Physics II: Electromagnetism

PH 102

Lecture-1

March-June 2022

- Vector and Scalar field
- Differential calculous of a scalar field
  - The gradient and its interpretation
- Differential calculous of a vector field
  - The Divergence
  - The curl
- Important Identies
- Summary

Instructor details

Welcome to PH102
(March - June 2022)

Instructors:

Dr. Gagan Kumar
Dr M.C.Kumar

Vector Calculus, Electrostatics

Prof. P. K. Giri
Prof. Bipul Bhuyan

Magnetostatics, Electrodynamics, Electromagnetic Waves

Contact details

Dr. M.C. Kumar

Department of Physics

IIT Guwahati

Guwahati-781039

Phone : +91 361 258 3566

Email : *mckumar@iitg.ac.in*

Office room : Top floor, Annex Building, Physics Department

Dr. Gagan Kumar

Department of Physics

IIT Guwahati

Guwahati-781039

Phone : +91 361 258 3557

Email : *gk@iitg.ac.in*

Office room : 2nd floor, Annex Building, Physics Department

Classes and Tutorials

Division	Batch	Timings
Division-I	ME, CE	Wednesday 11:00 - 12:00 Thursday 11:00 - 12:00
Division-II	BT, ECE, EEE	Wednesday 11:00 - 12:00 Thursday 11:00 - 12:00
Division – III	CSE, MC, EPH	Wednesday 16:00 - 17:00 Thursday 16:00 - 17:00
Division - IV	CL, CST, DS&AI	Wednesday 16:00 - 17:00 Thursday 16:00 - 17:00

Tutorials will be held on Tuesdays from 8:00 - 8:55

Prof. Bipul Bhuyan and Dr. M.C. Kumar : Division-I and III (Venue : Lecture Hall -L1)

Prof. P.K. Giri and Dr. Gagan Kumar : Division-II and IV (Venue : Lecture Hall -L3)

Tutorials

- Tutorials are for you to interact with a tutor
- A problem sheet will be given to you few days before the scheduled tutorial.
- You are expected to attempt and solve these problems before you come to tutorials
- If you find a problem to be difficult, please ask your respective tutor for guidances.
- Tutorials will be held offline in their respective venues.
- All the 19 tutorials groups (roll number wise) have already been created.

Tutorial Groups and venue

S.No.	Tutorial Group	Branch	Roll numbers	Venue	Tutor's name	TA's name
1	T1	ME	210103001 - 210103050	L1	MCK	Banashree Baishya
2	T2	ME	210103051 - 210103100	L2	PKG	Sumit Dey
3	T3	ME+CL	210103101 - 210104031	L3	BB	Subhangi K. Maurya
4	T4	CL	210104032 - 210104081	L4	GK	Rajnandan Choudhury Das
5	T5	CL+ECE	210104082 - 210102013	1005	Subhajit Barman	
6	T6	ECE	210102014 - 210102063	1G1	UR	Akanshu Chauhan
7	T7	ECE+BSBE	210102064 - 210106011	1G2	PA	Alolika Roy
8	T8	BSBE	210106012 - 210106061	2203	GSS	Nikhil Danny Babu
9	T9	BSBE+EEE	210106062 - 210108026	2204	Pamu	Mandira Das
10	T10	EEE+CSE	210108027 - 210101014	3102	AKSh	Debabrata Paul

Tutorial Groups

11	T11	CSE	210101015 - 210101064	3202	PKP	Sudeshna Madhual
12	T12	CSE+EP	210101065 - 210121001	4001	AKS	Sampreet Kalita
13	T13	EP	210121002 - 210121051	4005	SuBh	Mrinal Kanti Giri
14	T14	EP+MnC	210121052 - 210123033	4006	BRM	Partha Das
15	T15	MnC+CH	210123034 - 210107012	4G1	Asil	Sourav Pal
16	T16	CH	210107013 - 210107062	4G2	SBD	Monu Singh
17	T17	CH+CST	210107063 - 210122014	5102	SC	Angana Bhattacharya
18	T18	CST+	210122015 - 210122064	5201	Ambaresh Sahoo / Pritam Das	
19	T19	CST+DSAI +Backlog	210122065 - 210150020	5206	Sovan	Prantik Sarmah

Assessments

Events	Date	Time	Marks
Quiz 1	12th April, 2022	8:00 am - 8:55 am	15
Midsem	9th May 2022	2:00 pm - 4:00 pm	30
Quiz 2	31st May 2022	8:00 am - 8:55 am	15
Endsem	26th June 2022	2:00 pm - 5:00 pm	40

Syllabus

- **Vector analysis:** Gradient, Divergence and Curl; Line, Surface, and Volume integrals; Gauss's divergence theorem and Stokes' theorem in Cartesian; Spherical polar and Cylindrical polar coordinates; Dirac Delta function.

- **Electrostatics:** Gauss's law and its applications, Divergence and Curl of Electrostatic fields, Electrostatic Potential, Boundary conditions, Work and Energy, Conductors, Capacitors, Laplace's equation, Method of images, Boundary value problems in Cartesian Coordinate Systems, Dielectrics, Polarisation, Bound Charges, Electric displacement, Boundary conditions in dielectrics, Energy in dielectrics, Forces on dielectrics.

