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- Consider the problem (**P**),
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  subject to
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  where $\mathbf{a}_{B,m+1}^T$ and $\mathbf{a}_{N,m+1}^T$ are the components of $\mathbf{a}_{m+1}^T$ corresponding to the basic variables and nonbasic variables of $\mathbf{x}_0$, respectively.

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- The **inverse** of this new basis matrix is given by $\begin{bmatrix} B^{-1} & \mathbf{O} \\ -\mathbf{a}^T_{B,m+1}B^{-1} & 1 \end{bmatrix}$.

- The new row added to the (main) simplex table will be of the form

$$[-\mathbf{a}_{B,m+1}^T B^{-1}, 1] \begin{bmatrix} B & N & \mathbf{O} \\ \mathbf{a}_{B,m+1}^T & \mathbf{a}_{N,m+1}^T & 1 \end{bmatrix} =$$

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- Then **Dual Simplex** can be used either to obtain the new optimal solution or to conclude that new (P) is infeasible.

Changing entries in the coefficient matrix A:

- **Case 1:** The column corresponding to a **nonbasic variable** say $\tilde{\mathbf{a}}_j$ of the optimal solution is changed to $\tilde{\mathbf{a}}'_j$.

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- If $u_{j,(n+1)} = 0$, then it implies that $\{\tilde{\mathbf{a}}_1, \ldots, \tilde{\mathbf{a}}_{j-1}, \tilde{\mathbf{a}}'_j, \tilde{\mathbf{a}}_{j+1}, \ldots, \tilde{\mathbf{a}}_m\}$ is **LD** and we have to find an initial BFS for the changed problem.

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- If $u_{j,(n+1)} \neq 0$, then pivot on this element and make x_{n+1} enter the basis and x_j leave the basis.

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- If however the new $c_j - z_j$ values are all **nonnegative** and all the RHS entries remain **non negative**, then the table is **optimal** and the corresponding BFS is optimal for the new **(P)**.

- If the new table has all RHS entries nonnegative, but atleast one of the $c_j - z_j$ values is negative then use **Simplex** either to obtain the new optimal solution, or to conclude that the new **(P) does not** have an optimal solution.

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- If all the $c_j - z_j$ values are nonnegative but atleast one of the RHS entries in the simplex table is negative then use **Dual Simplex** to obtain the new optimal solution or to conclude that the new **(P)** has **no** feasible solution.

Artificial Variable Method to find an initial BFS of an LPP: Big- M method:

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- We get an initial BFS to the following problem **LP(M)** given by,
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- An initial BFS is $[\mathbf{O}_{1\times n}, w_1, \ldots, w_m]^T$.
- $\mathbf{x} \in$ **Fea(P)** $\Leftrightarrow$ $[\mathbf{x}^T, \mathbf{O}_{1\times m}]^T \in$ **Fea(LP(M))**.

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  Then $\mathbf{x}_*$ is an **optimal** BFS for **P**.

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  Then **P does not** have a **feasible solution.**
- **Case 2: LP(M) does not** have an **optimal** solution.

- **Case 2a:** In some iteration (or simplex table), there exists a k such that $c_k - z_k < 0$, the corresponding column $B^{-1}\tilde{\mathbf{a}}_k \leq \mathbf{O}$ and $\mathbf{w} = \mathbf{O}$ in the corresponding BFS for **LP(M)**.

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- **Case 2b:** In some iteration (or simplex table), $c_k - z_k = min\{c_j - z_j : c_j - z_j < 0\}$, $B^{-1}\tilde{\mathbf{a}}_k \le \mathbf{O}$ and $\mathbf{w} \neq \mathbf{O}$ in the corresponding BFS for **LP(M)**.

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- Then $\mathbf{d}_1$ is a **direction** of $Fea(\mathbf{P})$ and $\mathbf{c}^T\mathbf{d}_1 < 0$. Hence **P** does **not** have an **optimal** solution but **P** has a **feasible** solution.
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  Hence **P** does **not** have an **optimal** solution but **P** has a **feasible** solution.
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  Then **P** does **not** have a **feasible** solution.
- The above conclusion of **Case 2b** that **P** is **infeasible** may not be true if $c_k - z_k$ is **not** the **most negative** among the $c_j - z_j$ values.

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- **Remark 1:** If at some stage of solving the artificial variables **leave** the basis, then the corresponding BFS is a BFS of **P**.
  Then delete all columns corresponding to the **artificial variables** and continue.

