MA 322 Scientific Computing

Lab-13

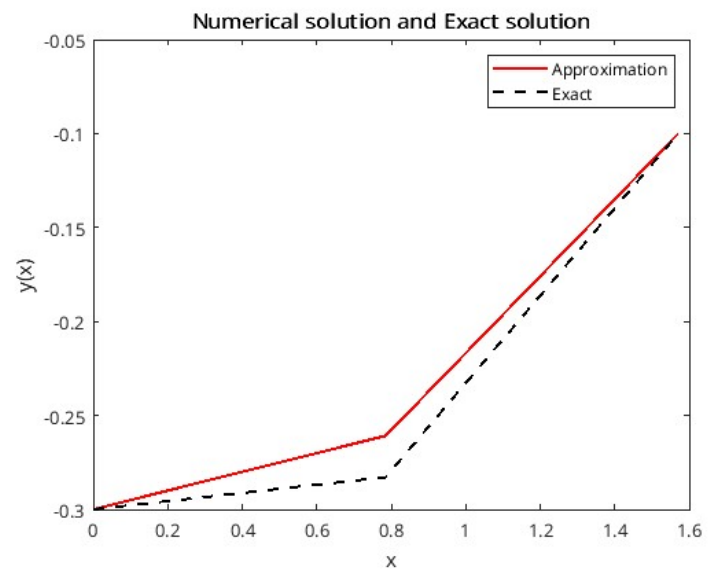**N K Sathvik**

210123041

Q1)

(a)

```
Using Linear Shooting method with h: 0.7854

     t          approx        exact        error
  _____     _____     _____     _____

       0          -0.3         -0.3            0
  0.7854       -0.2608     -0.28284        0.022
  1.5708          -0.1         -0.1            0
```
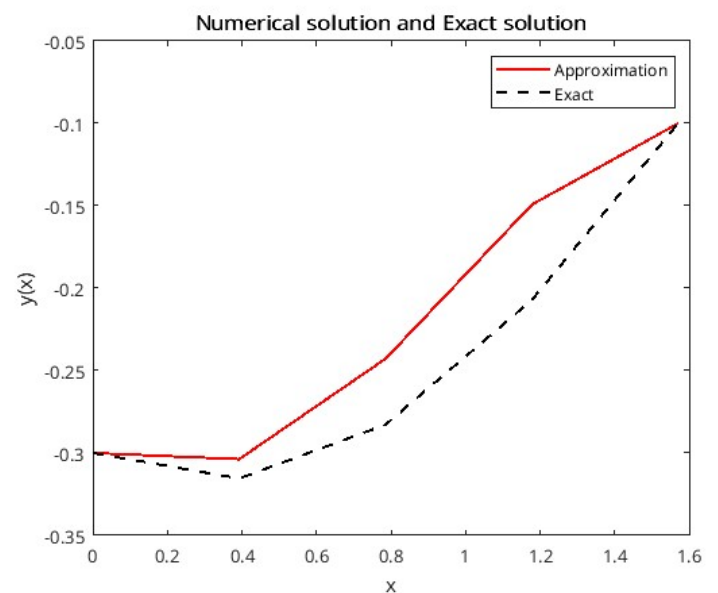

Numerical solution and Exact solution

(b)

```
Using Linear Shooting method with h: 0.3927

     t          approx        exact        error
  _____     _____     _____     _____

       0          -0.3         -0.3            0
  0.3927      -0.30376     -0.31543       0.0117
  0.7854      -0.24284     -0.28284         0.04
  1.1781      -0.14936     -0.20719       0.0578
  1.5708          -0.1         -0.1            0
```
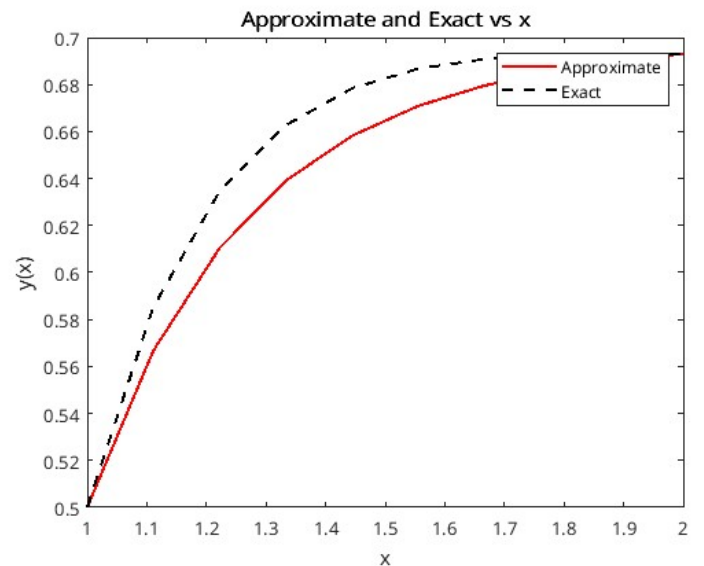

