Collective Decision Making

n players (odd)

ith player chooses action x_i ($x_i \geq 0$)

From all players → action player → $(x_1, x_2, \dots, x_n)$

(after sorting) ↓ Med(x) → Median will be chosen

ex: Budget chosen.

ith player has favourite policy x_i^* = (amount)

Res: For i, $x_i = x_i^*$ weakly dominates all other actions.

Preference of i.

$u_i(a) > u_i(a')$ iff $|x_i^* - Med(a)| < |x_i^* - Med(a')|$

[If Mean(a) would have been chosen, it's better to lie

$$\frac{x_i + \sum_{j \neq i} x_j}{n} = \bar{x} \text{ (policy)} = x_i^*$$

$$\Rightarrow x_i = n x_i^* - x'$$

He tells truth only if $x_i = x_i^*$

$\Rightarrow (n-1) x_i^* = x'$

$\Rightarrow x_i^* = \frac{x'}{n-1} = \frac{\sum_{j \neq i} x_j}{n-1}$]

1) $x_i > x_i^*$

Choose a player i, for n-1 players

$[x_1 \quad \underline{a} \quad \bar{a} \quad x_n]$

$x_i^* \quad x_i$

	$x_m(x_i^*)$	$x_m(x_i)$	Comment
$x_i^* < x_i \leq \underline{a}$	$\underline{a}$	$\underline{a}$	$x_i^* \sim_i x_i$
$x_i^* \leq \underline{a} < x_i \leq \bar{a}$	$\underline{a}$	x_i	$x_i^* \succ_i x_i$
$x_i^* \leq \underline{a} \leq \bar{a} < x_i$	$\underline{a}$	$\bar{a}$	$x_i^* \succ_i x_i$
$\underline{a} \leq x_i^* < x_i \leq \bar{a}$	x_i^*	x_i	$x_i^* \succ_i x_i$
$\underline{a} < x_i^* < \bar{a} < x_i$	x_i^*	$\bar{a}$	$x_i^* \succ_i x_i$
$\underline{a} < \bar{a} < x_i^* < x_i$	$\bar{a}$	$\bar{a}$	$x_i^* \sim_i x_i$

So, $x_i = x_i^*$ weakly dominat[es all other actions]

Solution concepts { Nash equilibrium
Dominance solvability

Max Min strategy

(Only applicable in two player zero-sum games)

ex:

		2			
		L	M	R	Min
1	T	2*	5	13*	2 (Payoffs of player 1)
	H	6	5.6**	10.5	$\boxed{5.6}$ → max(Player 1)
	L	6	4.5	1*	1
	B	10*	3	-2*	-2
Max		10	$\boxed{5.6}$	13	

(H,M) is NE ← min(Player 2)

$$\min_{a_1} \max_{a_2} u_2(a_1,a_2)$$
$$\max_{a_1} \min_{a_2} u_1(a_1,a_2)$$
> If they don't match, then no solution.

⇒ We may not always get solution, even if it's present.

Rationalisable actions

⇒ Some actions are never played, for a players, remaining actions are rationalisable actions.

ex:

		2			
		C_1	C_2	C_3	C_4
	R_1	0,7*	2,5	7*,0	0,1
	R_2	5,2	3*,3*	5,2	0,1
1	R_3	7*,0	2,5	0,7*	0,1
	R_4	0,0*	0,-2	0,0*	10*,-1

For 1: R_1 is BR to C_3
For 2: C_3 is BR to R_3
For 1: R_3 is BR to C_1
For 2: C_1 is BR to R_1

R_1, R_3 / $C_1\ C_3$ are rationalisable

For 1: R_2 is BR to C_2
For 2: C_2 is BR to R_2
(actions involving NE are rationalisable)

For 1: R_4 is BR to C_4
For 2: C_4 is BR to ϕ
R_4, C_4 are not rationalisable actions.

application of NE

Symmetric games

2 player games

⇒ off diagonal elements have symmetry. On diagonal they are equal.

PD →

	C	NC
C	1,1	3,0
NC	0,3	2,2

along diagonal payoff's of players are equal
⇒ off diagonal payoff's are flipped.

Focal point is not a mathematical concept.