Before midsem

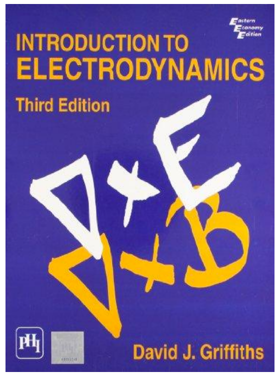
- Magnetostatics: Lorentz force, Biot-Savart and Ampere's laws and their applications, Divergence and Curl of Magnetostatic fields, Magnetic vector Potential, Force and torque on a magnetic dipole, Magnetic materials, Magnetization, Bound currents, Boundary conditions.

- Electrodynamics: Ohm's law, Motional EMF, Faraday's law, Lenz's law, Self and Mutual inductance, Energy stored in magnetic field, Maxwell's equations, Continuity Equation, Poynting Theorem, Wave solution of Maxwell Equations.

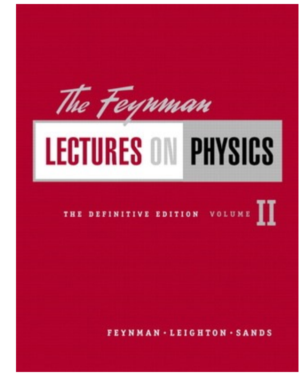
- Electromagnetic waves: Polarization, reflection & transmission at oblique incidences.

After midsem

Texts and References

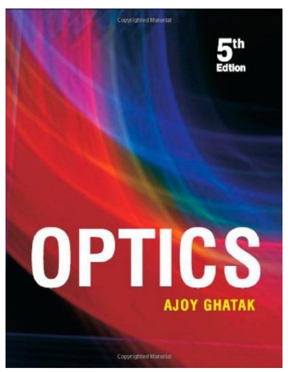


Introduction to electrodynamics : D. J. Griffiths, 3rd edition, Prentice Hall of India (2005)

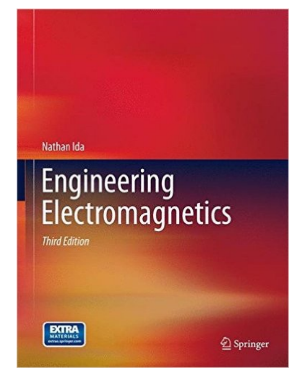


The Feynman lectures on Physics, Vol II: R. P. Feynman, R. B. Leighton and M. Sands, Narosa Publishing House (1998)

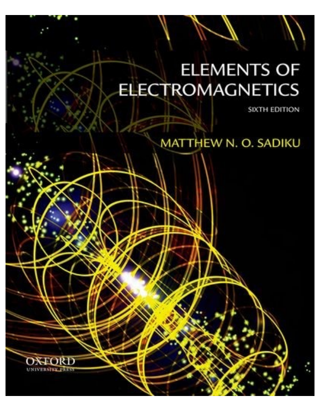
http://www.feynmanlectures.caltech.edu/



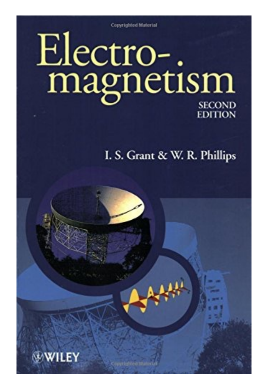
Optics : A. K. Ghatak, Tata McGraw Hill (2007)



Engineering Electromagnetics : N. Ida, Springer (2005)



Elements of Electromagnetics : M. N. O. Sadiku, Oxford (2005)



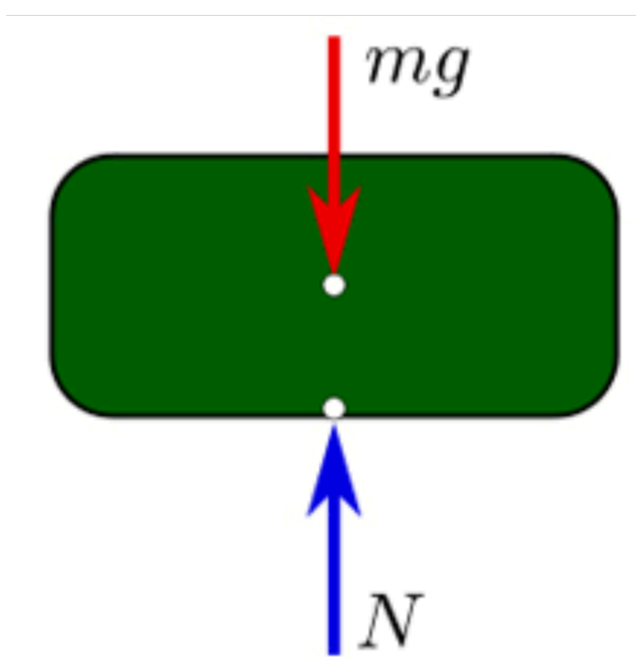
Electromagnetism : I. S. Grant and W. R. Phillips, John Wiley (1990)

