hat{y} v_0 e^{-x^2/\lambda^2}$



Field lines still point along *y* but the magnitude varies with *x*

$$\vec{\nabla} \times \vec{v} = -\hat{z} v_0 \frac{2x}{\lambda^2} e^{-x^2/\lambda^2}$$

Non-zero curl!



This can be understood by the non-zero torque on the paddle wheel in the field, although, there is no visible swirling of the field.

Second derivatives of vector fields

So far we had only first derivatives. Why not second derivatives?

Combinations which are possible

$$\vec{\nabla}.(\vec{\nabla}T) = \nabla_x(\nabla_x T) + \nabla_y(\nabla_y T) + \nabla_z(\nabla_z T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$\vec{\nabla}.(\vec{\nabla}T) = \vec{\nabla}.\vec{\nabla}T = \nabla^2 T$$

∇^2 : new operator (scalar) - "Laplacian"

Scalar field

$\vec{\nabla} \times (\vec{\nabla}T) = 0$ (why?) Theorem: If $\vec{\nabla} \times \vec{v} = 0$, there is a ϕ such that $\vec{v} = \vec{\nabla}\phi$

$\vec{\nabla}.(\vec{\nabla} \times \vec{v}) = 0$ (why?) Theorem: If $\vec{\nabla}.\vec{D} = 0$, there is a $\vec{C}$ such that $\vec{D} = \vec{\nabla} \times \vec{C}$

$\vec{\nabla}(\vec{\nabla}.\vec{v}) =$ a vector field.

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{v}) = \vec{\nabla}(\vec{\nabla}.\vec{v}) - \nabla^2 \vec{v}$$

Important identities involving Gradient , Divergence and Curl

$$\vec{\nabla}.(\vec{A}+\vec{B}) = \vec{\nabla}.\vec{A}+\vec{\nabla}.\vec{B}$$

$$\vec{\nabla}\times(\vec{A}+\vec{B}) = \vec{\nabla}\times\vec{A}+\vec{\nabla}\times\vec{B}$$

$$\vec{\nabla}(f+g) = \vec{\nabla}f+\vec{\nabla}g$$

$$\vec{\nabla}(fg) = f\vec{\nabla}g+g\vec{\nabla}f$$

$$\vec{\nabla}.(f\vec{A}) = f\vec{\nabla}.\vec{A}+\vec{A}.\vec{\nabla}f$$ Homework

$$\vec{\nabla}(\vec{A}.\vec{B}) = \vec{A}\times(\vec{\nabla}\times\vec{B})+\vec{B}\times(\vec{\nabla}\times\vec{A})+(\vec{A}.\vec{\nabla})\vec{B}+(\vec{B}.\vec{\nabla})\vec{A}$$

$$\vec{\nabla}.(\vec{A}\times\vec{B}) = \vec{B}.(\vec{\nabla}\times\vec{A})-\vec{A}.(\vec{\nabla}\times\vec{B})$$ Homework

$$\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}.\vec{\nabla})\vec{A}-(\vec{A}.\vec{\nabla})\vec{B}+\vec{A}(\vec{\nabla}.\vec{B})-\vec{B}(\vec{\nabla}.\vec{A})$$

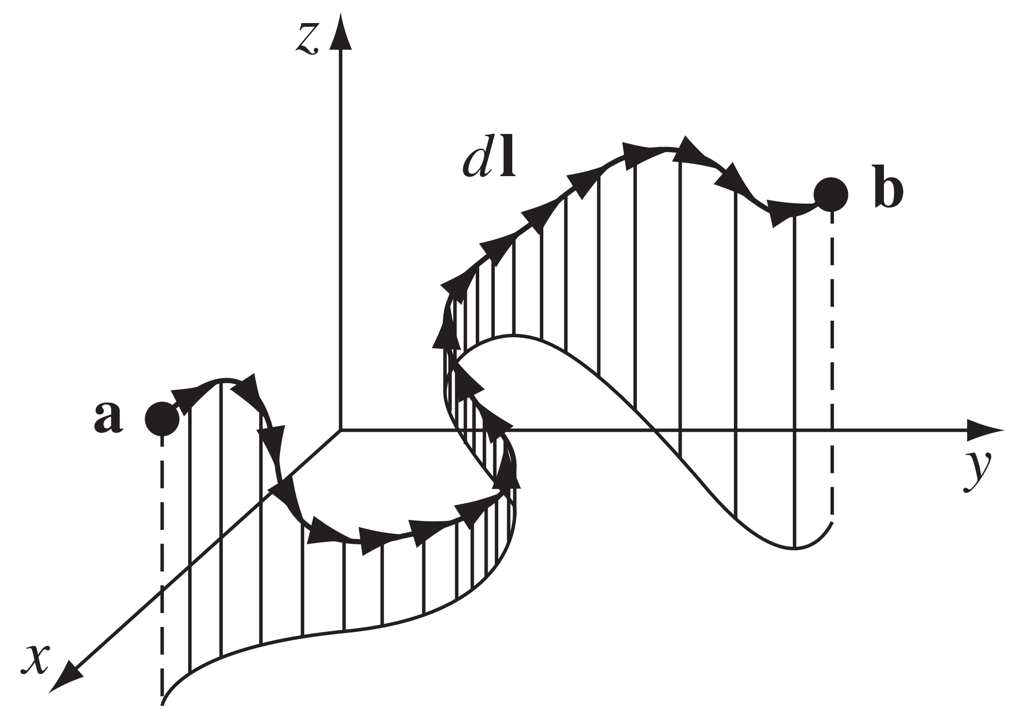
$$\vec{\nabla}\times(f\vec{A}) = f(\vec{\nabla}\times\vec{A})-\vec{A}\times\vec{\nabla}f$$ Homework

