Lecture-1 : A Gentle Introduction to the Linear Electric Circuits

Fundamental Quantities

- The voltage(V) between two ends of a path is the total energy required to move a unit electric charge along that path.
  - Measured in volt and denoted as V
- The current(I) is charge flow through a material in a unit time.
  - Measured in ampere and denoted as A

Energy, Voltage and Current

- 1 joule (J) Work is done in moving a 1 coulomb (C) charge from a point A to point B to have potential difference of 1 V.
  - 1 joule= 1 coulomb $\times$1 volt $\rightarrow$ 1 volt $= \frac{1\text{joule}}{1\text{coulomb}}$
  - 1 ampere $= \frac{1\text{coulomb}}{1\text{second}}$
  - 1 watt(W) $= \frac{1\text{joule}}{1\text{second}} \rightarrow \frac{1\text{coulomb}}{1\text{second}} \times 1\text{volt} \rightarrow 1W = 1V \times 1A$
- When Power is constant $VI =$ Constant.
  - But the relation between V and I is not linear.

Searching for Linearity

- Ohm's Law(An experimental law)- The current I flowing in a circuit is directly proportional to the applied potential difference V and inversely proportional to the resistance R in the circuit.[Can be justified using Maxwell's equation]
  - The unit of resistance is ohm denoted as Ω
- From Ohm's law $V = IR$
  - A linear relation between voltage and current.

Superposition Property

- Two variables x and $y = f(x)$ satisfy the property of superposition when the relation between them is homogeneous and additive.
- $y = f(x)$
  - is homogeneous when $f(ax) = af(x) = ay$ where a is any constant.
  - is additive when $f(x_1 + x_2) = f(x_1) + f(x_2) = y_1 + y_2$
- Example

$$y = kx$$

$$y(t) = k\frac{dx(t)}{dt}$$

$$y(t) = k\int_{-\infty}^{t} x(t)dt$$

Superposition Property Contd.

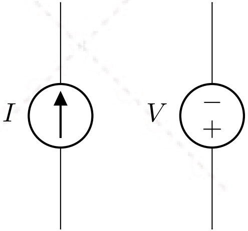
- $y = mx + c$ (where m and c are constants) is does not hold superposition property since homoginity and additivity cannot be applied.

Superposition on V and I

- Using Ohm's law superposition property can be ascertained.
    - homoginity $V_1 = I_1 R$ and $V_2 = I_2 R$
    - additivity $V_1 + V_2 = (I_1 + I_2)R$

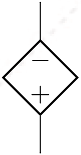
Independent Sources

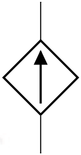
- Independent voltage sources- Maintains a constant voltage which is not affected by any other quantities.
  - An independent voltage source can never be shorted.
- Independent current sources- Maintains a constant current which is not affected by any other quantities.
  - An independent current source can never be opened.



Dependent Sources

- Dependent voltage source- The voltage across the source depends on other variables.
  - Represented as $V(V_1, V_2, \ldots, I_1, I_2, \ldots)$
- Dependent current source- The current through the source depends on other variables.
  - Represented as $I(V_1, V_2, \ldots, I_1, I_2, \ldots)$

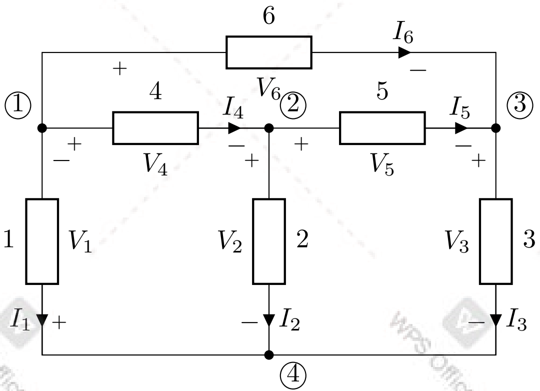
$V(V_1, V_2, \ldots, I_1, I_2, \ldots)$ 

 $I(V_1, V_2, \ldots, I_1, I_2, \ldots)$

Lecture-1.2 : Circuit Analysis

Circuit

- Circuit- Collection of devices such as sources and resistors(for the time being) in which terminals are connected together by conducting wires.
- These wires converge in Nodes denoted as n
- The devices are called Branches of the circuit denoted as b
- In the given circuit $n = 4$ and $b = 6$.



Circuit Analysis

- The circuit analysis problem is to find all the branch voltages and branch currents of the circuit.
- Suppose there are b branches then there are $2b$ variables.
  - b branch voltages and b branch currents
- Required $2b$ linearly independent equations
- The branch component value contributes up to b equations through Ohm's law
- The rest b equations are from the interconnection details of the circuit topological constrains
  - They depend only on how the elements are interconnected and will not depend on the nature or parameter value of the branches.

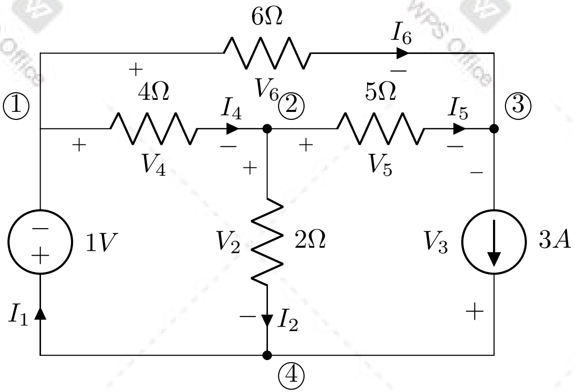
Convention of Direction and Polarity

- Current direction indicates the direction of flow of positive charge
- Voltage polarity indicates the relative potential between two points assigned $+$ to a higher potential point and $-$assigned to a lower potential point.

b Equations From Topological Constrains

- Kirchoff's Voltage Law (KVL): The algebraic sum of all voltages between successive nodes in a closed path in the circuit is equal to zero.
  - The physical basis of KVL is Amperes law which states that the line integral of the electric field along a closed path is zero, provided the loop does not enclose any changing magnetic field.
- Kirchoff's Current Law (KCL): The algebraic sum of the currents in all branches which converge to a common node is equal to zero.
  - The physical basis of KCL is law of charge conservation
- Using KVL $b - (n - 1)$ linearly independent equations can be formed
- Using KCL $n - 1$ linearly independent equations can be formed
- Combining both a set of b linearly independent equations

Sample circuit



b Equations From Topological Constrains Contd.

- Out of 12 variables 2 values are known.
  - The branch voltage $V_1 = 3V$ and branch current $I_5 = 2A$
- The values of the remaining 10 variables are yet to be calculated.
- Invoking Ohm's law
  - $2I_2 = V_2, \quad 4I_4 = V_4, \quad 5I_5 = V_5, \quad 6I_6 = V_6$
- Using KVL
  - Around the loop 1-2-4-1 $\quad V_4 + V_2 + 1 = 0$
  - Around the loop 2-3-4-2 $\quad V_5 - V_3 - V_2 = 0$
  - Around the loop 1-2-3-1 $\quad -V_4 - V_5 + V_6 = 0$
- Using KCL
  - On the node 1 $\quad I_1 - I_4 - I_6 = 0$
  - On the node 2 $\quad I_4 - I_5 - I_2 = 0$
  - On the node 3 $\quad I_6 + I_5 - 3 = 0$

Conclusion of Lecture 1

- The fundamental quantities V and I are introduced.
- A linear relation between them is derived.
- Using Ohm's law superposition property of V and I is established.
- The notions associated to independent and dependent sources are explained.
- The basic steps of circuits analysis is explained.
- Through topological constrains KVL and KCL are introduced
- The requirement of linear independent equations are discussed
- Using an example the construction of these linear independent equations is explained

Lecture-2.1 : The Systematic Procedures in Circuit Analysis-1
Nodal Analysis with Current Source

The Nodal Analysis

- Nodal Analysis uses the KVL and element equations to eliminate voltage variables.
- Reduces the number of relevant variables to $n-1$ node voltage variables.
- Using the KCL these node voltage variables are solved
- Conductance G is the inverse of resistance and unite is S
  - $G = \frac{1}{R} = \frac{I}{V}$

An Example for Nodal Analysis

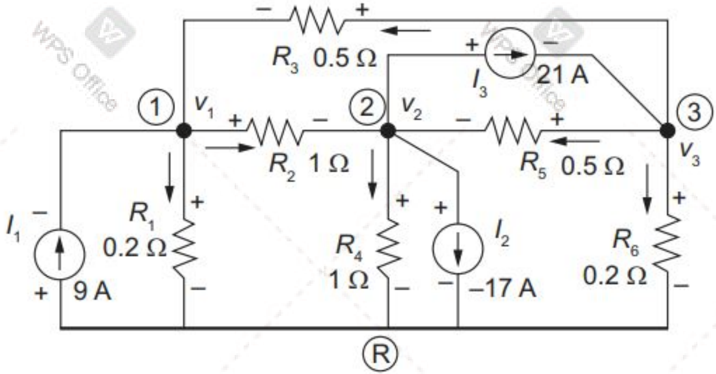


Figure 1: Example circuit with Resistors and Independent Current sources for Nodal Analysis

An Example for Nodal Analysis Contd.

- In this $b = 9, n = 4$.
- Independent current sources are I_1, I_2 and I_3.
- The node that has the largest number of elements connected to it is taken as the reference node and denoted as R.
- v_1, v_2 and v_3 are the node voltages with reference to R

Branch Voltages in Terms of Node Voltages

$$V_{R_1} = v_1 \quad V_{R_2} = v_1 - v_2 \quad V_{R_3} = v_3 - v_1$$
$$V_{R_4} = v_2 \quad V_{R_5} = v_3 - v_2 \quad V_{R_6} = v_3$$
$$V_{I_1} = -v_1 \quad V_{I_2} = v_2 \quad V_{I_3} = v_2 - v_3$$

$$I_{R_1} + I_{R_2} - I_{R_3} - I_1 = 0$$

- At node 1 $\dfrac{V_{R_1}}{R_1} + \dfrac{V_{R_2}}{R_2} - \dfrac{V_{R_3}}{R_3} - I_1 = 0$

  $$G_1 v_1 + G_2(v_1 - v_2) + G_3(v_1 - v_3) = I_1$$
- At node 2 $G_2(v_2 - v_1) + G_5(v_2 - v_3) + G_4 v_2 = -I_2 - I_3$
- At node 3 $G_3(v_3 - v_1) + G_5(v_3 - v_2) + G_6 v_3 = I_3$
- $G_1 = 5S \quad G_2 = 1S \quad G_3 = 2S$
  $G_4 = 1S \quad G_5 = 2S \quad G_6 = 5S$

Branch Voltages in Terms of Node Voltages Contd.

$$G_1v_1 + G_2(v_1 - v_2) + G_3(v_1 - v_3) = I_1$$
$$G_2(v_2 - v_1) + G_5(v_2 - v_3) + G_4v_2 = -I_2 - I_3$$
$$G_3(v_3 - v_1) + G_5(v_3 - v_2) + G_6v_3 = I_3$$

$$(G_1 + G_2 + G_3)v_1 - G_2v_2 - G_3v_3 = I_1$$
$$-G_2v_1 + (G_2 + G_3 + G_4)v_2 - G_5v_3 = -I_2 - I_3$$
$$-G_3v_1 - G_5v_2 + (G_3 + G_5 + G_6)v_3 = I_3$$

$$\begin{bmatrix} (G_1+G_2+G_3) & -G_2 & -G_3 \\ -G_2 & (G_2+G_4+G_5) & -G_5 \\ -G_3 & -G_5 & (G_3+G_5+G_6) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\boldsymbol{YV = CU}$$

$$\begin{bmatrix} 8 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ -17 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Node Voltage to Other Variables

$$
\boxed{
\begin{array}{ccc}
V_{R_1=v_1} & V_{R_2} = v_1 - v_2 & V_{R_3} = v_3 - v_1 \\
V_{R_4} = v_2 & V_{R_5} = v_3 - v_2 & V_{R_6} = v_3 \\
V_{I_1} = -v_1 & V_{I_2} = v_2 & V_{I_3} = v_2 - v_3 \\
\multicolumn{3}{c}{\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}^T}
\end{array}
}
$$

Steps Involved in Nodal Analysis

- When there are only resistors and independent current sources $[\boldsymbol{Y}]_{n-1\times n-1}[\boldsymbol{V}]_{n-1\times 1} = [\boldsymbol{C}]_{n-1\times n_{cs}}[\boldsymbol{U}]_{n_{cs}\times 1}$ where n_{cs} is the number of independent current sources.
- y_{ii} = sum of all conductances connected at ith node
- y_{ij} = negative of sum of all conductances connected between ith node and jth node
- $c_{ij} = \begin{cases} 0 & \text{when } j\text{th current source is not connected at } i\text{th node} \\ 1 & \text{when } j\text{th current source is delivering current to } i\text{th node} \\ -1 & \text{when } j\text{th current source is drawing current from } i\text{th node} \end{cases}$

Nodal Analysis With Dependent Current Source

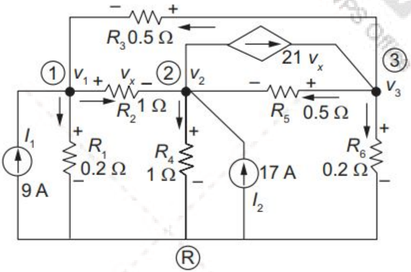


Figure 2: Example circuit of Nodal Analysis With Dependent Current Source

Nodal Analysis With Dependent Source Contd.

- In this $b = 9, n = 4$.
- Independent current sources are I_1 andI_2 and dependent current source $21V_x$.
- v_1, v_2 and v_3 are the node voltages with refernce to R
- $21V_x = 21V_{R_2} = 21(v_1 - v_2)$

  Node-1: $G_1v_1 + G_2(v_1 - v_2) + G_3(v_1 - v_3) = I_1$
- Node-2: $G_4v_2 + G_2(v_2 - v_1) + G_5(v_2 - v_3) + 21(v_1 - v_2) = I_2$

  Node-3: $G_6v_3 + G_3(v_3 - v_1) + G_5(v_3 - v_2) - 21(v_2 - v_1) = 0$
- $$\begin{bmatrix} (G_1 + G_2 + G_3) & -G_2 & -G_3 \\ -G_2 + 21 & (G_2 + G_4 + G_5 - 21) & -G_5 \\ -G_3 - 21 & -G_5 + 21 & (G_3 + G_5 + G_6) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$
- $$\begin{bmatrix} 8 & -1 & -2 \\ 20 & -17 & -2 \\ -23 & 19 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 9 \\ 17 \end{bmatrix} = \begin{bmatrix} 9 \\ 17 \\ 0 \end{bmatrix}$$

Lecture-2 : The Systematic Procedures in Circuit Analysis-1
Nodal Analysis with Voltage Sources

The Nodal Analysis with Independent Voltage Sources

- An independent voltage source can appear in three positions in a circuit.
  - Node to the reference node.
  - Between two nodes at which more than two elements are incident.
  - Connected in series with some resistor.

Independent Voltage Source Between Node and Node-R

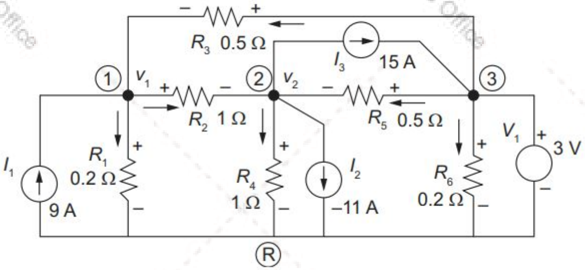


Figure 1: Example circuit of Independent Voltage Source.

Independent Voltage Source Between Node and Node-R Contd.

- $$G_1 v_1 + G_2(v_1 - v_2) + G_3(v_1 - V_1) = I_1$$
  $$G_4 v_2 + G_2(v_2 - v_1) + G_5(v_2 - V_1) = -I_2 - I_3$$
- In the matrix form
  $$\begin{bmatrix} G_1 + G_2 + G_3 & -G_2 \\ -G_2 & G_2 + G_4 + G_5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 + G_3 V_1 \\ -I_2 - I_3 + G_5 V_1 \end{bmatrix}$$
- $$= \begin{bmatrix} 1 & 0 & 0 & G_3 \\ 0 & -1 & -1 & G_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \end{bmatrix}$$
- $$YV = CU$$
  - $$[Y]_{(n-1-n_v)\times(n-1-n_v)}[V]_{(n-1-n_v)\times 1} = [I]_{(n-1-n_v)\times(n_i+n_v)}[U]_{(n_i+n_v)\times 1}$$

Independent Voltage Source Between Node and Node-R Contd.

- Substituting the values for conductances and source functions
  - $\begin{bmatrix} 8 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \end{bmatrix}$
- Solving for v_1 and v_2 to get $v_1 = 2\text{V}$, $v_2 = 1\text{V}$ and $v_3 = V_1 = 3\text{V}$

Independent Voltage Source Between Two Nodes

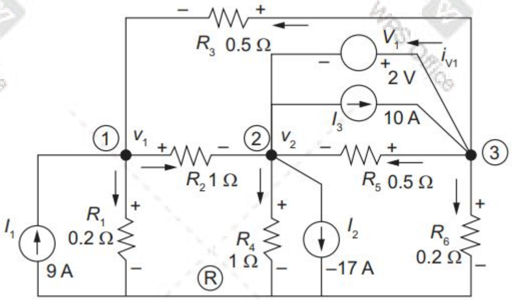


Figure 2: Example circuit of Independent Voltage Source Between Two Nodes.

Independent Voltage Source Between Two Nodes Contd.

- $v_3 = v_2 + V_1$[You can also take $v_2 = v_3 - V_1$]
- The resulting KCL equations are
-

$$
\begin{aligned}
\text{Node 1 :} & G_1 v_1 + G_2(v_1 - v_2) + G_3(v_1 - v_2 - V_1) = I_1 && (1)\\
\text{Node 2 :} & G_4 v_2 + G_2(v_2 - v_1) - G_5 V_1 - I_{V_1} = -I_2 - I_3 && (2)\\
\text{Node 3 :} & G_6(v_2 + V_1) + G_3(v_2 + V_1 - v_1) + G_5 V_1 + I_{V_1} = I_3 && (3)
\end{aligned}
$$

- Doing away with I_{V_1}
  - (2)+(3) gives

$$G_4 v_2 + G_2(v_2 - v_1) + G_6(v_2 + V_1) + G_3(v_2 + V_1 - v_1) = -I_2 \quad (4)$$

- (1) and (4) in the matrix form results in

Independent Voltage Source Between Two Nodes Contd.

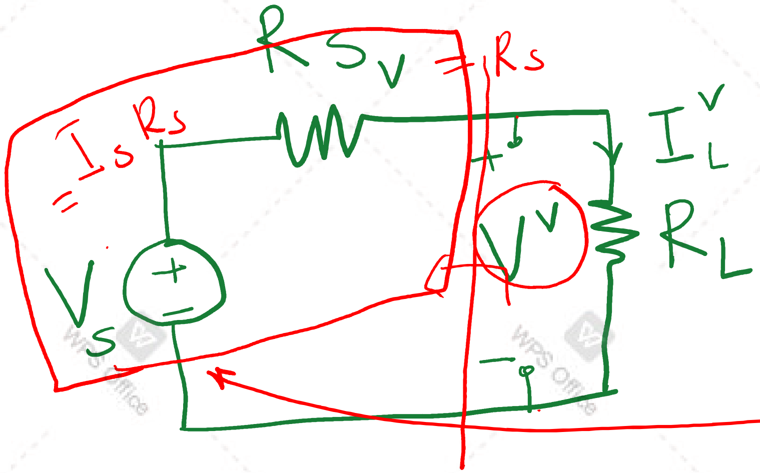
- Expressing in the Matrix form
$$\begin{bmatrix} G_1+G_2+G_3 & -(G_2+G_3) \\ -(G_2+G_3) & G_2+G_4+G_5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1+G_3V_1 \\ -(G_6+G_3)V_1-I_2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 & G_3 \\ 0 & -1 & 0 & -(G_6+G_3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ V_1 \end{bmatrix}$$
- $$YV = CU$$
- $\begin{bmatrix} 8 & -3 \\ -3 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 13 \\ 3 \end{bmatrix}$ to get $v_1 = 2\text{V}$ and $v_2 = 1\text{V}$.
- $$[Y]_{(n-1-n_v)\times(n-1-n_v)}[V]_{(n-1-n_v)\times 1} = [I]_{(n-1-n_v)\times(n_i+n_v)}[U]_{(n_i+n_v)\times 1}$$

Conclusions From Lecture 2

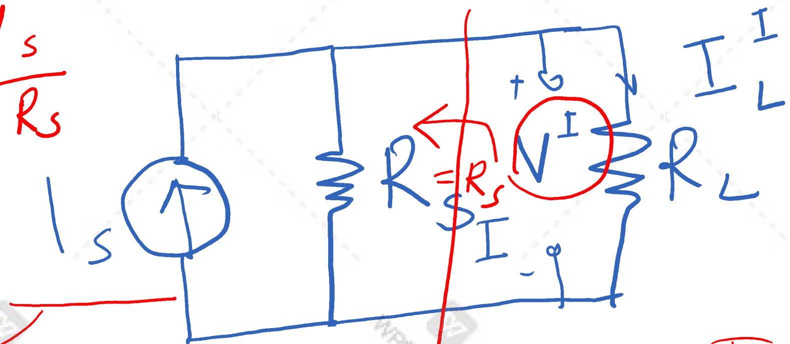
- In the first session, the nodal analysis with both independent and dependent current sources is discussed.
- In this session the nodal analysis in the presence of independent voltage sources along with the current sources is discussed.
- An independent voltage source can appear in three positions in a circuit. The nodal analysis for two configurations are discussed in this session.The remaining one will be done later.
- The core idea of the nodal analysis is
  - finding branch voltages in terms of node voltages
  - formulating KCL equations at each node other than reference node
  - expressing these equations in $\boldsymbol{YV} = \boldsymbol{CU}$ canonical form to get the node voltage vector as $\boldsymbol{V} = \boldsymbol{Y}^{-1}\boldsymbol{CU}$
  - From the node voltage, the values of the branch voltages are found out
  - Using Ohm's these branch voltages along with Ohm's law the branch currents can be found out

Lecture-3.1 The Systematic Procedures in Circuit Analysis-Nodal Analysis with Voltage Sources Contd.

Source Transformation Theorem



$I_s = \frac{V_s}{R_s}$



$I_L^I = \frac{I_s \times R_{S_I}}{R_{S_I} + R_L}$

$V^I = I_L^I R_L$

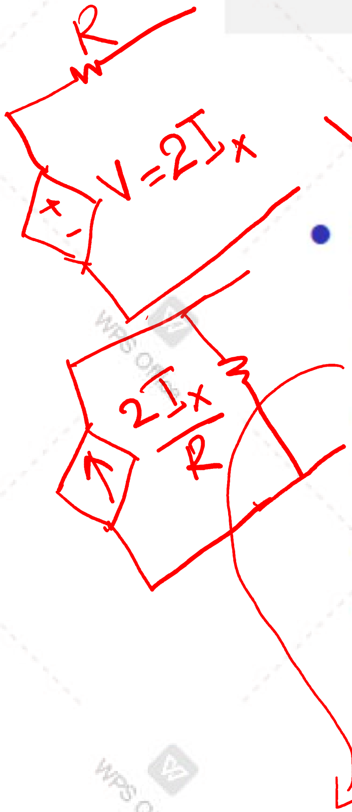
$$\boxed{V^V = \frac{V_S R_L}{R_L + R_{S_V}}} \quad (1)$$

$V^V \mathrel{!}= V^I$

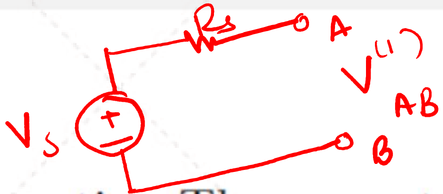
$$V^I = \frac{I_S R_{S_I} R_L}{R_L + R_{S_I}} \quad (2)$$

- Invoking conditions
  - $R_{S_V} = R_{S_I} = R_s$
  - $V_S = I_S R_S$

Source Transformation Theorem Contd.