Introduction: Scalar and Vector Fields

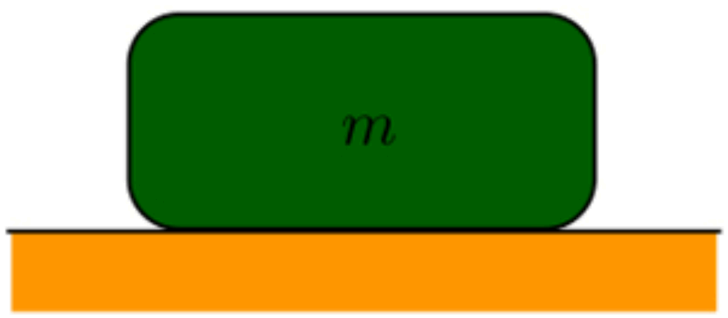
Vectors and Scalars

Vector: Quantities which have direction as well as magnitude

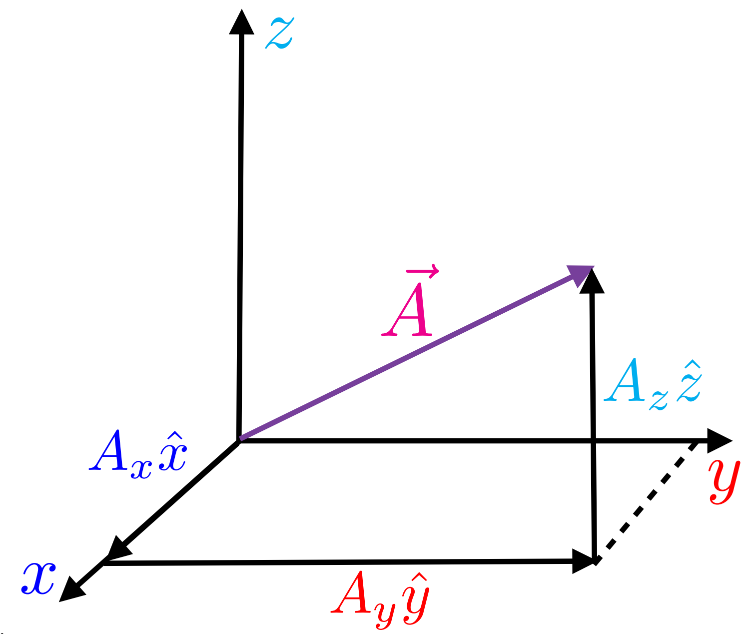
Scalar: Quantities which have no direction but magnitude



Vectors	Scalars
Position	Norm of position vector
Force	Work
Velocity	Temperature
Current Density	Charge Density
Momentum	Kinetic Energy
Area	Volume
Weight	Mass



An arbitrary vector **A** can be expanded in terms of basic vectors



$$\vec{A} = (A_x, A_y, A_z)$$

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

Important vector operations

$$\begin{aligned}
\vec{A}.\vec{B} &= \text{scalar} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta \\
\vec{A}\times\vec{B} &= \text{vector} = AB\sin\theta\hat{n} \\
(\vec{A}\times\vec{B})_z &= A_xB_y - A_yB_x \\
(\vec{A}\times\vec{B})_x &= A_yB_z - A_zB_y \\
(\vec{A}\times\vec{B})_y &= A_zB_x - A_xB_z
\end{aligned}$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Vectors Algebra (contd.)

Few more important vector operations

$$\begin{aligned} \vec{A} \times \vec{A} &= 0 \\ \vec{A}.(\vec{A} \times \vec{B}) &= 0 \\ \vec{A}.(\vec{B} \times \vec{C}) &= (\vec{A} \times \vec{B}).\vec{C} \end{aligned}$$

Triple product

- Scalar triple product $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B}),$

- Vector triple product $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}).$ | **BAC-CAB rule**

Higher vector products

$$\begin{aligned} (\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) &= (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}); \\ \mathbf{A} \times (\mathbf{B} \times (\mathbf{C} \times \mathbf{D})) &= \mathbf{B}(\mathbf{A} \cdot (\mathbf{C} \times \mathbf{D})) - (\mathbf{A} \cdot \mathbf{B})(\mathbf{C} \times \mathbf{D}). \end{aligned}$$

Levi Civita symbol

$$\vec{A}.\vec{B} = \sum_{i=1}^{3} A_i B_i \quad \text{also written as} \longrightarrow A_i B_i \qquad \text{(Einstein's summation convention)}$$

$$(= A_1 B_1 + A_2 B_2 + A_3 B_3) \qquad \boxed{\text{where } \; 1 \rightarrow x, \; 2 \rightarrow y, \; 3 \rightarrow z}$$

Tullio Levi-Civita

The cross product can be conveniently described by introducing a symbol ε_{ijk} (Levi Civtia)

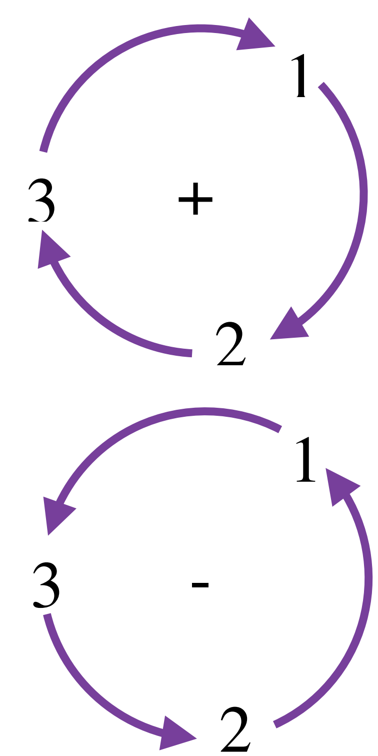
$j, k :$ dummy indices

$i :$ free index

$$\vec{A} \times \vec{B} = \sum_{j=1}^{3} \sum_{k=1}^{3} \varepsilon_{ijk} A_j B_k$$

where $\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$ and $\varepsilon_{132} = \varepsilon_{321} = \varepsilon_{213} = -1$ and all $\varepsilon_{iij} = \varepsilon_{iii} = 0$

$$\begin{aligned}
(\vec{A} \times \vec{B})_1 &= \varepsilon_{123} A_2 B_3 + \varepsilon_{132} A_3 B_2 = A_2 B_3 - A_3 B_2 \\
(\vec{A} \times \vec{B})_2 &= \varepsilon_{231} A_3 B_1 + \varepsilon_{213} A_1 B_3 = A_3 B_1 - A_1 B_3 \\
(\vec{A} \times \vec{B})_3 &= \varepsilon_{312} A_1 B_2 + \varepsilon_{321} A_2 B_1 = A_1 B_2 - A_2 B_1
\end{aligned}$$



Helps to simplify cross product calculations.