Numerical solution and Exact solution

Q2)

(a)

```
Using Central Difference method with N: 10

     t        approx       exact       error
  _____    _____    _____    _____

        1         0.5         0.5           0
   1.1111     0.57577     0.58536      0.0096
   1.2222     0.62196     0.63455      0.0126
   1.3333     0.65032     0.66268      0.0124
   1.4444     0.66773     0.67838      0.0106
   1.5556     0.67838     0.68673      0.0084
   1.6667     0.68487     0.69083       0.006
   1.7778     0.68885     0.69255      0.0037
   1.8889     0.69138     0.69308      0.0017
        2     0.69315     0.69315           0
```
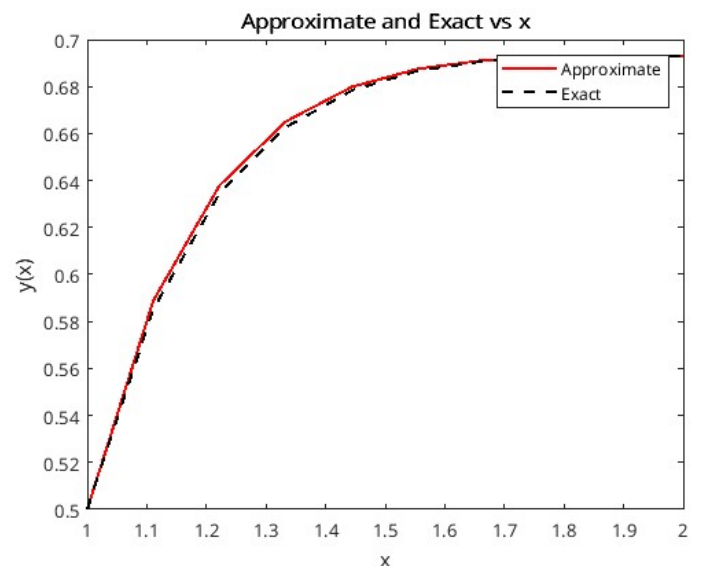

Approximate and Exact vs x
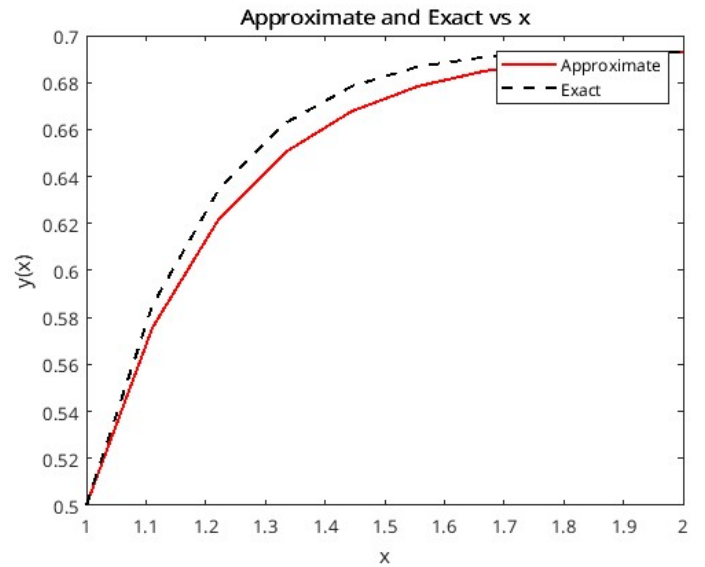
```
Using Forward Difference method with N: 10

     t        approx       exact       error
  _____    _____    _____    _____

        1         0.5         0.5           0
   1.1111     0.56679     0.58536      0.0186
   1.2222      0.6104     0.63455      0.0242
   1.3333     0.63912     0.66268      0.0236
   1.4444     0.65818     0.67838      0.0202
   1.5556     0.67092     0.68673      0.0158
   1.6667     0.67955     0.69083      0.0113
   1.7778     0.68553     0.69255       0.007
   1.8889     0.68984     0.69308      0.0032
        2     0.69315     0.69315           0
```