Line, Surface and Volume Integrals

Line Integrals

Line integral is an integral where the function to be integrated is evaluated along a path.

$$\int_{\mathbf{a},\mathcal{P}}^{\mathbf{b}} \mathbf{v} \cdot d\mathbf{l},$$



where $\mathbf{v}$ is a vector function, $d\mathbf{l}$ is the infinitesimal displacement vector

integral is to be carried out along a prescribed path $\mathcal{P}$ from point $\mathbf{a}$ to point $\mathbf{b}$

If the path is closed loop

$$\oint \mathbf{v} \cdot d\mathbf{l},$$

Line integrals are basically one dimensional integrals

Line Integrals (Example)

(i) Calculate the line integral of the function $\mathbf{v} = \mathbf{y^2\hat{x} + 2x(y+1)\hat{y}}$ from the point **a**=(1,1,0) to the point **b**=(2,2,0) along the paths (1) and (2) shown in the figure.

(ii) Calculate also the closed line integral from **a** to **b** via path (1) and back from **b** to **a** via path (2)

As always, $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$. Path (1) consists of two parts. Along the "horizontal" segment, $dy = dz = 0$, so

(i) $d\mathbf{l} = dx\,\hat{\mathbf{x}},\ y = 1,\ \mathbf{v}\cdot d\mathbf{l} = y^2\,dx = dx$, so $\int \mathbf{v}\cdot d\mathbf{l} = \int_1^2 dx = 1$.

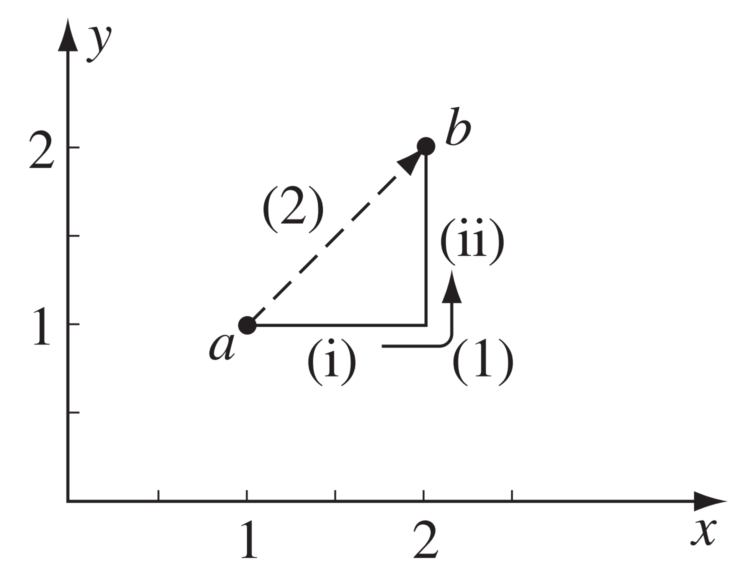
On the "vertical" stretch, $dx = dz = 0$, so

(ii) $d\mathbf{l} = dy\,\hat{\mathbf{y}},\ x = 2,\ \mathbf{v}\cdot d\mathbf{l} = 2x(y+1)\,dy = 4(y+1)\,dy$, so

$$\int \mathbf{v}\cdot d\mathbf{l} = 4\int_1^2 (y+1)\,dy = 10.$$

By path (1), then,

$$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{v}\cdot d\mathbf{l} = 1 + 10 = 11.$$



Line Integrals (Example)