$V^{(1)}_{AB} = V^{(2)}_{AB}$



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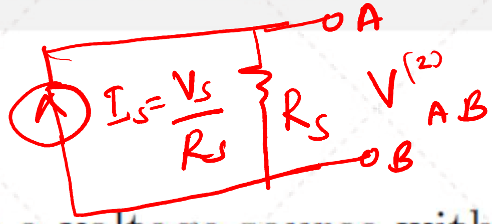


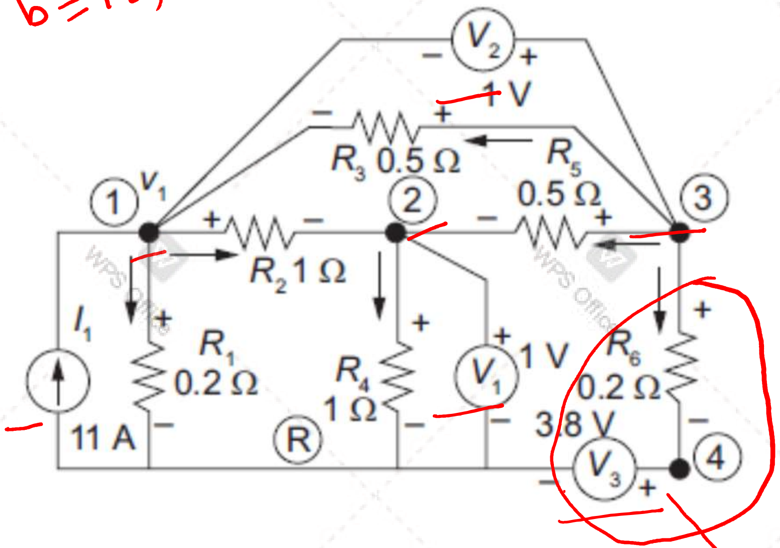
I_s, R_s

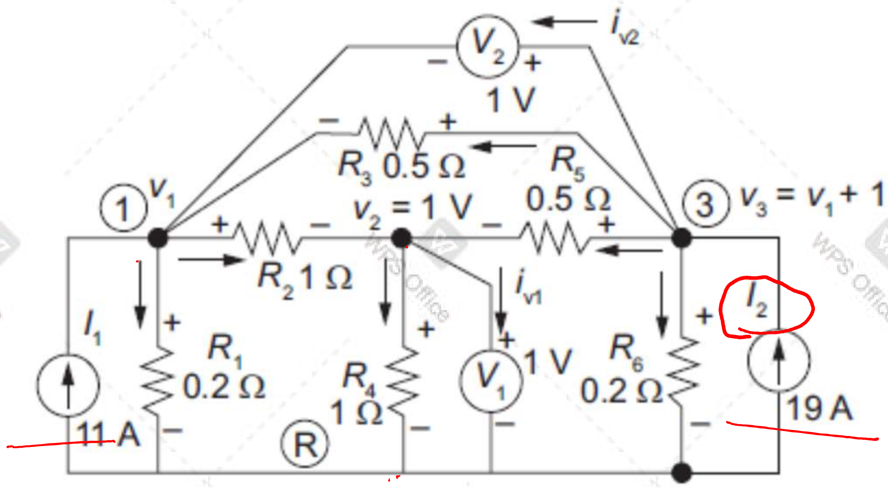
$V_s = I_s R_s$

- Source Transformation Theorem states that a voltage source with source function V_s in series with a resistance R_s can be replaced by a current source with source function $I_s = \frac{V_s}{R_s}$ in parallel with a resistance R_s without affecting any voltage/current/power variable external to the source. The direction of current source is such that current flows out of the terminal at which the positive of the voltage source is presently connected and vice versa.
  - [Can be proved using Ohm's law and Kirchoff's laws. A rigorous proof can be found in "Basic Circuit Theory", Charles A. Desoer, pages 654-658]
- This theorem is equally valid in the case of dependent sources. Hence, Source Transformation Theorem is applicable to the dependent sources as well.

Voltage Source in Series with Resistor

$b = 10,\ n = 5$





$$I_2 = \frac{V_3}{R_6} = \frac{3.8\text{V}}{0.2\,\Omega} = 19\text{ A}$$

Voltage Source in Series with Resistor Contd.

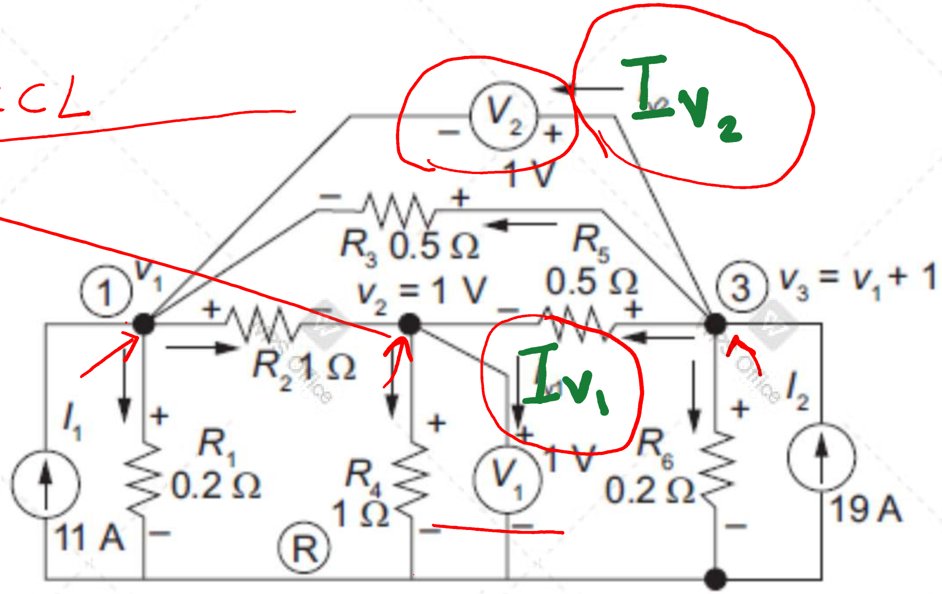
b = 10, n = 4

n = 4 , 4 − 1 = 3 KCL

v1, v2 and v3

- $v_2 = V_1$
- $v_3 = v_1 + V_2$

$(v_1 = v_3 - V_2)$



"An independent voltage source imposes a constraint on the node voltage variables and reduces their number by one."

v1, | V2 and ∀ | V1

- KCL at node-1

$$G_1 v_1 + G_2(v_1 - V_1) - G_3 V_2 - I_{V_2} = I_1 \qquad (3)$$

- KCL at node-2

$$G_2(v_2 - v_1) + G_4 v_2 + G_5(v_2 - v_3) + I_{V_1} = 0$$

- KCL at node-3

$$G_6(v_1 + V_2) + G_3 V_2 + G_5(v_1 + V_2 - V_1) + I_{V_2} = I_2 \qquad (4)$$

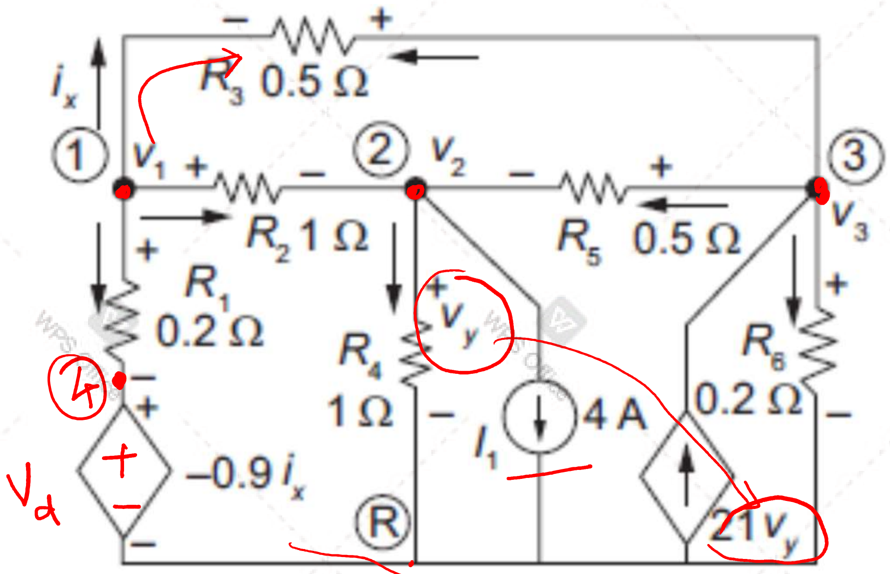
Voltage Source in Series with Resistor Contd.

$\backslash v_2$

- (3)+(4)
- $G_1 v_1 + G_2(v_1 - V_1) + G_6(v_1 + V_2) + G_5(v_1 + V_2 - V_1) = I_1 + I_2 = I_1 + G_6 V_3$
- $[G_1 + G_2 + G_5 + G_6]\,[v_1] = \begin{bmatrix} 1 & (G_2 + G_5) & -(G_5 + G_6) & G_6 \end{bmatrix} \begin{bmatrix} I_1 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix}$
- $\boldsymbol{YV = CU}$ $\quad [Y]_{n-1-n_v \times n-1-n_v} \, [V]_{n-1-n_v \times 1} = [C]_{n-1-n_v \times (n_I+n_v)} \, [U]_{(n_I+n_v) \times 1}$

$v_1 = 2V \quad \boxed{v_2 = 1V} \quad v_3 = 3V \quad I_{V_1} = 4A \quad I_{V_2} = -2A \quad I_{V_3} = -4A$

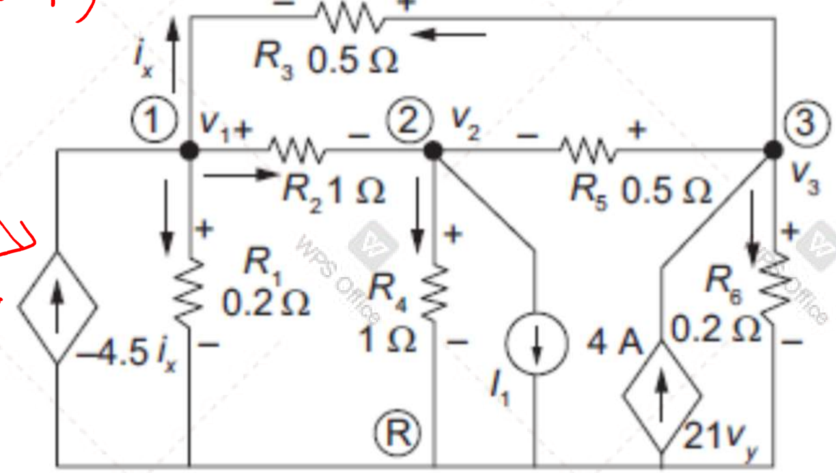
Nodal Analysis with Dependent Voltage Sources



$b = 9$

$$\frac{V_d}{R_1} = \frac{-0.9\, i_x}{0.2} = -4.5\, i_x$$

$b = 9,\ n = 4$

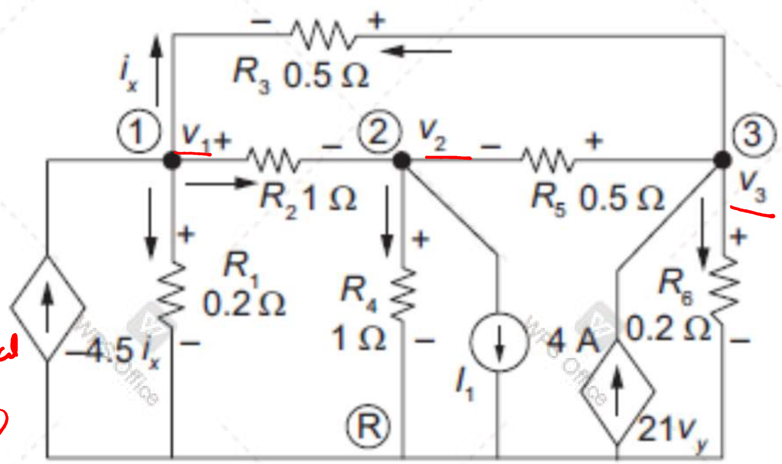


Nodal Analysis with Dependent Voltage Sources Contd.

$b = 9$, $n = 4 \Rightarrow v_1, v_2, v_3$

$3 \; (n-1)$ KCL

- At node-1
  - $G_1 v_1 + G_2(v_1 - v_2) + G_3(v_1 - v_3) + k_1 G_1 G_3 (v_1 - v_3) = 0$
- At node-2
  - $G_4 v_2 + G_2(v_2 - v_1) + G_5(v_2 - v_3) = -I_1$
- At node-3
  - $G_6 v_3 + G_3(v_3 - v_1) + G_5(v_3 - v_2) - k_2 v_2 = 0$



1 Independent cur.

2 Independent vol (a), (b), (c)

3 Dependent cur.

4 Depend vol.

$$\begin{bmatrix} G_1 + G_2 + G_3 + k_1 G_1 G_3 & -G_2 & -(G_3 + K_1 G_1 G_3) \\ -G_2 & G_2 + G_4 + G_5 & -G_5 \\ -G_3 & -(G_5 + k_2) & G_3 + G_5 + G_6 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \; [4]$$

Here $k_1 = -0.9$
$k_2 = 21$

$$YV = CU$$

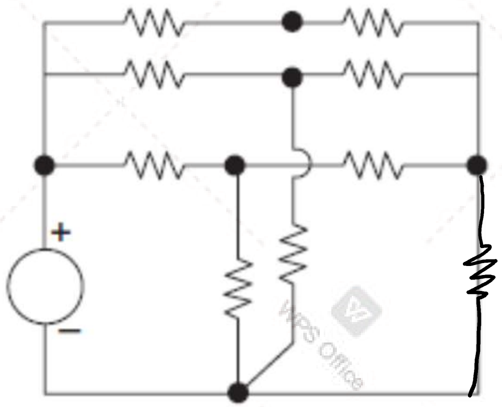
$$[Y]_{n-1 \times n-1} [V]_{n-1 \times 1} = [C]_{n-1 \times n_I} [U]_{n_I \times 1}$$

Lecture-3.2 : The Systematic Procedures in Circuit Analysis-II

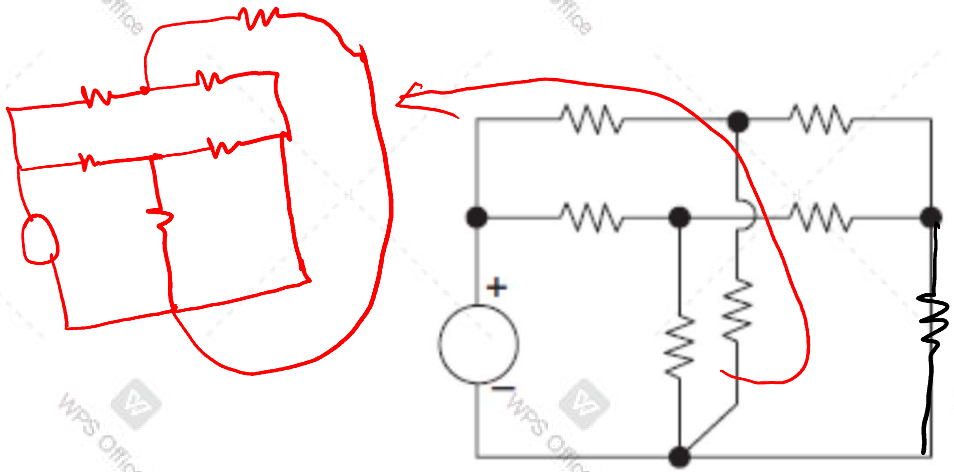
Mesh Analysis

Planar Network

- A planar network is one that can be drawn on a plane without any component crossing over another.



non-planar network.

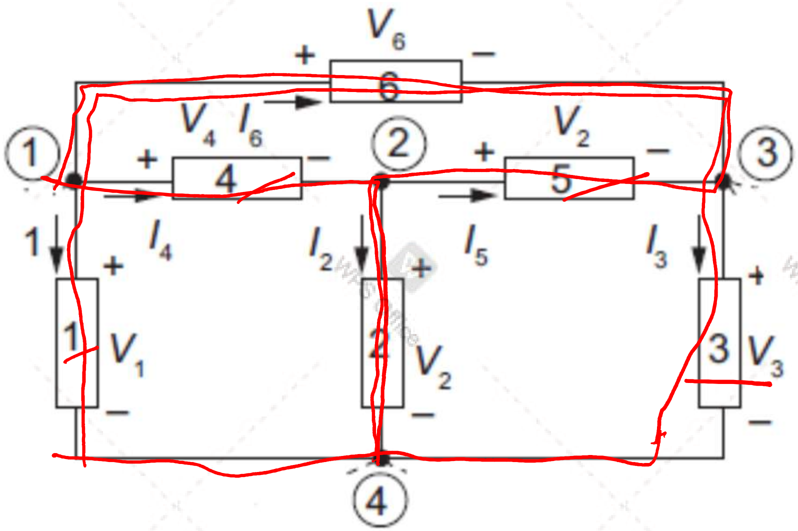


Planar network

Distinction between Loop and Mesh

- Loop- The closed paths comprising of elements and nodes are called loops.
- Mesh- A loop in a planar circuit that does not contain any other loop within it.

$b=6, n=4, l=7, m=3$



L1 — 142

L2 — 253

L3 — 456

L4 — 163

L5 — 1453

L6 — 2463

L7 — 1652

Interesting Facts about Circuit

- A circuit with n nodes, b elements and l loops will have
  - n KCL equations, l KVL equations and b element equations involving $2b$ element variables.
  - Only $n-1$ KCL equations of n will be linearly independent.
  - Only $b-n+1$ KVL equations of l such equations will be linearly independent.
- Any set of KCL equations written for $n-1$ nodes of the circuit will form a linearly independent set.
- However, an arbitrary of $b-n+1$ KVL equations drawn from the set of l, need not form a linearly independent set.

Mesh Analysis

- In a planar connected network containing b-elements and n-nodes, there will be exactly $b - n + 1$ meshes.
- The KVL equations for $b - n + 1$ meshes in a planar circuit will form a linearly independent set of equations.
  - [The proof for the above two statements can be seen in "Basic Circuit Theory", Desoer and Kuh, page numbers, 444-461]

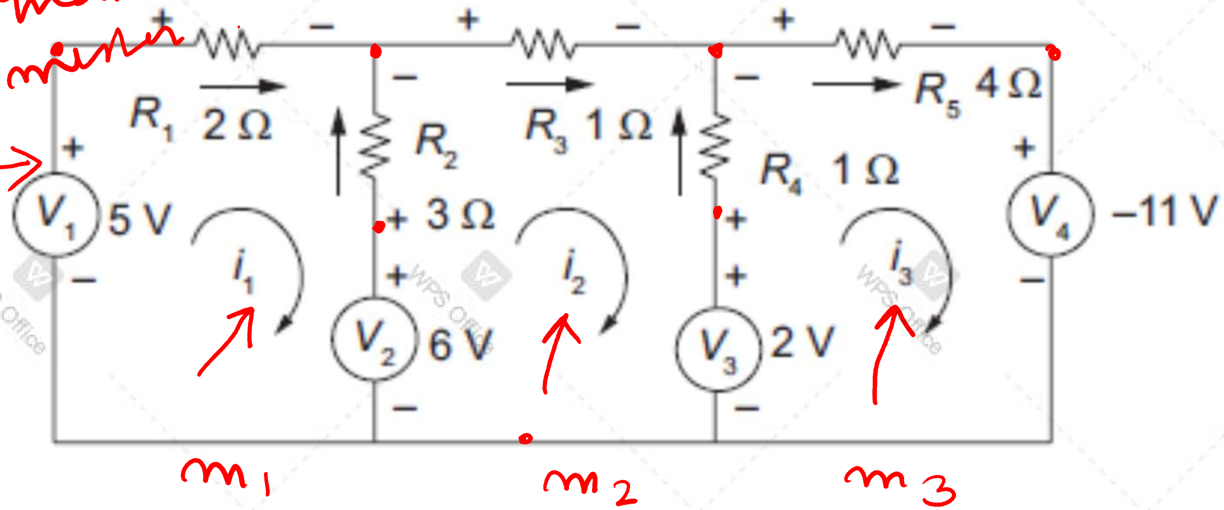
Mesh Analysis Contd.

- The branch currents will be expressed in terms of $\boxed{b-n+1}$ mesh currents using $n-1$ KCL equations
- Using $b-n+1$ KVL equations these $b-n+1$ mesh currents are solved

$i_1, i_2, \dots i_{b-n+1}$ → branch current will be found out → branch voltages will be computed

Mesh Analysis with Independent Voltage Sources

3 LI KVL equations
Ones these 3 meshes



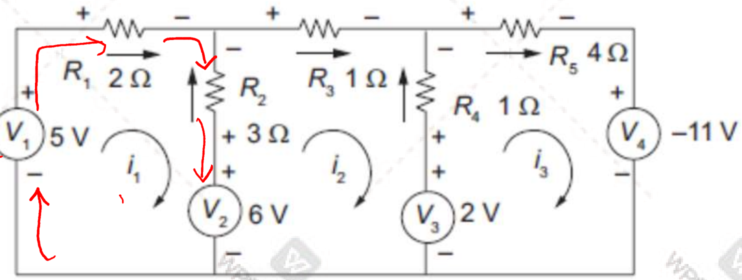
- $b = 9, n = 7$ $\quad 9 - 7 + 1 = 3$
- There are four independent voltage sources V_1, V_2, V_3 and V_4.
- The mesh currents are $\boxed{i_1, i_2 \text{ and } i_3}$
- $$\boxed{\begin{array}{lll} I_{V_1} = -i_1 & I_{R_1} = i_1 & I_{R_2} = i_2 - i_1 \\ I_{V_2} = i_2 - i_1 & I_{R_3} = i_2 & I_{R_4} = i_3 - i_2 \\ I_{V_3} = i_3 - i_2 & I_{R_5} = i_3 & I_{V_4} = i_3 \end{array}}$$

using $(n-1)$ KCL
6 KCL equations.

Mesh Analysis with Independent Voltage Sources Contd.

- $I_{V_1} = -i_1 \quad I_{R_1} = i_1 \quad I_{R_2} = i_2 - i_1$
  $I_{V_2} = i_2 - i_1 \quad I_{R_3} = i_2 \quad I_{R_4} = i_3 - i_2$
  $I_{V_3} = i_3 - i_2 \quad I_{R_5} = i_3 \quad I_{V_4} = i_3$

ohm's law branch voltages



- KVL at mesh-1
  - $-V_1 + R_1 i_1 - R_2(i_2 - i_1) + V_2 = 0 \quad \Rightarrow \quad (R_1 + R_2)i_1 - R_2 i_2 = V_1 - V_2$
- KVL at mesh-2
  - $R_2(i_2 - i_1) + R_3 i_2 - R_4(i_3 - i_2) = V_2 - V_3 \quad \Rightarrow \quad -R_2 i_1 + (R_2 + R_3 + R_4)i_2 - R_4 i_3 = V_2 - V_3$
- KVL at mesh-3
  - $R_4(i_3 - i_2) + R_5 i_3 = V_3 - V_4 \quad \Rightarrow \quad -R_4 i_2 + (R_4 + R_5)i_3 = V_3 - V_4$

Mesh Analysis with Independent Voltage Sources Contd.

- $$\begin{aligned} (R_1 + R_2)i_1 - R_2 i_2 &= V_1 - V_2 \\ -R_2 i_1 + (R_2 + R_3 + R_4)i_2 - R_4 i_3 &= V_2 - V_3 \\ -R_4 i_2 + (R_4 + R_5) i_3 &= V_3 - V_4 \end{aligned}$$

$I = Z^{-1} DU$

- $$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -R_2 & R_2 + R_3 + R_4 & -R_4 \\ 0 & -R_4 & (R_4 + R_5) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$
- $\boxed{ZI = DU}$
- $$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$Z :=$ Mesh Resistance matrix

$I :=$ Mesh current vector

$D :=$ Input matrix

$U :=$ Input source vector.