Approximate and Exact vs x

```
Using Backward Difference method with N: 10

     t        approx       exact       error
  _____    _____    _____    _____

        1         0.5         0.5           0
   1.1111      0.5889     0.58536      0.0035
   1.2222     0.63797     0.63455      0.0034
   1.3333     0.66512     0.66268      0.0024
   1.4444     0.67988     0.67838      0.0015
   1.5556     0.68756     0.68673      0.0008
   1.6667     0.69123     0.69083      0.0004
   1.7778     0.69273     0.69255      0.0002
   1.8889     0.69314     0.69308      0.0001
        2     0.69315     0.69315           0
```
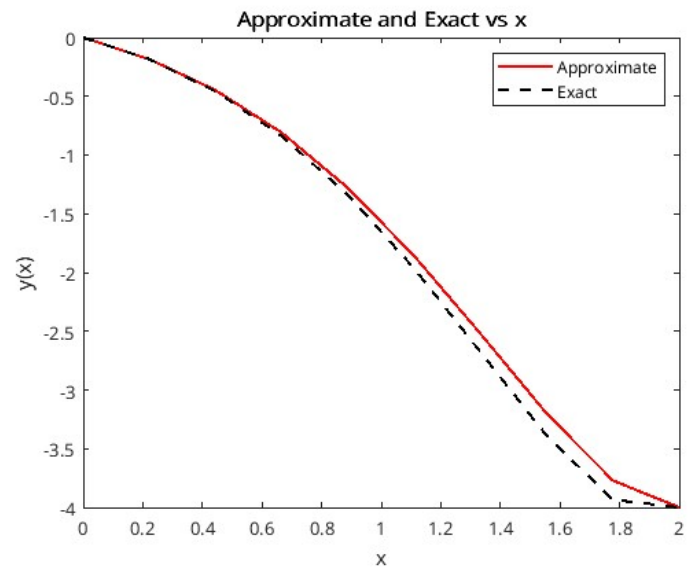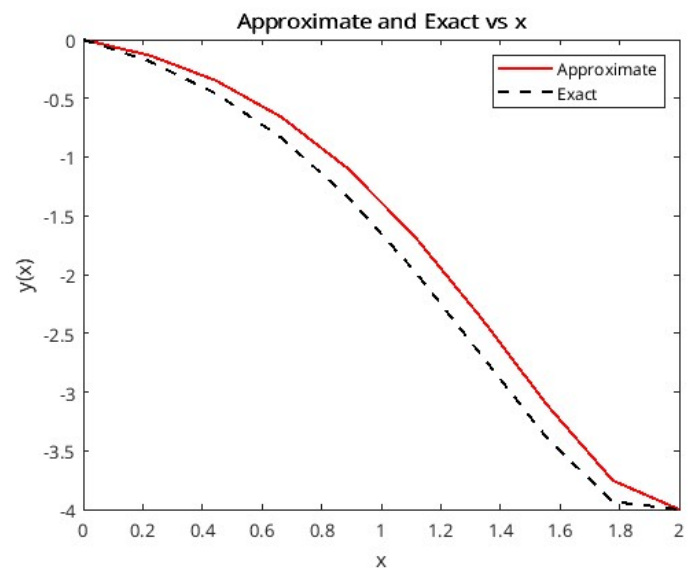