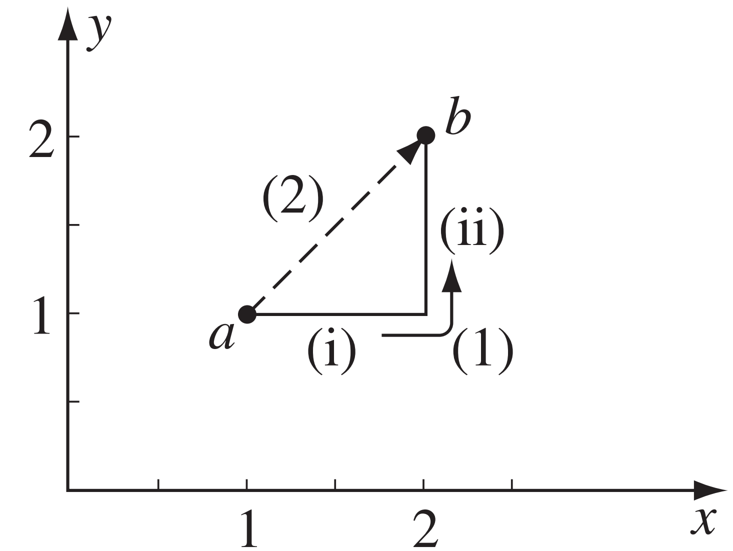
Meanwhile, on path (2) $x = y$, $dx = dy$, and $dz = 0$, so
$d\mathbf{l} = dx\,\hat{\mathbf{x}} + dx\,\hat{\mathbf{y}}$, $\mathbf{v}\cdot d\mathbf{l} = x^2\,dx + 2x(x+1)\,dx = (3x^2 + 2x)\,dx$,
and

$$\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{v}\cdot d\mathbf{l} = \int_1^2 (3x^2 + 2x)\,dx = (x^3 + x^2)\Big|_1^2 = 10.$$

(The strategy here is to get everything in terms of one variable; I could just as well have eliminated x in favor of y.)

For the loop that goes *out* (1) and *back* (2), then,

$$\oint \mathbf{v}\cdot d\mathbf{l} = 11 - 10 = 1.$$

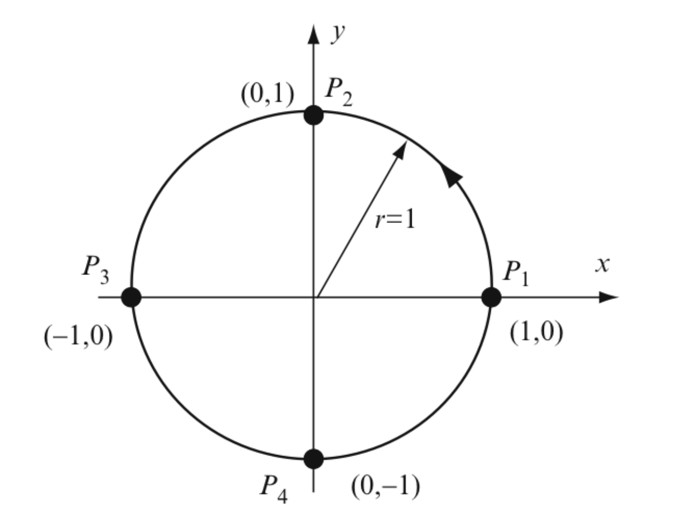


Line Integrals (example)

Circulation of a vector field. Consider a vector filed $\mathbf{A} = xy\hat{x} + (3x^2 + y)\hat{y}$

Calculate the circulation of this vector filed around the circular path : $x^2 + y^2 = 1$

Solution



The differential of length in the $x-y$ plane is $d\mathbf{l} = \hat{\mathbf{x}}dx + \hat{\mathbf{y}}dy$. The scalar product $\mathbf{A} \cdot d\mathbf{l}$ is

$$\mathbf{A} \cdot d\mathbf{l} = \left(\hat{\mathbf{x}}\,xy + \hat{\mathbf{y}}\left(3x^2 + y\right)\right) \cdot \left(\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}dy\right) = xydx + \left(3x^2 + y\right)dy$$

The circulation is now

$$\oint_L \mathbf{A} \cdot d\mathbf{l} = \oint_L \left[xydx + \left(3x^2 + y\right)dy\right]$$