⇒ $u_1(a_1,a_2) = u_2(a_2,a_1)$
If $a_1=a_2=a$ ⇒ $u_1(a,a) = u_2(a,a)$, Then diagonal elements will have same values.

Interaction between members of the same population (same action sets & preferences) that game is symmetric game.

ex: Prisoners dilemma

	C	NC
C	1,1	3,0
NC	0,3	2,2

BoS is not symmetric game

	B	O
B	2,1	0,0
O	0,0	1,2

Matching pennies is not symmetric game

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

stag hunt is ~~not~~ symmetric game

	St	H
St	3,3	1,2
H	2,1	2,2

Dawkhove is symmetric game

	A	P
A	0,0	3,1
P	1,3	2,2

NE is steady state & is repeated.

⇒ This is not applicable in symmetric games.

ex.

		2: a	b
1	a	0,0	1,1
	b	1,1	0,0

NE: (a,b), (b,a)

unilateral deviation is unprofitable, for others this is not the case.

This is symmetric game.

⇒ These can't be steady state.

⇒ In symmetric game, steady state should have same actions. 1,2 belong to same group i.e indistinguishable, so if actions are different NE is not steady i.e not symmetric NE

Symmetric NE :

a^* in a game with n players with $A_i = (A)$, is a symmetric NE if it is a NE and $a_i^* = a^*$, $\forall i$.

Oligopoly Market :

no. of sellers is small but not 1

1. Cournot Model

Equilibrium price · demand, supply = f(price)

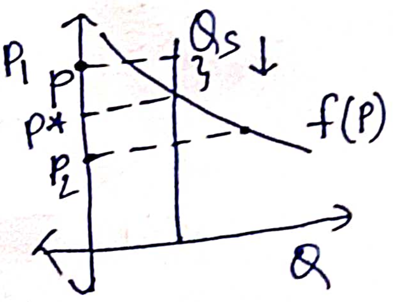
Demand Side :

demand function:

quantity demand as function of price

$Q^d = f(P)$

$f'(P) < 0$



Q^d – Quantity demanded

$P = f^{-1}(Q^d)$

$f^{-1\prime}(Q^d) < 0$

If demand should be decreased, price should be increased

Inverse demand function

Ss <u>Supply side</u> :

$n \geq 2$ firms

supply q_i $(i=1,2,\dots,n)$ $\Rightarrow$ $\sum_{i=1}^{n} q_i = Q^s$ (quantity supplied)

equilibrium condition $Q^d = Q^s$

$\cancel{P=Q^{-1}}$ $P^* = f^{-1}(Q^s)$

$= f^{-1}(Q)$

equilibrium price.

$\Rightarrow$ firms are players and they try to maximize their profit functions.

$\Rightarrow$ Institution through which purchase and selling is taking place – Market (abstract concept) (Transactions)

1. Firms are the players, n in number
2. $q_i \geq 0$ – action of ith players, $i=1,2,\dots,n$
3. Firms want to maximize their profits, π_i $i=1,2,\dots,n$.

$\pi_i(q_1,q_2,\dots,q_n) =$ Total Revenue$_i$ – Total cost$_i$

$= TR_i - TC_i$

$TC_i = C_i(q_i)$

$C_i'(\ ) > 0$

$\pi_i(q_1,q_2,\dots,q_n) = P \cdot q_i - C_i(q_i)$

$= P(Q)\cdot q_i - C_i(q_i)$ [$Q = \sum q_i$, $P(Q) \rightarrow$ inverse demand function]

$= P(q_1+q_2+\dots+q_n)q_i - C_i(q_i)$

$\Rightarrow$ Price is determined not by firms, it is determined by demand and supply.

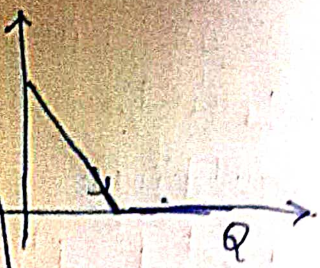
some other firm $\rightarrow Q\uparrow$ $P\downarrow \rightarrow$ so affecting our profit

$\pi(eq)$

$\Downarrow$

Prices are not set by producers, but total supply & demand are determining it.