Steps of Mesh Analysis with Independent Voltage Sources

- $$[\boldsymbol{Z}]_{b-n+1\times b-n+1}[\boldsymbol{I}]_{b-n+1\times 1} = [\boldsymbol{D}]_{b-n+1\times n_v}[\boldsymbol{U}]_{n_v\times 1}$$
- z_{ii} = sum of all resistances connected at ith mesh
- z_{ij} = negative of sum of all resistances connected between ith mesh and jth mesh
- $$d_{ij} = \begin{cases} 0 & \text{when } j\text{th voltage source is not present at } i\text{th mesh} \\ 1 & \text{when } j\text{th voltage source provides a voltage rise in } i\text{th mesh} \\ -1 & \text{when } j\text{th voltage source provides a voltage drop in } i\text{th mesh} \end{cases}$$

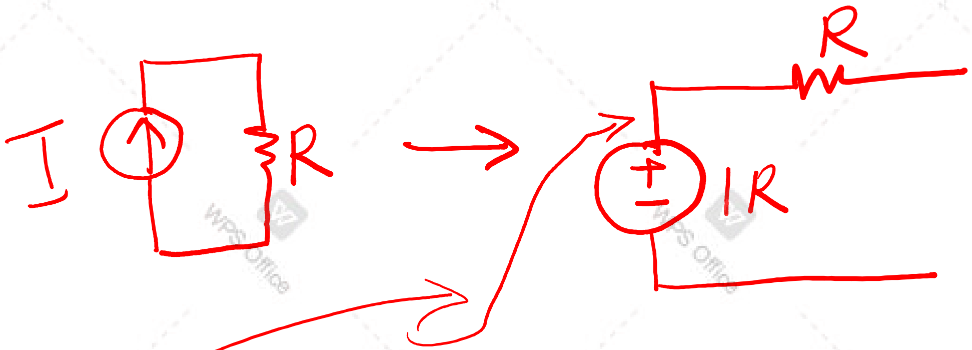
Conclusions of Lecture 3

- In the first part of the discussion source transformation theorem is introduced
- Using source transformation theorem the nodal analysis for the voltage sorce with series resistor is completed
- In the second part the notion of mesh in a circuit is introduced
- The mesh analysis involving independent voltage sources is completed

Lecture-4.1 The Systematic Procedures in Circuit Analysis-2

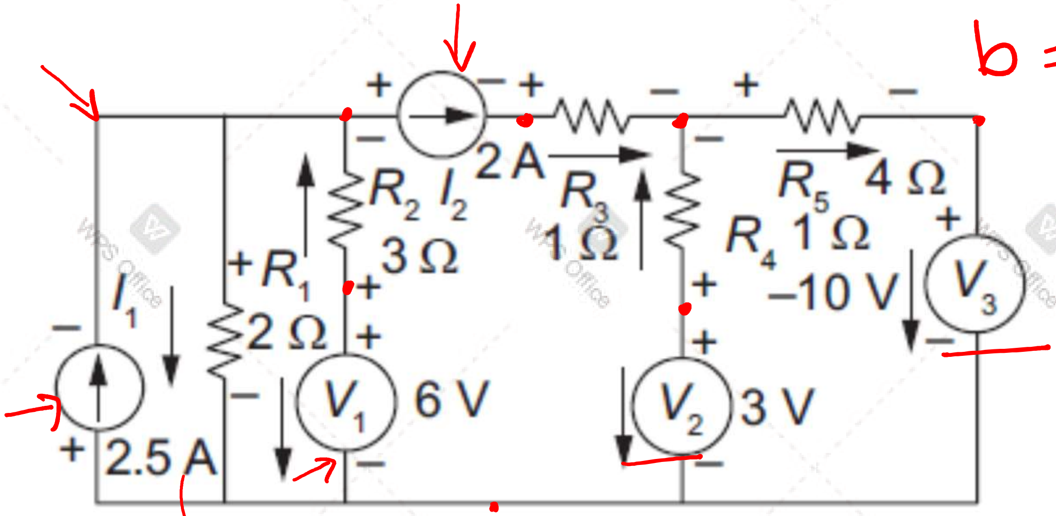
Mesh Analysis with Current Sources

Mesh Analysis with Independent Current Sources



- An independent current source can appear in three positions in a circuit
  - parallel with a resistor
  - series with another element
  - shared between two meshes

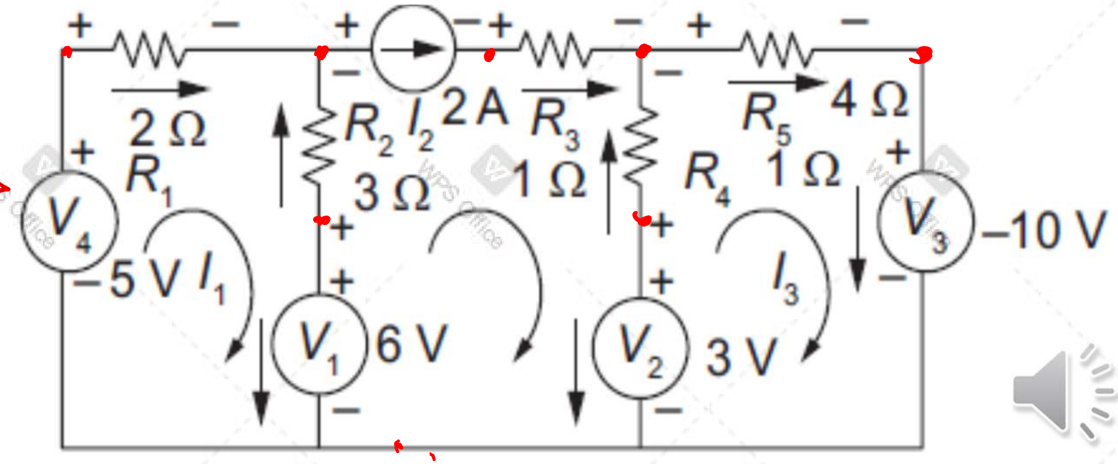
Mesh Analysis- Independent Current Source in Series with Another Element



$b = 10, n = 7, m = 4$

$b = 10, n = 8, m = 3$

$V_4 = I_1 R_1$

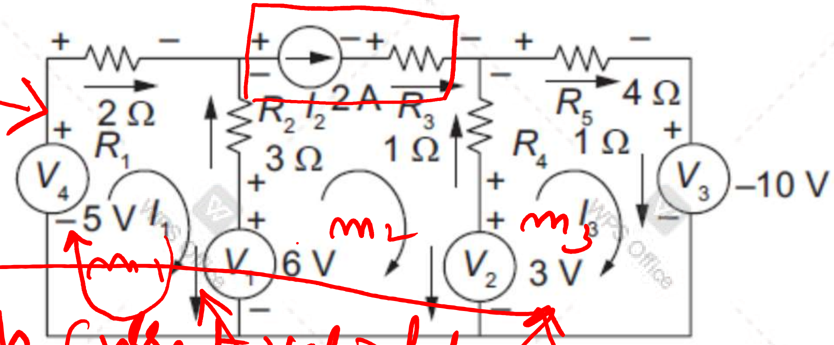


Mesh Analysis- Independent Current Source in Series with Another Element Contd.

l

7 KCL equation LI

- $b = 10$, $n = 8$ to get $\boxed{n - 1 = 7}$, $\boxed{b - n + 1 = 3}$
- The mesh current variables are $\boxed{i_1, i_2 \text{ and } i_3.}$



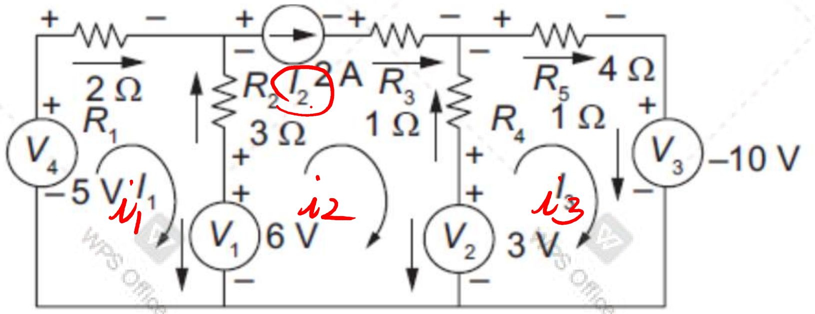
Express all the branch ~~currents~~ in terms of these mesh current variable.

- KVL at Mesh 1
  - $-V_4 + R_1 i_1 - R_2(i_2 - i_1) + V_1 = 0 \rightarrow \boxed{(R_1 + R_2) i_1 = V_4 + R_2 i_2 - V_1}$

With the help of $n-1 = 7$ KCL equation.

- KVL at Mesh 3
  - $-V_2 + R_4(i_3 - i_2) + R_5 i_3 + V_3 = 0 \rightarrow \boxed{(R_4 + R_5) i_3 = R_4 i_2 + V_2 - V_3}$

Mesh Analysis- Independent Current Source in Series with Another Element Contd.



- Here $\boxed{V_4 = I_1 R_1}$ and $\boxed{i_2 = I_2 = 2\text{A}}$ and plugging these values in the above equations

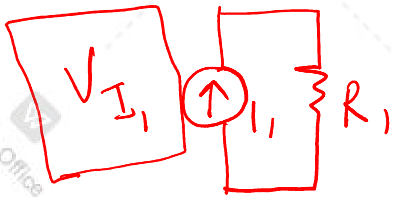
$$(R_1 + R_2)i_1 = V_4 + R_2 i_2 - V_1 \rightarrow \boxed{(R_1 + R_2)i_1 = R_1 I_1 + R_2 I_2 - V_1}$$

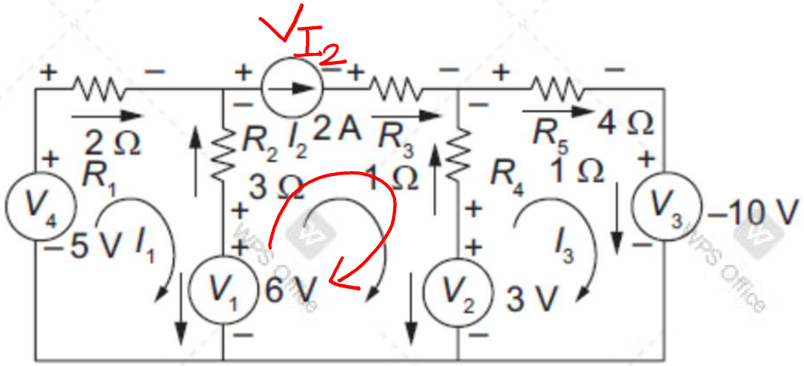
$$\boxed{(R_4 + R_5)i_3 = R_4 i_2 + V_2 - V_3} \rightarrow \boxed{(R_4 + R_5)i_3 = R_4 I_2 + V_2 - V_3}$$

- to get

- $$\begin{bmatrix} R_1 + R_2 & 0 \\ 0 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & -1 & 0 & 0 \\ 0 & R_4 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Mesh Analysis- Independent Current Source in Series with Another Element Contd.





- $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$ to get $i_1 = 1\text{A}$, $i_2 = 2\text{A}$ and $i_3 = 3\text{A}$.
- To find out the voltage across the current source I_2 the third KVL equation is used
  - $-V_1 + R_2(i_2 - i_1) + V_{I_2} + R_3 i_2 + R_4(i_2 - i_3) + V_2 = 0$

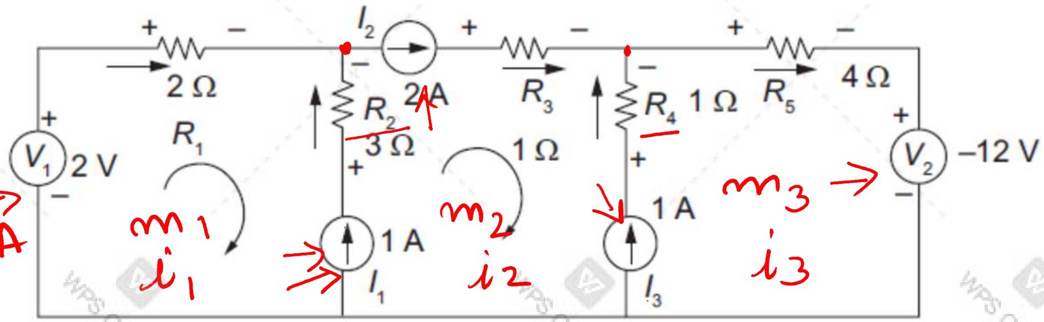
$V_{I_2} = -1$ V

Mesh Analysis- An Interesting Circuit

$i_2 = I_2 = 2A$

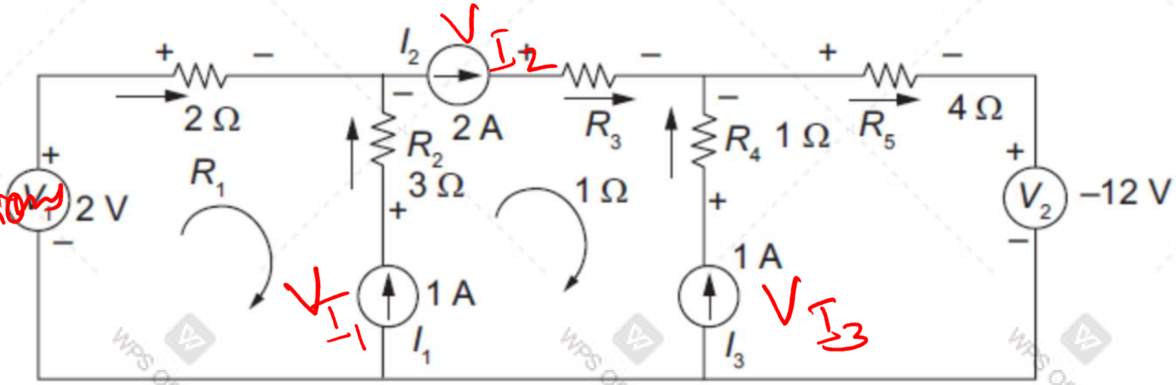
$i_1 = I_2 - I_1 = 1A$

$i_3 = I_2 + I_3 = 3A$



- $b = 10, n = 8$ to get $n - 1 = 7, b - n + 1 = 3$
- There are 2 independent voltage sources and 3 independent current sources.
- All i_1, i_2 and i_3 are constrained due to the presences of three independent current sources
  - $i_1 = 1\text{A}$, $i_2 = 2\text{A}$ and $i_3 = 3\text{A}$.

Mesh Analysis- An Interesting Circuit Contd.



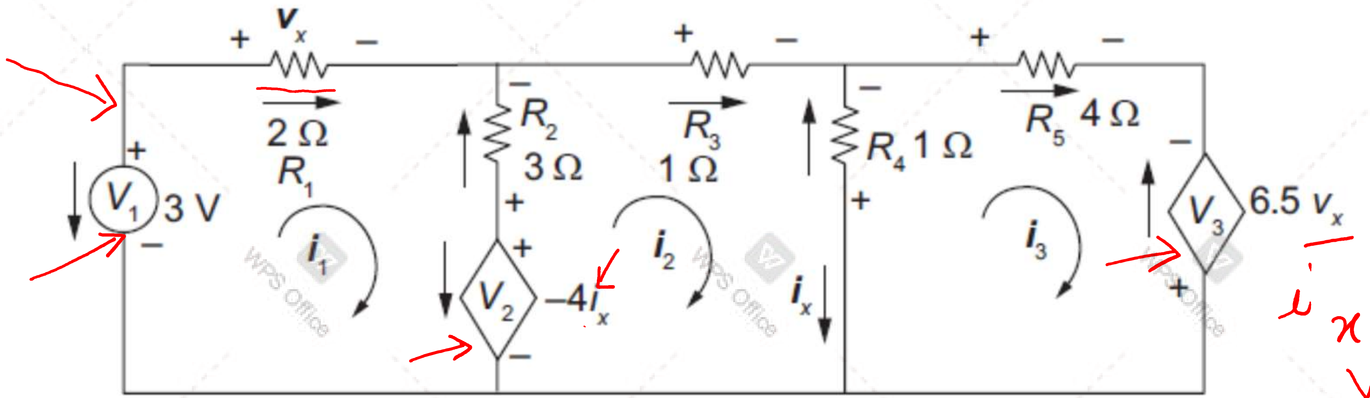
⇒ there are 3 KVL equations that are LI

- However there three other variables such as V_{I_1}, V_{I_2} and V_{I_3}
  - KVL at Mesh 1
    - $-V_1 + R_1 i_1 - R_2(i_2 - i_1) + V_{I_1} = 0$
  - KVL at Mesh 3
    - $V_{I_3} + R_4(i_3 - i_2) - R_5 i_3 + V_2 = 0$
  - KVL at Mesh2
    - $V_{I_1} + R_2(i_2 - i_1) + V_{I_2} + R_3 i_2 + R_4(i_2 - i_3) - V_{I_3} = 0$
  - $V_{I_1} = -3V \quad V_{I_2} = -2V \quad V_{I_3} = -1V$

Lecture-4.2 The Systematic Procedures in Circuit Analysis-2

Mesh Analysis with Dependent Sources

Mesh Analysis with Dependent Voltage Source



$i_x = (i_2 - i_3)$

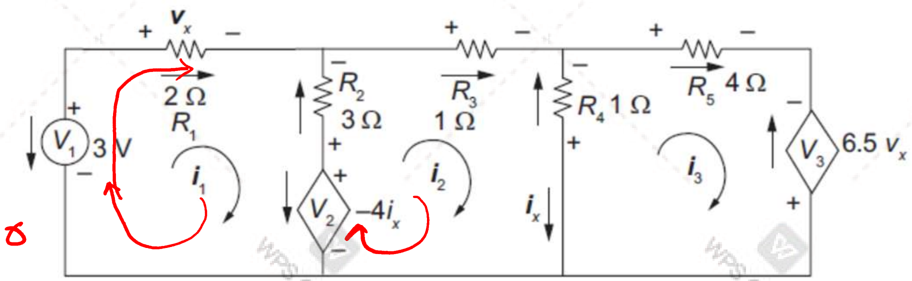
$V_x = i_1 R_1$

- $b = 8, n = 6$ to get $n - 1 = 5, b - n + 1 = 3$
- The mesh current variables are i_1, i_2 and i_3.
- Voltage sources
  - Independent voltage source: $V_1 = 3\text{V}$
  - Dependent voltage source:
    $V_2 = -4i_x \rightarrow \boxed{-4(i_2 - i_3)}$ $\boxed{V_3} = \boxed{6.5V_x} \rightarrow 6.5 i_1 R_1$

Mesh Analysis with Dependent Voltage Source Contd.

$n - 1 = 5$

3



$-V_1 + V_{R_1} - V_{R_2} + V_2 = 0$

- KVL at Mesh 1
  - $-V_1 + R_1 i_1 - R_2(i_2 - i_1) - 4i_x = 0$ $\rightarrow$ $\boxed{(R_1 + R_2)i_1 - (R_2 + 4)i_2 + 4i_3 = V_1}$

- KVL at Mesh 2 $-V_2 + V_{R_2} + V_{R_3} - V_{R_4} = 0$
  - $4i_x + R_2(i_2 - i_1) + R_3 i_2 - R_4(i_3 - i_2) = 0$ $\rightarrow$ $\boxed{R_2 i_1 + (R_2 + R_3 + R_4 + 4)i_2 - (R_4 + 4)i_3 = 0}$

$V_{R_4} + V_{R_5} - V_3 = 0$

- KVL at Mesh 3
  - $R_4(i_3 - i_2) + R_5 i_3 - V_3 = 0$ $\rightarrow$ $-6.5R_1 i_1 - R_4 i_2 + (R_4 + R_5)i_3 = 0$

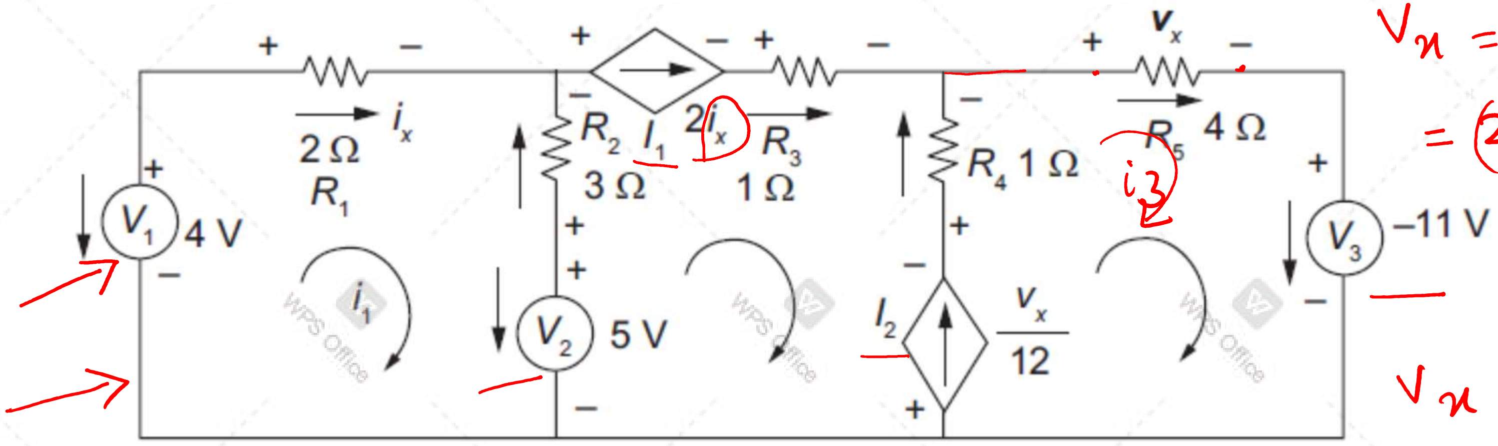
Mesh Analysis with Dependent Voltage Source Contd.

- $$\begin{bmatrix} R_1+R_2 & -R_2+4 & 4 \\ -R_2 & R_2+R_3+R_4+4 & -R_4+4 \\ -6.5R_1 & -R_4 & R_4+R_5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [V_1]$$

- $\boxed{\text{To get } i_1 = 1A \quad i_2 = 2A \quad i_3 = 3A}$ → branch currents

(Ohm's → branch voltages

Mesh Analysis with Dependent Current Source



$V_x = i_3 R_5$

$= \left(2 i_x + \frac{V_x}{12}\right) R_5$

$= \left(2 i_1 + \frac{V_x}{12}\right) R_5$

$V_x = 8 i_1 + \frac{V_x}{3}$

$3V_x - V_x = 24 i_1$

$V_x = 12 i_1$

I_1 is going to constrain i_2

$i_2 = I_1$

$i_3 = I_1 + I_2$

$I_1 = 2 i_1$

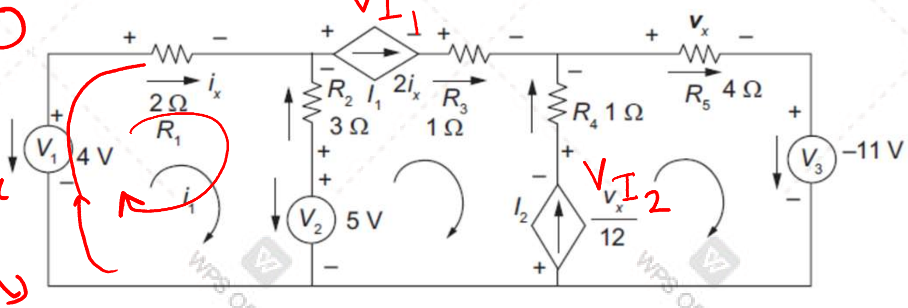
$i_x = i_1$

$I_2 = \frac{V_x}{12} = i_1$

- $b = 10, n = 8$ to get $n - 1 = 7, b - n + 1 = 3$
- The mesh current variables are i_1, i_2 and i_3.
- Independent voltage source: $V_1 = 3V \quad V_2 = 5V \quad V_3 = -11V$
- Dependent current source: $I_1 = 2i_x = 2i_1$ $\quad I_2 = \frac{V_x}{12}$

Mesh Analysis with Dependent Current Source Contd.

$-V_1 + V_{R_1} - V_{R_2} + V_2 = 0$



$i_2 = I_1 = 2i_x$

$2i_1$

- KVL at Mesh 1
  - $-V_1 + 2i_1 + R_2(i_1 - i_2) + V_2 = 0.$

    $+$ $2i_1$
  - $i_2 = I_1 \rightarrow i_2 = 2i_1$ and $i_3 = I_2 + I_1 \rightarrow 3i_1$
  - $i_1 = 1A \quad i_2 = 2A \quad i_3 = 3A$
- KVL at Mesh 3
  - $V_{I_2} + V_{R_4} + V_{R_5} + V_3 = 0$ to get $V_{I_2} = -2V$
- KVL at Mesh 2
  - $-V_2 + V_{R_2} + V_{I_1} + V_{R_3} - V_{R_4} - V_{I_2} = 0$ to get $V_{I_1} = -1V$

Conspectus of Nodal Analysis

- Determine the number of nodes in the circuit. Select one node as the reference node. Assign a node voltage between each non-reference node and the reference node. All node voltages are assumed positive with respect to the reference node. For an n node circuit, there are $n-1$ node voltages. As a result, $n-1$ linearly independent equations must be written to solve for the node voltages.
- Express all the branch voltages in terms of these node voltages using $b-n+1$ KVL equations. Find a constraint equation for each voltage source-independent or dependent-in the circuit in terms of the assigned node voltages using KVL.
- For each dependent sources, express the controlling variable in terms of the node voltages.
- Use KCL to formulate the remaining linearly independent equations and solve for the $\boxed{n-1-n_V}$ node voltages. The remaining KCL should be used to find the branch currents through the voltage sources.
- From those $n-1-n_V$ node voltages the branch voltages would be found out and then using Ohm's law the branch currents can be computed.

number of voltage source

Conspectus of Mesh Analysis

- Ascertain that the circuit is planar. Find the number of branches b and nodes n in the circuit. Identify the $b-n+1$ meshes in the circuit and epitomise them with the mesh current variables. To solve these mesh current variables $b-n+1$ linearly independent equations are required.
- Express all the branch currents in terms of these mesh current variables using $n-1$ KCL equations. Find a constraint equation for each current source-independent or dependent-in the circuit in terms of the assigned mesh current variables using KCL.
- For each dependent sources, express the controlling variable in terms of the mesh current variables.
- Use KVL to formulate the remaining linearly independent equations and solve for the $b-n+1-n_I$ mesh current variables. The remaining KVL should be used to find the branch voltages across the current sources.

  number current sources

- From those $b-n+1-n_I$ mesh currents the branch currents would be found out and then using Ohm's law the branch voltages can be computed.

Conclusions of this Topic

- This topic we have discussed two systematic procedures of solving the circuit analysis problem(only with resistor components!)
- In brevity circuit analysis problem is for an n node, b branch circuit involves the determination of b branch voltage variables and b branch current variables.
- Here we also have discussed source transformation theorem in which a voltages in series with a resistor R can be replaced with a current source in parallel with the resistor R and vice versa.
- Nodal Analysis is applicable to any circuit that has a unique solution whereas the mesh analysis is only applicable to the planar circuits.
- Both these methods are not efficient
  - To get the efficient methods elementary graph theory results are required. Thus emerged techniques are known as
    - Cut-set analysis
    - Fundamental circuit analysis

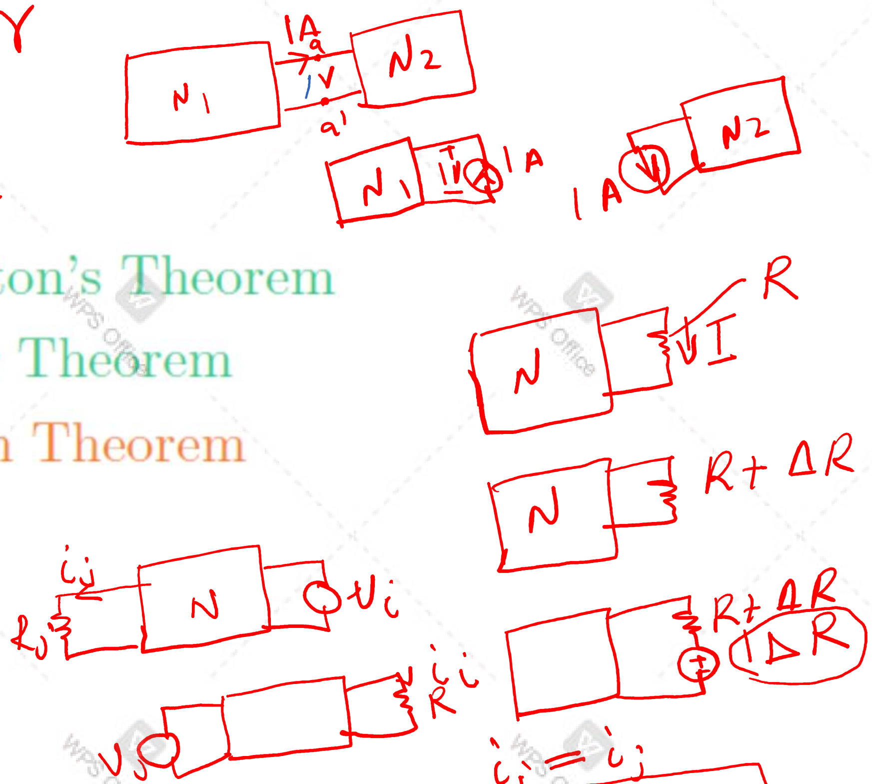
1. Suppose there are 6 node circuit contain 6 ~~current~~ voltage sources. whether it can be solvable.