Before this can be evaluated, we must make sure that integration is over a single variable. To do so, we use the equation of the circle and write

$$x = \left(1 - y^2\right)^{1/2}, \qquad y = \left(1 - x^2\right)^{1/2}$$

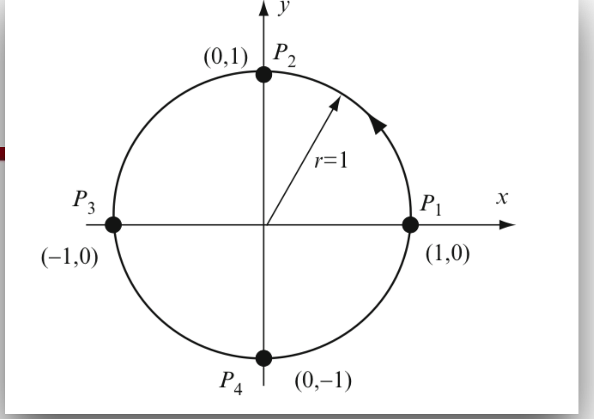
By substituting the first relation into the second term and the second into the first term under the integral, we have

$$\int_L \mathbf{A} \cdot d\mathbf{l} = \oint_L \left[x\left(1 - x^2\right)^{1/2} dx + \left(3\left(1 - y^2\right) + y\right)dy\right]$$

and each part of the integral is a function of a single variable. Now, we can separate these into four integrals:

$$\oint_L \mathbf{A} \cdot d\mathbf{l} = \int_{P_1}^{P_2} \mathbf{A} \cdot d\mathbf{l} + \int_{P_2}^{P_3} \mathbf{A} \cdot d\mathbf{l} + \int_{P_3}^{P_4} \mathbf{A} \cdot d\mathbf{l} + \int_{P_4}^{P_1} \mathbf{A} \cdot d\mathbf{l}$$

Line Integrals (example)



Evaluating each integral separately,

$$\int_{P_1}^{P_2} \mathbf{A}\cdot d\mathbf{l} = \int_{P_1}^{P_2} \left(x(1-x^2)^{1/2}dx + (3-3y^2+y)dy\right)$$

$$= \int_{x=1}^{x=0} x(1-x^2)^{1/2}dx + \int_{y=0}^{y=1}(3-3y^2+y)dy = -\frac{(1-x^2)^{3/2}}{3}\Bigg|_1^0 + \left(3y+\frac{y^2}{2}-y^3\right)\Bigg|_0^1 = \frac{13}{6}$$

Note that the other integrals are similar except for the limits of integration:

$$\int_{P_2}^{P_3} \mathbf{A}\cdot d\mathbf{l} = \int_{x=0}^{x=-1} x(1-x^2)^{1/2}dx + \int_{y=1}^{y=0}(3-3y^2+y)dy = -\frac{(1-x^2)^{3/2}}{3}\Bigg|_0^{-1} + \left(3y+\frac{y^2}{2}-y^3\right)\Bigg|_1^0 = -\frac{13}{6}$$

$$\int_{P_3}^{P_4} \mathbf{A}\cdot d\mathbf{l} = \int_{x=-0}^{x=0} x(1-x^2)^{1/2}dx + \int_{y=0}^{y=-1}(3-3y^2+y)dy = -\frac{(1-x^2)^{3/2}}{3}\Bigg|_{-1}^{0} + \left(3y+\frac{y^2}{2}-y^3\right)\Bigg|_0^{-1} = -\frac{11}{6}$$

$$\int_{P_1}^{P_1} \mathbf{A}\cdot d\mathbf{l} = \int_{x=0}^{x=1} x(1-x^2)^{1/2}dx + \int_{y=-1}^{y=0}(3-3y^2+y)dy = -\frac{(1-x^2)^{3/2}}{3}\Bigg|_0^{1} + \left(3y+\frac{y^2}{2}-y^3\right)\Bigg|_{-1}^{0} = \frac{11}{6}$$

The total circulation is the sum of the four circulations above. This gives

$$\oint_L \mathbf{A}\cdot d\mathbf{l} = 0$$

Conservative vector field

$$\int_a^b \vec{\nabla}\phi.d\vec{r} = \int_a^b \left(\hat{x}\frac{\partial\phi}{\partial x} + \hat{y}\frac{\partial\phi}{\partial y} + \hat{z}\frac{\partial\phi}{\partial z}\right).(dx\hat{x} + dy\hat{y} + dz\hat{z})$$

$$= \int_a^b \left(\frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz\right)$$

$$= \int_a^b d\phi = \phi(b) - \phi(a)$$

Fundamental theorem for gradients.
Will discuss later in detail.

Integral depends only on the end points, but not on the path chosen.