$TC_i = cq_i \quad , c>0$
(constant unit cost)



$P(Q) = \alpha - Q \quad \text{if } \alpha \geq Q$
$\quad\;\; = 0 \quad \text{otherwise}$

assumption $\alpha > c$ (production to be profitable)

α – willingness to pay

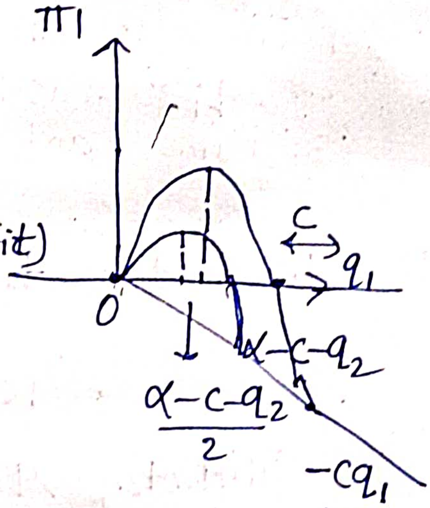
$n=2$

$\pi_1(q_1,q_2) = q_1(\alpha - q_1 - q_2) - cq_1 \quad (\text{if } q_1+q_2 \leq \alpha)$

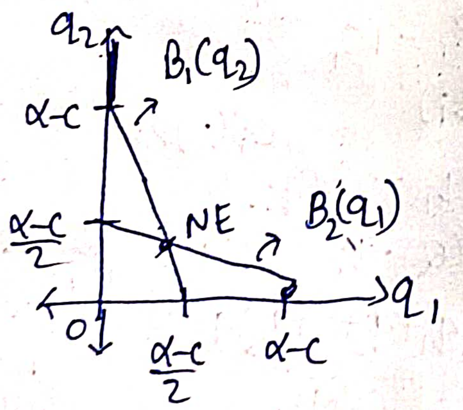
$\pi_2(q_1,q_2) = q_2(\alpha - q_1 - q_2) - cq_2$

$\pi_1(q_1,q_2) = q_1(\alpha - c - q_1 - q_2) \quad \text{if } \alpha \geq q_1+q_2$
$\quad = -cq_1 \quad \text{if } \alpha < q_1+q_2$

$B_1(q_2) = \frac{\alpha - c - q_2}{2} \quad \text{if } \alpha - c \geq q_2$
$\quad = 0 \quad \text{if } \alpha - c < q_2$
(negative profit)



q_2 is changing
as $q_2\uparrow$, optimal q_1 shifts left.
$\Downarrow$
(as $q_2\uparrow$, $q_1\downarrow$)



Price is decreased, So, Total revence is decreased, So, q_1 is decreased.

$B_2(q_1) = \frac{\alpha - c - q_1}{2} \quad \text{if } q_1 \leq \alpha - c$
$\quad = 0 \quad \text{if } q_1 > \alpha - c$

$q_2^* = q_1^* = \frac{\alpha - c}{3}$

$Q^* = \frac{2}{3}(\alpha - c)$

$P^* = \alpha - \frac{2}{3}(\alpha - c) = \frac{\alpha}{3} + \frac{2c}{3}$

$\Pi_1^* = \left(\frac{\alpha - c}{3}\right)^2 = \Pi_2^*$

α – willing to pay

$\frac{\partial q_i^*}{\partial \alpha} > 0$, $\frac{\partial P_i^*}{\partial \alpha} > 0$ (higher output produced at higher price)

$\frac{\partial \Pi_1^*}{\partial \alpha} > 0$

- technological improvement – c decreases.

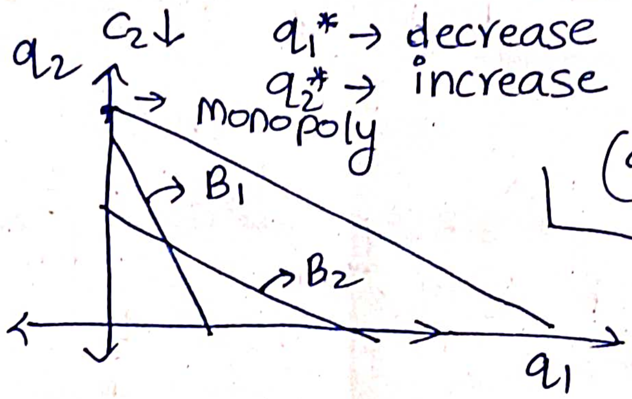
$\Rightarrow$ Profit $\uparrow$, profit part is given to buyers (Price is decreasing).

