max min &
min max not same

$\Rightarrow$

1 \ 2	x	y	z	min	max
a	2	3	2	2	2
b	0	5	4	0	
c	3	6	2	2	
max	3	6	4		
min	3				

$\Rightarrow$ no minmax solution

mixed strategy
– Nash equilibrium exists

Hotelling Model of electoral competition

$n \geq 2$

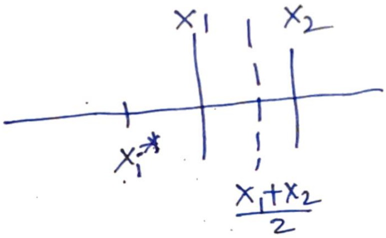
payoff: n if i wins out rightly
(in first places) $\leftarrow k$ if i ties with $(n-k)$ other candidates.
0 if she loses.

actions : policies : x_i (taxes, spendings)

payoff of voter $i = -|x_i^* - x_j|$

x_j = announcement of candidate j.

x_i^* – voter i's favourite policy.



find x_1 as closet

find x_2 as closest to their favourite policy.

m = median favourite policy position.

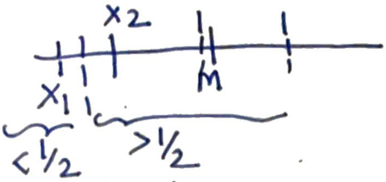
$$\int_0^m f(x)\,dx = \frac{1}{2}$$

$f(x)$ = pdf of favourite positions.

Corresponding voter – Median voter (can be many).

$n=2$ (candidates) Player.
Both want to win.

$x_1 = B_1(x_2)$



[x_2 has taken leftist position)

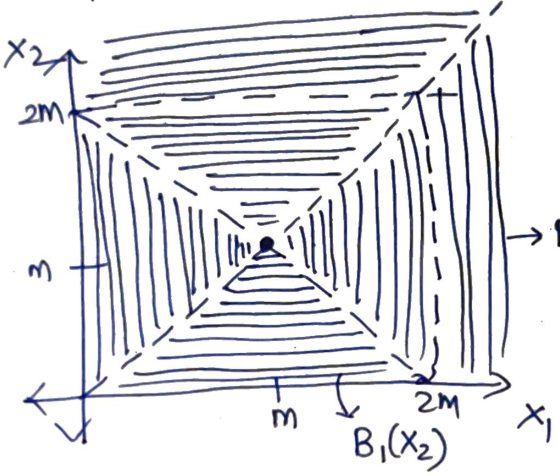
$\Rightarrow$ So, choose x_1 to the right of x_2

$\Rightarrow$ $x_1 > x_2$ ($x_1 = x_2$ not best response)

$\frac{x_1+x_2}{2} < m$

$\Rightarrow x_1 < 2m - x_2$

$x_1 = B_1(x_2) = \begin{cases} x_1: \; x_1 > x_2 \\ \quad \& \; x_1 < 2m - x_2 \\ \quad \text{if } x_2 < m \\ m: \text{ if } x_2 = m \\ x_1: \; 2m - x_2 < x_1 < x_2 \\ \quad \text{if } x_2 > m \end{cases}$



NE: (m,m) only intersection.

Conclu-sion $\Rightarrow$ No one is winning outrightly and decision is centrist.

In Reality, Policies are multidimensional, (here unidimensional)

$\Rightarrow$ No much difference between policies

Hotelling model of spatial competition

population $\rightarrow$ choose that mall which is closest.

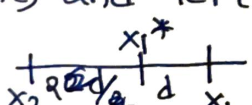
same as electrol competition

& NE is (m,m)

where $\int_0^m f(x)dx = 1/2$

assumptions:

voters care about rightist (to their right) and leftist (to their left) as same.



: preferences [same NE)

(dislike for left is more than dislike for right)

→ Candidates stay out

1. Win outright
2. Tie
3. Stayout
4. Loss

NE: (m, m)

suppose, ∃ a NE, where one candidate stays out, then he can choose m, then he will tie or win

⇒ If both stay out, If any one chooses to participate (with any policy) win.

(m, m) is NE

⇒ unilateral deviation is unprofitable, because choosing any other policy leads to loss (less payoff)

Any other is not N.E.

[choose N.E where atleast one doesnot choose m].

1. Win-loss

⇒ One who looses, can choose m, Then either he wins or tie.

(earlier one can't be m, as he won't loose, either wins or tie).

2. Tie.

↓

$x_1 = x_2$ or $\frac{x_1 + x_2}{2} = m$

⇒ any one can choose m and win.

n=3

⇒ If less than $\frac{1}{3}$ of citizens fav positions m, then no NE.

Cost of Contesting = 100 101 Voters 0 1 ... 100
Prize of winning = 200
$\bar{x}$ = position of winner
payoff of voter i = $-|x_i - \bar{x}|$ (Tie, then take average).
payoff of voter i if none contests (none win) = 0.

x_2 ___ x_i ___ x_1

vote for x_1 = Vote for x_2 (Probability for ith voter)

⇒ Median position = 50

1. 50 contests (voter 50) : N.E

for Voter 50 : ~~Pr~~ price − cost + payoff
payoff for voter 50 = 200 − 100 + $-|50-50|$ = 100.
(if contests)
(if he does not contest)
payoff for voter 50 = ~~200~~ 0

for voter 49 (he does not contest)
Payoff for voter 49 = −1
(if he contest) payoff for voter 49
49th voter gets → 50 votes (counting himself)
50th Voter gets → 51 votes
here payoff = −100 + $-|49-50|$
= −101 (unprofitable deviation)

⇒ Similarly, no other player contests (else less payoff).

⇒ <u>Median voter theorem</u>: Median voter becomes favourite policy

2. 49, 51 contest : NE [Tie 50.5, 50.5]
payoff (when contesting) : −1
⇒ If not contest = −2 policy = 50
If contest −5 { <49 ⇒ 51 wins & favourite policy far
50 ⇒ looses (only 1 vote)

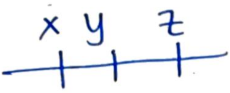
3. 48, 52 also NE

⋮

10, 90 not NE, as 50^{th} voter if contests wins/ties

8, 92 } not NE

10 30 → 30   30 70 → 40   70 90 → 30



Any voter's preference cannot be x, z, y
z, x, y

Q. odd no. of voters

Either $x \succ_i y$ or $y \succ_i x$

We say that position x^* is Condorcet winner if for many $y (\neq x^*)$ majority of voters prefer x^* over y. Pairwise x^* always gets majority voters.

Show that for any configuration of preferences, there is ~~atleast~~ atmost one condorcet winner.

(May be 1 or 0)

⇒ Proof by contradiction:

two or more → choose any 2

x^* y^* → prefer y^* over x^*

↓

prefer x^* over y^*

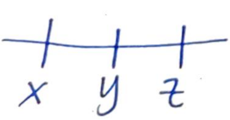
contradiction.

⇒ So, atmost one condorcet winner.

⇒ Example of 0 condorcet winner

odd no. of (finite) voters.

finite no. of x, y, z



$x \rightarrow$ x, y, z $y \rightarrow$ y, x, z $z \rightarrow$ z, y, x

x^*

ranking1   ~~x~~ y

$(x\ y)$ $(y\ z)$ (cycle then not condorcet)

$(x\ z)\ x$ → Then not Condorcet.

War of attrition

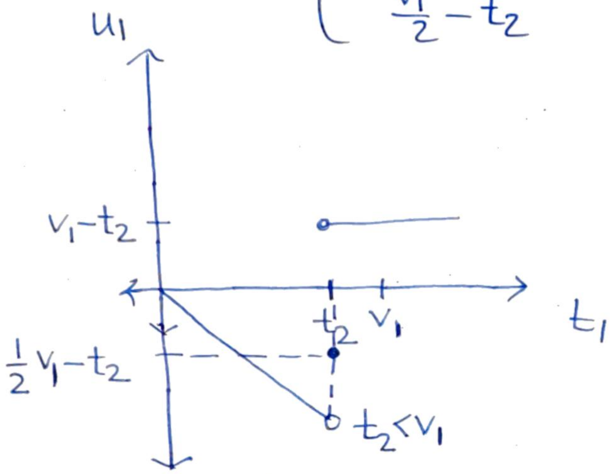
(Tensed competition situation)

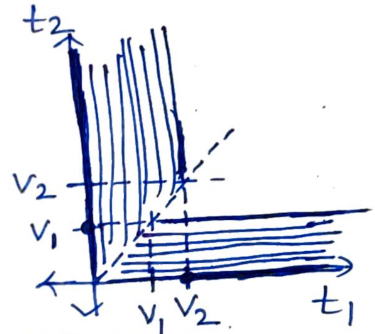
⇒ Two parties are engaged in a tough competitive position.