Approximate and Exact vs x

(b)

```
Using Central Difference method with N: 10

    t            approx         exact          error
  _____        _____        _____        _____

        0        -4.6259e-17          0              0
  0.22222        -0.18439        -0.18478         0.0004
  0.44444          -0.449        -0.45765         0.0087
  0.66667        -0.81103        -0.83916         0.0281
  0.88889         -1.2819         -1.3429          0.061
   1.1111         -1.8605         -1.9666         0.1061
   1.3333         -2.5221         -2.6776         0.1555
   1.5556         -3.2023         -3.3909         0.1887
   1.7778         -3.7716         -3.9347         0.1631
        2             -4             -4              0
```
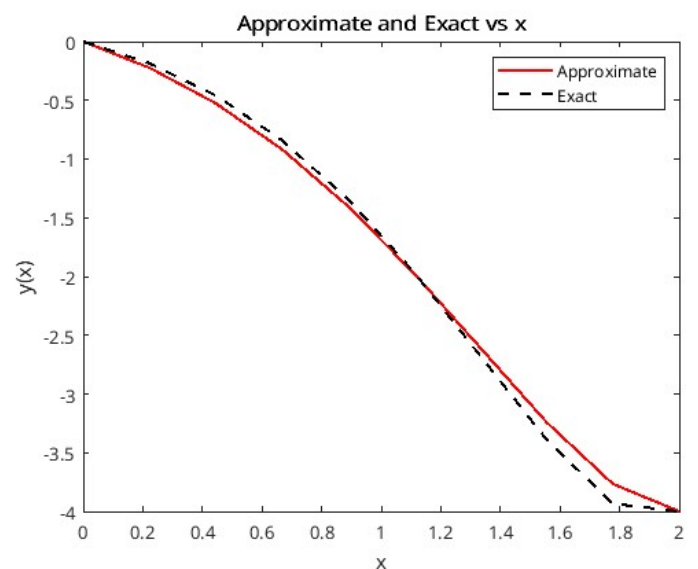

Approximate and Exact vs x

```
Using Forward Difference method with N: 10

    t            approx         exact          error
  _____        _____        _____        _____

        0              0             0              0
  0.22222        -0.13518        -0.18478         0.0496
  0.44444        -0.34779        -0.45765         0.1099
  0.66667        -0.66236        -0.83916         0.1768
  0.88889         -1.1004         -1.3429         0.2425
   1.1111         -1.6722         -1.9666         0.2944
   1.3333         -2.3627         -2.6776         0.3149
   1.5556         -3.1078         -3.3909         0.2832
   1.7778         -3.7547         -3.9347           0.18
        2             -4             -4              0
```


Approximate and Exact vs x

```
Using Backward Difference method with N: 10

    t            approx         exact          error
  _____        _____        _____        _____

        0              0             0              0
  0.22222        -0.22346        -0.18478         0.0387
  0.44444        -0.52516        -0.45765         0.0675
  0.66667        -0.91659        -0.83916         0.0774
  0.88889         -1.4026         -1.3429         0.0597
   1.1111         -1.9761         -1.9666         0.0095
   1.3333         -2.6093         -2.6776         0.0683
   1.5556         -3.2424         -3.3909         0.1485
   1.7778         -3.7665         -3.9347         0.1682
        2             -4             -4              0
```
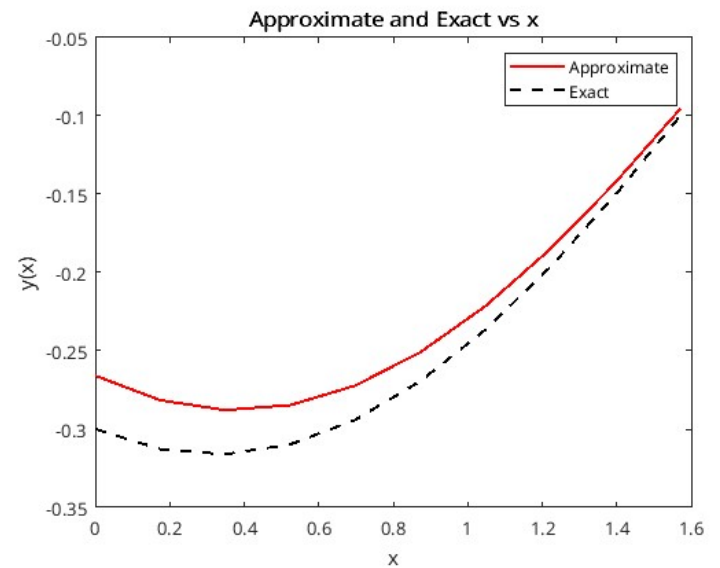