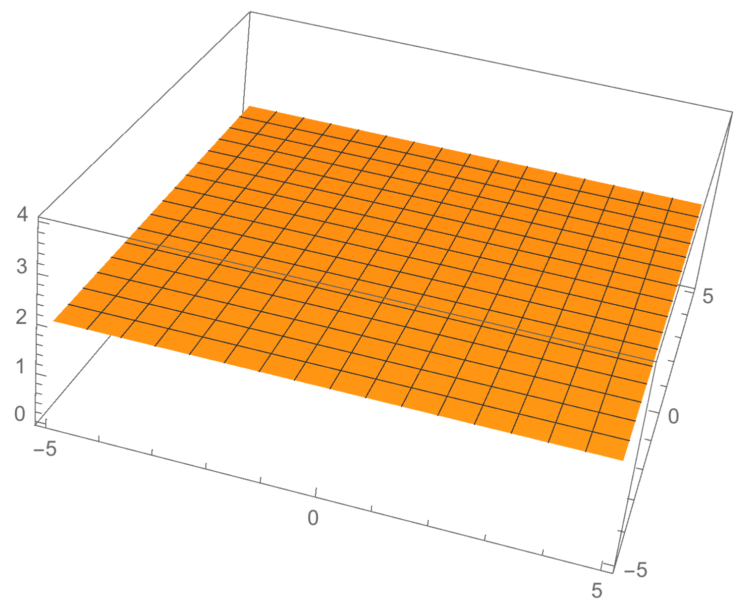
If a vector field can be written as $\vec{F} = \vec{\nabla}\phi$, then $\oint \vec{F}\cdot\vec{dl} = \oint dW = 0$ (integral form)

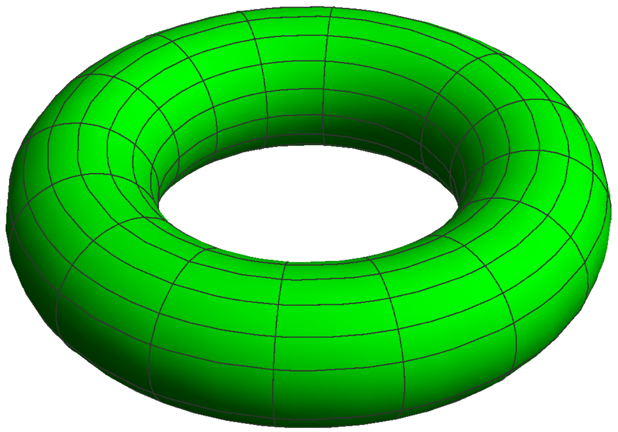
- Work done by such vector fields in a closed loop is zero.
- Gradient of a scalar function is always a **conservative field** $(\vec{F} = \vec{\nabla}\phi)$.
- We have the vector identify : $\vec{\nabla}\times(\vec{\nabla}\phi) = 0$ (Check it !!)
- **If** $\vec{F} = \vec{\nabla}\phi \implies \vec{\nabla}\times\vec{F} = 0$ (differential form)
- **Note :** *Every vector field can not be written as a gradient of some scalar field.*

Surface Integrals: How do we define a surface?

$z = f(x,y)$ or $x = f(y,z)$ or $y = f(x,z)$ is one of the standard form to represent surfaces.

Examples

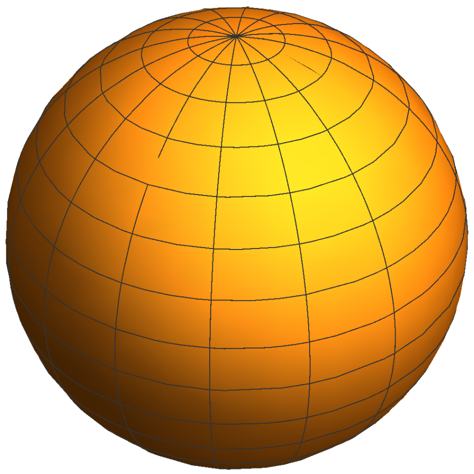




1. $z = \text{constant}$ is a plane parallel to xy plane.

2. $z^2 = a^2 - (c - \sqrt{x^2 + y^2})^2$ is a torus.