Lecture 5.1- Basic Circuit Theorems-1

Enumeration

Basic Circuit theory

- Superposition Theorem
- Thevenin's Theorem/Norton's Theorem
- Maximum Power Transfer Theorem
- Star-Delta Transformation Theorem
- Substitution Theorem
- Compensation Theorem
- Reciprocity Theorem
- Tellegen's theorem



$i_i = i_j$

$$\sum_{i=1}^{b} v_i i_i = 0$$

Superposition Theorem

- The response of any circuit variable in a multi-source resistive circuit containing n independent sources can be obtained by adding the responses of the same circuit variable in n single-source circuits with ith single-source circuit formed by keeping only ith independent source active and all the remaining independent sources deactivated.

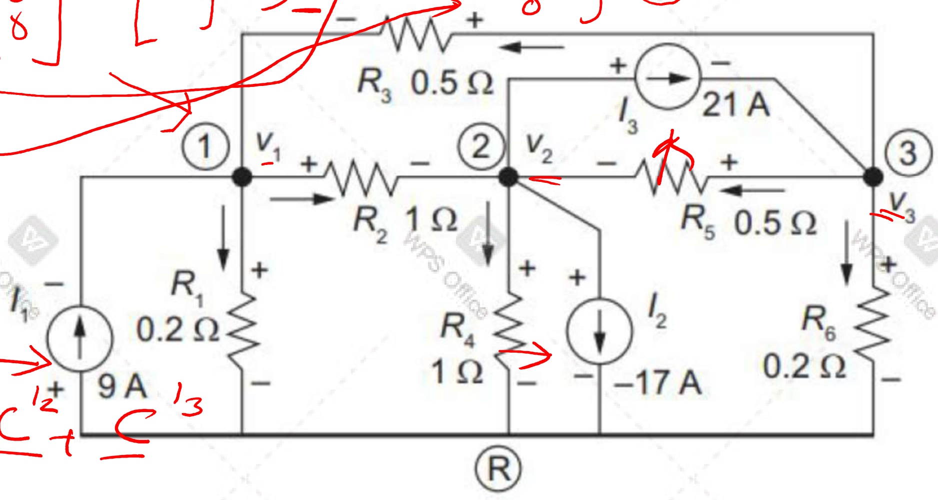
$\begin{bmatrix} 0 \\ 0 \\ 21 \end{bmatrix} \sim C'_3$

$\begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix} \to C'_1$

$\begin{bmatrix} 0 \\ -17 \\ 0 \end{bmatrix} \to C'_2$

$$\begin{bmatrix} 8 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ -17 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = C'_1 + C'_2 + C'_3$$



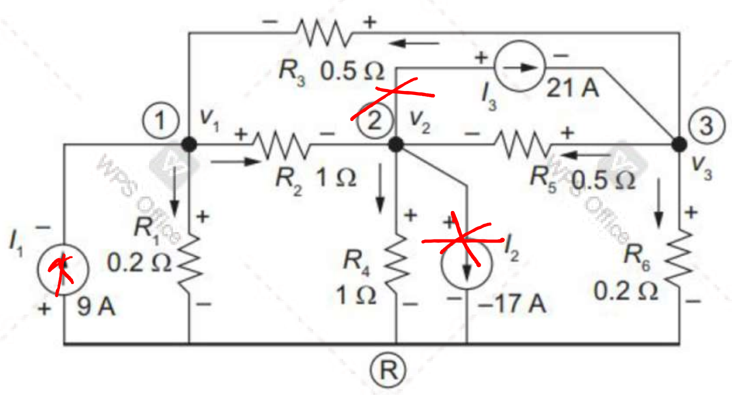
Superposition Theorem Contd.

$$\begin{bmatrix} 8 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1.29 \\ 0.52 \\ 0.40 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -17 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.99 \\ 5.18 \\ 1.37 \end{bmatrix}$$

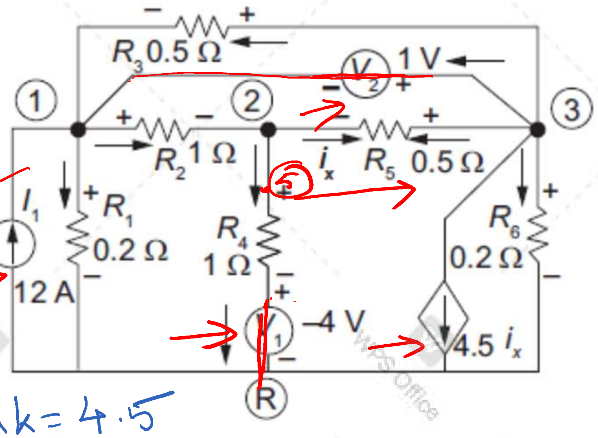
$$\begin{bmatrix} 8 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 9 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \overset{21}{17} \end{bmatrix} \rightarrow \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.28 \\ -4.7 \\ 1.22 \end{bmatrix}$$

$$\begin{bmatrix} 1.29 \\ 0.52 \\ 0.40 \end{bmatrix} + \begin{bmatrix} 0.99 \\ 5.18 \\ 1.37 \end{bmatrix} + \begin{bmatrix} -0.28 \\ -4.7 \\ 1.22 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$



Superposition Theorem with Dependent Sources

$$\begin{bmatrix}1.41\\1.06\end{bmatrix}+\begin{bmatrix}0.71\\-0.47\end{bmatrix}+\begin{bmatrix}-0.12\\0.41\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}=\begin{bmatrix}4&6\\-3&4\end{bmatrix}^{-1}\begin{bmatrix}1&0&2\\0&1&2\end{bmatrix}\begin{bmatrix}12\\-4\\1\end{bmatrix}$$



Y V = 1 U

k = 4.5

k = 4.5

$$\begin{bmatrix}G_1+G_2+G_6+(1-k)G_5 & (k-1)G_5-G_2\\ -G_2-G_5 & G_2+G_4+G_5\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}1 & 0 & (k-1)G_5-G_6\\ 0 & G_4 & G_5\end{bmatrix}\begin{bmatrix}I_1\\V_1\\V_2\end{bmatrix}$$

4 , Since 2 independ V_1 and V_2

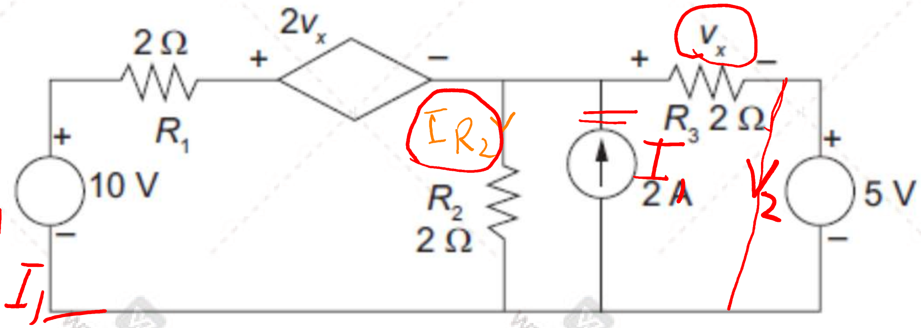
$$\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}4&6\\-3&4\end{bmatrix}^{-1}\begin{bmatrix}1&0&2\\0&1&2\end{bmatrix}\begin{bmatrix}12\\0\\0\end{bmatrix}\rightarrow\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}1.41\\1.06\end{bmatrix}$$

$$\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}4&6\\-3&4\end{bmatrix}^{-1}\begin{bmatrix}1&0&2\\0&1&2\end{bmatrix}\begin{bmatrix}0\\-4\\0\end{bmatrix}\rightarrow\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}0.71\\-0.47\end{bmatrix}$$

$$\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}4&6\\-3&4\end{bmatrix}^{-1}\begin{bmatrix}1&0&2\\0&1&2\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}\rightarrow\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}-0.12\\0.41\end{bmatrix}$$

Solving Circuits Using Superposition Theorem Contd.

I_{R_2} ?

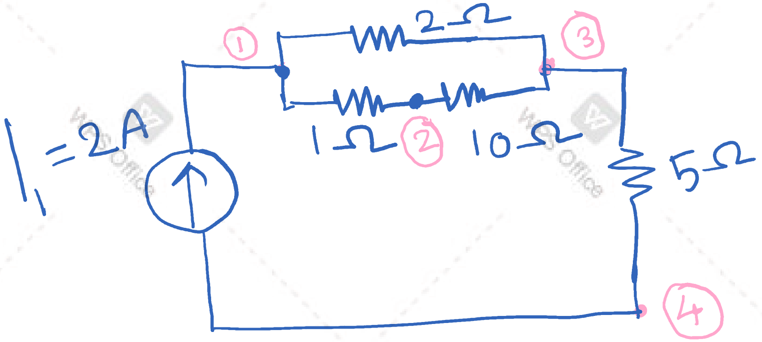


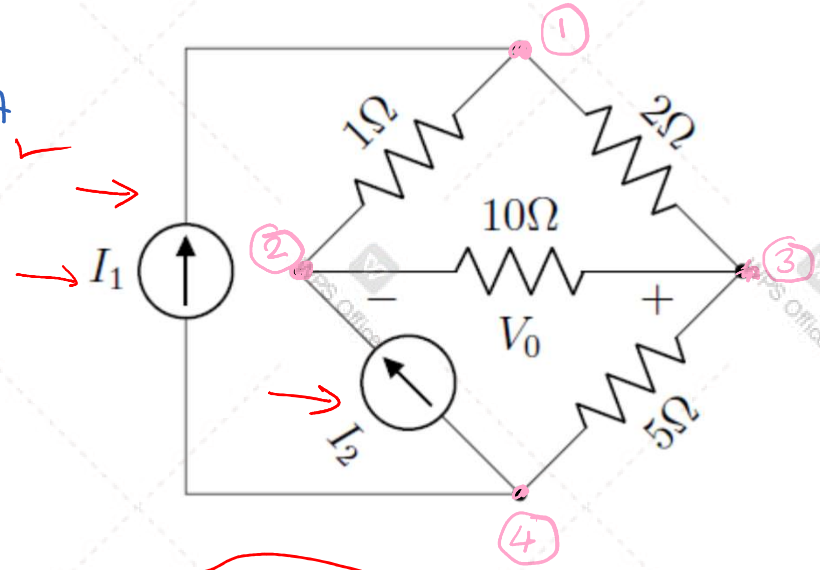
$$I_{R_2} = I_{R_2}^{V_1} + I_{R_2}^{V_2} + I_{R_2}^{I_1}$$

Solving Circuits Using Superposition Theorem

- When I_1 is ON and I_2 is OFF

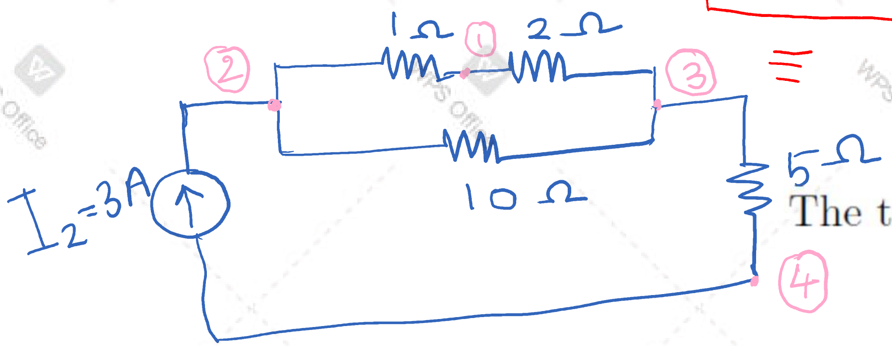
The current through the 10Ω resistor is $I_1^{10} = -2 \times \frac{2}{13}$ A





- When I_1 is OFF and I_2 is ON

The current through the 10Ω resistor is $I_2^{10} = -3 \times \frac{3}{13}$ A



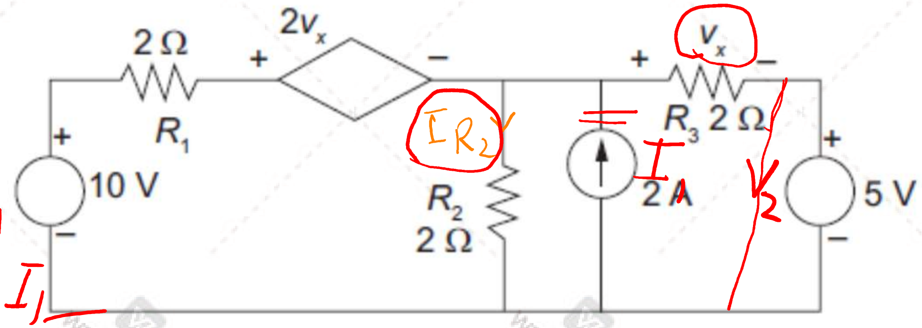
the voltage drop $v_0 = -1 \times 10 = -10$V.

The total current when both I_1 and I_2 are ON $I^{10} = \frac{-4}{13} + \frac{-9}{13}$ A

$= -1$ A

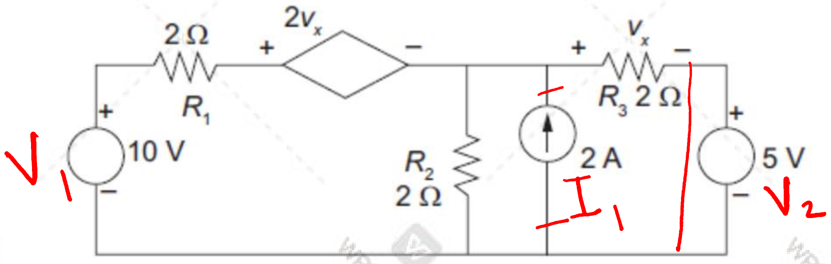
Solving Circuits Using Superposition Theorem Contd.

I_{R_2} ?

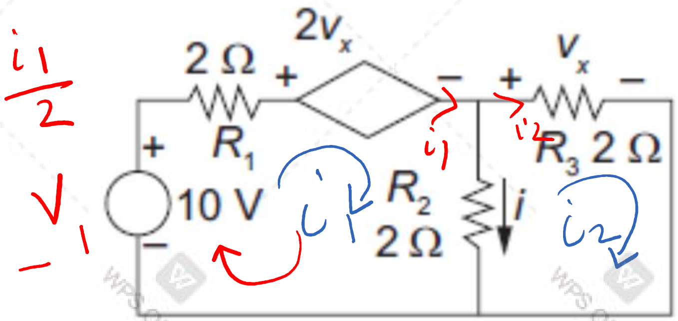


$$I_{R_2} = I_{R_2}^{V_1} + I_{R_2}^{V_2} + I_{R_2}^{I_1}$$

Solving Circuits Using Superposition Theorem Contd.



$i_2 = \frac{i_1}{2}$



When only V_1 is active

$I_{R_2} = \frac{i_1}{2}$

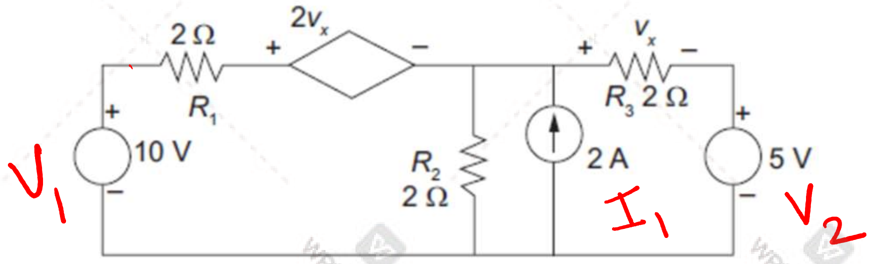
$V_x = i_2 R_3 = \frac{i_1}{2} \times 2 = i_1$ V

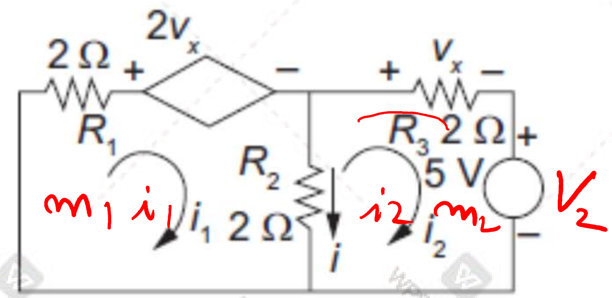
KVL over mesh 1

- $-V_1 + V_{R_1} + 2V_x + V_{R_2} = 0 \rightarrow -10 + 2i_1 + 2i_1 + i_1 = 0 \Rightarrow i_1 = 2\text{A}$

$$\boxed{I_{R_2}^{V_1} = 1\text{A}}$$

Solving Circuits Using Superposition Theorem Contd.





$I_{R_2}^{V_2} = (i_1 - i_2)$

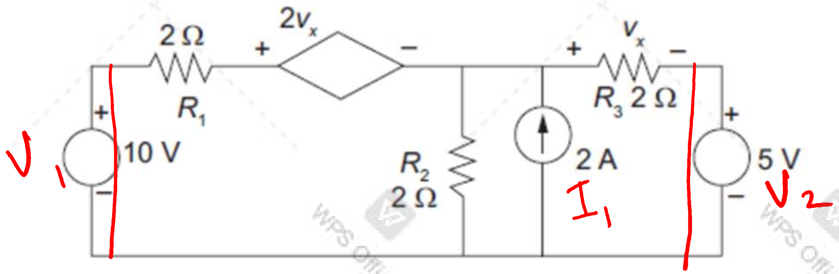
When only V_2 is active

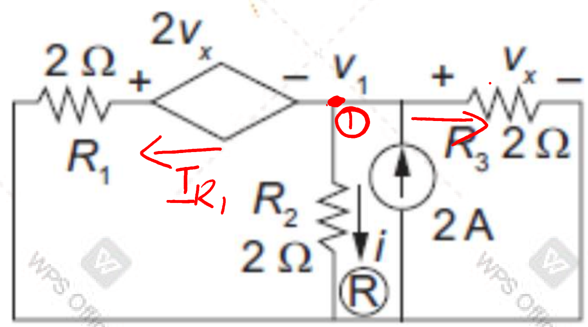
$V_x = 2i_2$

- KVL over mesh 1
  - $$V_{R_1} + 2V_x + V_{R_2} = 0 \rightarrow 2i_1 + 4i_2 + 2(i_1 - i_2) = 0 \qquad (1)$$
- KVL over mesh 2
  - $$-V_{R_2} + V_{R_3} + 5 = 0 \rightarrow 2(i_2 - i_1) + 2i_2 + 5 = 0 \qquad (2)$$

From (1) and (2) $I_{R_2}^{V_2} = i_1 - i_2 = 1.5\text{A}$

Solving Circuits Using Superposition Theorem Contd.





$V_x = v_1$

When only I_1 is present

KCL at node 1

$\boxed{I_{R_1} + I_{R_2} + I_{R_3} = 2} \rightarrow 0.5 \times 3v_1 + 0.5v_1 + 0.5v_1 = 2 \Rightarrow v_1 = 0.8\text{V}$

$I_{R_2}^{I_1} = 0.5v_1 = 0.4\text{A}$

Total current $I_{R_2} = \boxed{I_{R_2}^{V_1} + I_{R_2}^{V_2} + I_{R_2}^{I_1}} = 1 + 1.5 + 0.4 = 2.9\text{A}.$

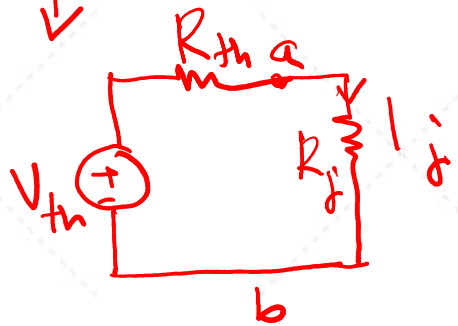
Thevenin's Theorem/Norton's Theorem

R_j

v_j, I_j

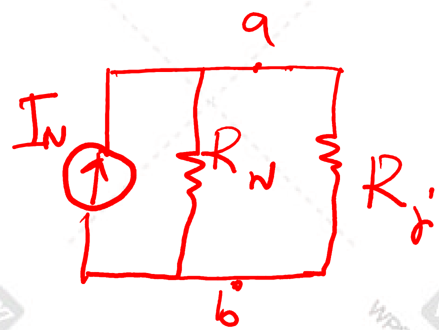
$R_j :=$ The load of the circuit

$V_{ab} = V_j$

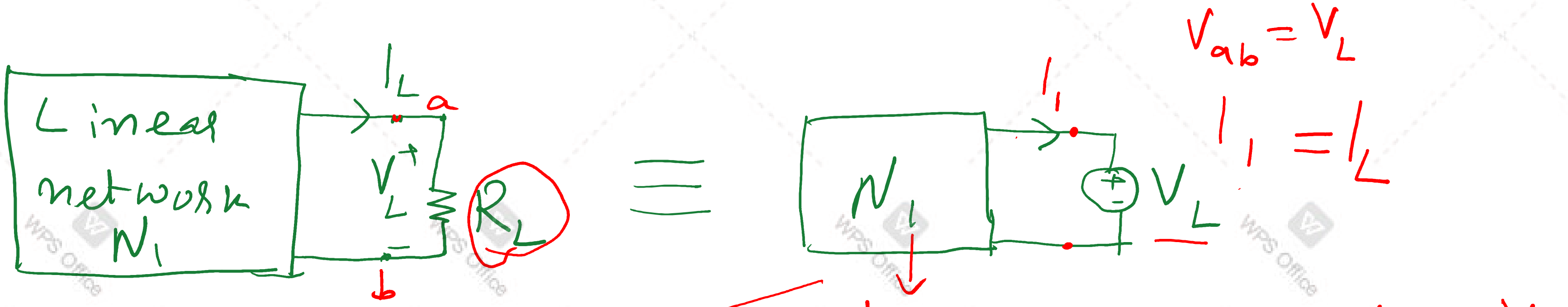


$V_{ab} = V_j$

$V_{ab} = V_j$



Thevenin's Theorem/Norton's Theorem Contd.



N_1 is unchanged, $V_{ab} = V_L$

The current

$$\boxed{I_L} = \underbrace{a_1V_1 + a_2V_2 + \ldots + a_{n_V}V_{n_V} + b_1I_1 + b_2I_2 + \ldots + a_{n_I}I_{n_I}}_{I_{SC}} + \boxed{G_0V_L}$$

$$\boxed{I_L = I_{SC} + G_0V_L}$$

? ?