Approximate and Exact vs x

(c)

```
Using Central Difference method with N: 10

        t           approx          exact          error
     _____      _____       _____       _____

        0          -0.25205          -0.3          0.048
     0.17453       -0.26775       -0.31281        0.0451
     0.34907       -0.27411       -0.31611         0.042
      0.5236       -0.27106       -0.30981        0.0387
     0.69813       -0.25883       -0.29409        0.0353
     0.87266       -0.23785       -0.26944        0.0316
      1.0472       -0.20883        -0.2366        0.0278
      1.2217       -0.17265       -0.19658        0.0239
      1.3963        -0.1304       -0.15058        0.0202
      1.5708      -0.083275          -0.1         0.0167
```
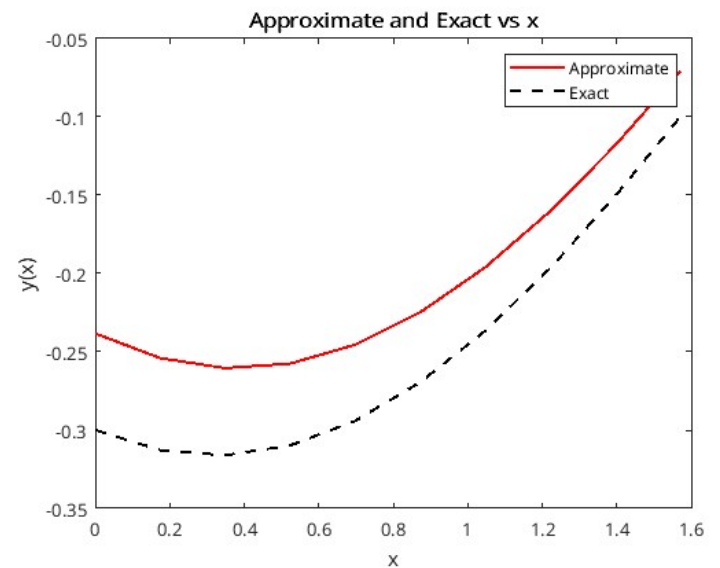

Approximate and Exact vs x
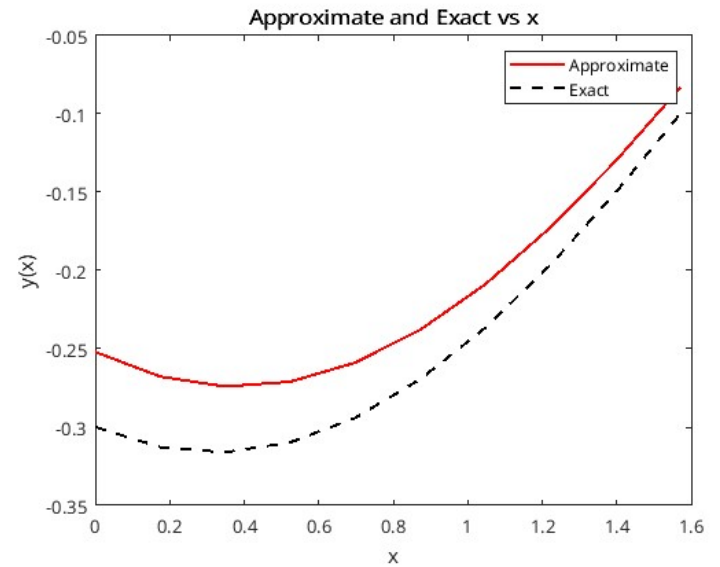
```
Using Forward Difference method with N: 10

        t           approx          exact          error
     _____      _____       _____       _____

        0          -0.26588          -0.3         0.0341
     0.17453       -0.28159       -0.31281        0.0312
     0.34907       -0.28788       -0.31611        0.0282
      0.5236       -0.28469       -0.30981        0.0251
     0.69813       -0.27223       -0.29409        0.0219
     0.87266       -0.25095       -0.26944        0.0185
      1.0472       -0.22159        -0.2366         0.015
      1.2217       -0.18509       -0.19658        0.0115
      1.3963       -0.14262       -0.15058         0.008
      1.5708      -0.095492          -0.1         0.0045
```