Another way to represent: $f(x,y,z) = \text{constant}$



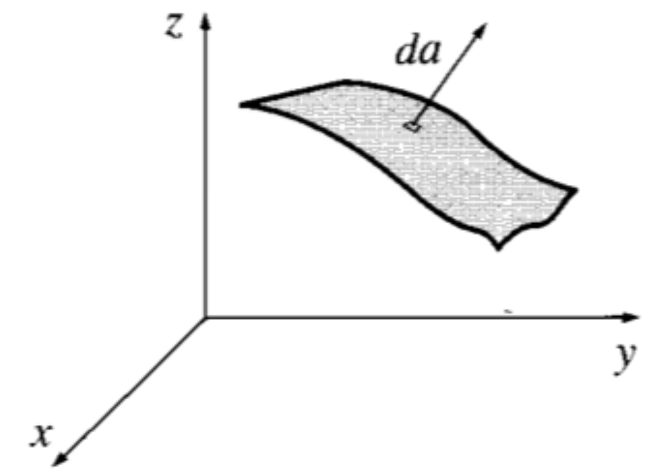
The sphere

$x^2 + y^2 + z^2 = a^2$

Surface integrals

Surface integral is an expression of the form

$$\int_S \mathbf{v} \cdot d\mathbf{a}$$



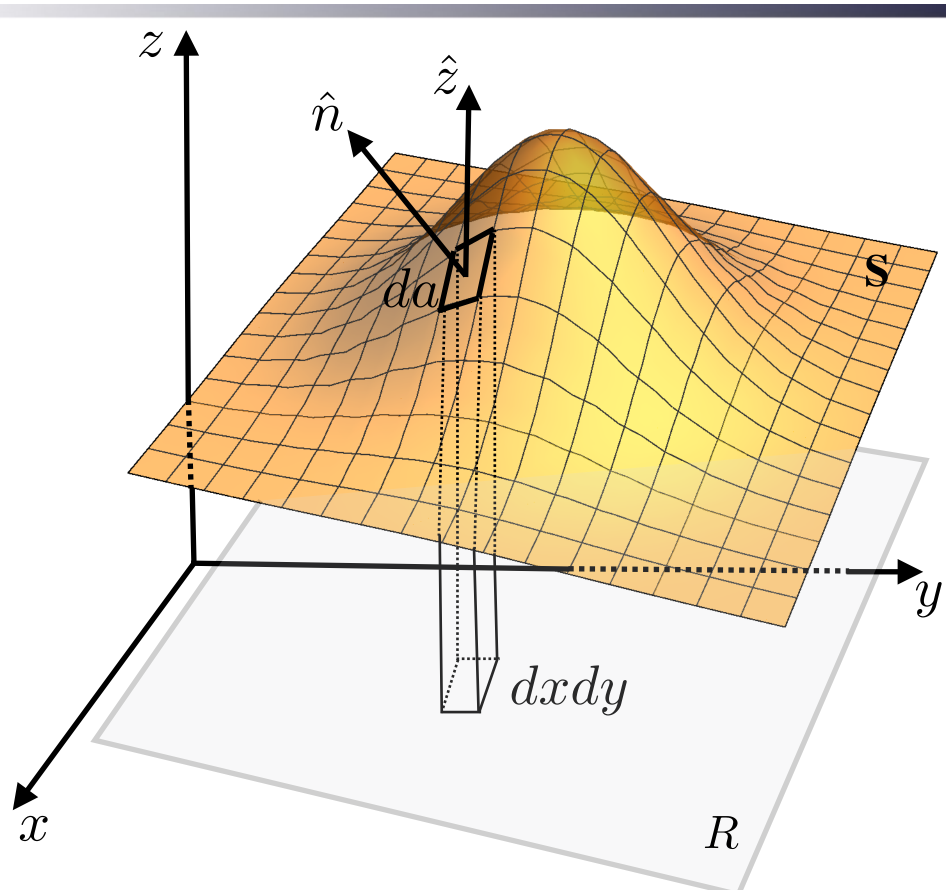
Here, $\mathbf{v}$ is some vector function and $d\mathbf{a}$ is an infinitesimal patch of area

If this surface is a closed surface

$$\oint \mathbf{v} \cdot d\mathbf{a}$$

Closed surface integration gives the total or net flux through a closed surface

Elementary area on a surface



- Let S be a smooth surface: $z = f(x, y)$.
- Project it on xy-plane: R be the projection.
- Choose an elementary area da on S and let $\hat{n}$ be a unit vector perpendicular to it.
- Projection of da on xy-plane is $dxdy$.

$$\therefore dxdy = |\hat{n}.\hat{z}|\ da$$

$$da = \frac{dxdy}{|\hat{n}.\hat{z}|}$$

- Hence we can denote da as vector area

$$d\vec{a} = \left(\frac{dxdy}{|\hat{n}.\hat{z}|}\right)\hat{n} = \hat{n}da$$

For an open two-sided surface, the "outward" normal shows the direction for the surface. Open surfaces are bounded by curves and "outward" normal is defined by the right hand rule-if the bounding curve is traversed in the direction of rotation of a right handed screw, the direction in which the head of screw moves is the direction of outward normal.

Concept of area as a vector

Imagine a tiny area (like a postage stamp) in 3 dimensions at some location $\vec{r}$. What can I do to specify it?

- how big it is?

da square meters (say).

- in which plane it lies?

in the xy plane (say)

it lies perpendicular to z axis

$\Longrightarrow$ A vector $d\vec{a}$, of magnitude da and direction along the z axis can be associated with this area.

But, there are two ways to draw $\perp$ to xy plane: up or down the z axis.

To further specify the area, to make it an oriented one, we draw arrows that run around the perimeter of the area in one of the two possible directions.