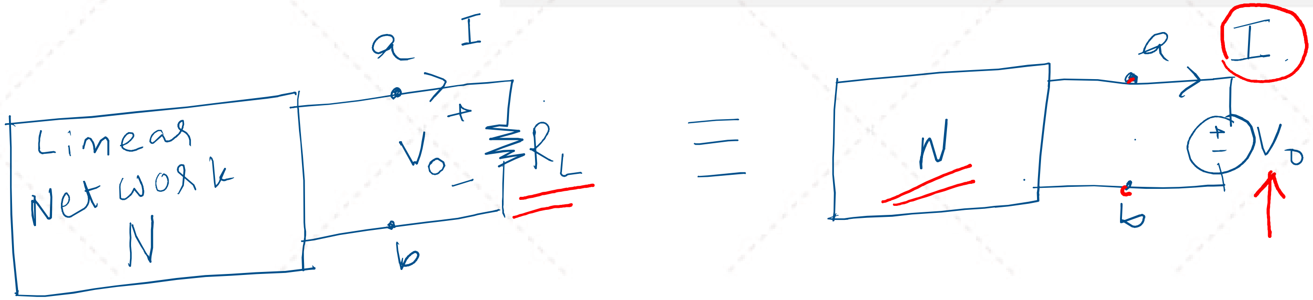
Conclusions of Lecture 5

- This lecture has introduced with some circuit theorems that form an indispensable tool set in circuit analysis.
- In a linear network containing multiple independent sources, the principle of superposition allows us to compute any current or voltage in the network as the algebraic sum of the individual contributions of each source acting alone.
- Please note that superposition is a linear property and does not apply to nonlinear functions such as power.
- A gentle introduction of Thevenin's theorem/ Norton's theorem is also a part of today's discussion

Lecture 6- Basic Circuit Theorems-2

Thevenin’s Theorem/ Norton’s Theorem

Justification of Thevenin's Theorem



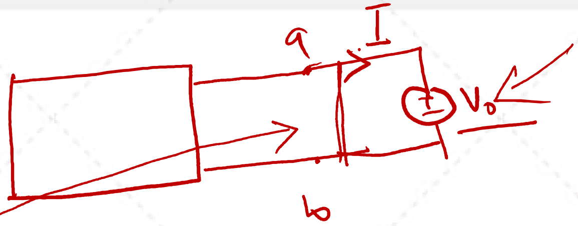
$V_o = V_{ab}$

$$I = a_1 V_{S_1} + a_2 V_{S_2} + \ldots + a_{n_V} V_{S_{n_V}} + b_1 I_{S_1} + b_2 I_{S_2} + \ldots + b_{n_I} I_{S_{n_I}} - a_0 V_o$$

$$= \underbrace{\sum_{k=1}^{n_V} a_k V_{S_k} + \sum_{k=1}^{n_I} b_k I_{S_k}}_{I_{sc}} - a_0 V_o$$

$$= I_{sc} - I_o$$

Justification of Thevenin's Theorem



$$I = \underbrace{\sum_{k=1}^{n_V} a_k \cancel{V_{S_k}}^{\,0} + \sum_{k=1}^{n_I} b_k \cancel{I_{S_k}}^{\,0}}_{I_{sc}} - a_0 \cancel{V_o}$$

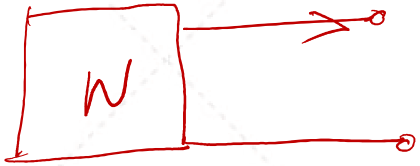
$$\boxed{= I_{sc} - I_o}$$

$$\boxed{V_o = 0} \Rightarrow \boxed{I = I_{sc}}$$

$$\boxed{I_{sc} = 0} \Rightarrow I = -I_0 = -a_0 V_0 = \frac{-V_o}{R_{Th}}$$

$\to R_{th}$

Using superposition theorem $\boxed{I = I_{sc} - \frac{V_o}{R_{Th}}}$



$$\boxed{I = 0} \Rightarrow V_o = V_{oc} \to \boxed{I_{sc} = \frac{V_{oc}}{R_{Th}}}$$

$$I = \frac{V_{oc}}{R_{Th}} - \frac{V_o}{R_{Th}} \Rightarrow \boxed{V_o = V_{oc} - R_{Th} I}$$

$$I = I_{sc} - I_o$$

$$= \frac{V_{oc}}{R_{th}} - \frac{V_o}{R_{th}}$$

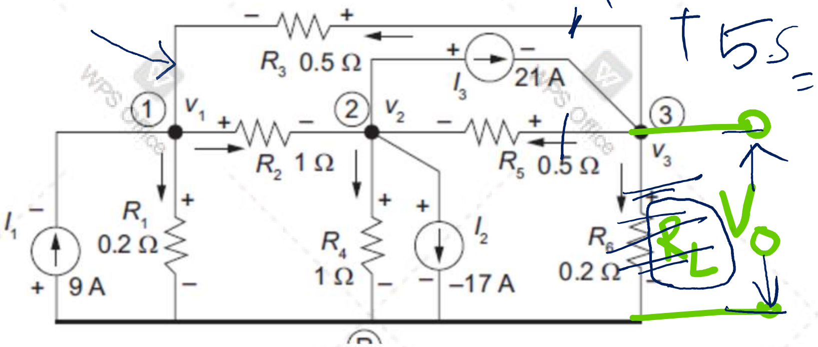
Finding R_{Th} - Thevenin's resistance

$$\boxed{R_{Th} = \frac{V_{oc}}{I_{sc}}}$$

$$\begin{bmatrix} 8 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 9 \\ -17 \\ 21 \end{bmatrix} \rightarrow \boxed{v_3 = V_{oc} = 9.83V}$$

2S + 2S + 5S = 9S

$\boxed{v_3 = V_{oc}}$



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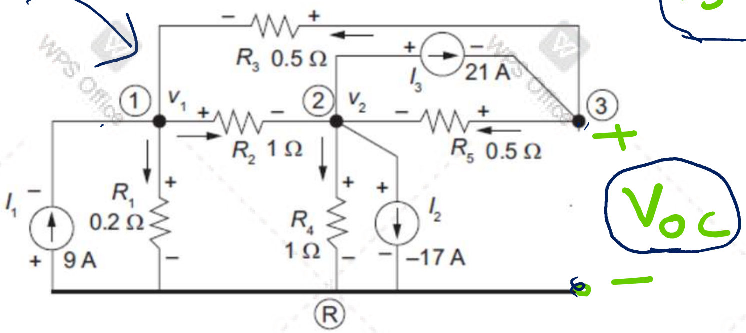
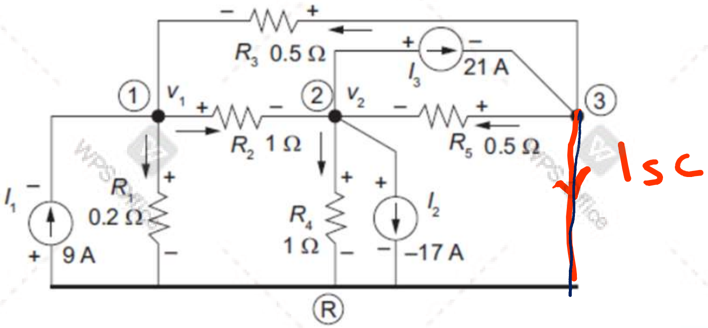


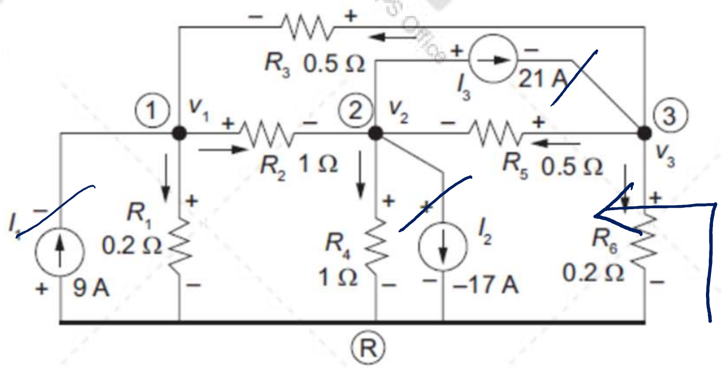


$$\begin{bmatrix} 8 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ -17 \\ 21 \end{bmatrix} \text{ to get } \boxed{I_{sc} = 21.58A}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{9.83}{21.58} = \boxed{0.4541\Omega}$$

Finding R_{Th} - Thevenin's resistance

R_{Th} is the equivalent resistance looking back into circuit from the output terminals with all independent sources in the circuit made zero.



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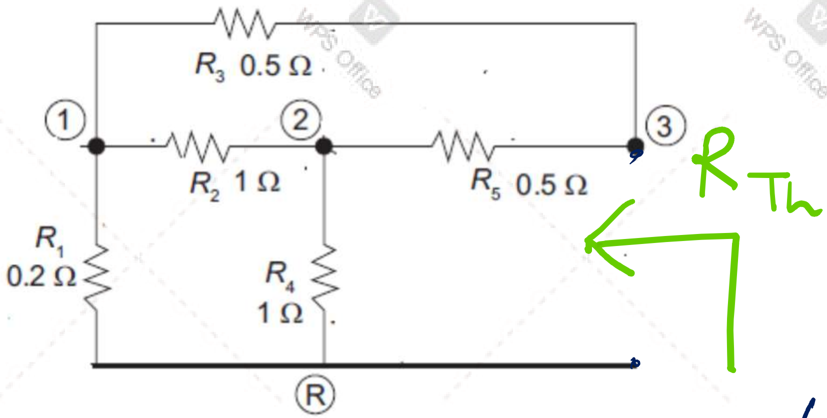


$$R_{Th} = 0.451\,\Omega = \frac{V_{oc}}{I_{sc}}.$$

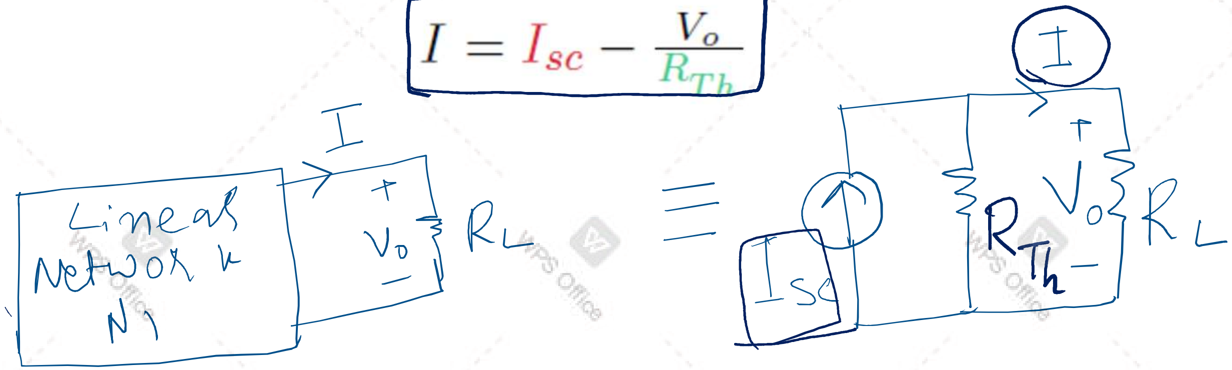
Thevenin's Theorem- A Simplified Statement

$$I = \frac{V_{oc}}{R_{Th}} - \frac{V_o}{R_{Th}} \Rightarrow \boxed{V_o = V_{oc} - R_{Th} I}$$

Let a network with unique solution be represented as interconnection between network N_1 and a load resistance R_L. Then, the network N_1 may be replaced by an independent voltage source of value V_{oc} in series with a resistance R_{Th} without affecting any voltage(accross) or current(flowing through) R_L.

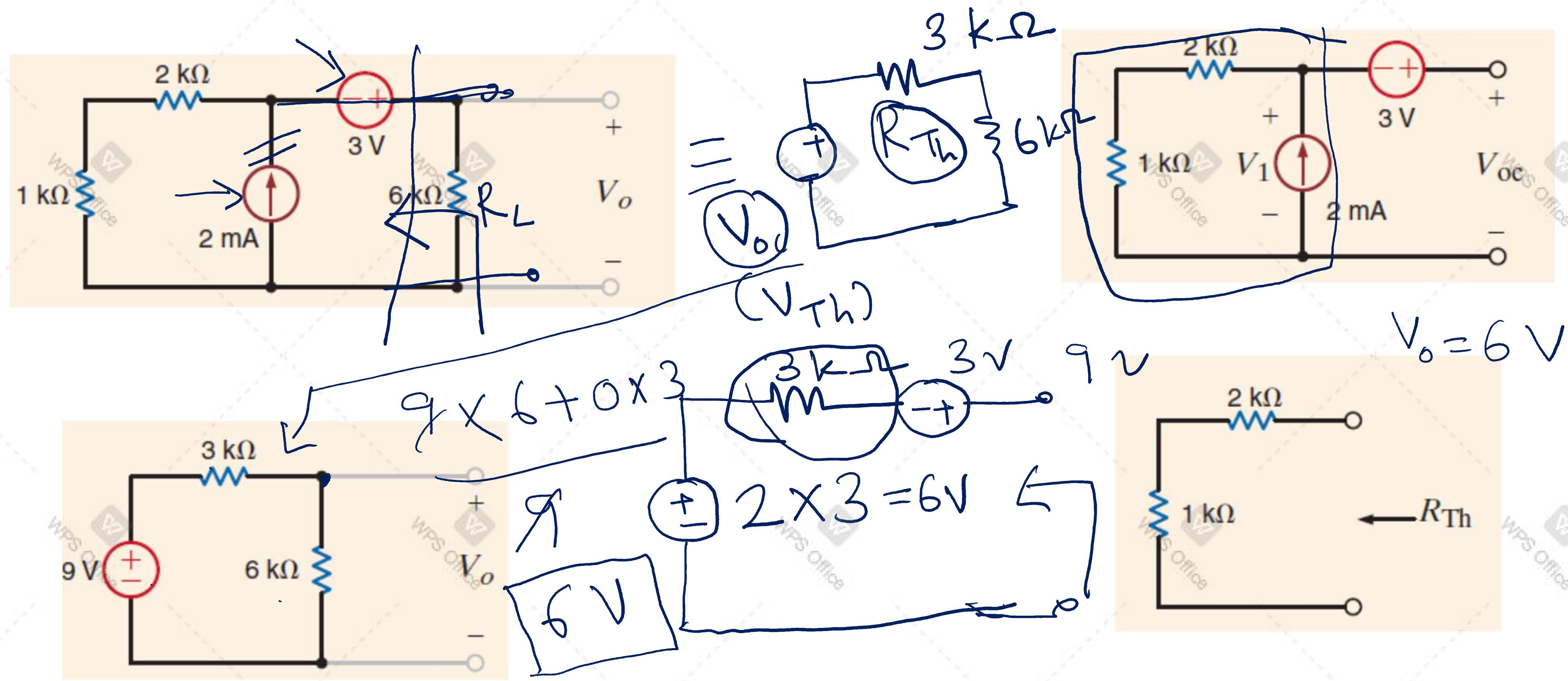
Norton's Theorem- A Simplified Statement

$$\boxed{I = I_{sc} - \frac{V_o}{R_{Th}}}$$

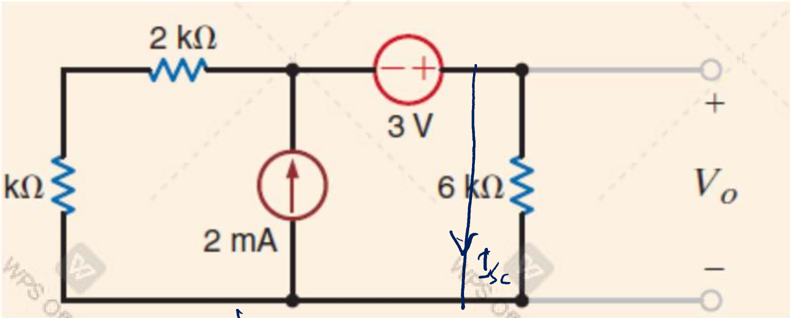


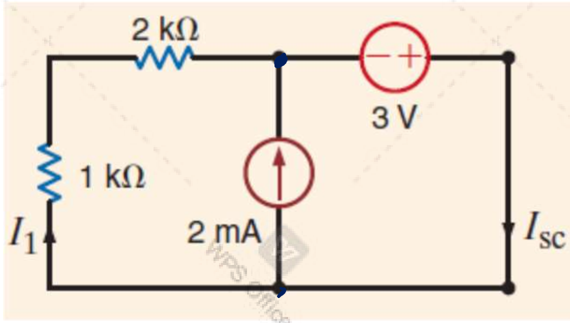
Let a network with unique solution be represented as interconnection between network N_1 and a load resistance R_L. Then, the network N_1 may be replaced by an independent current source of value I_{sc} in parallel with a resistance R_{Th} without affecting any voltage(accross) or current(flowing through) R_L.

Thevenin's Theorem- Circuit with Only Independent Sources



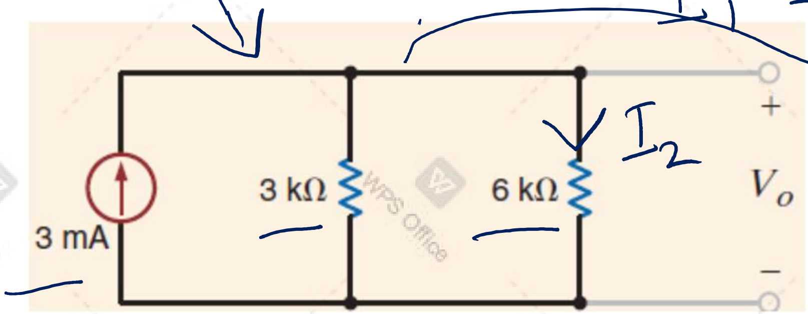
Norton's Theorem- Circuit with Only Independent Sources

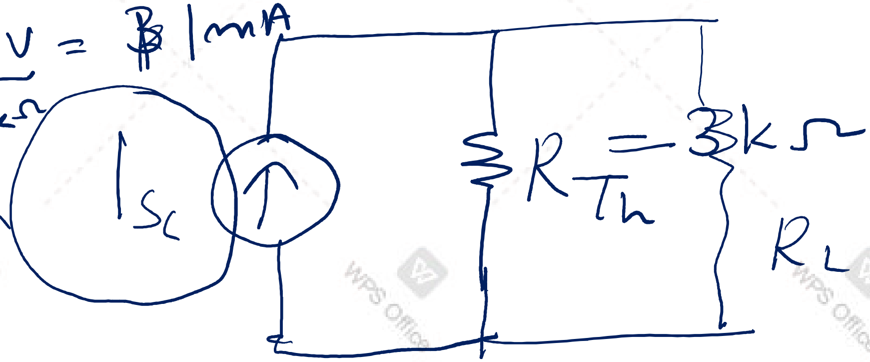




$$I_{sc} = I_1 + 2\,mA = 3\,mA$$

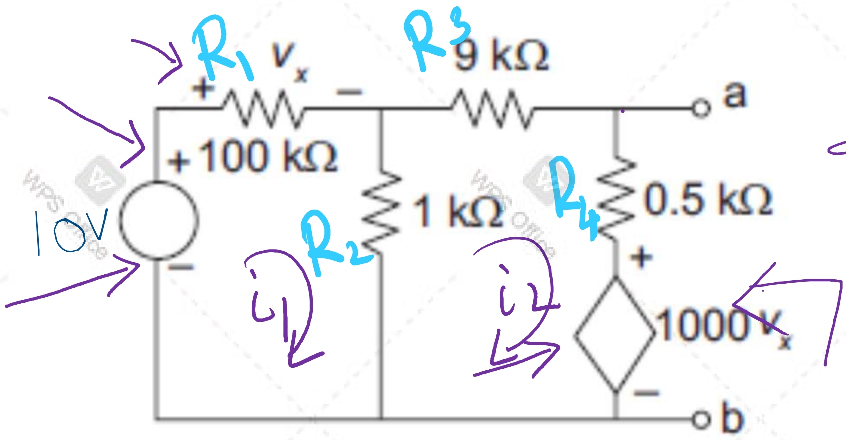
$$I_1 = \frac{3V}{3k\Omega} = 1\,mA$$

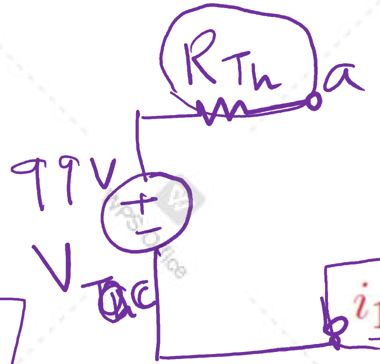




$$V_o = I_2 \times 6 = 3 \times \frac{3}{9} \times 6 = 6\,V$$

Thevenin's Theorem- Circuit with Independent as well as Dependent Sources





$V_{oc} = V_{R_4} + 1000 V_x$

$V_x = 100 i_1$

$R_{Th} = \dfrac{V_{oc}}{I_{sc}}$

$\boxed{i_1 = 0.001\text{A},\ i_2 = -9.8\text{A}}$

$\boxed{V_{oc} = 0.5 i_2 + 100 i_1 \times 1000 = 99\text{V}}$

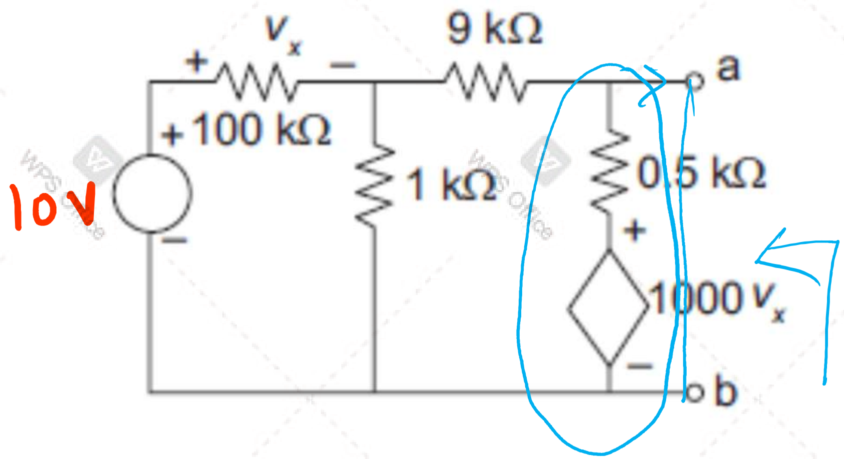
KVL at mesh 1

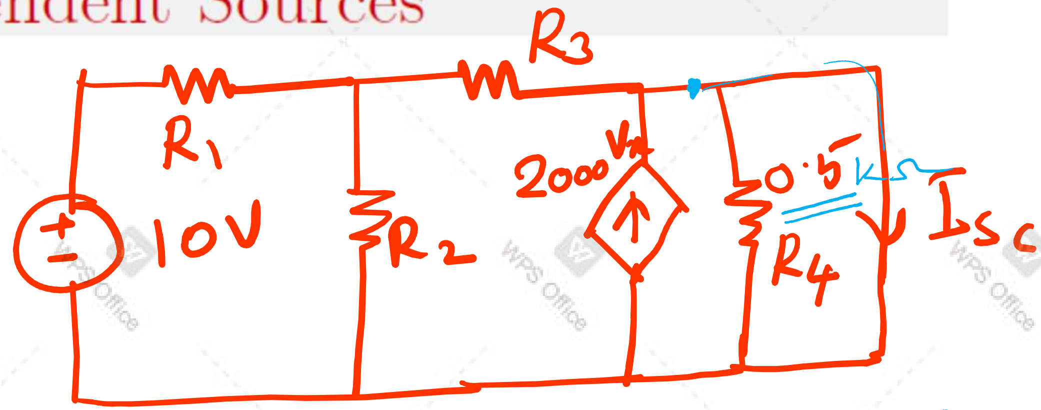
- $-V_1 + V_{R_1} + V_{R_2} = 0 \rightarrow 100 i_1 + (i_1 - i_2) = 10$

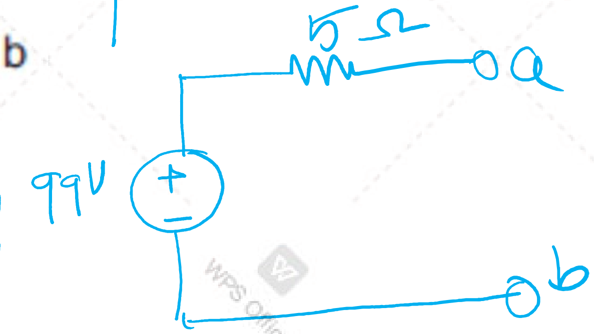
KVL at mesh 2

- $-V_{R_2} + V_{R_3} + V_{R_4} + 1000 V_x = 0 \rightarrow \quad (i_2 - i_1) + 9 i_2 + 0.5 i_2 + 1000 \times 100 i_1 = 0$

Thevenin's Theorem- Circuit with Independent as well as Dependent Sources







$$\boxed{R_{Th} = \frac{V_{oc}}{I_{sc}}}$$

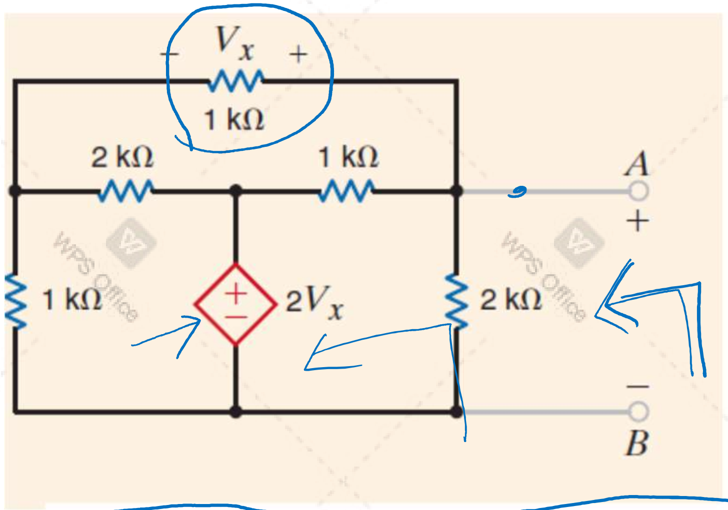
$$\boxed{V_x = \frac{10}{101} \times 100 \approx 10\text{V}}$$

$$\boxed{I_{sc} = \frac{10}{101}\frac{1}{10} + 500V_x \approx 20\text{A}}$$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} \approx 5\Omega$$

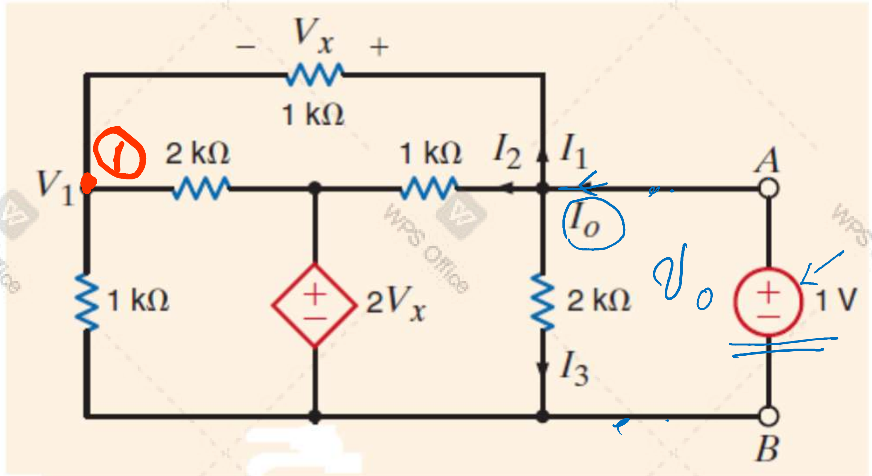
$$= \frac{99}{20}$$

Thevenin's Theorem- Circuit With Only Dependent Sources



I

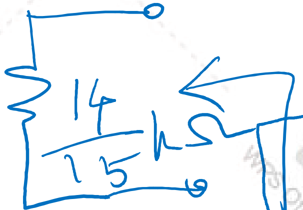
$\frac{V_{oc}}{R_{Th} = I_{sc}}$



$V_{oc} = 0, I_{sc} = 0 \Rightarrow R_{Th}$ as indeterminate

$R_{Th} = \frac{1}{I_o}$

$I_o = I_1 + I_2 + I_3$



$R_{Th} = \frac{v_o}{1A}$

$V_x = 1 - v_1$

KCL at node 1

$$\frac{V_1}{1k} + \frac{V_1 - 2V_x}{2k} + \frac{V_1 - 1}{1k} = 0 \to V_1 = \frac{4}{7}, V_x = \frac{3}{7}$$

$I_1 = \frac{3}{7}mA, I_2 = \frac{1}{7}mA, I_3 = \frac{1}{2}mA$ to get $I_o = \frac{15}{14}mA$

$R_{Th} = \frac{14}{15}k\Omega$

The Summary of Thevenin's Theorem

- Remove the load resistance and find the voltage across the open-circuit terminals, V_{oc}.
- Determine R_{Th} of the network at the open terminals with the load removed. Three different types of circuits may be encountered in determining the resistance, R_{Th}
  - Suppose the circuit contains only independent sources, they are made zero by replacing the voltage sources with short circuits and the current sources with open circuits. R_{Th} is then found by computing the resistance of the purely resistive network at the open terminals.
  - Suppose the circuit contains both independent and dependent sources, the open-circuit terminals are shorted and I_{sc} between these terminals is determined. The ratio of the V_{oc} to the I_{sc} is R_{Th}.
  - Suppose the circuit contains only dependent sources, an independent voltage or current source is applied at the open terminals and the corresponding current or voltage at these terminals is measured. The voltage/current ratio at the terminals is R_{Th}. Since there is no energy source, V_{oc} as well as I_{sc} are zero in this case.
- Suppose the load is now connected to the Thevenin equivalent circuit, consisting of V_{oc} in series with R_{Th} the desired solution can be obtained.