1. 2 players
2. Actions for player i → $t_i \geq 0$. (times of concede) (for which duration he contests).
3. $v_i > 0$, valuation of player i for the object(s) (ex: Territory, market share)

Payoff

$$u_1(t_1,t_2) = \begin{cases} v_1 - t_2 & , \ t_1 > t_2 \\ -t_1 & , \ t_1 < t_2 \\ \cancel{\frac{v_1-(t_1+t_2)}{2}} & , \ \cancel{t_1 = t_2} \\ \frac{v_1}{2} - t_2 & , \ t_1 = t_2 \end{cases}$$

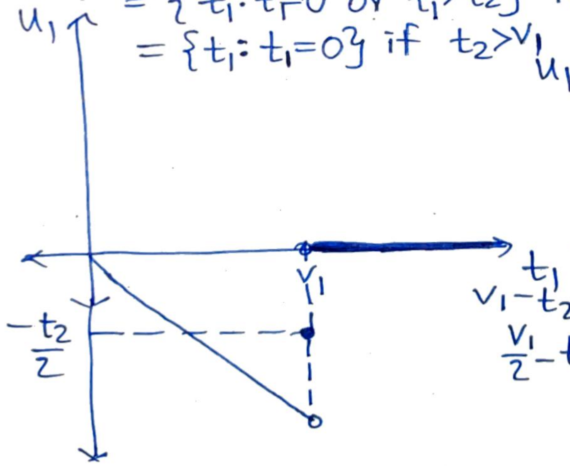




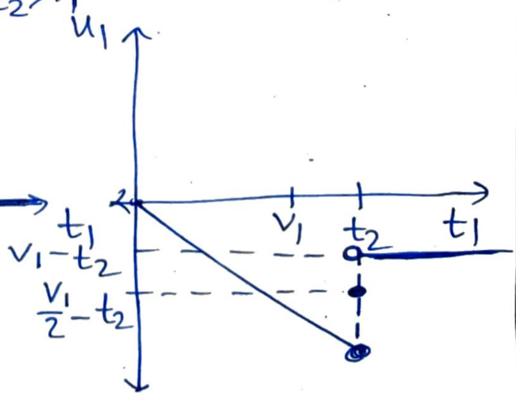
NE: $(t,0)$ $t \geq v_2$
$(0,t)$ $t \geq v_1$

(WLOG assume $v_1 < v_2$)

$B_1(t_2) = \{t_1 : t_1 > t_2\}$ if $t_2 < v_1$
$= \{t_1 : t_1 = 0 \text{ or } t_1 > t_2\}$ if $t_2 = v_1$
$= \{t_1 : t_1 = 0\}$ if $t_2 > v_1$



$t_2 = v_1$



$t_2 > v_1$

NE: $[t_2=0,\ t_1 \geq v_2]$ (Player 1 gets v_1)
(Player 2 gets 0)

$[t_1=0,\ t_2 \geq v_1]$ (Player 2 gets v_2)
(Player 1 gets 0)

⇒ None of the equilibria, there is fight, One of the party gives up and other gets highest possible payoff.

⇒ No equilibrium, where players choose same action (No symmetric NE)

⇒ $v_1 > v_2$ or $v_2 > v_1$ or $v_1 = v_2$ doesnot matter How much players value the prize, doesnot matter.

ex: Two countries fighting over a territory, t-money spent to continue war.

79.2: time spent is not valued. Object in dispute looses value as time passes. ex: Icecream, players fighting over it.

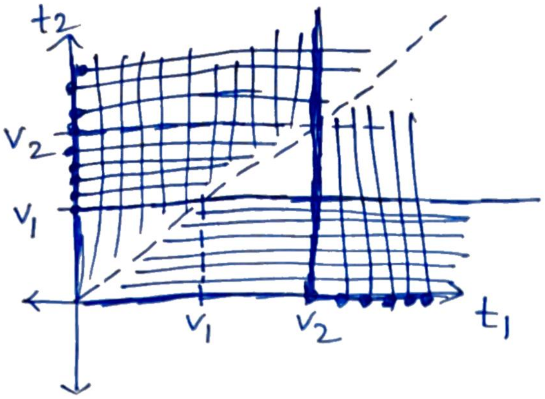
Value for i $= v_i - t$ (after t time spent)

$= \frac{1}{2}(v_i - t)$ (if tie)

2 Players.
Actions: times
Payoff functions:

$$u_1(t_1,t_2) = \begin{cases} 0 & \text{if } t_1 < t_2 \\ \frac{1}{2}(v_1 - t_2) & \text{if } t_1 = t_2 \\ v_1 - t_2 & \text{if } t_1 > t_2 \end{cases}$$

$$B_1(t_2) = \begin{cases} \{t_1 : t_1 > t_2\} & \text{if } t_2 < v_1 \\ \cancel{\{t_1 = 0 \text{ or } t_1 > t_2\} \text{ if } t_2 = v_1} & \\ \{t_1 : t_1 \geq 0\} & \text{if } t_2 = v_1 \\ \{t_1 : t_1 < t_2\} & \text{if } t_2 > v_1 \end{cases}$$



~~$t_1 \geq t_2$~~
$t_1 \geq v_2$
$t_2 \leq v_1$

WLOG assume $v_1 < v_2$

NE: $[\, t_1 \geq v_2$ and $t_2 \leq v_1]$ → ($t_2 \leq v_1$ else Player 1 doesn't get object)

$[\, t_2 \geq v_1$ and $t_1 \leq v_2]$
and $t_2 > t_1$

~~[t~~

(If he doesnot get object also, he can still try).

AUCTIONS

Auction is a process through which prices of objects are determined. bid submit
ex: object - contract with player

Auctioneer - Authority that conducts Auction.
ex: BCCI

No. of valuable objects is not more. unique object
Tender - not same as auctions.
right to explore - auction
frequency spectrum

Bidder - Those entities which want to get object.
They submit bids.

Two kinds of auction
1. English auction (reserve price) (stopped when no body outbids final price)
2. Dutch auction
   → Start's from highest price
   ↳ Tries to find out right price.

(price discover)

Valuation of objects by bidders > 0
$V_i > 0$.

$v_1 > v_2 > v_3 > \dots > v_n > 0$ (1,2,3,...,n are assigned as such)
More valuation, less no. assigned to them.

→ Game with perfect knowledge, people know other's valuation. (No Tie assume)

bids → $(b_1, b_2, \dots, b_n)$ → price announced as bidder to auction.

Second-price sealed bid auction

⇒ Unique player with highest b_i gets object, he pays the bid which is highest among rest of the player's bids.

⇒ Sealed bid, as bid's are not known to each other.

⇒ Mimics normal auction, English auction where player's don't know each other's bids.

On, Tie, person who values the object highest gets

Players : n bidders
Actions : Bids, $b_1, b_2, \dots, b_n$ $b_i \geq 0$
Payoffs:
$$\bar{b}_i = \max_{j \neq i} b_j \quad (j = 1, 2, \dots, n)$$

$u_i = v_i - \bar{b}$ if $b_i > \bar{b}$ (or) if $b_i = \bar{b}$ and i is least index among those bidding b_i.

$= 0$ otherwise.

NE: $b_i = v_i \ \forall i$

$(b_1, b_2, \dots, b_n) = (v_1, v_2, \dots, v_n)$.

pay-offs $= (v_1 - v_2, 0, \dots, 0)$.

1 → if $> v_1 \Rightarrow v_1 - v_2$
if $v_2 \leq \ \leq v_1 \Rightarrow v_1 - v_2$
if $\leq v_2 \Rightarrow 0$.
} no positive deviation.

other → if $< v_i \Rightarrow 0$.
if $v_i < \ \leq v_1 \Rightarrow 0$.
if $> v_1 \Rightarrow v_j - v_1 < 0$
} no positive deviation.

NE: $(v_2, v_1, 0, \dots, 0)$

payoffs $\to (0, v_2 - v_2, 0, \dots, 0)$.

Player2 $\Rightarrow > v_1 \Rightarrow$ same 0.

$\Rightarrow v_2 < \;\leq v_1 \Rightarrow$ same 0.