Levi Civita: An example

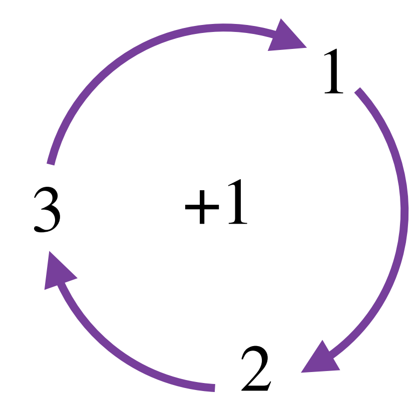
1. ε_{ijk} has $3 \times 3 \times 3 = 27$ components: 3 components are equal to 1, 3 are equal to -1 and rest 21 are zero.

2. $\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$, where $\delta_{ij} = 1$ if $i = j$ and 0 if $i \neq j$.

An example:

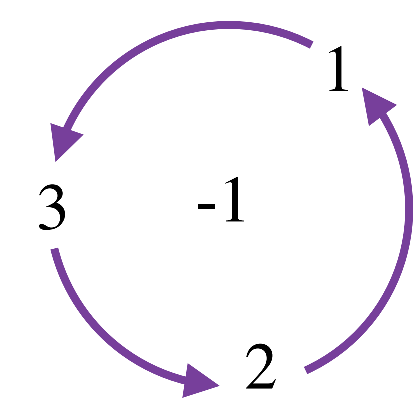
Show that $\quad \vec{A}.(\vec{B} \times \vec{C}) = (\vec{A} \times \vec{B}).\vec{C}$

$$\text{LHS} = A_i \varepsilon_{ijk} B_j C_k = \varepsilon_{ijk} A_i B_j C_k$$



By cycling the indices, we get

$$\varepsilon_{ijk} A_i B_j C_k = \varepsilon_{kij} C_k A_i B_j = \varepsilon_{jki} B_j C_k Ai.$$



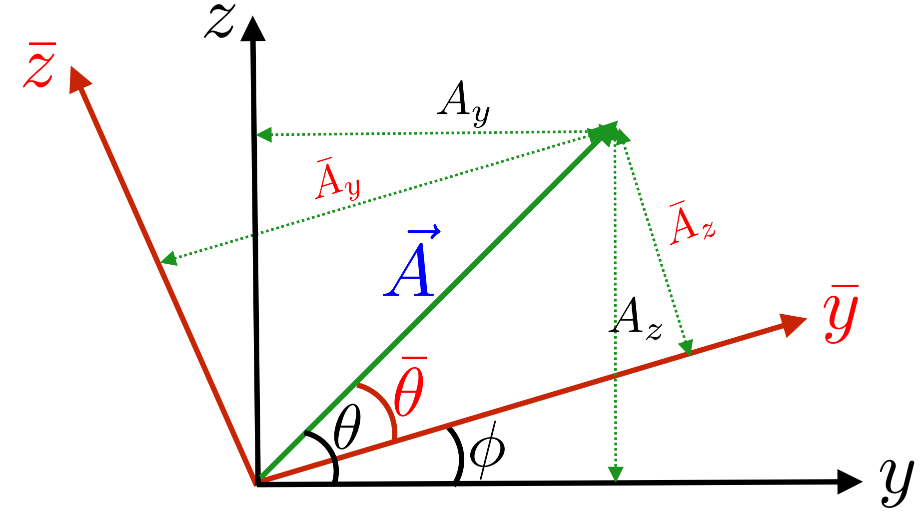
Hence

$$\vec{A}.(\vec{B} \times \vec{C}) = \vec{C}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{C} \times \vec{A})$$

Vector transformation

Components of a vector has to transform "properly" under a coordinate change.

Suppose $\bar{x}, \bar{y}, \bar{z}$ coordinate system is rotated by angle ϕ relative to x, y, z about a common axis $x = \bar{x}$ which is perpendicular to the page.



$$A_y = A\cos\theta, \;\; A_z = A\sin\theta$$

$$\bar{A}_y = A\cos\bar{\theta} = A\cos(\theta-\phi) = A(\cos\theta\cos\phi + \sin\theta\sin\phi) = \cos\phi A_y + \sin\phi A_z$$

$$\bar{A}_z = A\sin\bar{\theta} = A\sin(\theta-\phi) = A(\sin\theta\cos\phi - \cos\theta\sin\phi) = -\sin\phi A_y + \cos\phi A_z$$

In a compact form $$\begin{pmatrix} \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_y \\ A_z \end{pmatrix}$$

A scalar remains invariant under a change in coord. system

In general, for rotation about arbitrary axis in 3-D

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \Longrightarrow \bar{A}_i = \sum_{j=1}^{3} R_{ij} A_j$$