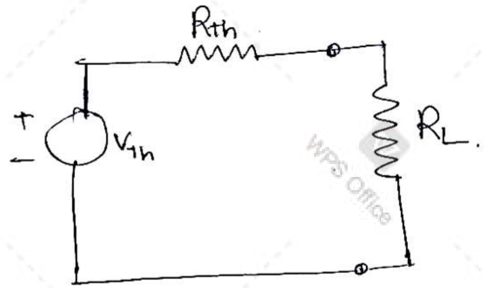
Conclusions of Lecture 6

- A qualitative justification for Thevenin's theorem is explained at the beginning of today's lecture.
- We have formulated Thevenin's model for three different scenarios.
- Using Thevenin's theorem a network n can be split into n_1(which should be linear) and n_2(need not to be linear). Then the network n_1 can be replaced with a voltage source V_{oc} in series with a resistor R_{Th} without affecting any voltage or current variable in the network n_2.
- Thevenin's theorem would be very helpful when the load network undergoes constant change.

Lecture-7: Basic Circuit Theorems-3

Maximum Power Transfer Theorem

Maximum Power Transfer Theorem



$$P_{RL} = I_{RL}^2 R_L$$

$$= \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 R_L.$$

$$= \frac{V_{th}^2 R_L}{(R_L + R_{th})^2}.$$

Maximum Power Transfer Theorem

$$\frac{dP_{RL}}{dR_L} = \frac{(V_{th})^2 \times \left[(R_L + R_{th})^2 - 2R_L(R_L + R_{th})\right]}{(R_L + R_{th})^2}$$

$$= \frac{V_{th}^2 \left[R_{th}^2 - R_L^2\right]}{(R_L + R_{th})^2}$$

$$\frac{dP_{RL}}{dR_L} = 0 \Rightarrow R_{th} = R_L$$

Maximum Power Transfer Theorem

The power delivered by a linear circuit containing independent DC sources is a maximum of $\frac{V_{oc} I_{sc}}{4}$ when it is delivering $\frac{I_{sc}}{2}$ to the load, where V_{oc} is the open-circuit voltage in its Thevenin's equivalent and I_{sc} is the short-circuit current in its Norton's equivalent.

Maximum Power Transfer Theorem Contd.

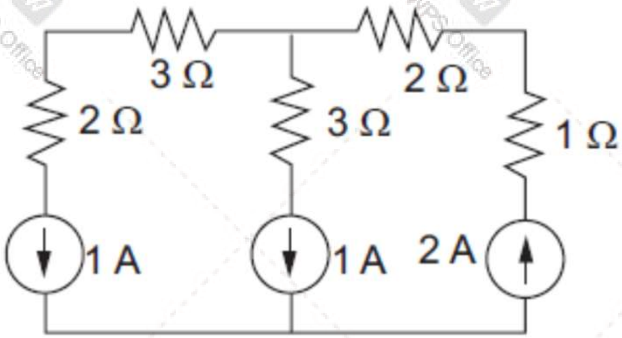
When the load circuit is a single resistor of value R_L then the condition for maximum power transfer reduces to $R_L = R_{Th}$ and the maximum power transferred will be $\frac{V_{oc}^2}{4R_L}$ W.

Conclusions From Basic Circuit Theorems

- These theorems are very useful since they are applicable to very large classes of practical circuits.
- The principal assumption underlying all these network theorems is the uniqueness of the solution for the network under consideration.
- The superposition theorem as well as Thevenin's theorem apply to all linear networks i.e. both time invariant and time varying whereas maximum transfer theorem is mainly applied to linear time invariant system.

A Question

Can the circuit be solved uniquely?



Lecture-7.2: AC Circuit Analysis

Classification of Two Terminal Elements

Lumped and Distributed Elements

An element is classified as a lumped element if the net effect of electrical phenomena taking place within that element can be described in terms of only its terminal voltage and current variables, irrespective of its internal details and geometry.

If the electrical description of an element calls for voltage and current variables that are functions of space variables over the element, the element is called as a distributed element.

Classification of Two Terminal Elements

Linear and Non-linear Elements

A two-terminal element is linear suppose its v-i relationship satisfies the principle of superposition.

- $v(t) = Ri(t) \quad v(t) = L\frac{di(t)}{dt} \quad v(t) = \frac{1}{C}\int\limits_{-\infty}^{t} i(t)dt$

A two-terminal element is non-linear suppose its v-i relationship does not satisfy the principle of superposition.

A two-terminal ideal independent voltage source is described by the relations $v(t) = E(t)$ (an independently specified function of time) and $i(t)$ is arbitrary, which is a non-linear relationship.

Classification of Two Terminal Elements

Bilateral and Non-bilateral Elements

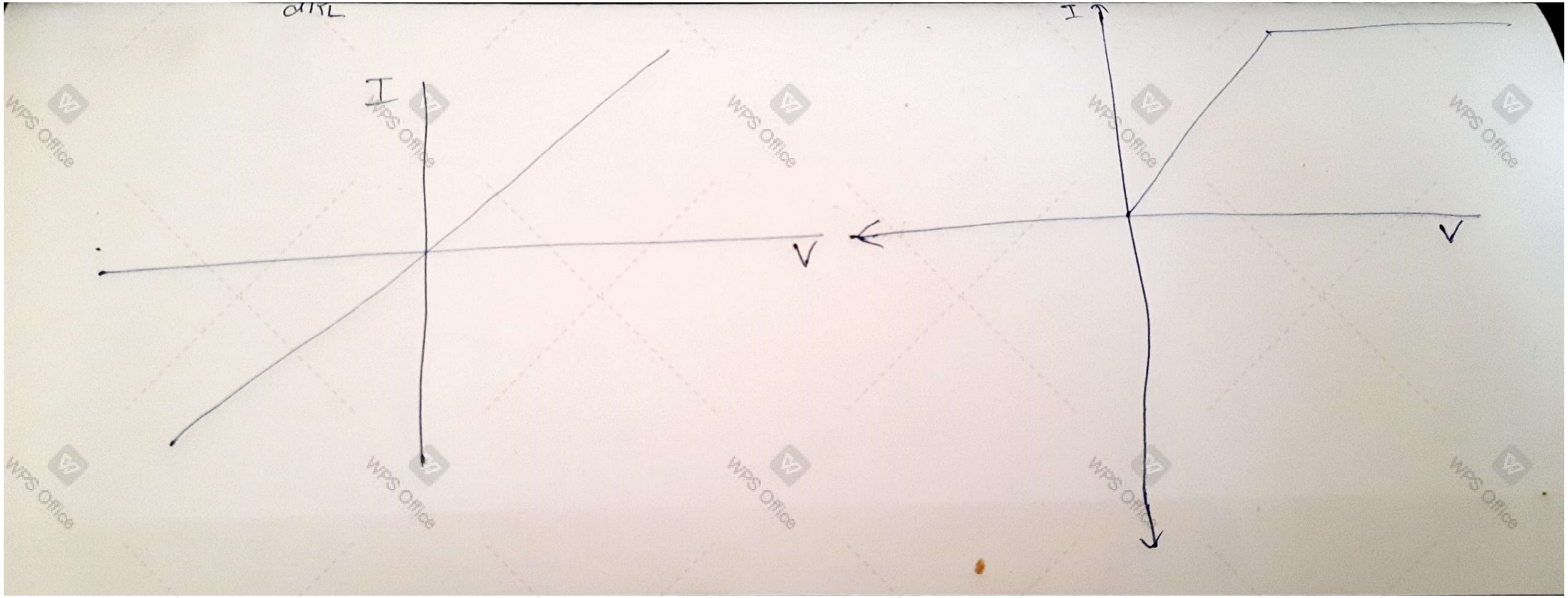
An element with a voltage-current relation that is odd-symmetric about the vertical axis in the v-i plane is called as a bilateral element.

A linear two-terminal element will always be bilateral.

An element with a voltage-current relation that is not odd-symmetric about the vertical axis in the v-i plane is called as a non-bilateral element.

A diode is a non-bilateral element.

Classification of Two Terminal Elements



Classification of Two Terminal Elements

Passive and Active Elements

An element is called a passive element suppose the energy delivered to it is always non-negative for any t and for any possible terminal voltage-current conditions of the device.

An element is called a active element suppose the energy delivered to it is not always non-negative for any t and for any possible terminal voltage-current conditions of the device.

Classification of Two Terminal Elements

Time-invariant and Time-variant elements

An element is time-invariant suppose the values of parameters that characterise it are independent of time.

An element is time-variant suppose the values of parameters that characterise it are dependent of time.

The Inductor

An inductor is a two-terminal device, which develops a voltage drop that consists of only induced voltage due to time-varying magnetic fields.

$\Phi(t) = Li(t) \rightarrow v(t) = L\frac{di(t)}{dt}$ [The voltage across inductor is proportional to the rate of change of current through it.]

$i(t) = \frac{1}{L}\int\limits_{-\infty}^{t} v(t)dt$ [The current through the inductor is proportional to the area under the voltage waveform]

Instantaneous current in an inductor cannot be predicted from instantaneous value of voltage across it.

$$i(t) = \frac{1}{L}\int\limits_{-\infty}^{t} v(t)dt = \underbrace{\frac{1}{L}\int_{-\infty}^{0} v(t)dt}_{I_0} + \frac{1}{L}\int\limits_{0}^{t} v(t)dt$$

I_0

$\frac{V\text{-}sec}{H} = A$

$I(t) = \frac{V(t)}{R}$

Superposition Property on Inductor

Checking for homogeneity

- $i(t) = I_0 + \frac{\alpha}{L}\int\limits_0^t v(t)dt \neq \alpha i(t)$

Checking for additivity

$$i_1(t) + i_2(t) = I_0 + \frac{1}{L}\int\limits_0^t v_1(t)dt + I_0 + \frac{1}{L}\int\limits_0^t v_2(t)dt \neq I_0 + \frac{1}{L}\int\limits_0^t (v_1(t) + v_2(t))dt$$

An inductor with zero initial current is a linear electrical element

Energy Storage in an Inductor

An arbitrary $v(t)$ is applied across an inductor from $t = 0$ causing a current $i(t)$ through it.

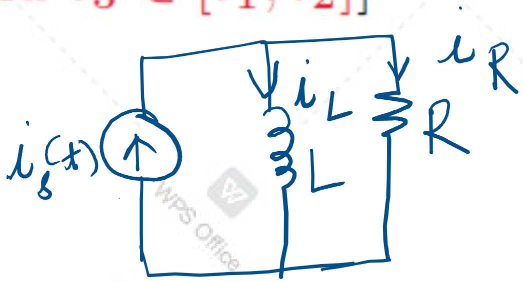
The source delivers power and energy to the inductor in this process.

$$\boxed{E_L(t) - E_L(0)} = \int_0^t \boxed{v(t)i(t)}\,dt = \qquad \boxed{\tfrac{1}{2} L\, i^2(t)}$$

$$L\int_0^t i(t)\frac{di(t)}{dt}dt = L\int_0^t i(t)di(t) = \frac{L}{2}\int_0^t d\left(i(t)\right)^2 = \boxed{\frac{L\left(i(t)^2 - i(0)^2\right)}{2}}$$

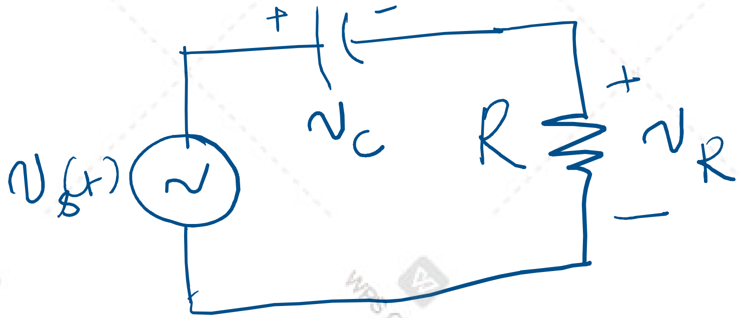
Abrupt Change of i_L Through L and v_C Across C

- $L \times \Delta i_L = \int_{t_1}^{t_2} v(t)dt = v(t_3)\Delta t$ [MVT and $t_3 \in [t_1, t_2]$]



$$v_L = L\frac{di_L(t)}{dt}$$

Current through an inductor cannot change abruptly.



- $C \times \Delta v_c = \int_{t_1}^{t_2} i(t)dt = i(t_3)\Delta t$

$$i_C = C\frac{dv_C(t)}{dt}$$

Voltage across a capacitor cannot change abruptly.

Response of a First Order Linear System

$\frac{dx(t)}{dt} + bx(t) = f(t)$ and let $x_p(t)$ satisfies this equation.

$\frac{dx(t)}{dt} + bx(t) = 0$ has $x_c(t)$ as its solution.

$$x(t) = x_p(t) + x_c(t)$$

$x_p(t)$ is called particular integral solution or forced response

$x_c(t)$ is called complementary solution or natural response

Suppose $f(t) = A$ then $x(t) = \frac{A}{b} + K_2 e^{-bt}$

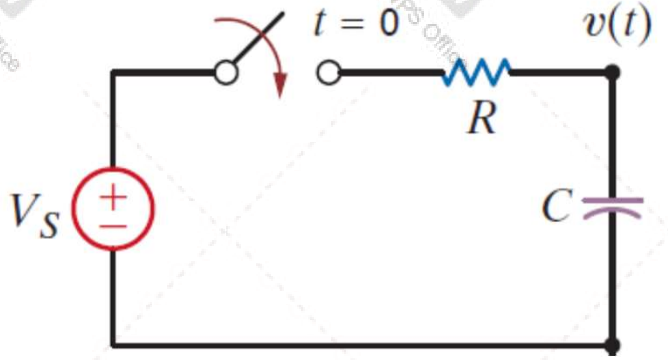
$K_1 = \frac{A}{b}$

$\tau = \frac{1}{b}$

The general form can be seen as $x(t)$

- $$x(t) = K_1 + K_2 e^{-t/\tau}$$

Series RC and RL circuits



At $t = 0$ the switch closes and for $t > 0$

- $C\frac{dv_c(t)}{dt} + \frac{v_c(t)-V_s}{R} = 0 \longrightarrow \frac{dv_c(t)}{dt} + \frac{v_c(t)}{RC} = \frac{V_s}{RC}$ where $A = \frac{V_s}{RC}, b = \frac{1}{RC}$

$$v_c(t) = K_1 + K_2 e^{-t/\tau} \qquad K_1 = \frac{A}{b} \rightarrow \frac{V_s}{RC} \div \frac{1}{RC} \rightarrow V_s \qquad \tau = \frac{1}{b} \rightarrow \tau = RC$$

$$v_c(t) = V_s + K_2 e^{-t/RC}$$

At $t = 0$

- $v_c(0) = V_s + K_2 \rightarrow K_2 = v_c(0) - V_s$

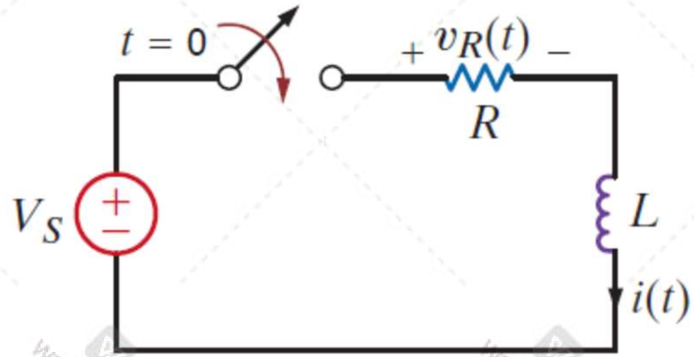
$$v_c(t) = \underbrace{V_s}_{\text{steady state}} + \underbrace{(v_c(0) - V_s)e^{-tRC}}_{\text{transient}}$$

$e^{-t/RC}$

$$= \underbrace{V_s\left(1 - e^{-t/RC}\right)}_{\text{ZSR}} + \underbrace{v_c(0)e^{-tRC}}_{\text{ZIR}}$$

$e^{-t/RC}$

Series RC and RL circuits



At $t = 0$ the switch closes and for $t > 0$

- $\frac{di_L(t)}{dt} + \frac{R}{L} i_L(t) = \frac{V_s}{L}$ where $A = \frac{V_s}{L}, b = \frac{R}{L}$

$i_L(t) = K_1 + K_2 e^{-t/\tau}$ where $K_1 = \frac{V_s}{R}, \tau = \frac{L}{R}$

to get $i_L(t) = \frac{V_s}{R} + K_2 e^{-t\frac{R}{L}}$

At $t = 0$

- $i_L(0) = \frac{V_s}{R} + K_2 \rightarrow K_2 = i_L(0) - \frac{V_s}{R}$

$$i_L(t) = \underbrace{\frac{V_s}{R}}_{\text{steady state}} + \underbrace{\left(i_L(0) - \frac{V_s}{R}\right) e^{-t\frac{R}{L}}}_{\text{transient}}$$

$$= \underbrace{\frac{V_s}{R}\left(1 - e^{-t\frac{R}{L}}\right)}_{\text{ZSR}} + \underbrace{i_L(0) e^{-t\frac{R}{L}}}_{\text{ZIR}}$$

Conclusions From Lecture 8

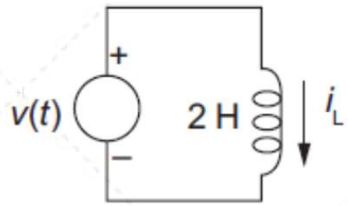
- Circuits containing energy storage elements have memory in time-domain. They can be described by linear ordinary differential equations with constant coefficients.
- We focussed on series RC and RL circuits in today's discussion. The dynamics of both RC and RL circuits are described by a first order linear differential equation.
- The past history of the capacitor charging(inductor charging) is contained in a single initial condition specification in an RC (RL) circuit.
- The solution of the dynamic equation that is characterised by the first order linear differential equation can be seen as total response = steady state response+ transientresponse as well as total response = ZSR+ZIR where ZSR corresponds to the particular integral value and ZIR corresponds to the complementary solution.

Lecture-8.1 : AC Circuit Analysis

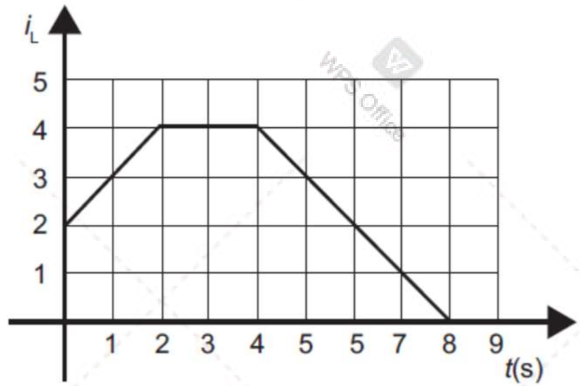
C, L, RL, RC and RLC Circuits

Examples in Inductor Charging

The current in an inductor of 2H is as shown in the diagram.

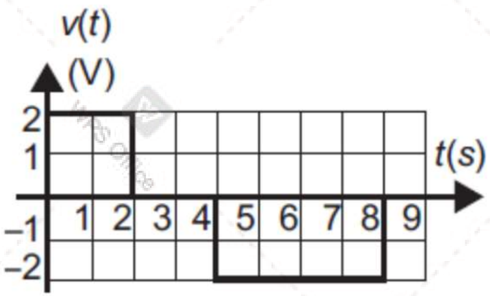


Plot $v(t), p(t)$ and $E(t)$ from 0 sec to 9 sec.

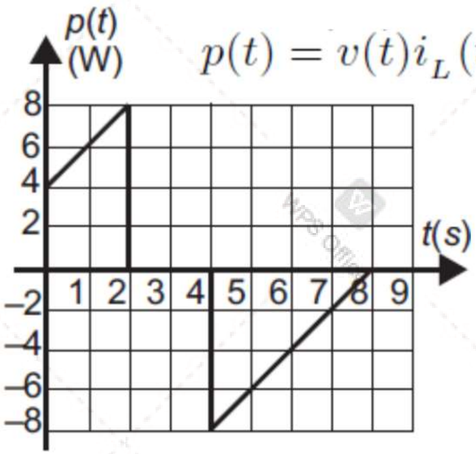


$v(t) = L\frac{di(t)}{dt}$ to get

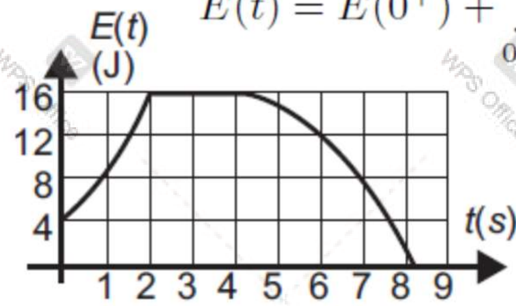
$$v(t)|_{(0,2)} = 2V \quad v(t)|_{(2,4)} = 0V \quad v(t)|_{(4,8)} = -2V$$



$p(t) = v(t)i_L(t)$

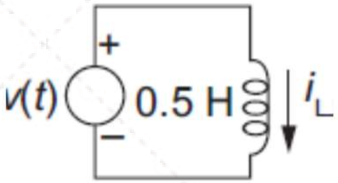


$$E(t) = E(0^+) + \int_{0^+}^{t} p(t)dt$$



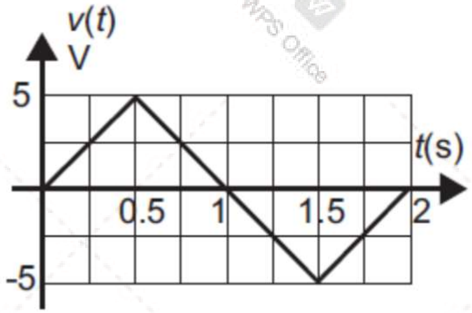
Examples in Inductor Charging

The pulse voltage waveform as shown is applied to an inductor of 0.5 H with initial current I_0 of 1 A.



Find the inductor current and energy stored at $t = 1$ s, $t = 2$ s and $t = 3$ s.

$E(t) = \frac{1}{2} L i^2(t)$



$$i(t) = I_0 + \frac{1}{L}\int_0^t v(t)dt \longrightarrow \quad i(1) = 6A \quad i(2) = I_0 = 1A \quad i(3) = I_0 = 1A$$

What is the net energy delivered to the inductance in the time interval [0, 1],[1, 2] and [0, 2] seconds?

$$E(t_2) - E(t_1) = \frac{1}{2}L\left(i^2(t_2) - i^2(t_1)\right) \longrightarrow$$

$$E(1) - E(0) = 9 - 0.25 = 8.75J$$
$$E(2) - E(1) = 0.25 - 9 = -8.75J$$
$$E(2) - E(0) = 0.25 - 0.25 = 0$$

The Capacitor

A capacitor is a two-terminal device, which will have a current proportional to the rate of change of voltage across it.

$$Q(t) = Cv(t) \rightarrow i(t) = C\frac{dv(t)}{dt}$$

$v(t) = \frac{1}{C}\int\limits_{-\infty}^{t} i(t)dt$ [The voltage across the capacitor is proportional to the area under the current waveform]

Instantaneous voltage across a capacitor cannot be predicted from instantaneous value of current through it.