$\Rightarrow \;\leq v_2 \Rightarrow$ doesnot get $\to$ still 0.

others $\Rightarrow$ $< v_1 \Rightarrow 0$ (lost)

$= v_1 \Rightarrow 0$ (gets) (player 1)

$> v_1 \Rightarrow 0$ (gets)

$< v_1 \Rightarrow 0$ (lost) (others)

$\cancel{\geq} v_1 \Rightarrow 0$ (lost)

$> v_1 \Rightarrow \cancel{0}$ (gets) (−ve)

~~Other player (not 1) get object in NE $\Rightarrow$ gets 0 payoff~~

NE: $(0, 0, \dots, v_i, 0, \dots, 0) \Rightarrow b_i \geq v_1$ & all other $b_j \leq v_i$

for ith $\to$ any positive $\Rightarrow v_i - 0$. } no profitable deviation.

if 0 $\Rightarrow 0$

others $\to$ get only when outbit v_1

$\Rightarrow$ Player 1 $\Rightarrow v_1 - v_1 = 0$

$\Rightarrow > v_1 \Rightarrow -ve$

other $2, 3, \dots, i-1 \Rightarrow v_1 \Rightarrow -ve$

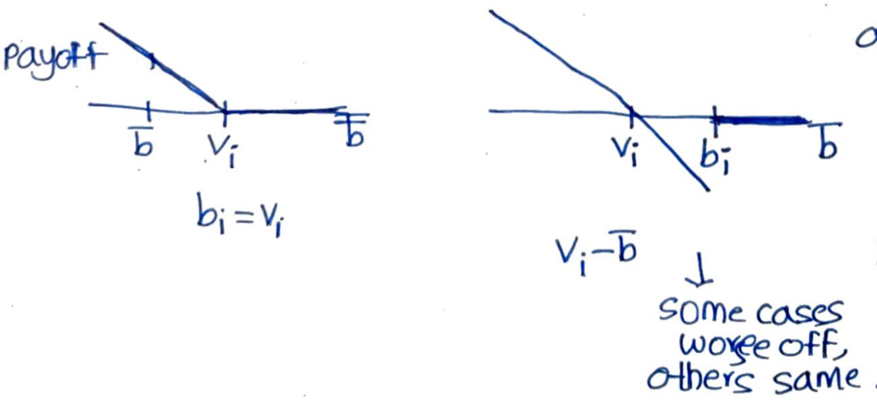
$\Rightarrow > v_1 \Rightarrow -ve$

other $i+1, \dots, n \Rightarrow > v_1 \Rightarrow -ve$

$\Rightarrow v_1 = 0 \Rightarrow$ else

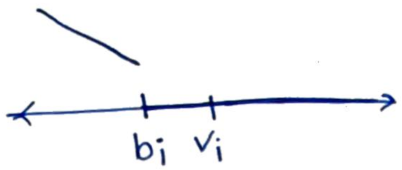
$\Rightarrow$ other players slightly less than v_i

$v_1 = b_2$ is weakly dominated action of 2. by $b_2 = v_2$



any bid more than valuation is weakly dominated by bid = valuation

→ also weakly dominated by action $v_i = b_i$



$b_i < v_i$

$b_i = v_i$ action weakly dominating all other actions.

for $n=2$

$\Rightarrow u_1(b_1, b_2) = v_1 - b_2$ if $b_1 \geq b_2$

$= 0$ if $b_1 < b_2$

$u_2(b_1, b_2) = v_2 - b_1$ if $b_2 > b_1$

$= 0$ else

like war of attrition

First price sealed bid auction

Submit bid within dates, and winner bears his own bid (not second highest bid) i.e $v_i - b_i$. he doesnot know second highest bid as price is coming from large value. It mimics dutch auction.

$u_i = v_i - b_i$, if $b_i > \bar{b}$

or $b_i = \bar{b}$ and i is the least index among those who bid $\bar{b}$.

$= 0$ otherwise.

$(v_1, v_2, \ldots, v_n) \Rightarrow$ all payoffs $= 0$

for player 1 $\Rightarrow b_1 = v_2$ wins

payoff $= v_1 - v_2$

↓

positive deviation, so, not NE.

$(v_2, v_2, v_3, .., v_n) \Rightarrow$ NE

$\Rightarrow$ player 1 $\Rightarrow$ no better

$\Rightarrow$ others $> v_2 \Rightarrow -ve$

$\}$ No positive deviation.

$\Rightarrow$ Here Player 1 always wins object in all NE's.

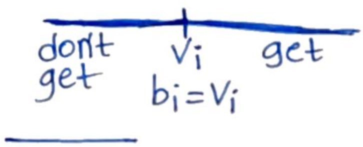
$\Rightarrow$ Any other player must/can bid atmost v_i, but player 1 bid more than v_i, get positive payoff.

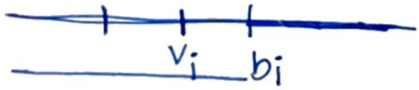
$\Rightarrow$ Player 1 can always outbid ith player.

1
87.1 q. Show NE $\rightleftarrows$ 1. Two highest bids are same.
2. One of these bids is submitted by 1
3. Highest bids are atleast v_2 & atmost v_1.

→ If not satisfied, not N.E.

1. Highest bidder, can decrease to $b_j+\varepsilon$ and get positive deviation.
3. Player 1 get's so, can't be $>v_1$
$<v_2 \Rightarrow$ Player 2 can bid highest $b+\varepsilon$ and get positive payoff.





$b_i \not> v_i$

v_i-b_i

$b_i \not< v_i$

$b_i>v_i$ is weakly dominated by $b_i=v_i$

$b_i=v_i$ is weakly dominated by $b_i<v_i$

$b_1<b_2<v_i$

no relation of weak domination b/w each other.

$v_2 \leq b^* \leq v_1$

one of the highest bidder (not player 1) is bidding weakly dominated action (Player 1 also, if $b^*=v_1$).

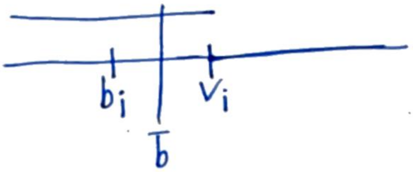
⇒ If bids are not continuous, These result's don't hold. (ε smallest granular unit, below which we can't go).

$b_j \leq v_j-\varepsilon \quad j=3,4,\dots,n$ (not weakly dominated).

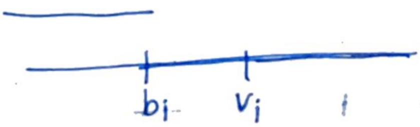
$(v_2-\varepsilon, v_2-\varepsilon, b_3, \dots, b_n) \xrightarrow{\text{as } \varepsilon\to 0} (v_2, v_2, b_3, \dots, b_n)$

[second price sealed bid → $(v_1, v_2, \dots, v_n)$] ↔ revenue equivalent.

In Third price sealed bid auction,
v_i weakly dominates $b_i < v_i$
doesnot weakly dominate $b_i > v_i$.

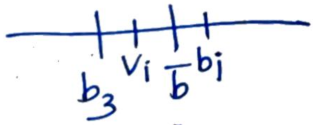


$b_i = v_i$



$b_i < v_i$

$v_i - b_3$
if $\bar{b} < b_i$
$\Rightarrow$ same payoff



$b_i \rightarrow$ $v_i - b_3$
$v_i - b_3$
more than v_i, is strictly better than bidding $= v_i$.

$\Rightarrow$ Single unit good auction above.
$\Rightarrow$ Multi unit goods auctions
$b_1 = (b_1^1, b_1^2, \ldots, b_1^m)$ m units are auctioned, Submit bids for each unit.

$\Rightarrow$ Rule 1: Discriminatory auction [replication first price sealed bid auction]
- pays his own price highest bidder, in each unit.

lobbying groups
$\rightarrow$ influence government policies for their benefit.
$\downarrow$
All pay auction
- Highest bidder get policy in his favour.

like war of attrition $\leftarrow$ $\left\{ \begin{array}{l} u_i = -b_i \ (\bar{b} > b_i) \\ \quad = v_i - b_i \ (\bar{b} < b_i) \end{array} \right.$

90.2: Waiting in a line
Movie hall - cap - 100
people who want to watch > 100. = 200.
First come First ser-vice,
5:30 PM
$v_1 > v_2 > \ldots > v_{200}$ [in time]

$v_i - t_i$ | If same t_i, decide on index bids only on one unit.

$t_1 = t_2 = \dots = t_{100} = v_{100}$ other 0.

(Highest waiting person can reduce)

$t^* \leq v_{105}$ (for positive payoff).

t^* or $t^* + \varepsilon \rightarrow$ positive payoff (first 100).