A vector is any set of 3 components that has the above transformation property

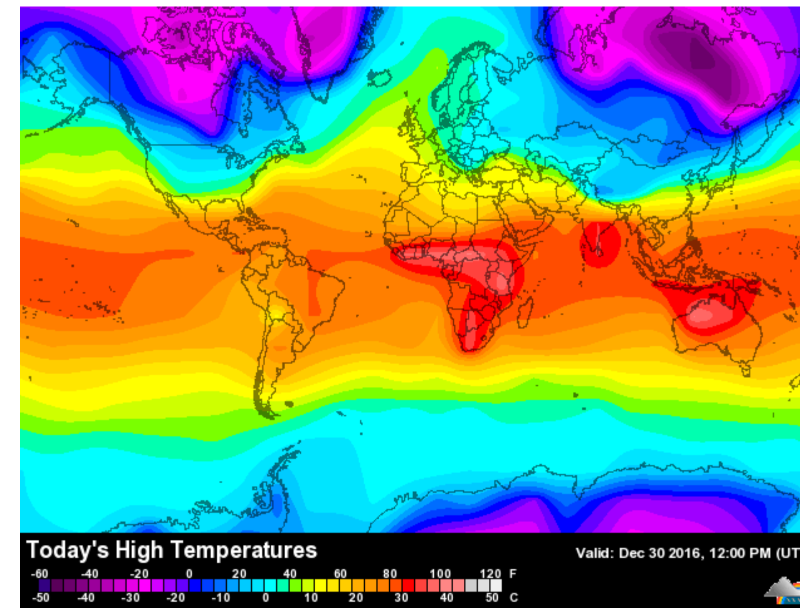
Concept of a field

Field is physical quantity (scalar or a vector) that has a value at each point in space or time.

Scalar field:

We can associate a number at each and every point in space.

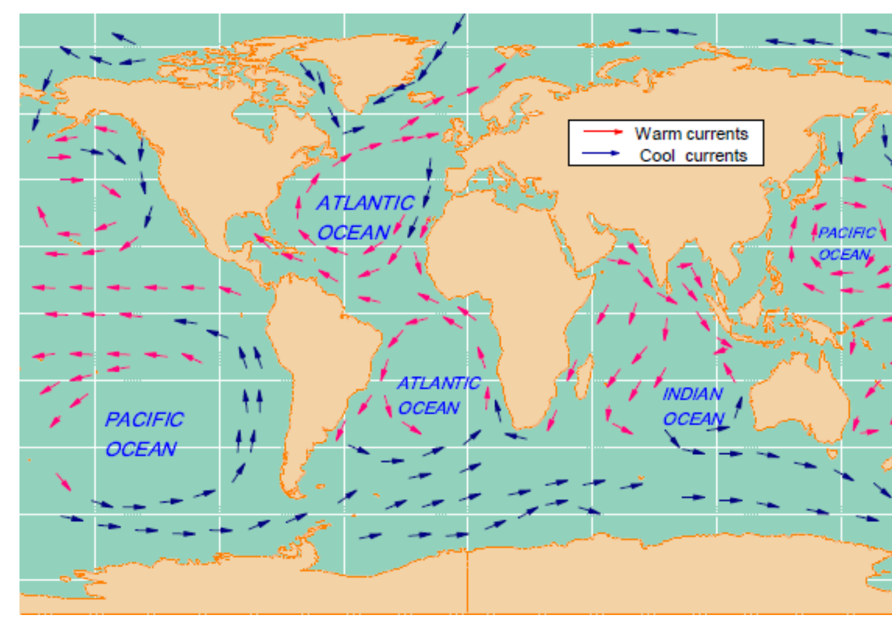
Example: We can associate a temperature (a number) at every point in this room. The field is the temperature field and it is a scalar field because the field quantity "temperature" is a scalar. Similarly density, potential energy etc…



Vector field:

We can associate a vector at each and every point in space.

Example: Gravitational force field, electric & magnetic field, ocean current etc…