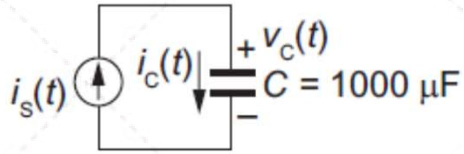
$$v(t) = \frac{1}{C}\int\limits_{-\infty}^{t} i(t)dt = \underbrace{\frac{1}{C}\int\limits_{-\infty}^{0} i(t)dt}_{V_0} + \frac{1}{C}\int\limits_{0}^{t} i(t)dt$$

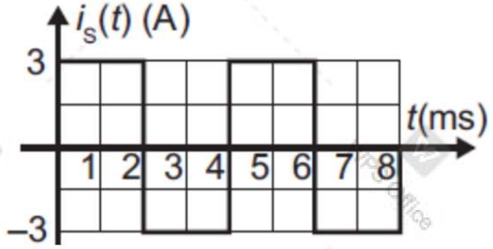
The energy delivered to a capacitor by a source at t is

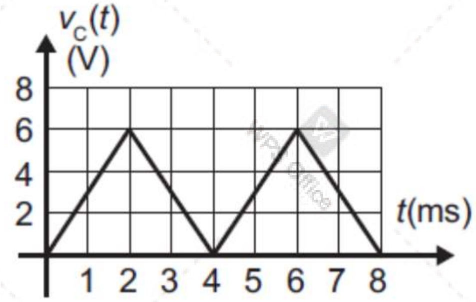
- $E_C(t) = \int\limits_{0}^{t} v(t)i(t)dt = L\int\limits_{0}^{t} v(t)\frac{dv(t)}{dt}dt = C\int\limits_{0}^{t} v(t)dv(t) = \frac{C}{2}\int\limits_{0}^{t} d\left(v(t)\right)^2 = \frac{C(v(t))^2}{2}$

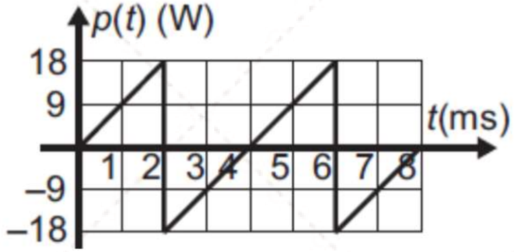
Examples in Capacitor Charging

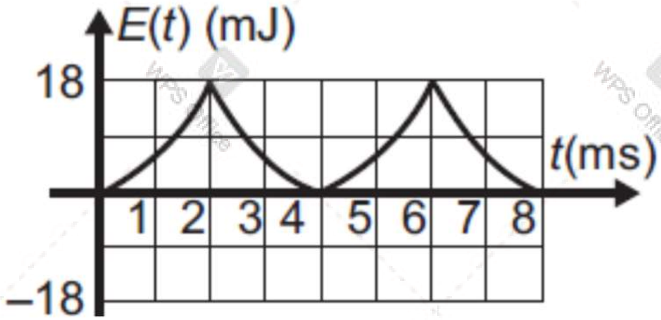
The voltage across a $1000\mu F$ capacitor with zero initial voltage is shown here.







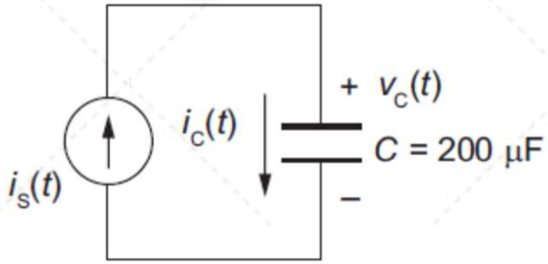




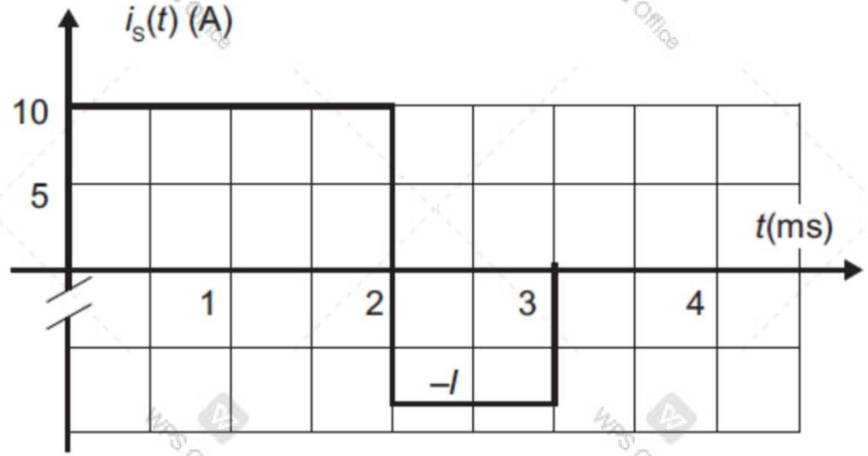
Examples in Capacitor Charging

Initial voltage at $t = 0$ across the $200\mu F$ capacitor is $-50\,\text{V}$.

Find the voltage across and energy stored in it at 2 ms.



$$v_c(2) = V_0 + \frac{1}{200\times 10^{-6}} \times \int_0^{2} 10\,dt = -50 + 100 = 50$$



Lecture-9.1: AC circuit Analysis-2

RL and RC Circuits

Step-by-step Approach to Solve First Order Linear Circuits for $f(t) = A$

Step 1- Assume a solution for the variable $x(t)$ of the form
$x(t) = K_1 + K_2 e^{-t/\tau}$

Step 2- Draw the circuit at $t = 0-$ with the capacitor replaced by an open circuit or the inductor replaced by a short circuit. Solve for the voltage across the capacitor, $v_C(0-)$ or the current through the inductor, $i_L(0-)$ prior to switch action.

Step 3- Draw the circuit valid for with the switches in their new positions. Replace a capacitor with a voltage source $v_C(0+) = v_C(0-)$ or an inductor with a current source of value $i_L(0+) = i_L(0-)$. Solve for the initial value of the variable $x(0+)$.

Step-by-step Approach to Solve First Order Linear Circuits for $f(t) = A$

Step 4- Draw the steady state equivalent of the circuit by replacing the capacitor by an open circuit or the inductor by a short circuit to get $x(\infty)$.

Step 5- To obtain the time constant τ reduce the entire circuit to a simple series circuit containing a voltage source, resistor, and a storage element (i.e.capacitor or inductor) by forming a simple Thevenin equivalent circuit at the terminals of the storage element to get $\tau = R_{Th}C$ and for a circuit containing an inductor it is $\tau = \frac{L}{R_{Th}}$.

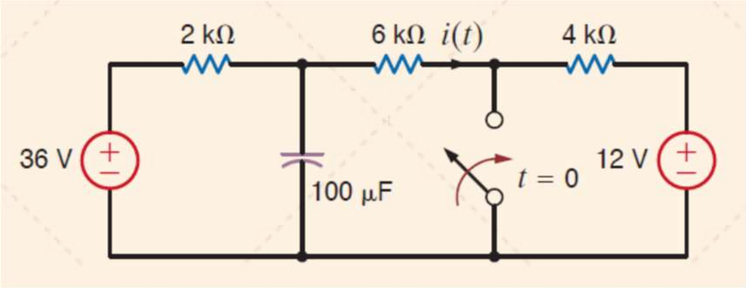
Step 6- Using the results of steps 3, 4, and 5, we can evaluate the constants in step 1 as

$$x(0+) = K_1 + K_2$$

$$x(\infty) = K_1 \rightarrow K_2 = x(0+) - x(\infty)$$

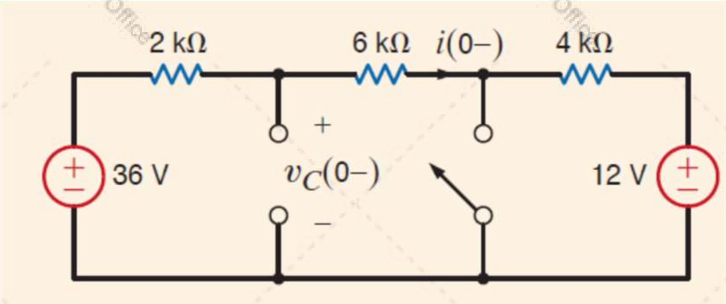
$$x(t) = x(\infty) + [x(0+) - x(\infty)]e^{-t/\tau}$$

Examples of First Order Linear Circuits

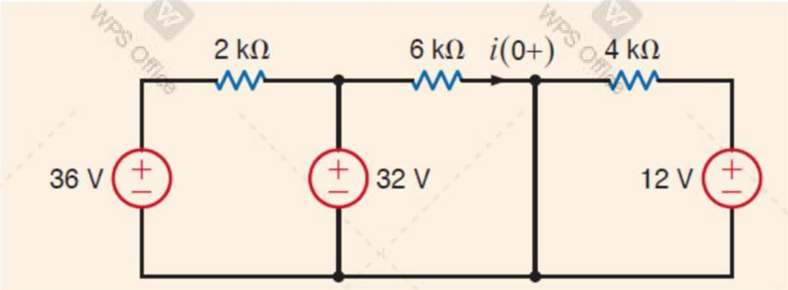


step 1- The solution in the canonical form. $i(t) = K_1 + K_2 e^{-t/\tau}$
[Alternatively $i(t) = i(\infty) + (i(0+) - i(\infty))\, e^{-t/\tau}$]

step 2- Find the initial condion. $v_C(0-) = \frac{36\times10+12\times2}{12} = 32\text{V}$



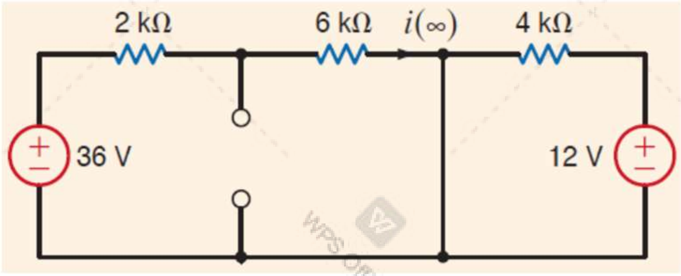
step 3- Find the transient response. Since $v_C(0-) = v_C(0+)$,



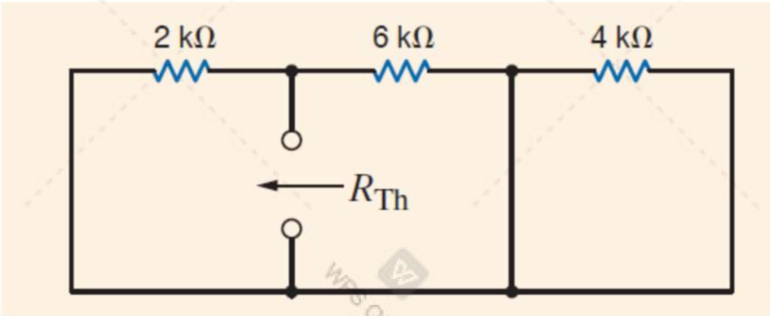
$$i(0+) = \frac{32}{6K} = \frac{16}{3}\text{mA}$$

Examples of First Order Linear Circuits

step 4- Find the steady state response. $i(\infty) = \frac{36}{2k+6k} = \frac{9}{2}\text{mA}$



step 5- Find the time constant $\tau = R_{Th}C$.

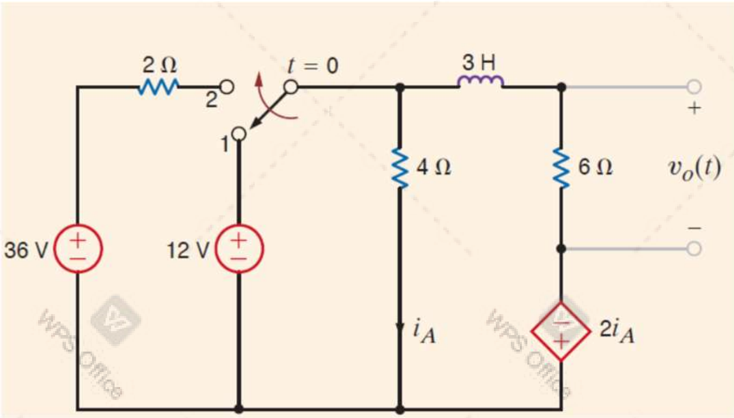


$R_{Th} = \frac{2k \times 6k}{2k+6k} = \frac{3}{2}k$ and $\tau = R_{Th}C = 0.15\text{sec.}$

step 6- Total response

$$i(t) = i(\infty) + (i(0+) - i(\infty))\, e^{-t/\tau} = \frac{9}{2} + (\frac{16}{3} - \frac{9}{2})e^{-t/0.15}\text{mA}$$

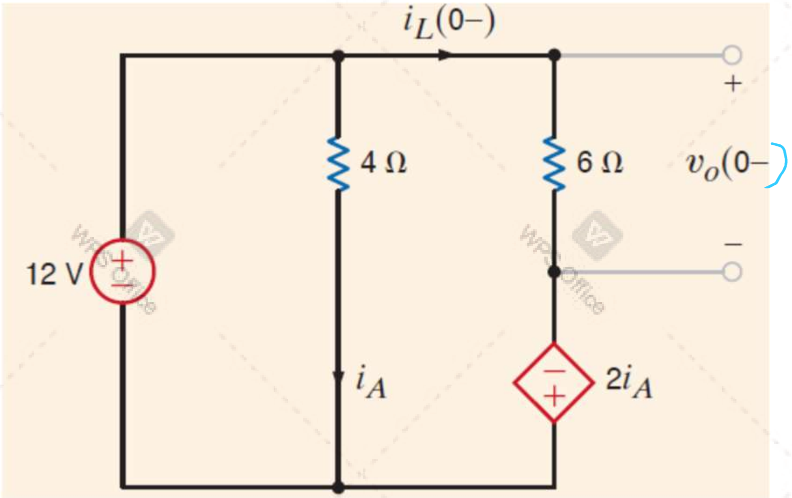
Examples of First Order Linear Circuits



step 1- The solution in the canonical form. $v(t) = K_1 + K_2 e^{-t/\tau}$

[Alternatively $v(t) = v(\infty) + (v(0+) - v(\infty))\, e^{-t/\tau}$]

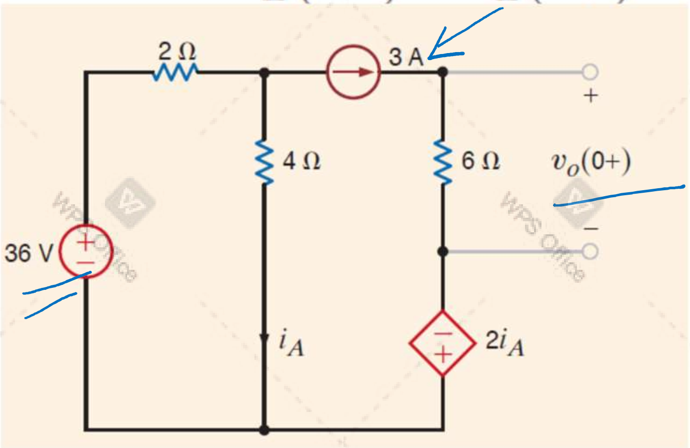
step 2- Find the initial condion



to get $i_L(0-) = \frac{12+2i_A}{6} = 3\text{A}$

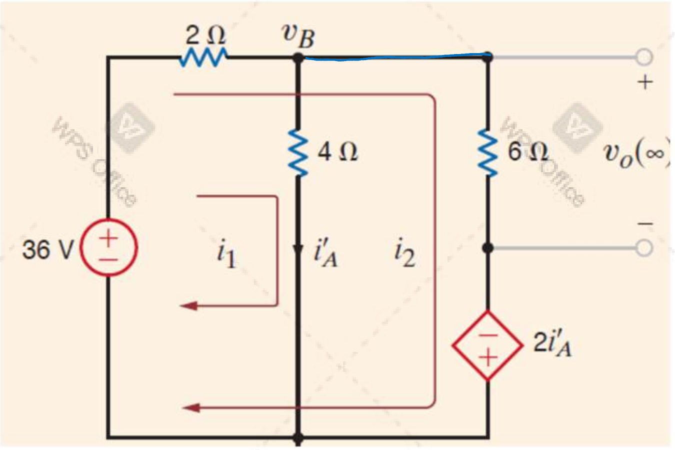
Examples of First Order Linear Circuits

step 3- Find a part of transient response. Since $i_L(0-) = i_L(0+) = 3\text{A}$ to get $v(0+) = \boxed{3 \times 6 = 18\text{V}}$



Home work - 2.

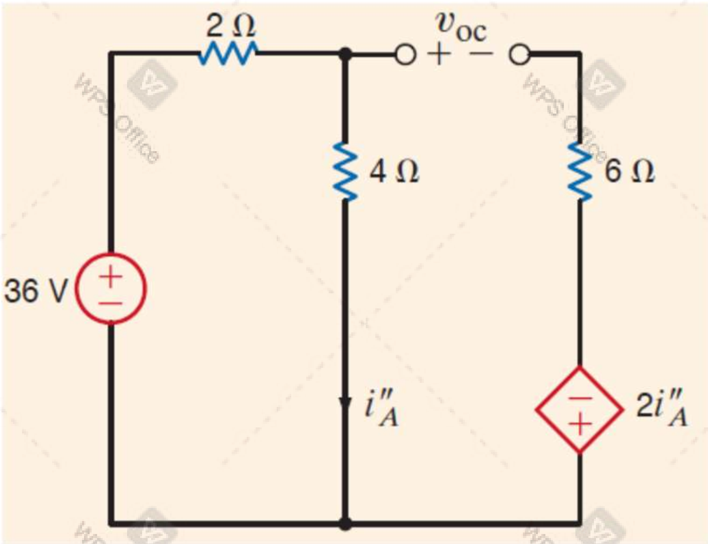
step 4- Find the steady state response. $\boxed{v(\infty) = 27\text{V}}$

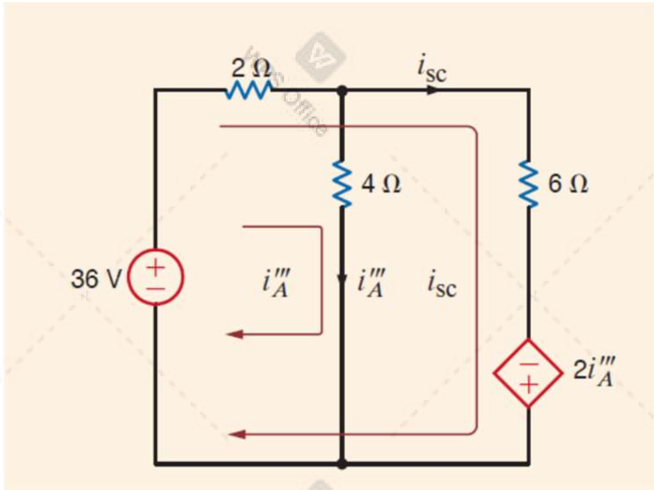


Homework- 3.

Examples of First Order Linear Circuits

step 5- Find the time constant. Since there is a dependent source $R_{Th} = \frac{V_{oc}}{I_{sc}} = 8\Omega$ to get $\tau = \frac{L}{R_{Th}} = \frac{8}{3}$sec.





step 6- Total response

$$v(t) = v(\infty) + (v(0+) - v(\infty))\, e^{-t/\tau} = 27 + (18 - 27)e^{-\frac{8}{3}t}\text{mA}$$

Compendium of the Discussion on First Order Linear Circuits

- Circuits containing energy storage elements have memory in time-domain and their dynamics can be characterised by linear ordinary differential equations with constant coefficients.
- To solve these circuits two factors should be known beforehand
  - time instant at which the forcing function is connected to the circuits
  - the initial conditions in the circuits.
- The total solution can expressed as
  - steady state response+transient response
  - ZSR+ZIR
- ZIR of a linear time-invariant circuit is independent of the forcing function and depends only on circuit parameters and nature of interconnections.
- The transient response largely depends on the time constant that is also a function of circuit parameters and the nature of interconnections. The smaller value of time constant indicates that the transient response decays faster. The value of the time constant can be found by determining the Thevenin equivalent resistance at the terminals of the storage element.
- The current in an RL circuit at $t = 0-$ and $t = 0+$(or voltage in ~~an~~ the RC circuits at $t = 0-$ and $t = 0+$) will be the same suppose the circuit does not contain impulse source.

Lecture-9.2: AC Circuit Analysis

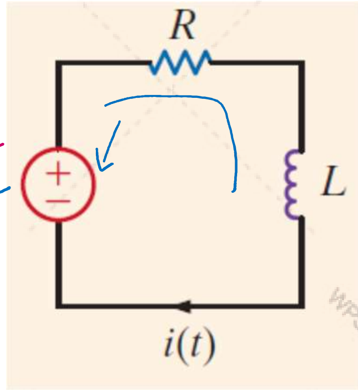
Sinusoidal Steady-State Analysis

Sinusoidal Forcing Functions

A, ω, ϕ

$$L\frac{di}{dt} + Ri(t) = V_m \cos \omega t$$

$V_m \cos \omega t$



$$i(t) = \frac{RV_m}{R^2 + \omega^2 L^2} \cos \omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin \omega t$$

$$= A \cos(\omega t + \phi)$$

$R > 0, \; L > 0$

where $A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$ and $\phi = -\tan^{-1} \frac{\omega L}{R}$

$0 < \phi < \frac{\pi}{2}$

$$\boxed{i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1} \frac{\omega L}{R}\right)}$$

Complex Forcing Function

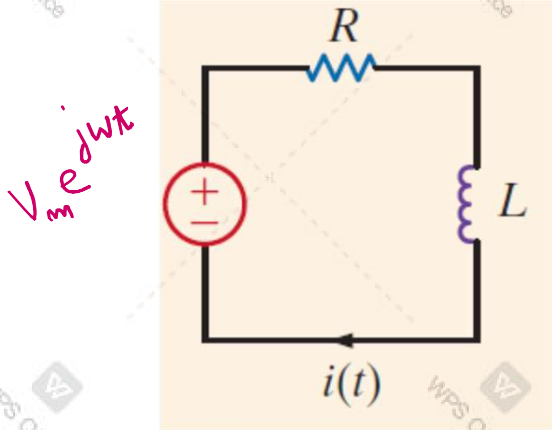
- Euler's identity
  - $e^{j\omega t} = \cos\omega t + j\sin\omega t$

- Suppose $v(t) = V_m e^{j\omega t} = V_m \cos\omega t + jV_m \sin\omega t$

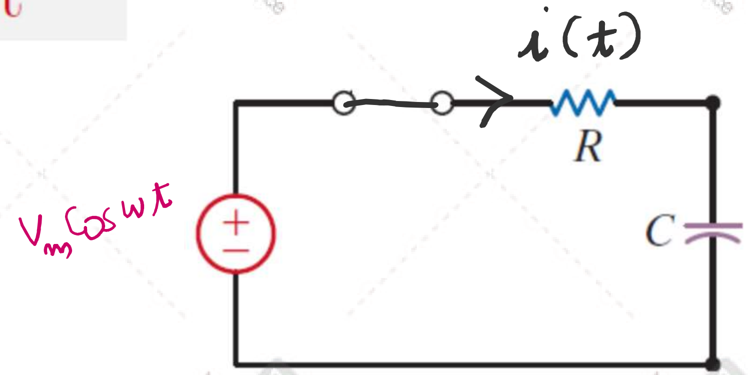
$$i(t) = I_m \cos(\omega t + \phi) + jI_m \sin(\omega t + \phi)$$
$$= I_m e^{(j\omega t + \phi)}$$

$$RI_m e^{(j\omega t+\phi)} + L\frac{d}{dt}\left(I_m e^{(j\omega t+\phi)}\right) = V_m e^{j\omega t}$$ to get
$$RI_m e^{(j\omega t+\phi)} + j\omega L I_m e^{(j\omega t+\phi)} = V_m e^{j\omega t}$$

$$I_m e^{j\phi} = \frac{V_m}{R + j\omega L}$$
$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} e^{j\left(-tan^{-1}\frac{\omega L}{R}\right)}$$



In Series RC circuit

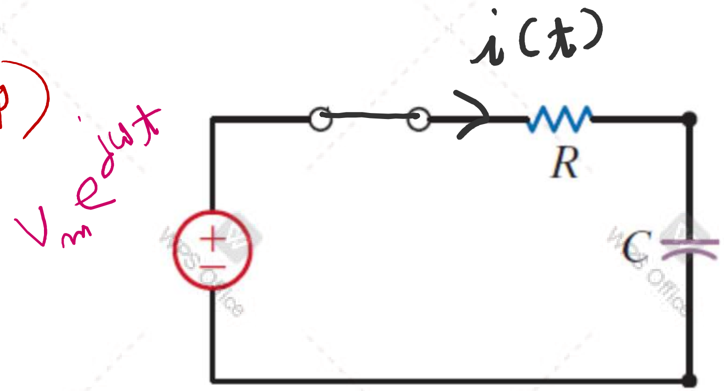


Input is a Sinusoidal Function

- $Ri(t) + \frac{1}{C}\int i(t)dt = V_m \cos \omega t$
- $R\frac{di(t)}{dt} + \frac{i(t)}{C} = -V_m \omega \sin \omega t$
- $i(t) = \frac{V_m}{\sqrt{R^2+\left(\frac{1}{\omega c}\right)^2}} \cos\left(\omega t + \tan^{-1}\frac{1}{\omega RC}\right)$

- Input is a complex exponential function

$i(t) = I_m e^{j(\omega t + \phi)}$



- $$I_m e^{j\phi} = \frac{V_m}{R + \frac{1}{j\omega C}}$$
  $$= \frac{V_m}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} e^{j\left(\tan^{-1}\frac{1}{\omega RC}\right)}$$

Impedance

- Impedance is defined as the ratio of the phasor voltage $\boldsymbol{V}$ to the phasor current $\boldsymbol{I}$.

$$\boldsymbol{Z} = \frac{\mathbf{V}}{\boldsymbol{I}} = \frac{V_m\angle\theta_v}{I_m\angle\theta_i} = \frac{V_m}{I_m}\underline{/\theta_v - \theta_i} = Z\underline{/\theta_z}$$

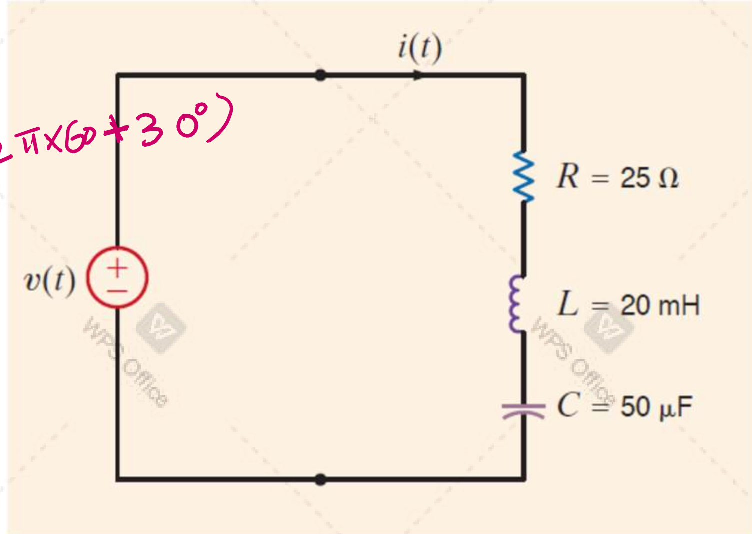
$$\boldsymbol{Z}(\omega) = R + jX(\omega)$$