$c_1 > c_2$ firm 2 has advantage. 58.1

$\Rightarrow$ firm 2 produces more in eq $\frac{\alpha - c_2}{3}$

$c_2 \downarrow$ $q_1^* \to$ decrease
$q_2^* \to$ increase (If q_2^* increases, q_1^* decreases)

(q_1^* will become 0, at some point)



(not o case)
Q^* increases
P^* decreases.

q.

$n = 2$

$C_1(q_1) = 10 + 2q_1$

$C_2(q_2) = 12 + 8q_2$

$P = 50 - 2Q$

(10, 12 – fixed costs of production)
(2, 8 – marginal costs)

$\Pi_1(q_1, q_2) = q_1(50 - 2(q_1 + q_2)) - (10 + 2q_1)$

$\Pi_2(q_1, q_2) = q_2(50 - 2(q_1 + q_2)) - (12 + 8q_2)$

$\frac{\partial \Pi_1}{\partial q_1} = 0 \Rightarrow q_1 = \frac{1}{2}(24 - q_2) = B_1(q_2)$ —(1) if $q_2 \leq 24$ else 0.

$\frac{\partial \Pi_2}{\partial q_2} = 0 \Rightarrow q_2 = \frac{1}{2}(21 - q_1) = B_2(q_1)$ —(2) if $q_1 \leq 21$ else 0.

from (1), (2)

$q_1^* = 9, \; q_2^* = 6.$

$P^* = 50 - 2(15)$
$= \underline{20}$

$\Pi_1^* = 9(20) - (10 + 18)$
$= 180 - 28 = \underline{152}$

$\Pi_2^* = \underline{60}$

59.2

$n=2$

$C_i(q_i) = 0$, if $q_i = 0$

$= f + cq_i$, if $q_i > 0$

$c \geq 0, f > 0$

$P = \alpha - Q$, $\alpha > c$

π_1, π_2

$\frac{\partial \pi_1}{\partial q_1} \; \frac{\partial \pi_2}{\partial q_2}$

Solve to get q_1, q_2

$q_1 = B_1(q_2) = \frac{\alpha - c - q_2}{2}$ (f is not affecti

$\pi_1 = (\alpha - (q_1+q_2))q_1 - cq_1 - f$

for some, q_2, it could be -ve.

$\Rightarrow \pi_1(B_1(q_2), q_2) = \left(\frac{\alpha - c - q_2}{2}\right)^2 - f = 0$

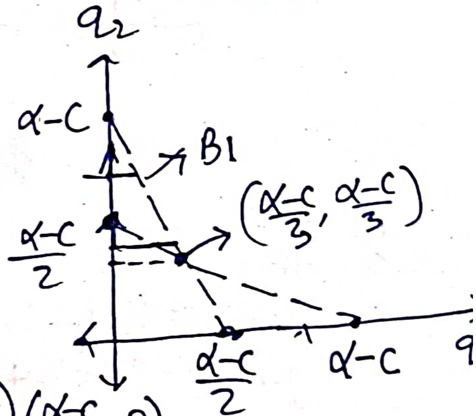
If $f = \left(\frac{\alpha - c - q_2}{2}\right)^2$, $\pi_1 = 0$

If $f > \left(\frac{\alpha - c - q_2}{2}\right)^2 \Rightarrow q_1 = 0 = B_1(q_2)$

$\Rightarrow (\alpha - c - q_2)^2 \cancel{<} = 2^2 f$.

$\Rightarrow \bar{q}_2 \cancel{<} = \alpha - c - 2\sqrt{f}$

$\Rightarrow$ If $q_2 > \bar{q}_2$, Then $\pi_1 < 0$ for given best response function.