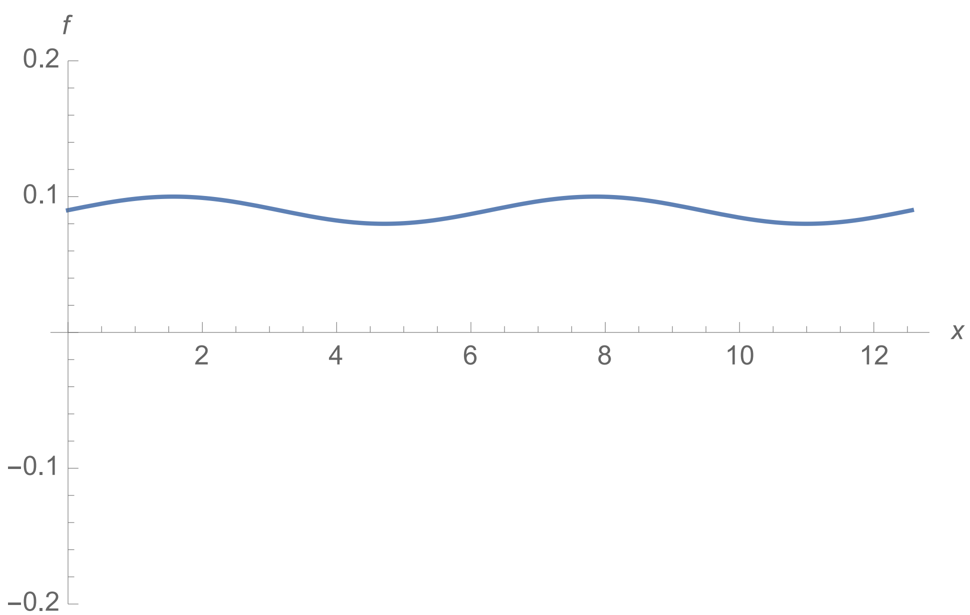


Differential Calculus of the scalar field

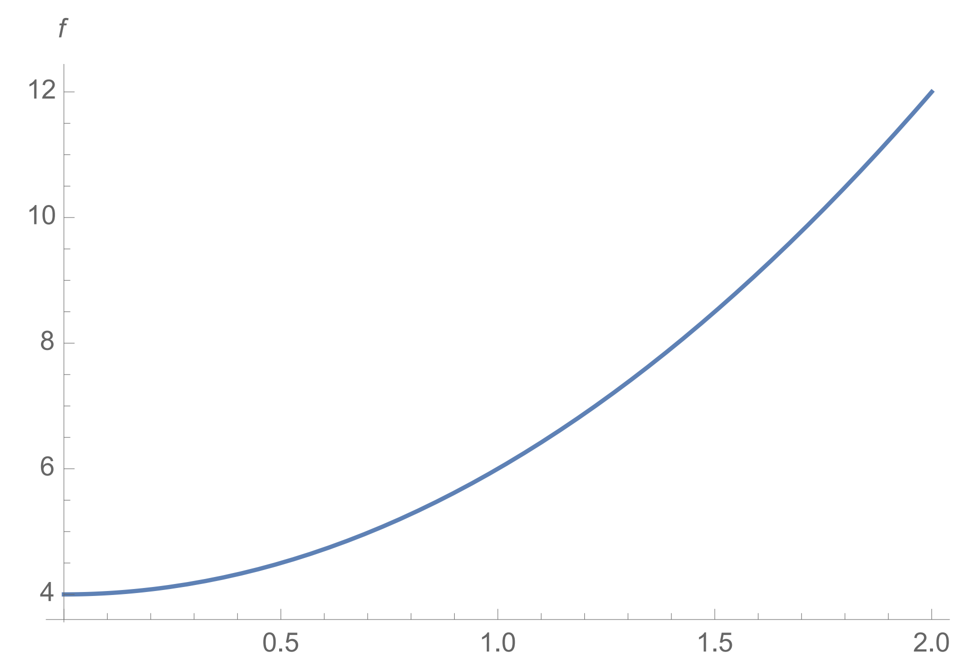
We already know the definition of ordinary derivative of a function $f(x)$ of a single variable x

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Tells us how rapidly the function $f(x)$ varies when we change the argument x by a tiny amount Δx.



The function varies slowly with x
The derivative is small



The function varies rapidly with x
The derivative is large

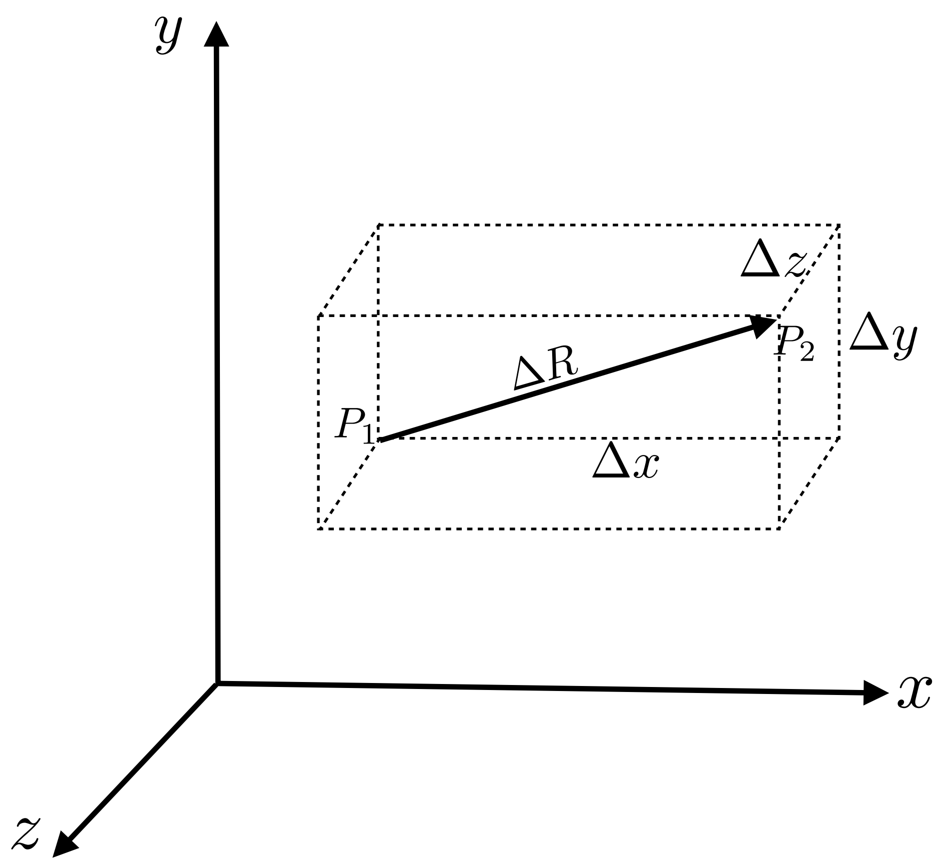
Derivative df/dx is the slope of the function.