Area vector will point in the direction following the right hand thumb rule.

Only a planar area can be represented as a vector. Non-Planar areas like a hemisphere can not be represented by a single vector.

The use of right hand rule in defining areas might remind you of the cross product and indeed that is true as we will see soon.

Surface integrals (Example)

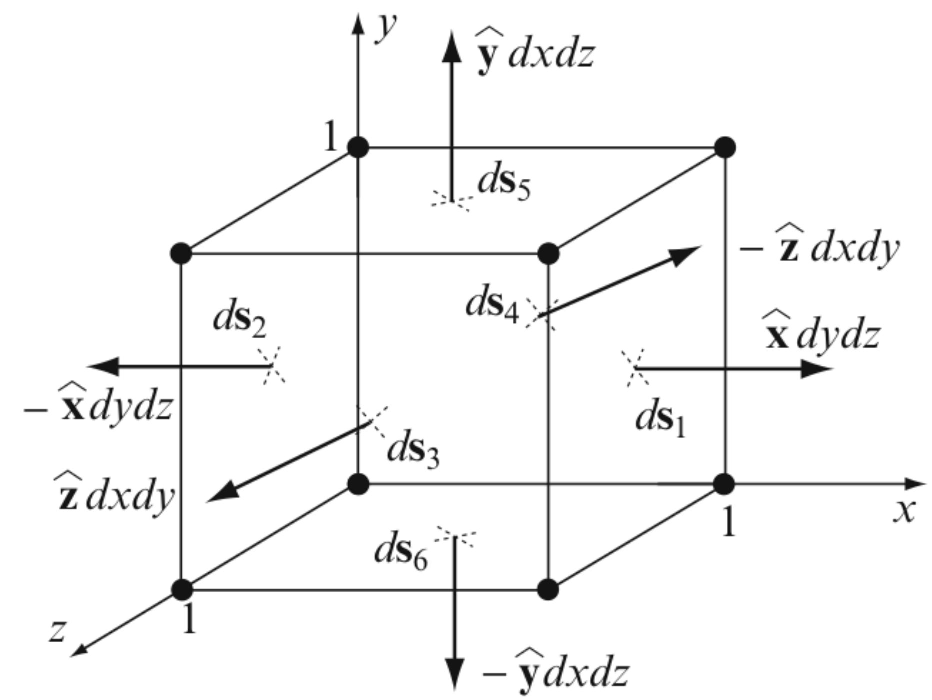
Closed Surface integral

Given a vector $A = 2xz\ \hat{x} + 2zx\ \hat{y} - yz\ \hat{z}$, calculate the closed surface integral of it over the surface defined by a cube. The cube occupies the space between $0 \leq x, y, z, \leq 1$.

Solution

the differentials of surface $d\mathbf{s}$ are

$$d\mathbf{s}_1 = \hat{\mathbf{x}}dydz, \quad d\mathbf{s}_2 = -\hat{\mathbf{x}}dydz$$
$$d\mathbf{s}_3 = \hat{\mathbf{z}}dxdy, \quad d\mathbf{s}_4 = -\hat{\mathbf{z}}dxdy$$
$$d\mathbf{s}_5 = \hat{\mathbf{y}}dxdz, \quad d\mathbf{s}_6 = -\hat{\mathbf{y}}dxdz$$



The surface integral is now written as

$$\oint_s \mathbf{A} \cdot d\mathbf{s} = \int_{s_1} \mathbf{A} \cdot d\mathbf{s}_1 + \int_{s_2} \mathbf{A} \cdot d\mathbf{s}_2 + \int_{s_3} \mathbf{A} \cdot d\mathbf{s}_3 + \int_{s_4} \mathbf{A} \cdot d\mathbf{s}_4 + \int_{s_5} \mathbf{A} \cdot d\mathbf{s}_5 + \int_{s_6} \mathbf{A} \cdot d\mathbf{s}_6$$

Surface integrals (Example contd.)

Each term is evaluated separately. On side 1,

$$\int_{s_1} \mathbf{A} \cdot d\mathbf{s}_1 = \int_{s_1} (\hat{\mathbf{x}}\, 2xz + \hat{\mathbf{y}}\, 2zx - \hat{\mathbf{z}}\, yz) \cdot (\hat{\mathbf{x}} dydz) = \int_{s_1} 2xzdydz$$

To perform the integration, we set $x = 1$. Separating the surface integral into an integral over y and one over z, we get

$$\int_{s_1} \mathbf{A} \cdot d\mathbf{s}_1 = \int_{y=0}^{y=1} \left[\int_{z=0}^{z=1} 2z\, dz \right] dy = 2 \int_{y=0}^{y=1} \left[\frac{z^2}{2} \bigg|_{z=0}^{z=1} \right] dy = \int_{y=0}^{y=1} dy = y \bigg|_{y=0}^{y=1} = 1$$