If $\bar{q}_2 \sim \left(\frac{\alpha - c}{2}, \alpha - c\right)$

$\bar{q}_1 \sim \left(\frac{\alpha - c}{2}, \alpha - c\right)$ (symmetric)

NE is same

If $\bar{q}_2 \sim \left(\frac{\alpha - c}{3}, \frac{\alpha - c}{2}\right)$

NE $\rightarrow \left(\frac{\alpha-c}{3}, \frac{\alpha-c}{3}\right) \left(0, \frac{\alpha-c}{2}\right) \left(\frac{\alpha-c}{2}, 0\right)$

If $\bar{q}_2 \sim \left(0, \frac{\alpha - c}{3}\right)$ NE $\rightarrow \left(0, \frac{\alpha-c}{2}\right)\left(\frac{\alpha-c}{2}, 0\right)$

If $\bar{q}_2 < 0 \rightarrow$ NE $\rightarrow (0,0)$

(more f)

Increase marketshare

$c_i(q_i) = cq_i$

$s_1 = \frac{q_1}{q_1+q_2}$ [→ Increase if $q_1 \uparrow$] → no losses, but not maximizing profit.

firm 1 → maximises s_1 (subj to not making losses)
firm 2 → maximises Π_2

$\cancel{B_1(q_2) =}$

$B_2(q_1) = \frac{\alpha - c - q_1}{2}$ if $q_2 \leq \alpha - c$

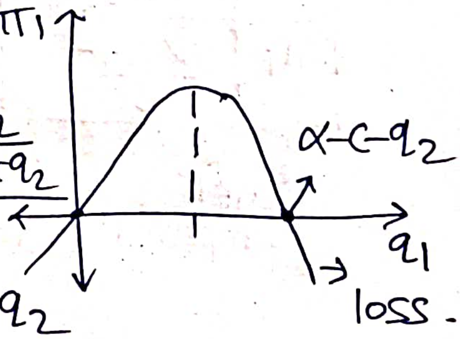
$= 0$ if $\alpha - c < q_2$

$\Pi_1(q_1, q_2) = q_1(\alpha - q_1 - q_2) - cq_1$

$\max \frac{q_1}{q_1+q_2}$

$\frac{\partial}{\partial q_1}\left[\frac{q_1}{q_1+q_2}\right]$ $1 - \frac{q_2}{q_1+q_2}$

$\frac{ba' - ab'}{b^2}$



$B_1(q_2) = \alpha - c - q_2$

$= (q_1+q_2)(1) - 1\, q_1(1)\, \cancel{q_2}$ $\frac{q_2}{(q_1+q_2)^2}$

$q_1 = B_1(q_2) = \alpha - c - q_2$

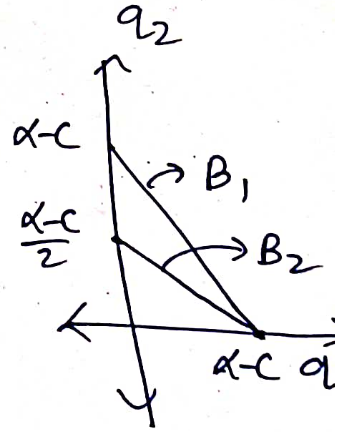
$q_2 = \frac{\alpha - c - q_1}{2}$

$= \alpha - c - \left(\frac{\alpha - c - q_1}{2}\right)$

$2q_1 = \alpha - c + q_1$

$\Rightarrow q_1 = \alpha - c$

$q_2 = 0$



NE → $(\alpha - c, 0)$ → monopoly

$\Rightarrow p^* = \cancel{\alpha - \alpha}\ \alpha - (\alpha - c) = c$

$\Rightarrow \Pi_i^* = 0$ [aim for firm 1, to not make losses]

⇒ If both want to maximize s_i ⇒ all points on line are NE.

Collusion

⇒ competitors come to agreement, to fix supplies $\left(\frac{Q}{2}\right)$ & profit shared equally.

$\Pi = q_1 P - cq_1 + q_2 P - cq_2$

$= P(q_1 + q_2) - c(q_1 + q_2) = PQ - CQ$

$= Q(\alpha - Q) - CQ$

$\frac{\partial \Pi}{\partial Q} = 0 \Rightarrow Q^{*c} = \frac{\alpha - c}{2}$

$q_i^{*c} = \frac{\alpha - c}{4}$, $p^c = \frac{\alpha + c}{2}$,

$\Pi^c = \frac{(\alpha - c)^2}{8} > \Pi_i^* = \frac{(\alpha - c)^2}{9}$ (NE)

(Collusion)