The Gradient

Suppose we have a function of three variables, say the temperature, $T = T(x, y, z)$,

T depends upon three variables and therefore the concept of derivation that depends upon three variable has to be generalised.

Think about the temperature field at P_1 and P_2 separated by $\Delta\vec{R}$



$$\Delta T = T_2 - T_1 \longrightarrow \text{Scalar}$$

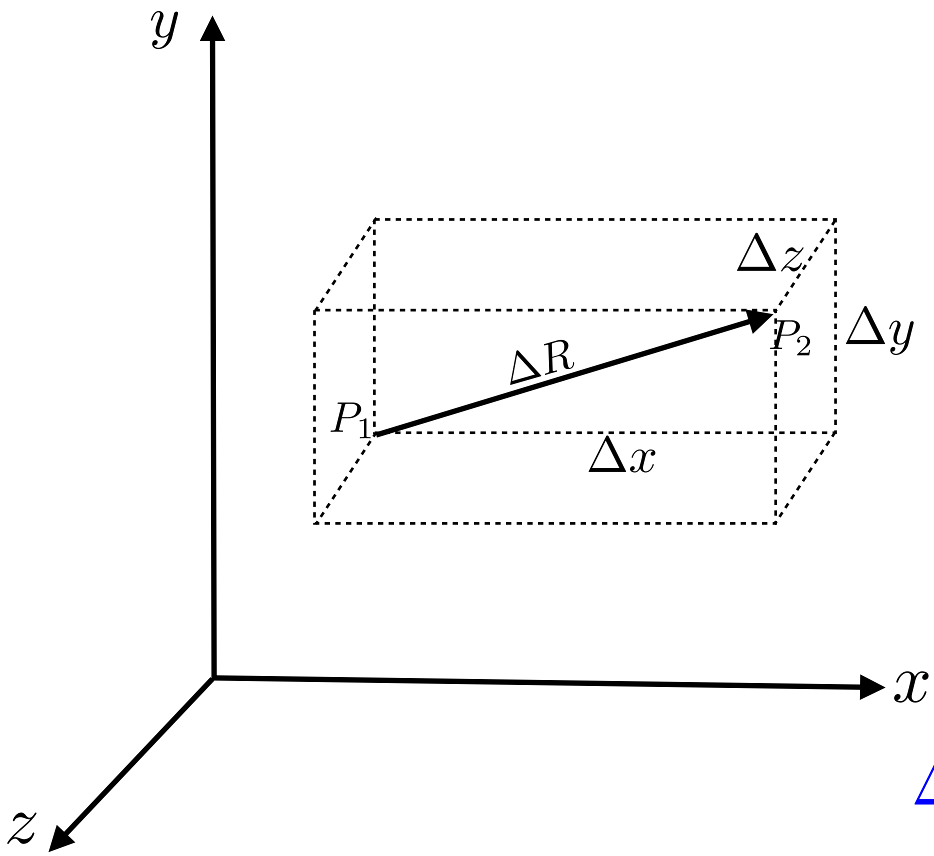
In certain coord. system $T_1 = T(x, y, z)$ and $T_2 = T(x + \Delta x, y + \Delta y, z + \Delta z)$.

$\Delta x, \Delta y,$ & Δz are components of the vector $\Delta\vec{R}$.

$$\Delta T = \frac{\partial T}{\partial x}\Delta x + \frac{\partial T}{\partial y}\Delta y + \frac{\partial T}{\partial z}\Delta z$$

(Theorem on "partial derivatives": true in the limit $\Delta x, \Delta y, \Delta z$ tends to zero)

The Gradient



$$\Delta T = \frac{\partial T}{\partial x}\Delta x + \frac{\partial T}{\partial y}\Delta y + \frac{\partial T}{\partial z}\Delta z$$

Scalar

Components of vector $\Delta\vec{R}$

Must be x, y, z component
of another vector

$$\Delta T = \left(\hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z}\right).(\Delta x\hat{x} + \Delta y\hat{y} + \Delta z\hat{z})$$

$$\Delta T = \vec{\nabla}T.\Delta\vec{R}$$

We call this new vector Gradient of T (or "del-T", or "grad-T"). The symbol $\vec{\nabla}$ (called "del") is supposed to remind us of differentiation!

Important: The quantity $\vec{\nabla}T$ is a vector.

$\Delta T = \vec{\nabla}T.\Delta\vec{R} \longrightarrow$ says that the difference in temperature between two nearby points is the dot product of the gradient of T and the vector displacement between the points.

The Gradient: Geometrical interpretation

$\Delta T = \vec{\nabla} T . \Delta \vec{R} = |\vec{\nabla} T||\Delta \vec{R}| \cos\theta$ $\qquad$ θ is the angle between $\vec{\nabla} T$ and $\Delta \vec{R}$.

If we fix magnitude $|\Delta \vec{R}|$ and search around in various directions (i.e. vary θ), the maximum change in T occurs when $\theta = 0$ (i.e. $\cos\theta = 1$).