$\Rightarrow \quad t^* = v_{100}$

$t_{101}, \dots < v_{100}$.

$\Rightarrow \quad v_{100} - \varepsilon \Rightarrow$ one of the first 100, still get better payoff.

$\Rightarrow$ $\boxed{t^* (= v_{100}) \text{ (for } i = 1, 2, \dots, 100) \text{ and } t_j = t^* \text{ (for atleast one of } j = 101, \dots, 200)}$

$\Rightarrow \quad v_{101} \leq t^* \leq v_{100} \Rightarrow$ (all these are NE).

(If $< v_{101} \Rightarrow$ can choose (v_{101}) $t^* + \varepsilon$ and positive)

(If $> v_{100} \Rightarrow$ v_{100} get's ~~neg g~~ negative payoff)

[All people don't know other's valuations, so, this behaviour is not observed in real life].

Accident Laws

→ Not intentional, Loss $= L > 0$

party which causes accident = Injurer

Loss is on victim.

Other party is victim.

How much proportion of L is borne by Injurer and how much by Victim. — Law

Injurer is not liable to pay any | Injurer is completely liable to pay → Extremes.

a_i : care taken by player i to avoid the accident ≥ 0.

$L(a_1, a_2) \rightarrow$ decreasing function.

more care, Then probability of accident decrease (L is like expectation)

$\rho(a_1,a_2)$ = proportion of loss borne by injurer.

$$u_1(a_1,a_2) = -a_1 - \rho(a_1,a_2)\,L(a_1,a_2)$$
$$u_2(a_1,a_2) = -a_2 - (1-\rho(a_1,a_2))\,L(a_1,a_2)$$

Negligance with contributory negligance : (law)
(X_1,X_2) = threshold care levels for 1 & 2.

$$\rho = 1 \text{ if } a_1 < X_1 \text{ and } a_2 \geq X_2$$
$$\rho = 0 \text{ if } a_1 \geq X_1 \text{ or } a_2 < X_2$$

$a_1 = X_1$ and $a_2 = X_2 \Rightarrow \rho = 0$ $\quad$ X_1 very large & X_2 very small $\rightarrow$ victim favour

$X_1 \to \infty,\ X_2 = 0 \rightarrow$ $\boxed{\text{Strict liability}}$

$X_1 \to$ large, $X_2 = 0 \rightarrow$ $\boxed{\text{Pure negligence}}$

Suppose, $(\bar{a}_1,\bar{a}_2)$ is socially desirable. $\rightarrow$ Is this NE? (pareto optimal)

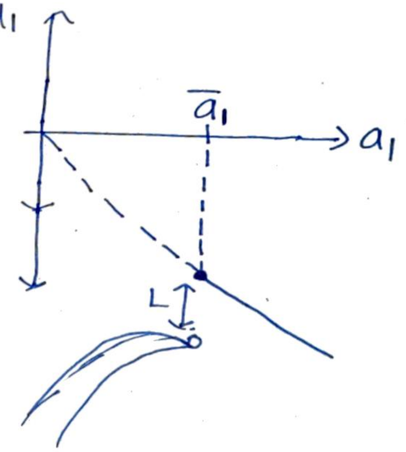
Yes If $X_1 = \bar{a}_1,\ X_2 = \bar{a}_2$ (This is unique NE)

$$\bar{a}_1 \in B_1(\bar{a}_2)$$
$$\bar{a}_2 \in B_2(\bar{a}_1)$$

$$u_1 = -a_1 - L(a_1,\bar{a}_2) \quad \text{if } a_1 < \bar{a}_1$$
$$= -a_1 \quad , \text{ if } a_1 \geq \bar{a}_1$$

~~$u_2 = -a_2$~~

$(B_1(\bar{a}_2) = \bar{a}_1)$



a_1 decreases but L increases

★ $(\bar{a}_1,\bar{a}_2)$ maximises $u_1+u_2 = -a_1 - a_2 - L(a_1,a_2)$

$a_1 = \bar{a}_1$ maximises

$\quad \Downarrow \rightarrow -a_1 - L(a_1,\bar{a}_2)$

$\Rightarrow$ So, It has max at $\bar{a}_1 \Rightarrow$ but here it is greater now $(-\bar{a}_1) \Rightarrow$ best response.

So, $B_1(\bar{a}_2) = \bar{a}_1$

Similarly $B_2(\bar{a}_1) = \bar{a}_2$

$\Rightarrow$ If player 1 reduces even a little, he have to pay entire loss.

$$u_2 = -a_2 - L(\bar{a}_1, a_2)$$

$$\Downarrow$$

$B_2(\bar{a}_1) = \bar{a}_2$ [from socially desirable condition]

Mixed strategy Nash Equilibrium $\rightarrow$ Stochastic steady state.

populations of players. (at each player-pick)

pop$|^n$ pl.1 pop$|^n$ pl.2 NE: (a_1^*, a_2^*)

pl.1 $\Leftrightarrow$ pl.2

a_1^* a_2^*

at each play

Best response functions intersection

$\Rightarrow$ Restrictive

$A_1 = (a_1^*, b_1^*, c_1^*)$ (1,0,0) in NE. (earlier)

probability distribution over action set

$A_2 = (a_2^*, b_2^*, c_2^*)$ (1,0,0) in NE

Mixing the actions is allowed.

Steady state, and probability distribution of players remain same. $\rightarrow$ Mixed strategy NE

A_1 $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ A_2 $(\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$

according to P_2's distribution over ~~ø~~ $A_2 \rightarrow A_1$ with given distribution is best response and vice-versa

(1,0,0) (1,0,0) $\rightarrow$ Pure strategy NE

MSNE PSNE

Theorem:
atleast one MSNE always exists
While NE may not exist sometimes.

ex
Matching pennies

		2	
		H	T
1	H	$1^*,-1$	$-1,1^*$
	T	$-1,1^*$	$1^*,-1$

If match, P_1 wins
else P_2 wins

(no PSNE)

$\Rightarrow$ MSNE : Unique NE at $\underbrace{\overset{H\ \ T}{(\tfrac{1}{2},\tfrac{1}{2})}}_{\text{Probabilities of P1}}$ & $\underbrace{\overset{H\ \ T}{(\tfrac{1}{2},\tfrac{1}{2})}}_{\text{Probabilities of }P_2.}$

P_1 POV :
~~Play~~ P_1 chooses $\overset{H\ \ T}{(p, 1-p)}$ and P_2 chooses $\underbrace{(\tfrac{1}{2},\tfrac{1}{2})}_{\alpha_2}$

Probability that player gets 1
(He would like to maximize this probability)

$= P(HH \cup TT)$
$= P(H \cap H) + P(T \cap T)$
$= P(H)P(H) + P(T)P(T)$
$= \tfrac{1}{2}p + \tfrac{1}{2}(1-p)$
$= \tfrac{1}{2} \rightarrow$ Independent of p
$\forall p \rightarrow$ getting 1Rs remains same
$\Rightarrow$ So, $p=\tfrac{1}{2}$ is also optimal.

$B_1(\alpha_2) = (p, 1-p) \ \forall p$
$\ni (\tfrac{1}{2},\tfrac{1}{2})$

Similarly, if P_1 chooses $\underbrace{(\tfrac{1}{2},\tfrac{1}{2})}_{\alpha_1}$, P_2 choosing $(\tfrac{1}{2},\tfrac{1}{2})$ is also it's best response.

$B_2(\alpha_1) = (p, p) \ \forall p$
$= (\tfrac{1}{2},\tfrac{1}{2})$

(α_1,α_2) of P1 & P2

(α_1,α_2) is MSNE.

$\Rightarrow$ let $\alpha_1 = (p, 1-p)$ $\Rightarrow$ Pr(P1 gains Rs1)
$\alpha_2 = (q, 1-q)$
$= pq + (1-p)(1-q)$
$= 2pq + 1 - p - q$
$= p(2q-1) + (1-q)$

(No NE, as no intersections)

If $q > \tfrac{1}{2}$, $p \rightarrow 1$ $\rightarrow q=0$ (no NE if $q>\tfrac{1}{2}$)

$\rightarrow$ (Here $q=0$, is optimal)

If $q < \frac{1}{2}$, $p \to 0 \to q \to 1$ no NE if $q > \frac{1}{2}$

If $q = \frac{1}{2}$, ~~any p~~ $p > \frac{1}{2} \to q \to 0$ } no NE
$p < \frac{1}{2} \to q \to 1$ }
$p = \frac{1}{2} \to$ any q ($q = \frac{1}{2}$ allowed)

⇒ Similarly, if $p \neq \frac{1}{2}$ not NE
if $q \neq \frac{1}{2}$ not NE

$(p,q) = (\frac{1}{2}, \frac{1}{2})$ is NE, proved before.