Approximate and Exact vs x

```
Using Backward Difference method with N: 10

        t           approx          exact          error
     _____      _____       _____       _____

        0          -0.23834          -0.3         0.0617
     0.17453       -0.25405       -0.31281        0.0588
     0.34907       -0.26046       -0.31611        0.0556
      0.5236       -0.25755       -0.30981        0.0523
     0.69813       -0.24552       -0.29409        0.0486
     0.87266       -0.22482       -0.26944        0.0446
      1.0472        -0.1961        -0.2366        0.0405
      1.2217       -0.16021       -0.19658        0.0364
      1.3963       -0.11815       -0.15058        0.0324
      1.5708      -0.071023          -0.1          0.029
```
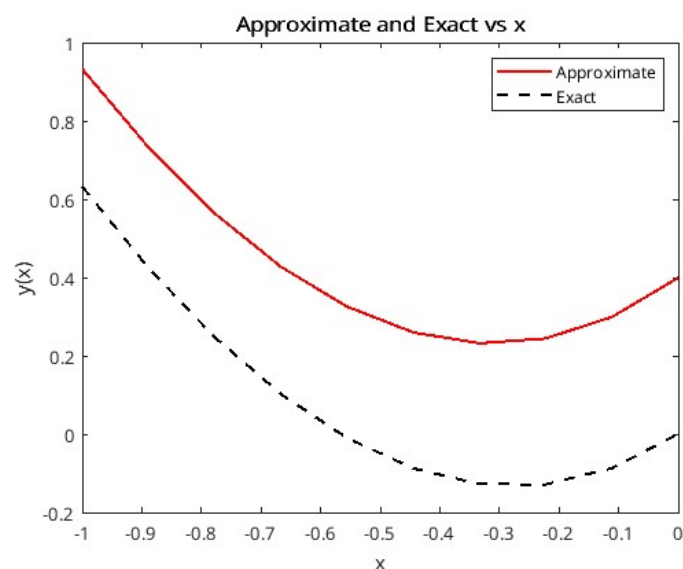

Approximate and Exact vs x

(d)

```
Using Central Difference method with N: 10

        t         approx         exact         error
     _____      _____      _____      _____

          -1       0.98135       0.63212        0.3492
     -0.88889       0.78135       0.42469        0.3567
     -0.77778       0.61203       0.24761        0.3644
     -0.66667       0.47501       0.10217        0.3728
     -0.55556       0.37204      -0.01011        0.3822
     -0.44444       0.30503     -0.087438        0.3925
     -0.33333       0.27609      -0.12773        0.4038
     -0.22222        0.2875      -0.12856        0.4161
     -0.11111        0.3418     -0.087081        0.4289
           0        0.4418             0        0.4418
```
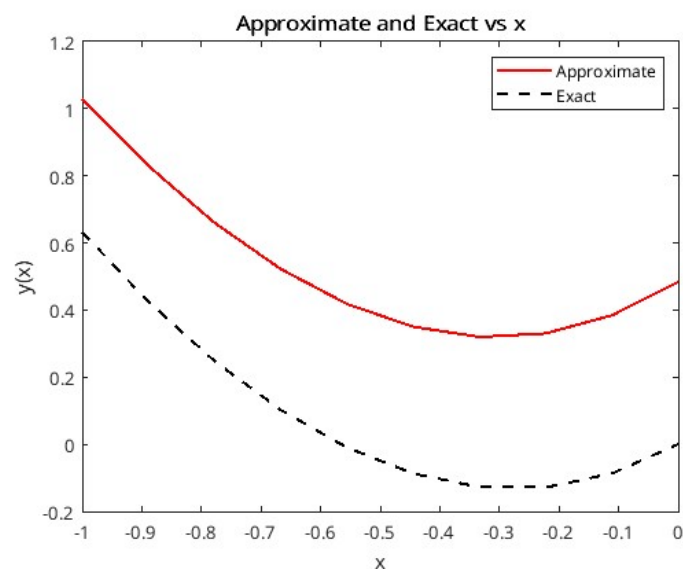