	Collusion	NE
q_i	$\frac{\alpha - c}{4}$	$\frac{\alpha - c}{3}$
Q	$\frac{\alpha - c}{2}$	$\frac{2}{3}(\alpha - c)$
p	$\frac{\alpha + c}{2}$	$\frac{\alpha + 2c}{3}$
Π_i	$\frac{(\alpha - c)^2}{8}$	$\frac{(\alpha - c)^2}{9}$

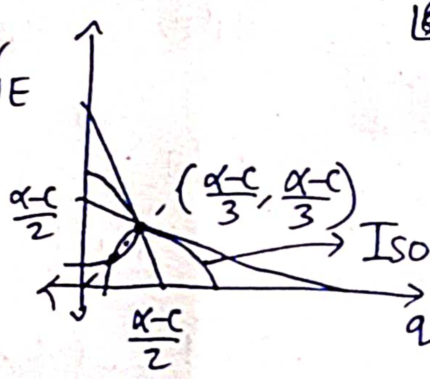
Paretosuperior compared to NE

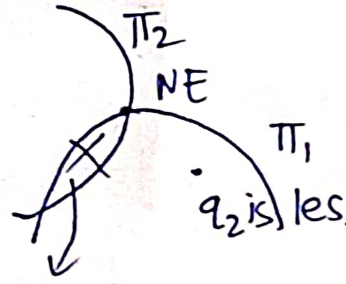
Overproducing in NE compared to when profit was higher.

$B_1\left(\frac{\alpha - c}{4}\right) \neq \frac{\alpha - c}{4}$

$= \frac{3}{8}(\alpha - c)$

(Profit is higher if q_1 is this)





q_2 is less, other ~~profit~~ produce less, 1's profit increases.

$\Downarrow$

$\boxed{q_1(\alpha - c - q_1 - q_2)}$

both firms, profit is more

for n players:

$q_i^* = \frac{1}{n+1}(\alpha - c)$

$Q^* = \frac{n}{n+1}(\alpha - c)$

$p^* = \frac{\alpha + nc}{n+1}$

(Perfect competition $n \to \infty$)

($p^* \to c$ as $n \to \infty$)

$\Rightarrow$ Through competetion price is reduced & profit approaches 0.

BERTRAND OLIGOPOLY (except agricultural)
(goods are same)

n firms decide price.
($n \geq 2$)

$p_i \geq 0$ is set by firm i

$\Rightarrow$ No customers if price is even slightly higher than least price

$\Downarrow$

entire market
(profit of other ~~parts~~ firms $=0$)

(if k firms have set least price, $\frac{1}{k}$(market) for each of these firms).

$\pi_i(p_1, p_2, \dots, p_n) = p_i \frac{D(p_i)}{k} - c_i\left(\frac{D(p_i)}{k}\right)$ if $p_i = \min(p_1, \dots)$ and there are k such firms with min prices

D (quantity demand) $= D(p)$

$D'(p) < 0$

$\pi_i(p_1, p_2, \dots, p_n) = \cancel{p_i D(p_i)}\ 0$, if $p_i > \min(p_1, p_2, \dots, p_n)$

$\Rightarrow c_i(q_i) = cq_i$

$\Rightarrow$ If $p_i < c \rightarrow$ Negative profit, but entire market is captured. (then cater to that market)

Suppose,

$n = 2$

$D = \alpha - p, \ \alpha > 0.$

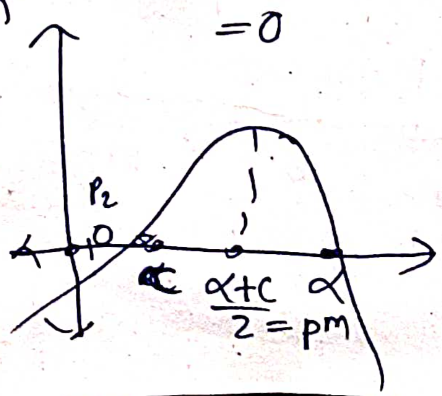
$c_i(q_i) = cq_i, \ \alpha > c > 0.$

$\pi_1(p_1, p_2) = p_1\left(\frac{\alpha - p_1}{2}\right) - c\left(\frac{\alpha - p_1}{2}\right)$ if $p_1 = p_2$

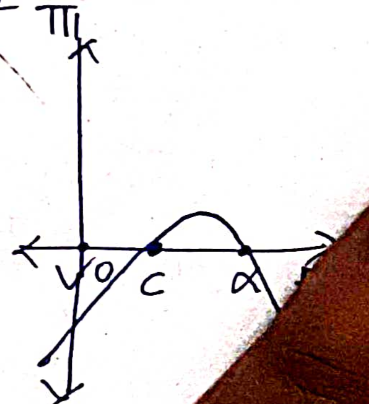
$= \frac{1}{2}\underbrace{(\alpha - p_1)}_{\text{(size of market, demand)}}(p_1 - c)$

$= (\alpha - p_1)(p_1 - c)$ if $p_1 < p_2$

$= 0$ else.