⇒ So, This is unique NE.

Decision under Uncertainity, some probability by which actions are played.

Events A B
probability distribution over A & B

P1	
A	p (maximize p)
B	$1-p$

$(p, 1-p)$
→ lotteries [probability distribution over events]

⇒ maximize probability associated with preferred event

(Before gaining 1Rs & loosing 1Rs are only 2 events).

If 3 or more events ex:

Risk lovers ~~might be~~ $l_1 \succ l_2$
Risk Aversers ~~mig~~ $l_2 \succ l_1$

	A	B	C
	+50	0	-50.
lotteries 1	1/4	0	3/4
2	0	1	0

⇒ Theory of expected Utility (von Neumann & Morgenster)

$x_1, x_2, \ldots, x_n \to$ events (like action profils)

⇒ $u_i(x_i)$ [If certain events] [utility function]

l_1, l_2 lotteries, l_1 ~~is~~: $(\alpha_{11}, \alpha_{12}, \ldots, \alpha_{1n})$
l_2 : $(\alpha_{21}, \alpha_{22}, \ldots, \alpha_{2n})$

$\alpha_{ij} \geq 0$
$\sum_i \alpha_{ij} = 1$

$l_1 \succ_j l_2$ if

$$\Rightarrow \sum_{i=1}^{n} \alpha_{1i} u_j(x_i) > \sum_{i=1}^{n} \alpha_{2i} u_j(x_i)$$

⇒ vN-M utility functions: $u_j(\ )$

unlike payoff functions, these are non-ordinal functions (else inequalities will change)

PD:

1 \ 2	NC	C
NC	2,2	0,3
C	3,0	1,1

l_1 : (NC,NC) event with certainty, other's 0.

l_2 : (C,NC), (C,C) happen with probability ½ each.

$$\Rightarrow \text{Player 1} \Rightarrow U_1(l_1) = 1(2) + 0(3) + 0(0) + 0(1) = 2$$

$$\text{Player 1} \Rightarrow U_1(l_2) = 0(2) + \tfrac{1}{2}(3) + 0(0) + \tfrac{1}{2}(1) = 2$$

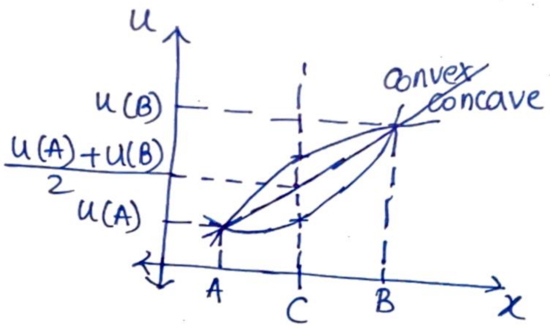
$u_1 \to$ defined over certain outcome.

$U_1(l_1) = 1 \cdot u_1(NC,NC) + 0 . = 1 \cdot 2 = 2$

$U_2(l_2) = 2 \qquad l_1 \sim l_2$

If $u_1(NC,NC) = 2.5$ (ordinal preference matches)

but $l_1 \succ l_2$, as preference over l_1, l_2 changes, $u_i()$ must be non-ordinal.



$l_1 = (\overset{A}{½}, \overset{B}{½})$

$U(l_1) = u(A) + u(B)$

C: $\frac{A}{2} + \frac{B}{2}$

Based on $u(C) \to$ people are divided into 3 categories.

concave $\to u(C) > \frac{1}{2}u(A) + \frac{1}{2}u(B)$

$u(\frac{1}{2}A + \frac{1}{2}B) > \frac{1}{2}u(A) + \frac{1}{2}u(B)$.

$\Rightarrow C\ (1) = l_2$

$l_2 \succ l_1$

⇒ Risk averse person → when C is chosen with certainty, payoff is more

⇒ Concavity → degree of risk aversion.

Risk lovers

$\Rightarrow u(C) < \frac{1}{2}u(A) + \frac{u(B)}{2}$

$u\left(\frac{A}{2}+\frac{B}{2}\right) < \frac{u(A)+u(B)}{2}$

Risk Neutral Individual

$u(C) = \frac{1}{2}u(A) + \frac{1}{2}u(B)$

$u\left(\frac{A}{2}+\frac{B}{2}\right) = \frac{1}{2}u(A) + \frac{1}{2}u(B)$.

$\Rightarrow$ These depend on condition ($u(\,)$s) Insurance $\rightarrow$ Risk averse

* mixed strategy is probability distribution over set of actions.

Mixed strategy :

For player i, α_i is a mixed strategy, a probability distribution over her set of actions,

$\alpha_i(a_j)$, for $a_j \in A_i$, is a probability such that

$0 \leq \alpha_i(a_j) \leq 1$

$\sum_{\forall a_j \in A_j} \alpha_i(a_j) = 1$

Collection of mixed strategies : Mixed strategy profile.

: $\alpha : (\alpha_1, \alpha_2, \dots, \alpha_n)$ $\alpha_i \rightarrow$ Mixed strategy

$\rightarrow$ This produces a lottery.

$\rightarrow$ A mixed strategy profile generates a probability distribution over set of outcomes – a lottery (Probability distribution over set of events).

ex:

$$\begin{array}{ccccc} & & & 2 & \\ & & a_2 & b_2 & \\ 1 & a_1 & & & p \\ & b_1 & & & 1-p \\ & & q & 1-q & \end{array}$$

$\alpha_1 : (p, 1-p)$ $\alpha_2 : (q, 1-q)$.

(α_1, α_2) :

prob.	outcome
pq	(a_1, a_2)
$p(1-q)$	(a_1, b_2)
$(1-p)q$	(b_1, a_2)
$(1-p)(1-q)$	(b_1, b_2)

Mixed strategy Nash equilibrium: If for each i, $U_i(\alpha^*) \geq U_i(\alpha_i, \alpha_{-i}^*)$ $\forall \alpha_i$ of i, then α^* is MSNE. $U_i(\alpha)$ is i's expected payoff from mixed strategy profile α. → (collection of mixed strategies)

(α_i^*) change i's mixed strategy, keeping others' mixed strategies same (α_{-i}^*).

expected payoff can't increase if MSNE

(No unilateral positive deviation).

ex: Matching pennies.

		2: H	T	
1	H	1,-1	-1,1	p
	T	-1,1	1,-1	1-p
		q	1-q	

$E_1(H, \alpha_2) = q + (1-q)(-1) = 2q-1$

$E_1(T, \alpha_2) = q(-1) + (1-q)(1) = 1-2q$

$U_1(\alpha_1, \alpha_2) = p\, E_1(H, \alpha_2) + (1-p)\, E_1(T, \alpha_2)$

$= p(2q-1) + (1-p)(1-2q)$

$B_1(q) = \{1\}$ if $q > 1/2$

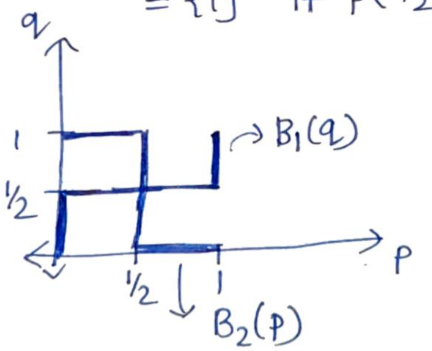
$= \{p : 0 \leq p \leq 1\}$ if $q = 1/2$

$= \{0\}$ if $q < 1/2$

$B_2(p) = \{0\}$ if $p > 1/2$

$= \{q : 0 \leq q \leq 1\}$ if $p = 1/2$

$= \{1\}$ if $p < 1/2$



one point of intersection, unique nash equilibrium at $p = 1/2, q = 1/2$.

⇒ 3 actions, can't use this method

ex: BoS.