- **Example 1 :** Consider the problem **P**,
  Minimize $x_1 - x_2$
  subject to
  $2x_1 + x_2 \geq 4$
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- By adding variables we get the following problem,
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- The (**Big-M**) is used which provides an initial BFS of **P** for Simplex algorithm.

- Consider the modified problem
  Minimize $x_1 - x_2 + Mw$
  subject to
  $2x_1 + x_2 - s_1 + w = 4$
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  Here w is called the artificial variable and cost M associated with it is **very large**.
- The initial table corresponding to the basic variables w and s_2 is given below.

$c_j - z_j$	$1 - 2M$	$-M - 1$	M	0	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
w	2	1	-1	0	1	4
s_2	1	-1	0	1	0	1

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w	2	1	-1	0	1	4
s_2	1	-1	0	1	0	1

- x_1 will be the **entering variable** and s_2 will be the **leaving variable** for the next table.

-

$c_j - z_j$	0	$-3M$	M	$2M-1$	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
w	0	3	-1	-2	1	2
x_1	1	-1	0	1	0	1

-

$c_j - z_j$	0	$-3M$	M	$2M-1$	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
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w	0	3	-1	-2	1	2
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$c_j - z_j$	0	0	0	-1	M	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
x_2	0	1				$\frac{2}{3}$
x_1	1	0				$1+\frac{2}{3}$

- Continue with the BFS corresponding to the basic vectors x_1 and x_2 and drop the column corresponding to the artificial variable w from all future calculations.

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- **Example 2:** Consider the problem,
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  We consider the corresponding modified problem
- Minimize $x_1 + x_2 + Mw$
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$c_j - z_j$	$1-3M$	$1-5M$	0	M	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
s_1	1	2	1	0	0	2
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$c_j - z_j$	$\frac{1}{2}(1-M)$	0	$\frac{1}{2}(5M-1)$	M	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
x_2	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	1
w	$\frac{1}{2}$	0	$-\frac{5}{2}$	-1	1	10

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$c_j - z_j$	0	$M-1$	$3M-1$	M	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
x_1	1	2	1	0	0	2
w	0	-1	-3	-1	1	9

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$c_j - z_j$	0	$M-1$	$3M-1$	M	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{s}_1$	$B^{-1}\mathbf{s}_2$	$B^{-1}\mathbf{w}$	$B^{-1}\mathbf{b}$
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- Since the above optimal table has an artificial variable taking positive value hence the original problem (without the artificial variable) has no feasible solution.

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The initial table is given by:

$c_j - z_j$	1	1	0	0	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{e}_1$	$B^{-1}\mathbf{e}_2$	$B^{-1}\mathbf{b}$
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s_1	1	2	1	0	2
s_2	-3	-5	0	1	-15

Here s_2 is the leaving variable and x_2 the entering variable.

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$c_j - z_j$	$\frac{2}{5}$	0	0	$\frac{1}{5}$	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{e}_1$	$B^{-1}\mathbf{e}_2$	$B^{-1}\mathbf{b}$
s_1	$-\frac{1}{5}$	0	1	$\frac{2}{5}$	-4
x_2	$\frac{3}{5}$	1	0	$-\frac{1}{5}$	3

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$c_j - z_j$	$\frac{2}{5}$	0	0	$\frac{1}{5}$	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{e}_1$	$B^{-1}\mathbf{e}_2$	$B^{-1}\mathbf{b}$
s_1	$-\frac{1}{5}$	0	1	$\frac{2}{5}$	-4
x_2	$\frac{3}{5}$	1	0	$-\frac{1}{5}$	3

- Here s_1 is the leaving variable and x_1 the entering variable. The next table is given by:

$c_j - z_j$	0	0			
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{e}_1$	$B^{-1}\mathbf{e}_2$	$B^{-1}\mathbf{b}$
x_1	1	0	-5	-2	20
x_2	0	1	3	1	-9

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$c_j - z_j$	$\frac{2}{5}$	0	0	$\frac{1}{5}$	
	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{e}_1$	$B^{-1}\mathbf{e}_2$	$B^{-1}\mathbf{b}$
s_1	$-\frac{1}{5}$	0	1	$\frac{2}{5}$	-4
x_2	$\frac{3}{5}$	1	0	$-\frac{1}{5}$	3

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	$B^{-1}\tilde{\mathbf{a}}_1$	$B^{-1}\tilde{\mathbf{a}}_2$	$B^{-1}\mathbf{e}_1$	$B^{-1}\mathbf{e}_2$	$B^{-1}\mathbf{b}$
x_1	1	0	-5	-2	20
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Hence clearly the problem is infeasible.