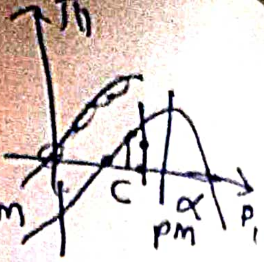


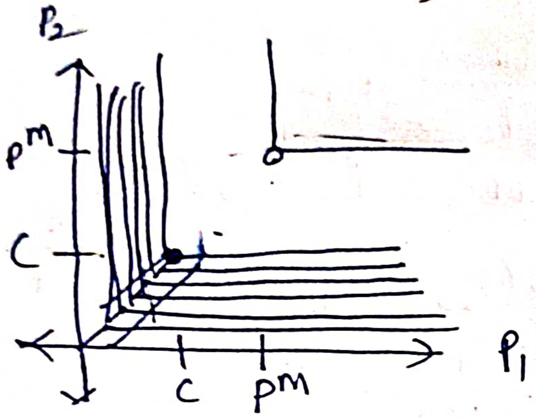
m → monopoly



$P_1 = B_1(P_2) = \{P_1 : P_1 > P_2\}$ if $P_2 < C$
$= \{P_1 : P_1 \geq P_2\}$ if $P_2 = C$
$= \phi$ if $p^m \geq P_2 > C$
$= \{P_1 : P_1 = p^m\}$ if $P_2 > p^m$

(because P_1 is chosen as continuous variable)





(C,C)
(not strict NE)

NE → (C,C) → No positive profit for firms.
↳ price undercutting, quote price very less ($\geq C$) because of competetion.

Show (C,C) is a NE & no other (P_1, P_2) is NE.
↓
$\pi_i = 0$
encrease price, still $\pi_i = 0$
decrease price, loss which is worse, no profitable deviation is possible, So, it is NE

⇒ C is weakly dominating P_1 when $P_2 < C$. for firm (where $P_1 < C$)

No other (P_1, P_2) is NE

- $P_1 = P_2$
  - $P_1 < C$ ⇓ just slightly higher than P_2, profit = 0
  - $P_1 > C$ ⇓ just slightly less than P_2, profit is better
- $P_1 < P_2$
  - $P_1 < C$: firm1 has negative profit, so better deviation (even slightly increasing is better) → $\frac{P_1 + P_2}{2}$
  - $P_1 = C$ ↓ firm1 has profitable deviation
  - $P_1 > C$ ↓ bot

raise. Then can if ($P_1 < P^M$)

ex:

$\alpha > c+1$, $c>0$, an integer Profits are descrete

(c,c) No profitable deviation. so, it is NE.

$(c+1, c+1)$ is NE → No profitable deviation, so, it is NE.

↓ $\frac{1}{2}(1)(\alpha-(c+1)) > 0$ $\frac{1}{2}(2)(\alpha-(c+2))$.

ex: $n \geq 3$

Set of NE = $(P_1, P_2, \dots, P_n)$ st prices $\underset{\forall i}{P_i \geq c}$ and atleast two prices=c

A B

$B \Rightarrow A$: At B, $\pi_j = 0$. no profitable deviation for any firms.

$\sim B \Rightarrow \sim A$

$\Rightarrow \not\exists P_i > c$ $\forall i$ → two cases. all equal X
~~atte~~ else X

→ atleast one price $P_i < c$ → make $c \Rightarrow$ zero

→ One price=c → This can increase.

$\Rightarrow c_1 \neq c_2$ (No NE) $c_1 < c_2$ c_2, c_2 $c_1\, c_1$

can decrease increase

⇓

Here if firm1 gets total market Then no need to decrease (to increase profit), Then this is NE.