1 \ 2	B	O	
B	2,1	0,0	p
O	0,0	1,2	1-p
	q	1-q	

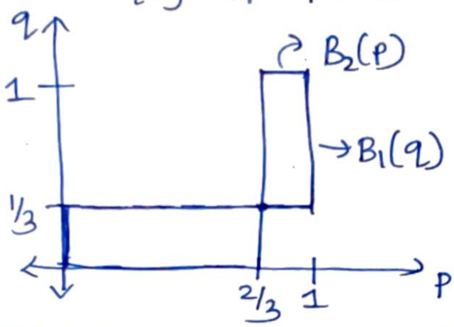
$$E_1(B,\alpha_2) = q(2) + (1-q)(0) = 2q$$
$$E_1(O,\alpha_2) = q(0) + (1-q)(1) = 1-q$$

$\left(q=\frac{1}{3}\begin{pmatrix}2/3\\2/3\end{pmatrix}\right)$

$$U_1(\alpha,\alpha_2) = p(2q) + (1-p)(1-q).$$

$$B_1(q) = \{1\} \text{ if } q > \tfrac{1}{3}$$
$$= \{p : 0 \leq p \leq 1\} \text{ if } q = \tfrac{1}{3}$$
$$= \{0\} \text{ if } q < \tfrac{1}{3}.$$

$$B_2(p) = \{1\} \text{ if } p > \tfrac{2}{3}$$
$$= \{q : 0 \leq q \leq 1\} \text{ if } p = \tfrac{2}{3}$$
$$= \{0\} \text{ if } p < \tfrac{2}{3}.$$



3 MSNE: 1. $p=0, q=0$ ✓ (NE) (PSNE)
2. $p=\frac{2}{3}, q=\frac{1}{3}$
3. $p=1, q=1$ ✓ (NE) (PSNE)

(For all, any no. of players & action)

⇒ If all action's probabilities are non-zero for this player, expected payoffs of each action is same.

In MSNE

$p\,E_1(H,\alpha_2) + (1-p)\,E_1(T,\alpha_2)$

If one of them is higher, other's probability becomes zero.

for some players,

⇒ If some actions, positive probability, then those expected payoff from these ~~ex~~ actions, must be same.

↓ This same expected payoff of each action is expected payoff (Total).

In MSNE $\Rightarrow$ $\left\{ \text{Expected payoffs of other actions with 0 probability} \leq \text{Expected payoffs of actions with positive probability} \right.$

ex:

		2: L	R
1	T	1,2	2,5*
	M	3*,2*	0,0
	B	0,1	4*,2*

→ Strictly dominated action is not played in any equilibrium
→ eliminate that row and solve for 2x2 game.

		2: L	R
1	T	1	1
	M	4	0
	B	0	3

(Only Player 1's payoffs)

a_i' strictly dominates a_i'' iff

$$u_i(a_i', a_{-i}) > u_i(a_i'', a_{-i}) \quad \forall a_{-i} \in A_{-i}$$

T is strictly dominated by $\alpha_1 : (0, \frac{1}{2}, \frac{1}{2})$

$$U_1(\alpha_1, L) = \frac{1}{2}(4) = 2 > 1 = u_1(T, L)$$
$$U_1(\alpha_1, R) = \frac{1}{2}(3) = 1.5 > 1 = u_1(T, R)$$

⇒ Player i's mixed strategy α_i strictly dominates her action a_i' iff

$$U_i(\alpha_i, a_{-i}) > u_i(a_i', a_{-i}), \quad \forall a_{-i}$$

(also will be satisfied for mixed strategies of other player, trivially).

⇒ Player i's mixed strategy α_i ~~strictly~~ weakly dominates her action a_i' iff

$$U_i(\alpha_i, a_{-i}) \geq u_i(a_i', a_{-i}), \quad \forall a_{-i} \text{ and}$$
$$U_i(\alpha_i, a_{-i}) > u_i(a_i', a_{-i}) \text{ for atleast one } a_{-i}$$

→ T is strictly dominated by $(0, \frac{1}{2.5}, 1 - \frac{1}{2.5})$

all mixed strategies that dominate T. $0, \frac{1}{2.5}, 1-\frac{1}{2.5}$

$\frac{1.5}{2.5} = \frac{3}{5} \times 4 = \frac{\cancel{12}}{8}$ 2.5

⇒ $(0, p, 1-p)$

$U_1(\alpha_1, L) = 3p > 1 = u_1(T, L)$

$U_1(\alpha_1, R) = 4(1-p) > 2 = u_1(T, R)$

$p > \frac{1}{3}$

$p < \frac{1}{2}$

$p \in (\frac{1}{2}, \frac{1}{3})$

ex:

			2 L	B	
1	M		3,2	0,0	p
	B		0,1	4,2	1-p
			q	1-q	

$\left.\begin{array}{l} p=1, q=1 \\ p=0, q=0 \end{array}\right\}$ PSNE

$U_1(M, \alpha_2) = U_1(B, \alpha_2)$

$\Rightarrow \quad 3q = 4(1-q)$

$\Rightarrow \quad q = \frac{4}{7}$

$U_2(L, \alpha_1) = U_2(R, \alpha_1)$

$\Rightarrow 2p + (1-p) = 2(1-p)$

$2p = 1-p$

$\Rightarrow \quad p = 1/3$

$\left. p = \frac{1}{3}, q = \frac{4}{7} \right\}$ MSNE

En Extensive games with perfect information

Strategy games - don't know actions of other players, even previous actions

→ Sequential games

→ Completely aware of actions taken by other players ex: chess.

1. Player set
2. Preferences
3. Terminal historys. (like sequence of actions)
4. Player functions.

how the game can change with time, it's structure

→ At any stage (of Terminal history) we should know, which player plays.

4 gives identity of player

Ex: ENTRY GAME

Incumbent (someone already in power, monopoly)
Challenger (whether to go into market or not)

Starting point - sequential

→ Entry (In)

→ Do not Enter (out)

→ Fight

→ Accomodate

3

3 Terminal histories
→ out
→ in fight
→ in accomodate

Player function:

assigns player any sequence [illegible]

$P(\phi) = Ch$ (first mover)

⇒ for non-terminal history, we need [illegible] should play after this history

$P(In) = Inc$

Preferences:

These are defined over terminal [illegible] (i.e ended)/outcomes (ordinal payoff function)

$u_{ch}(In, Accomodate) = 2$

$u_{ch}(In, Fight) = 0$

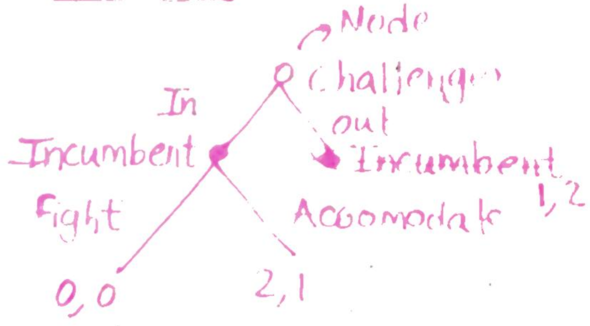
$u_{ch}(out) = 1$

$u_{Inc}(out) = 2$

$u_{Inc}(In, Fight) = 0$

$u_{Inc}(In, Accomodate) = 1$

Game Tree



⇒ Action set keeps changing

1. Terminal history:

Any Terminal history is a sequence of actions

$(a^1, a^2, \ldots, a^m)$ is a terminal history. m can be infinite.

$(a^1, a^2, \ldots, a^m, \ldots)$ is an infinite terminal history

⇒ If in a game, if all terminal histories are finite, such game is called finite horizon game (chess also by rules)

⇒ Finite horizon game and no. of terminal histories is finite, then game is called finite game

ex: choosing price of product is infinite game, but could be finite horizon game.