  - $Z = \sqrt{R^2 + X^2}$ and $\theta_z = \tan^{-1}\frac{X}{R}$

Impedance

$$\boldsymbol{Z}_R = 25\Omega$$
$$\boldsymbol{Z}_L = j\omega L = j(2\pi \times 60)(20 \times 10^{-3}) = j7.54\Omega$$
$$\boldsymbol{Z}_C = \frac{-j}{\omega c} = \frac{-j}{(2\pi\times 60)(50\times 10^{-6})} = -j53.05\Omega$$

v(t)=50 cos (2π×60+30°)



$$\boldsymbol{Z} = \boldsymbol{Z}_R + \boldsymbol{Z}_L + \boldsymbol{Z}_C = 25 - j45.51\Omega$$

$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}} = \frac{50\angle 30^\circ}{25 - j45.51} = \frac{50\angle 30^\circ}{51.93\angle -61.22^\circ} = 0.96\underline{/91.22^\circ}\,\mathrm{A}$$
$$\boldsymbol{I} = 0.96\cos(377t + 91.22^\circ)$$

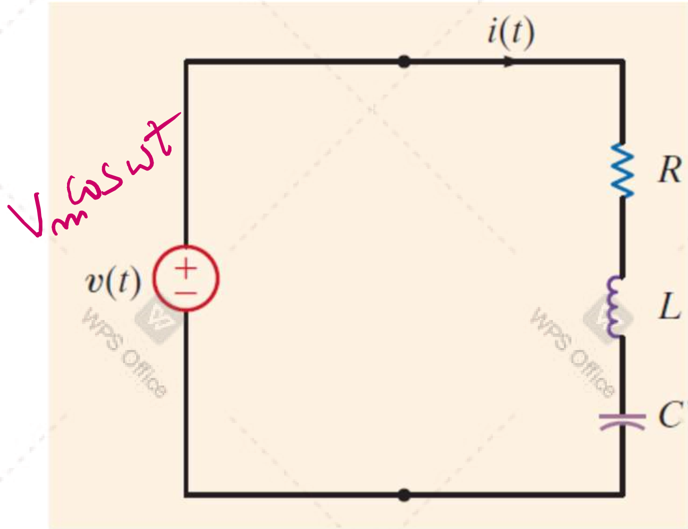
Series RLC Circuit

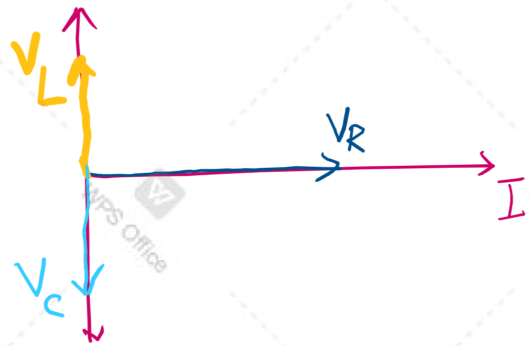
$$\boldsymbol{I} = \frac{\boldsymbol{V}}{\boldsymbol{Z}}$$

$$v(t) = V_m \cos \omega t \text{ and } i(t) = \frac{V_m \cos \omega t}{\boldsymbol{Z}}$$

$$\begin{aligned}\boldsymbol{Z} &= R + j\omega L - j\frac{1}{\omega C}\\ &= \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \angle \tan^{-1}\left(\frac{1}{\omega C R} - \frac{\omega L}{R}\right)\end{aligned}$$

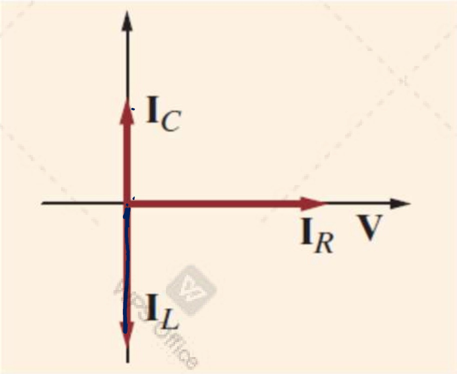
$$i(t) = \frac{V_m}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \cos\left(\omega t + \tan^{-1}\left(\frac{1}{\omega C R} - \frac{\omega L}{R}\right)\right)$$

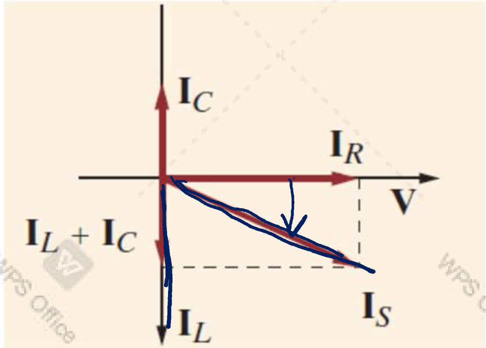


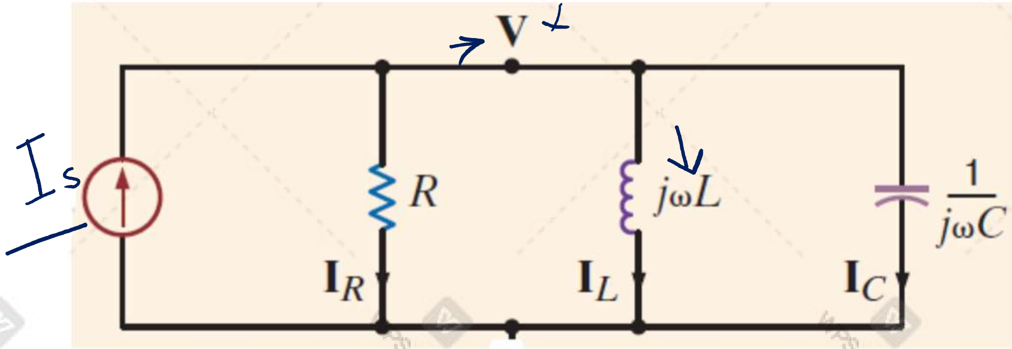


$$v(t) = i(t)R + i(t)j\omega L + \frac{i(t)}{j\omega C}$$

Parallel RLC Circuit





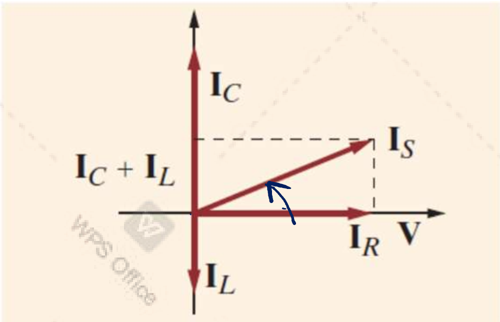


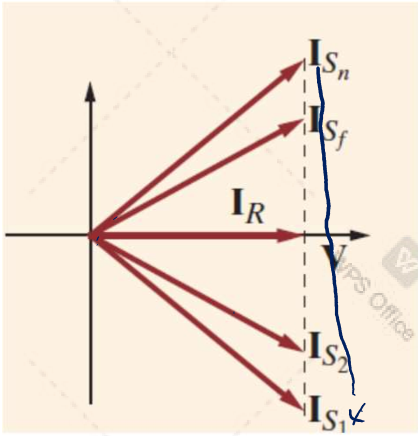
$$I_s = I_R + I_L + I_C$$

$$= \frac{V}{R} + \frac{V}{j\omega L} + j\omega C \times V$$

ω

$\omega \to 0 \to \infty$



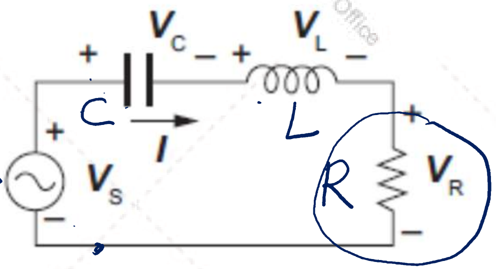


Lecture-10.1: AC Circuit Analysis

RLC Circuits

Series Resonant Circuits

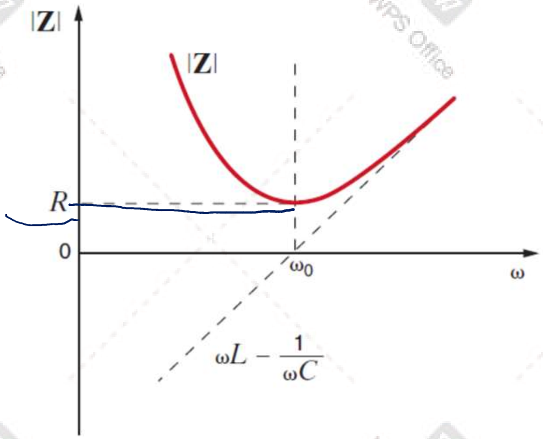
$\omega = \omega_0$

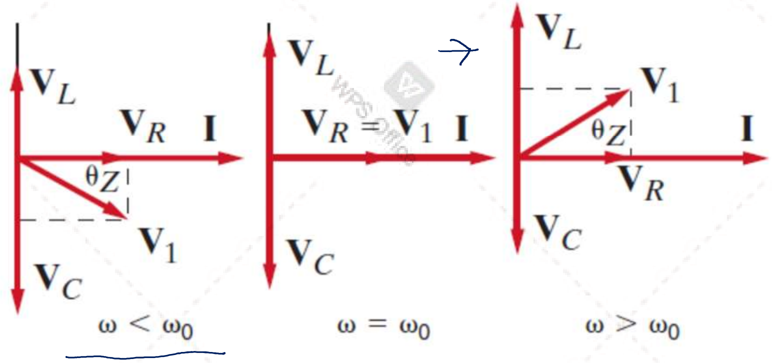


$$\boldsymbol{Z}(j\omega) = R + j\omega L - j\frac{1}{\omega C}$$

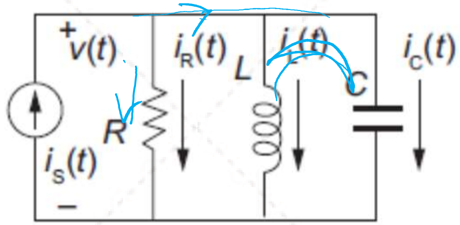
The impedance would become purely resistive when $\omega L = \frac{1}{\omega C}$ and the value of ω that satisfies $\omega_0 = \frac{1}{\sqrt{LC}}$

- $\boldsymbol{Z}(j\omega_0) = R$





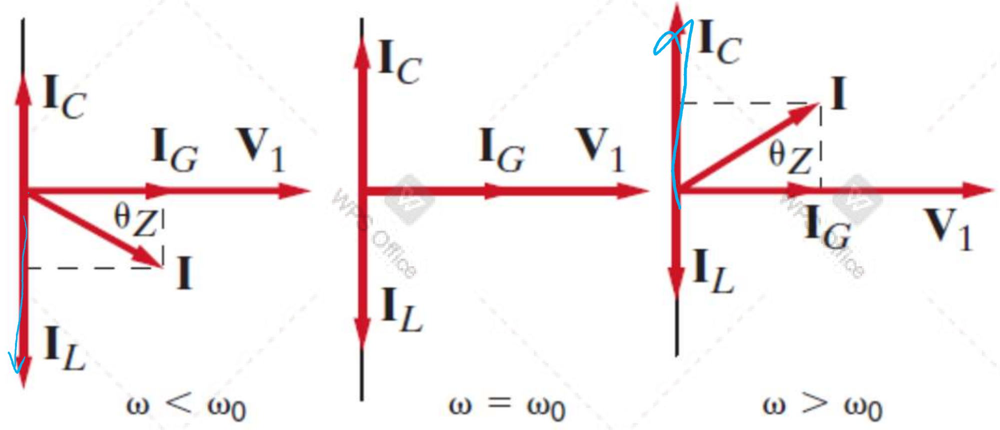
Parallel Resonant Circuits

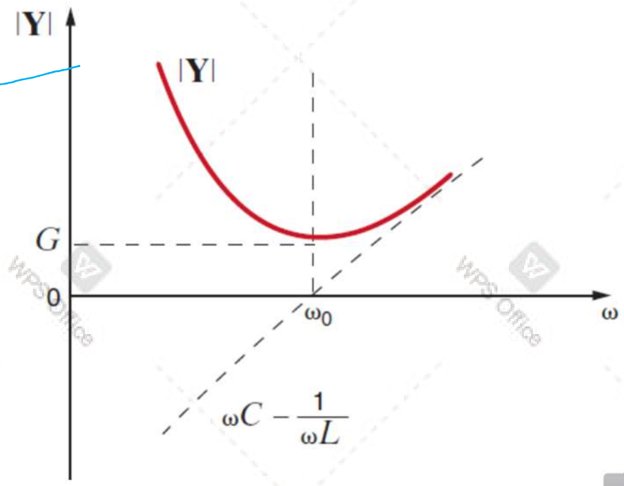


$$Y(j\omega) = G + j\omega C - j\frac{1}{\omega L}$$

The admittance would become purely conductance when $\omega C = \frac{1}{\omega L}$ and the value of ω that satisfies $\omega_0 = \frac{1}{\sqrt{LC}}$

- $Y(j\omega_0) = G$





Conclusions From the Discussions on Sinusoidal Steady State Analysis

- Sinusoidal steadystate in a dynamic circuit is a state when all the response variables contain only one component with a sinusoidal waveshape that has the same frequency as that of the sinusoidal forcing function applied. The response will, in general, have a phase difference with respect to input.
- The ratio of voltage phasor to current phasor of an element is called its phasor impedance.
- In an RLC circuit excited by a single sinusoidal voltage source (current source) across a pair of terminals, resonance is the sinusoidal steady-state condition under which the current drawn at the terminals (voltage appearing across the terminals) is in phase with the source voltage (current).
- Both series and parallel RLC circuit are resonant at $\omega_0 = \frac{1}{\sqrt{LC}}$. At that frequency, the impedance of the series RLC circuit is a minimum and is resistive. On the other hand in a parallel RLC circuit the impedance of the circuit is a maximum and is also resistive.

The Sequel

- The nodal analysis and mesh analysis techniques developed for memoryless circuits apply to the phasor equivalent circuits with no change, except that impedance Z takes the place of resistance R and admittance Y takes the place of conductance G. All circuit theorems, except the maximum power transfer theorem, apply to phasor equivalent circuits without modification.

Lecture-10.2: AC Circuit Analysis

Power Energy Relations

Instantaneous Power

The instantaneous power delivered to a two-terminal element is given by $p(t) = v(t)i(t)$, where $v(t)$ and $i(t)$ are the voltage across the element and current through the element as per passive sign convention.

$$p(t) = \frac{dE(t)}{dt} = v(t)i(t)\mathrm{W} \quad E(t) = \int_{-\infty}^{t} p(t)dt = \int_{-\infty}^{t} v(t)i(t)dt\mathrm{J}$$

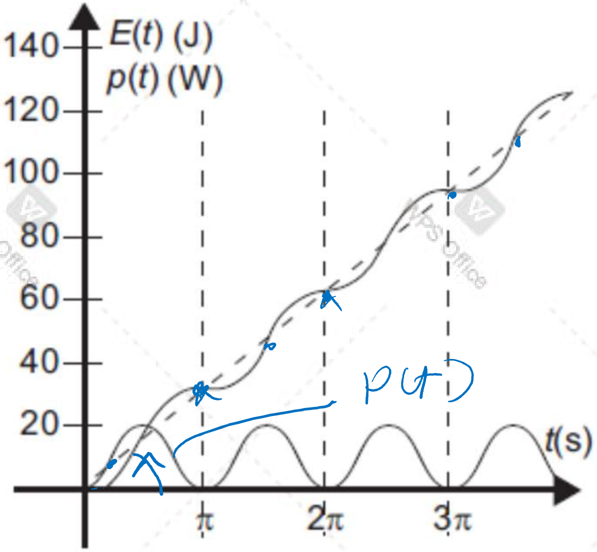
Instantaneous Power

$\int_0^T v(t)dt = 0$

Let $v(t) = 10\sqrt{2}\sin t$V is applied to a 10Ω resistor from $t = 0$ onwards. Plot $p(t)$ and $E(t)$ for $t \geqslant 0$.

$$p(t) = v(t)i(t) = \frac{v^2(t)}{R} = 20\sin^2 t = 10 - 10\cos 2t$$

$$E(t) = \int_0^t (10 - 10\cos 2t)dt = 10t - 5\sin 2t$$



$$\boxed{\text{Average power over a cycle} = \frac{\int_0^\pi (10-10\cos 2t)dt}{\pi} = 10\text{W}}$$

Cycle Average Power and Average Power

- The Cycle Average Power in the context of periodic waveforms is defined as the cycle average of instantaneous power over a cycle of instantaneous power and is denoted by P.

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{2}{T}\int_0^{0.5T} v(t)i(t)\,dt$$

- The Average Power P_{av} delivered during the interval $[t_1, t_2]$ is the value of constant power that would have delivered the same amount of energy to the element in the interval between t_1 to t_2.

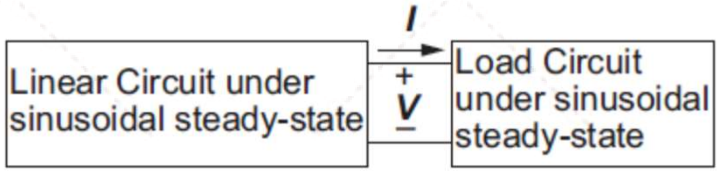
  - $P_{av} = \frac{1}{(t_1 - t_2)} \int_{t_1}^{t_2} v(t)i(t)dt$

Effective Value of Periodic Waveforms

- The effectiveness of a given voltage or current waveform by calculating the cyclic average power that will be delivered to a resistance R and comparing it with that of a DC voltage or current, which will result in same average power in R.
  $P = \frac{1}{T}\int\limits_0^T x^2(t)dt = (X_{\text{rms}})^2$
- For sinusoidal waveforms $V_{\text{rms}} = \sqrt{\frac{2\pi}{\omega}\int\limits_0^{\frac{2\pi}{\omega}} V_m^2 \sin^2 \omega t dt} = \frac{V_m}{\sqrt{2}}$
- Let $v(t) = v_1(t) + v_2(t) + \ldots + v_n(t)$ be a composite waveform comprising n distinct frequency sinusoidal waveforms.
  $V_{\text{rms}}^2 = V_{1_{\text{rms}}}^2 + V_{2_{\text{rms}}}^2 + \ldots + V_{n_{\text{rms}}}^2$

Maximum Power Transfer Theorem for Sinusoidal Steady-State Condition

$\boldsymbol{V_{oc}} = V_{ocm}\angle 0^\circ, \boldsymbol{I} = I_m\angle\phi, \boldsymbol{Z} = Z\angle\theta$



$$v_{oc}(t) = V_{ocm}\cos\omega t$$
$$i(t) = I_m\cos(\omega t + \phi)$$
$$v(t) = V_{ocm}\cos\omega t - ZI_m\cos(\omega t + \phi + \theta)$$

The average power delivered to the load circuit is the cycle average of $v(t) \times i(t)$. $P = \frac{2\pi}{\omega}\int\limits_0^{\frac{2\pi}{\omega}} v(t)i(t)dt$ to get $0.5V_{ocm}I_m\cos\phi - 0.5ZI_m^2\cos\theta$

Maximum Power Transfer Theorem for Sinusoidal Steady-State Condition

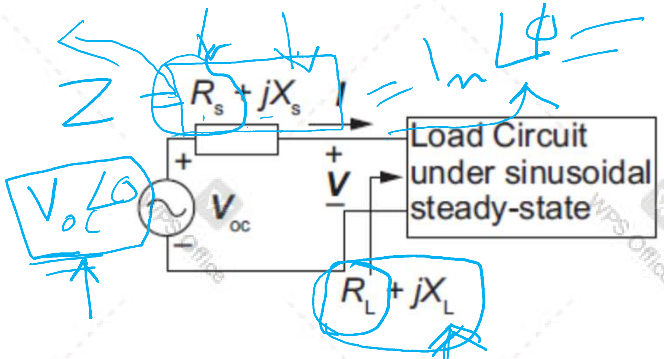
$$\boxed{P_L = 0.5V_{ocm}I_m\cos\phi - 0.5ZI_m^2\cos\theta}$$

$$\frac{\partial P_L}{\partial I_m} = 0 \Rightarrow \boxed{0.5V_{ocm}\cos\phi - ZI_m\cos\theta = 0}$$

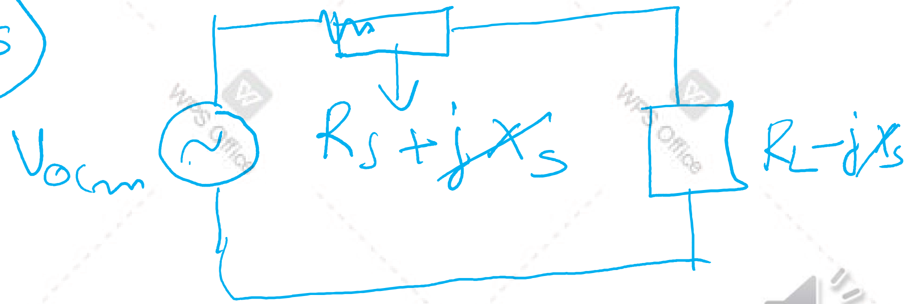
$$\frac{\partial P_L}{\partial \phi} = 0 \Rightarrow 0.5V_{ocm}I_m\sin\phi = 0 \Rightarrow \phi = 0$$

$$I_m = \frac{V_{ocm}}{2Z\cos\theta} = \frac{V_{ocm}}{2R_s}$$

$$R_L = R_s$$



$$-jX_L = +jX_s$$



Maximum Power Transfer Theorem for Sinusoidal Steady-State Condition

Maximum average power is transferred to a load circuit from a power delivery circuit under the sinusoidal steady-state when the load impedance, $\boldsymbol{Z}_L = R_L + jX_L$, is the conjugate of Thevenin's impedance, $\boldsymbol{Z}_S = R_S + jX_S$ of the power delivery circuit. Therefore $R_L = R_S$ and $X_L = -X_S$ are the required conditions. The maximum average power transferred under this condition will be $P_{L\,\max} = \frac{V_{ocm}^2}{4R_S} = \frac{V_{rms}^2}{2R_S}$.

Complex Power, Apprent Power and Power Factor

Apparent Power carried by a sinusoidal voltage of rms value V_{rms} and a sinusoidal current of rms value I_{rms} is defined as the actual power that will be carried by a DC voltage of the same effective value and a DC current of the same effective value and it is expressed as in the unit of VA.

- Apprent Power $= V_{rms} \times I_{rms}$.

The component in the direction of voltage phasor is the in-phase component and this component will carry active power.

- Active Power $= V_{rms} \times I_{rms} \cos\theta$.

- Power factor $= \frac{\text{Active Power}}{\text{Apprent Power}} = \cos\theta$
- Complex Power$= V_{rms} \times I^*_{rms}$

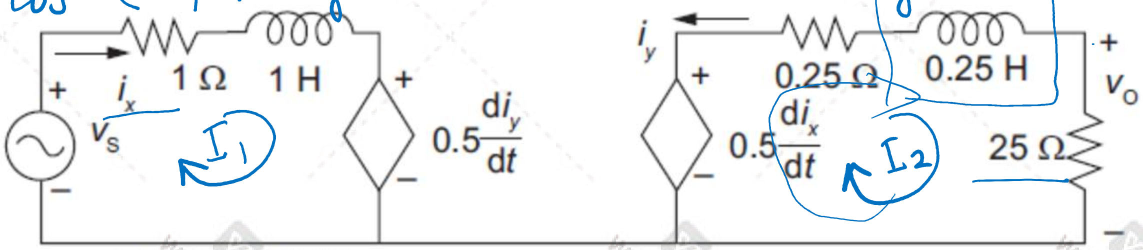
Example Circuit

$\omega = 100$

$= 0.5 \times 300 \times 4.18 \times \cos(-44.72)$

$= 445.4\,W$

$\frac{d}{dt} \equiv j\omega$



$v_s = 300 \cos 100t$ V

$0.5 \times j \times 100$

$$-300\angle 0^\circ + I_1 + j100I_1 + j50(-I_2) = 0$$
$$-j50I_1 + 0.25I_2 + j25I_2 + 25I_2 = 0$$

$I_1 = 4.18\angle -44.72^\circ$

$I_2 = 5.88\angle 0.57^\circ$

$V_o = 25 \times 5.88\angle 0.57^\circ = 147\angle 0.57$

$v_o(t) = 147\cos(100t + 0.57)$

$i_1(t) = 4.18\cos(100t - 44.72)$

$i_2(t) = 5.88\cos(100t + 0.57)$

Output power $= 0.5 \times (5.88)^2 \times 25 = 432.2\,W$

Conclusions from Power and Energy Discussions

- Suppose the current and voltage are sinusoidal functions of time, the instantaneous power is equal to the cycle average power plus a sinusoidal term that has a frequency twice that of the sinusoidal function.
- To obtain the maximum average power transfer to a load, the load impedance should be chosen equal to the complex conjugate of the Thevinin equivalent impedance representing the power delivery circuit.
- The rms value of a periodic waveform is introduced as a means of measuring the effectiveness of a source in delivering power to a resistive load.
- Apparent power is defined as the product $V_{rms}I_{rms}$. The power factor is defined as the ratio of the average power to the apparent power.
- The complex power S can be written as $S = P + jQ$, where P is the average power and Q is the reactive power.