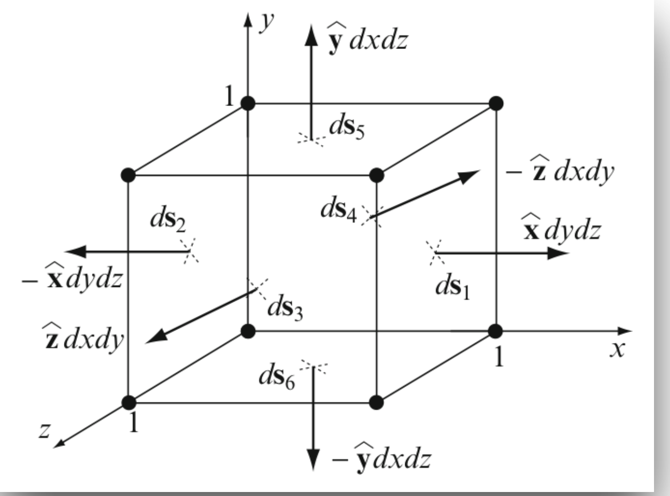
On side 2, the situation is identical, but $x = 0$ and $d\mathbf{s}_2 = -d\mathbf{s}_1$. Thus,

$$\int_{s_2} \mathbf{A} \cdot d\mathbf{s}_2 = -\int_{s_2} 2xzdydz = 0$$

On side 3, $z = 1$ and the integral is

$$\begin{aligned} \int_{s_3} \mathbf{A} \cdot d\mathbf{s}_3 &= \int_{s_3} (\hat{\mathbf{x}}\, 2xz + \hat{\mathbf{y}}\, 2zx - \hat{\mathbf{z}}\, yz) \cdot (\hat{\mathbf{z}} dxdy) \\ &= -\int_{s_3} yzdxdy = -\int_{x=0}^{x=1} \left[\int_{y=0}^{y=1} y\, dy \right] dx = -\int_{x=0}^{x=1} \left[\frac{y^2}{2} \bigg|_{y=0}^{y=1} \right] dx = -\int_{x=0}^{x=1} \frac{dx}{2} = -\frac{x}{2} \bigg|_{x=0}^{x=1} = -\frac{1}{2} \end{aligned}$$

Surface integrals (Example contd.)



On side 4, $z = 0$ and $d\mathbf{s}_4 = -d\mathbf{s}_3$. Therefore, the contribution of this side is zero:

$$\int_{S_4} \mathbf{A} \cdot d\mathbf{s}_4 = \int_{S_4} yz dx dy = 0$$

On side 5, $y = 1$:

$$\int_{S_5} \mathbf{A} \cdot d\mathbf{s}_5 = \int_{S_5} (\hat{\mathbf{x}}\, 2xz + \hat{\mathbf{y}}\, 2zx - \hat{\mathbf{z}}\, yz) \cdot (\hat{\mathbf{y}} dxdz) = \int_{x=0}^{1} \int_{z=0}^{1} 2zx dx dz = \frac{1}{2}$$

On side 6, $y = 0$:

$$\int_{S_6} \mathbf{A} \cdot d\mathbf{s}_6 = \int_{S_6} (\hat{\mathbf{x}}\, 2xz + \hat{\mathbf{y}}\, 2zx - \hat{\mathbf{z}}\, yz) \cdot (-\hat{\mathbf{y}} dxdz) = -\int_{x=0}^{1} \int_{z=0}^{1} 2zx dx dz = -\frac{1}{2}$$

The result is the sum of all six contributions:

$$\oint_{S} \mathbf{A} \cdot d\mathbf{s} = 1 + 0 - \frac{1}{2} + 0 + \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$$

Summary

- Divergence of a vector field is denoted by $\vec{\nabla}.\vec{v}$ and is a scalar. It gives the outward flux of the field around a point.

- Curl of a vector field is a vector field and denoted by $\vec{\nabla} \times \vec{v}$ and non-zero curl implies rotational flow in case of a velocity field of a fluid.

Line integral

$$\boxed{\int_{\mathbf{a}\mathcal{P}}^{\mathbf{b}} \mathbf{v} \cdot d\mathbf{l},}$$

If the path is closed loop

$$\boxed{\oint \mathbf{v} \cdot d\mathbf{l}.}$$

Surface integral

$$\boxed{\int_{\mathcal{S}} \mathbf{v} \cdot d\mathbf{a},}$$

Closed Surface integral

$$\boxed{\oint \mathbf{v} \cdot d\mathbf{a},}$$

Thank You