2. Non terminal history:

If $(a^1, a^2, \ldots, a^m)$ is a terminal history, then

ϕ, $(a^1, a^2, \ldots, a^k)$ $1 \leq k \leq m$, are non terminal histories

Backward Induction.

roll-back equilibrium)

156.2
c: Carl choses person who makes first choice.
Rosa/ Ernesto

choose venue for a party
Berlin/ Havana
(Choosing in a sequential manner)

$\text{both}_b >$ Rosa prefers both choosing h > than when both choose different

Ernesto $\Rightarrow$ $\text{both}_h > \text{both}_b >$ different

Carls $\Rrightarrow$

1. Players: Karl, Rosa, Ernesto.

2. Terminal histories:

$(R, X, Y), (E, X, Y)$ X = Berlin, Havana $\rightarrow 8$
Y = Berlin, Havana
R = Rosa
E = Ernesto

3. Player functions:

$$P(\phi) = \text{Karl}$$
$$P(R) = P(E, B) = P(E, H) = R$$
$$P(E) = P(R, B) = P(R, H) = E$$

4. Preferences:

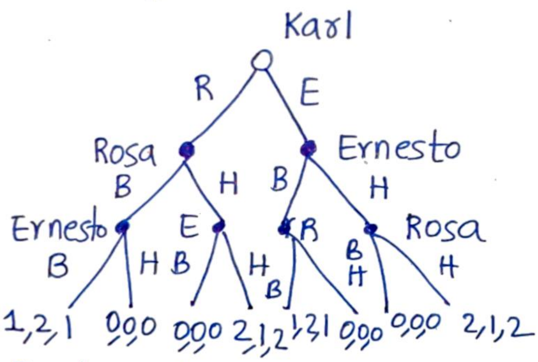
$X = R, E$

$u_1(X, H, H) = 2$
$u_1(X, B, B) = 1$
$u_1(X, H, B) = 0$
$u_1(X, B, H) = 0$

$u_3(X, H, H) = 2$
$u_3(X, B, B) = 1$
$u_3(X, H, B) = 0$
$u_3(X, B, H) = 0$

$1 = K$
$2 = R$
$3 = E$

$u_2(X, B, B) = 2$
$u_2(X, H, H) = 1$
$u_2(X, H, B) = 0$
$u_2(X, B, H) = 0$



⇒ Backward Induction gives Havana outcome as equilibrium. (E, H, H)

<u>A strategy</u>:

defined for a particular player.
Player $i \longrightarrow$ strategy S_i.
⇒ S_i contains his action, it specifies after each non-terminal history and he is the player, his action.

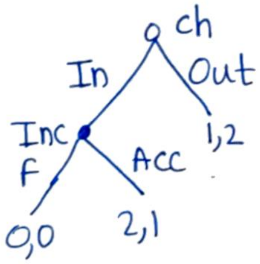
⇒ A player's strategy specifies her action for every non-terminal history, after which it is her turn to make a move.

h: non-terminal history, $P(h) = i$

$s_i(h) \cancel{=} \in A_i(h)$

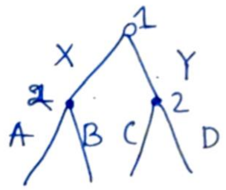
↓

might be single action / combination of different actions.



$s_{ch} = (In), (Out)$

$s_{Inc} = (F), (Acc)$



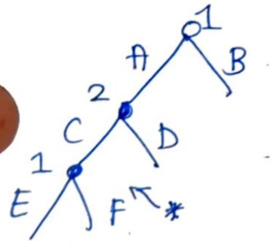
$s_2 = (A,C), (A,D), (B,C), (B,D)$

↓

left to right

⇒ strategy is similar to action in strategic game

⇒ combine strategies of different players – strategy profile.



s_1: $\cancel{(B)}$, (A,E), (A,F), (B,E), (B,F)

↓

If ~~pl~~ game comes to *, player 1 chooses E, F

⇒ for every non-terminal history, give strategy

⇒ ~~ti~~ Taking care of continguencies

strategy profile : $(s_1, s_2, \ldots, s_n) = s$

$\downarrow$

generates terminal history

$s_1 = (X), (Y)$

$s = (Y, AC)$

$P(\phi) = 1$

$s_1(\phi) = Y$

$P(Y) = 2$

$s_2(Y) = C \quad s_2 = AC$

Terminal history : $(Y, C) \rightarrow$ outcome of strategy profile.

$s \rightarrow h$ (terminal history)

$=$ outcome $= O$.

$\Rightarrow u_i(O(s))$

strategy, s_i

$\downarrow$

strategy profile, s

$\downarrow$

Outcome, $O(s)$

$\downarrow$

Payoff, $u_i(O(s))$

$\Rightarrow$ The strategy profile s^* is a <u>Nash equibrium</u> in an extensive game if

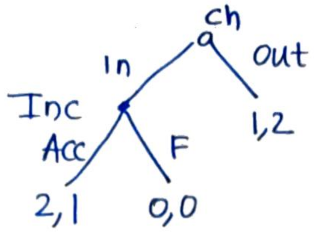
$$\forall i, \ \forall s_i \in S_i \ , \ u_i(O(s^*)) \geq u_i(O(s_i, s_{-i}^*))$$

strategy as action — same definition as in strategic game.

$\Rightarrow$ Construct the <u>strategic form</u> of an extensive game:

- Players : same as players in extensive game
- Actions : Action set of a player is set of all strategies in extensive game
- Preferences : Payoff from an action profile is payoff from outcome generated by strategic ~~g~~ profile in extensive game.

ex: Entry game



$S_{ch} : ((In), (Out)) \rightarrow A_{ch}$

$S_{Inc} : (Acc, F) \rightarrow A_{Inc}$

(In, Acc) ⟶ (In, acc)
action profile   strategy profile

⇒ In (Out, F), F is like threat given to challenger.

Ch \ Inc	Acc	F
In	2*,1*	0,0
Out	1,2*	1*,2*

NE : (In, Acc), (Out, F)

Terminal history : (Out)

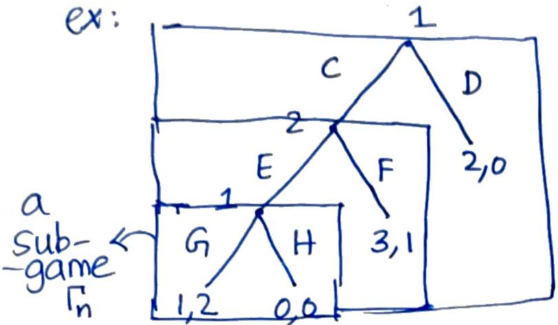
⇒ If actions are sequential, NE is not a robust.
NE is ~~stable~~ steady, based on belief, action (optimal) is chosen. It also gets validated, so repeated.

(Out, F) F doesnot occur.
So no belief is formed.
So, Foundation of this NE is not robust.
Unable to interpret it as steady state.

(ch)
Sometimes he chooses In by experiment/Mistake, then Incumbent chooses Accomodate (as its optimal) (empty threat F)

Hence, this NE is difficult to justify as steady state outcome. This is not steady state.
Not perturbed steady state.

ex:



strategic form:

$A_1 : S_1 : (CG, CH, DG, DH)$

$A_2 : S_2 : (E, F)$

Pereferences are repres-ented by following payoff Table.

1 \ 2	E	F
CG	1,2*	3*,1
CH	0,0	3*,1*
DG	2*,0*	2,0*
DH	2*,0*	2,0*

NE : (CH, F), (DG, E), (DH, E).

Action/strategy profile

↓

generates outcomes.

↓ Outcomes

(CH, F) ⟶ (C, F)

(DG, E) ⟶ (D)

(DH, E) ⟶ (D)

Subgame Perfect Equilibrium (SPE)

Non-terminal-history, part of the game that comes after this is subgame.

Γ = game

Γ_h = subgame, where h is non-terminal history.

⇒ Game is a subgame of itself. others are proper-subgame.

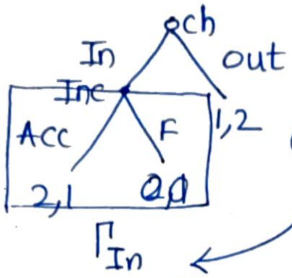
Subgame Γ_h

1. Player set : same as Γ's
2. Terminal histories
3. Player functions : same as Γ's
4. Preferences: same as Γ's. $(h \succ_i h')$

(h, h') is terminal history in Γ, then h' is terminal history in Γ_h.

⇒ SPE generates equilibrium in every possible subgame.
↳ generates optimal outcome/action in every possible subgame.

ex:



(Out, F)

↳ This strategy profile is suboptimal.

$O_{In}(Out, F) = F$ (Suppose In is being played, then F will be played).
↳ O: In, F

But $O_{In}(Out, Acc) = Acc$; O: In, F

$u_{Inc}(In, F) = 0 \to$ Not optimal as $\to u_{Inc}(In, Acc) = 1$

$\therefore$ (Out, F) is not SPE.

s is a strategy profile $\to O(s)$
h is a non-terminal history, not necessarily consistent with s.
$O_h(s)$: outcome that is generated if players stick to s, even if h has occured.

$s = (Out, F)$, $h = In$
$O_{In}(out, F) = In, F$

⇒ s^* is a subgame perfect equilibrium if $\forall h$ & i s.t $P(h) = i$
$$u_i(O_h(s^*)) \geq u_i(O_h(s_i, s_{-i}^*)), \forall s_i \in S_i$$

s^* is NE if $\forall i$, $u_i(O(s^*)) \geq u_i(O(s_i, s_{-i}^*))$ (i.e $h = \phi$)
So, SPE is also NE.