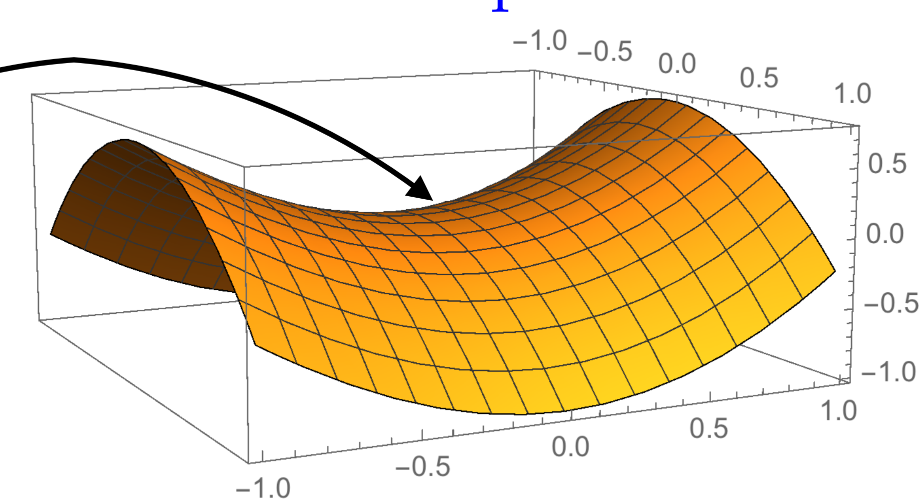
i.e. for a fixed distance $|\Delta \vec{R}|$, ΔT is greatest when you move in the same direction as $\vec{\nabla} T$.

The gradient $\vec{\nabla} T$ points in the direction of maximum increase of the scalar function T.

The magnitude $|\vec{\nabla} T|$ gives the slope (rate of change of $T(x, y, z)$) along this maximal direction.

$\vec{\nabla} T = 0$ at a point $(x, y, z) \implies$ maximum or minimum or saddle point.

Saddle point



Analogous to the situation for functions of one variable!

The Gradient: Geometrical meaning

Gradient of a scalar function T is always normal to the level surfaces or level curves.

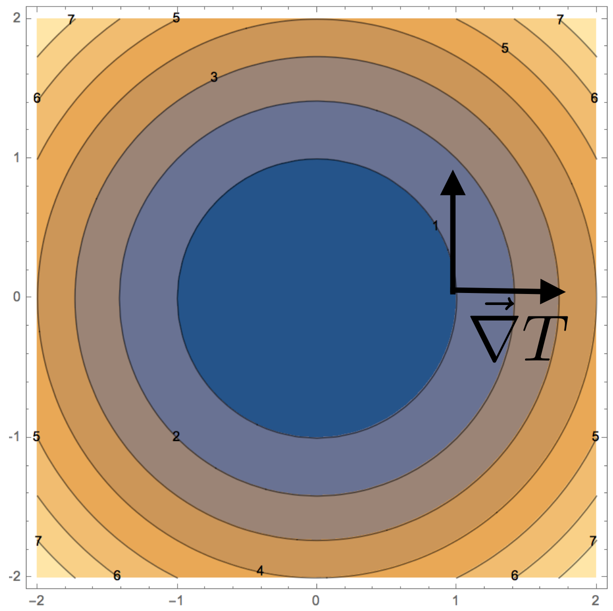
Level curve : parametrised by a variable t, which varies from point to point on the curve. Example of such a parameter for the circle is angle θ, so that $x = R\cos\theta, y = R\sin\theta$, where R (fixed) is the radius and $0 < \theta < 2\pi$ is the polar angle.

Position vector on the level curve $\vec{r}(t) = x(t)\hat{x} + y(t)\hat{y} + 0\hat{z}$

Equation of the level curve $T(x(t), y(t), z(t) = 0) = \text{constant} \implies \frac{\mathrm{dT}}{\mathrm{dt}} = 0$

Tangent to the curve $\vec{r}\,' = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y} + 0\hat{z}$

$$\frac{dT}{dt} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} = \vec{\nabla}T.\vec{r}\,' = 0$$



Which shows that the gradient is normal to the level curve.

Example:

Suppose the height of the hill (in feet) behind the CC is $h(x,y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$, where y is the distance (in miles) north, x the distance west of the IITG admin. building Where is the top of the hill and how high is the hill? How steep is the slope 1 mile north and 1 mile west of admin building?

$$\begin{aligned}\vec{\nabla} h(x,y) &= \frac{\partial h}{\partial x}\hat{x} + \frac{\partial h}{\partial y}\hat{y} \\ &= 10(2y - 6x - 18)\hat{x} + 10(2x - 8y + 28)\hat{y}\end{aligned}$$

If the hilltop is located at $(\bar{x}, \bar{y})$, then $\vec{\nabla} h|_{(\bar{x},\bar{y})} = 0$

(condition for maxima)

This gives $$\begin{aligned} 2\bar{y} - 6\bar{x} - 18 &= 0 \\ 2\bar{x} - 8\bar{y} + 28 &= 0. \end{aligned} \implies \bar{x} = -2 \text{ and } \bar{y} = 3$$

The hill top is 3 miles north and 2 miles east of the admin building.

Height $h(-2,3) = 720$ feet.

The slope at a point 1 mile west and 1 mile north of admin building will be found from the magnitude of $\vec{\nabla} h(x,y)$ at $x = 1, y = 1$, which gives $\vec{\nabla} h(1,1) = 220(-\hat{x} + \hat{y})$. Slope $\implies 220\sqrt{2}$ ft/mile.

Summary

- The gradient of a scalar field is defined as $\vec{\nabla} T = (\hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z})$.
- $\vec{\nabla} T$ is a vector field and it points in the direction of maximum increase of $T(x, y, z)$.
- $\vec{\nabla} T$ is perpendicular to level curves/surfaces.
- The scalar field T has extremum/saddle point if $\vec{\nabla} T = 0$.

Thank You