$s = (Out, F) \to$ Not SPE
$h = In$

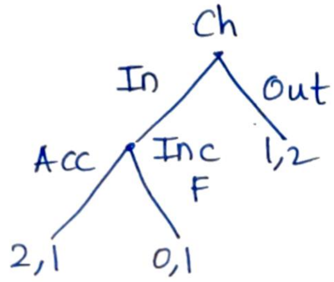
$u_{Inc}(O_{In}(s)) = u_{Inc}(In, F) = 0$

$u_{Inc}(O_{In}(s')) = u_{Inc}(In, Acc) = 1$ (Incumbent's strategy is changed, giving better payoff).
s': (out, Acc)

⇒ Along all possible paths, optimal outcomes/action is chosen.

(Out, F) is not perturbed steady state.

SPE's are perturbed steady steate. Even if they deviate, actions taken by all players are optimal.

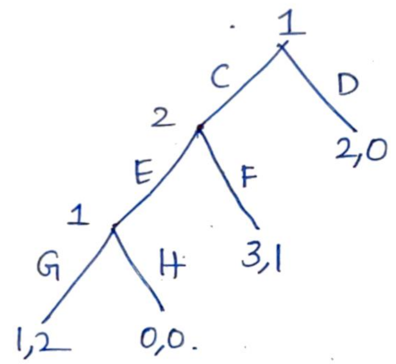


Inc

Ch	Acc	F
In	2*, 1*	0, 1*
Out	1, 2*	1*, 2*

(In, Acc), (Out, F). > Both are NE's. & both are SPE's.

ex:



NE : (CH, F), (DG, E), (DH, E)

(CH, F) → h = (C, E), if player 1 follows his strategy he gets 0
if he plays G, he get 1.
not optimal, So not SPE.

(DG, E) → If Player 2 deviates he gets less ~~hist~~ payoff.
↳ for ϕ, no need ~~n~~ to check, as its NE.
↓
SPE

(DH, E) → not SPE, as player 1 can choose G, in h = (C, E).

In a finite horizon game, the backward induction method will give same solution as SPE.

unit length subgame

Subgame that follows $h: (C, E)$, $P(h) = 1$

Optimal action of 1 after (C, E) : G

subgame that follows $h : (C)$, $P(h) = 2$

Optimal action of 2 after (C) : E

Subgame that follows $h : (\phi)$, $P(h) = 1$

Optimal action of 1 after ϕ : D

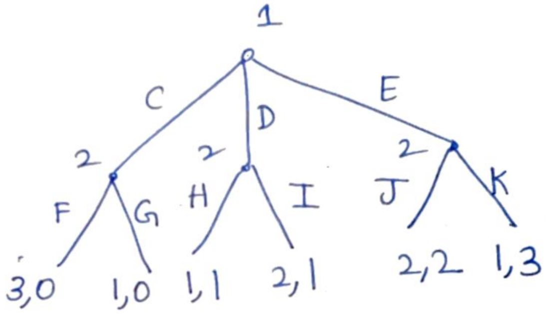
$s^* : (DG, E)$

$\uparrow \quad \uparrow$

$s_1^* \quad s_2^*$

Player could be indifferent.

ex:



2 1.5 (C, FG, HI, K)

(C, FHK)

(C, FIK)

~~(C, GIK)~~

~~(C, GHK)~~

~~(D, FIK)~~

3×8 → Payoff table.

h = C, P(h)=2, optimal actions : F, G

h = D, P(h)=2, optimal actions : H, I

h = E, P(h)=2, optimal actions : K

↳ optimal strategy of 2 : (FHK, FIK, GHK, GIK)

↓

with each find optimal strategy of 1

FHK : C

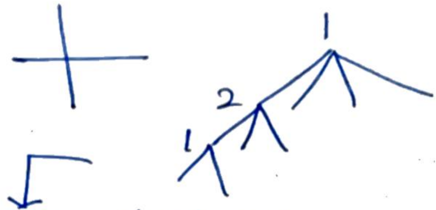
FIK : C

GHK : C, D, E

GIK : D

6 → SPE's.

ex: Tic-Tac-Toe



$4 \times 3 \times 2$ 24 terminal histories.

chess:

$W \to 20$
$\times$
$B \to 20.$

400

Finite horizon game & hence can be solved through backward induction

10^{120} Terminal histories are possible.

10^{100} years by computer

177.2

Rotten kid problem:

C (child) PA (parent)

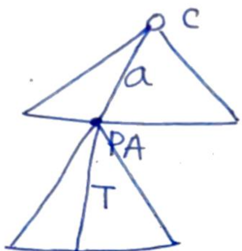
action : a $\longrightarrow$ p(a) (income of p).

c (income) = c(a)

→ child is ~~cornereced~~ concered of his income.

→ Parents also care about children's income.

T: Transfer from PA to C.



$(c(a)+T,\ p(a)-T)$ → payoff of children is $\min(c(a)+T,\ p(a)-T)$

subgame perfect ⇒ child maximizes his & parents income.

$\Gamma(a)$: PA's optimal action

$$c(a)+T = p(a)-T$$

$$T = \frac{p(a)-c(a)}{2}$$

~~$\Gamma(a)$: C's optimal acti~~

$\Gamma(\phi)$: C's optimal action

$$\max_a \left[c(a)+T\right]$$

$$\Rightarrow = \max_a \left[\tfrac{1}{2}(p(a)+c(a))\right]$$

Stackelberg Duopoly

output levels are decided sequential

1. leader
2. Follower

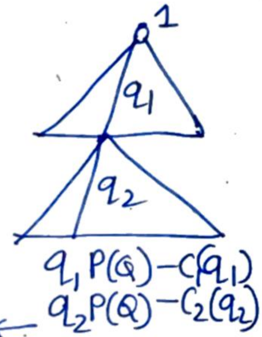
$P = \alpha - Q$	$q_i^* = \frac{\alpha - c}{3}$
$C(q_i) = cq_i$	$\pi_i^* = \frac{(\alpha-c)^2}{9}$

Cournot

Terminal histories : (q_1, q_2), $q_i \geq 0$

Player function : $P(\phi) = 1$, $P(q_1) = 2$

Preferences : π_1 & π_2 given by game tree.



$\Gamma(q_1)$: max π_2 wrt q_2.

$$b_2(q_1) = \frac{\alpha - c - q_1}{2} \text{ if } q_1 \leq \alpha - c$$
$$= 0 \quad \text{if } q_1 > \alpha - c.$$

$\Gamma(\phi)$:

max π_1 wrt q_1

$$\max_{q_1} \pi_1(q_1, q_2) = \max_{q_1} \pi_1\left(q_1, \frac{\alpha - c - q_1}{2}\right)$$
$$= \max_{q_1} \left[\tfrac{1}{2} q_1 (\alpha - c - q_1)\right]$$

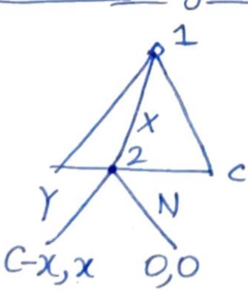
$\Rightarrow q_1^s = \frac{\alpha - c}{2} \rightarrow$ leader producing more

$\Rightarrow q_2^s = \frac{\alpha - c}{4} \rightarrow$ te follower producing less.

First mover advantage.

$$\pi_1^s = \frac{(\alpha - c)^2}{8}$$
$$\pi_2^s = \frac{(\alpha - c)^2}{16}.$$

Ultamatum game



x is continuous $0 \leq x \leq c$.

SPE:

S_2^* : Y for all $x \geq 0 \rightarrow S_{21}^*$

Y for all $x > 0$, N for $x = 0$ $\rightarrow S_{22}^*$

$S_{21}^* \rightarrow x = 0$

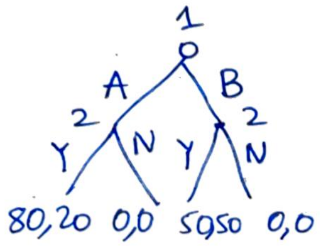
$S_{22}^* \rightarrow$ no optimal Strategy for player 1.

Only SPE: $\underset{S_1}{X=0}$ & $\underset{S_2}{Y \text{ for all } x \geq 0}$.

If x is discrete:

$\downarrow$

Other SPE: smallest x non-zero & Y for all x>0 & N for x=0.



~~A Y Y~~ $\underset{SPE}{=}$ (A, (YY))

	14	16	19
AY	12	11	7
AN	7	4	5
BY	21	12	15 → receprocity/sense of fairness
BN	1	0	1