Approximate and Exact vs x
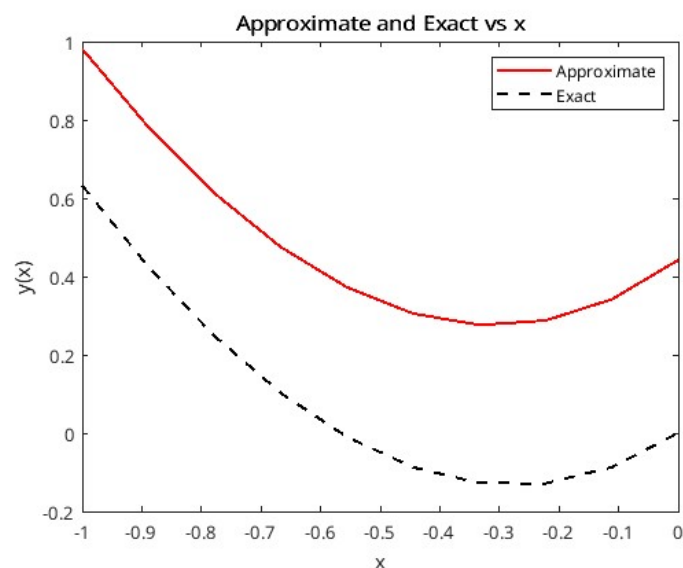
```
Using Forward Difference method with N: 10

        t         approx         exact         error
     _____      _____      _____      _____

          -1        0.9326       0.63212        0.3005
     -0.88889        0.7326       0.42469        0.3079
     -0.77778       0.56371       0.24761        0.3161
     -0.66667       0.42746       0.10217        0.3253
     -0.55556       0.32552      -0.01011        0.3356
     -0.44444       0.25968     -0.087438        0.3471
     -0.33333       0.23189      -0.12773        0.3596
     -0.22222       0.24427      -0.12856        0.3728
     -0.11111       0.29917     -0.087081        0.3862
           0       0.39917             0        0.3992
```


Approximate and Exact vs x

```
Using Backward Difference method with N: 10

        t         approx         exact         error
     _____      _____      _____      _____

          -1        1.0295       0.63212        0.3973
     -0.88889       0.82946       0.42469        0.4048
     -0.77778       0.65974       0.24761        0.4121
     -0.66667       0.52199       0.10217        0.4198
     -0.55556       0.41804      -0.01011        0.4281
     -0.44444       0.34992     -0.087438        0.4374
     -0.33333       0.31986      -0.12773        0.4476
     -0.22222       0.33033      -0.12856        0.4589
     -0.11111       0.38405     -0.087081        0.4711
           0       0.48405             0         0.484
```
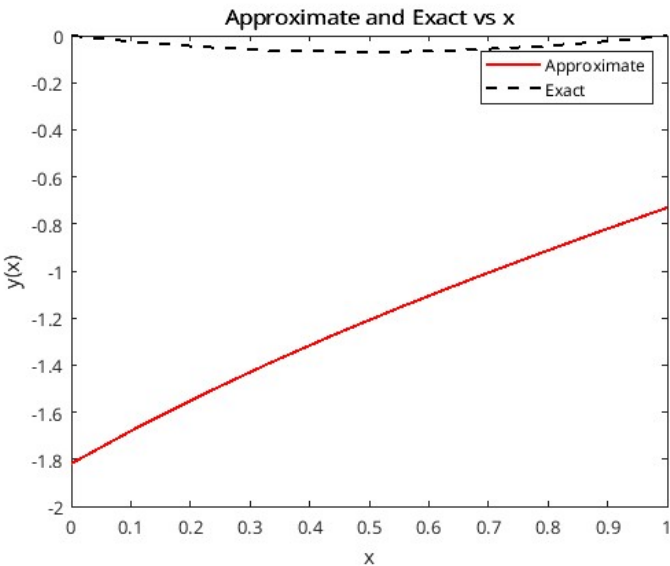

Approximate and Exact vs x

(e)

Using Central Difference method with N: 10

t	approx	exact	error
0	-1.8185	0	1.8185
0.11111	-1.6649	-0.027794	1.6371
0.22222	-1.523	-0.048491	1.4745
0.33333	-1.391	-0.062047	1.329
0.44444	-1.2674	-0.068507	1.1989
0.55556	-1.1507	-0.067984	1.0827
0.66667	-1.0397	-0.060647	0.979
0.77778	-0.93332	-0.046706	0.8866
0.88889	-0.83076	-0.026403	0.8044
1	-0.73121	0	0.7312

Using Forward Difference method with N: 10

t	approx	exact	error
0	-1.3955	0	1.3955
0.11111	-1.2841	-0.027794	1.2563
0.22222	-1.1797	-0.048491	1.1312
0.33333	-1.081	-0.062047	1.0189
0.44444	-0.9869	-0.068507	0.9184
0.55556	-0.8966	-0.067984	0.8286
0.66667	-0.80925	-0.060647	0.7486
0.77778	-0.72415	-0.046706	0.6774
0.88889	-0.64073	-0.026403	0.6143
1	-0.55846	0	0.5585
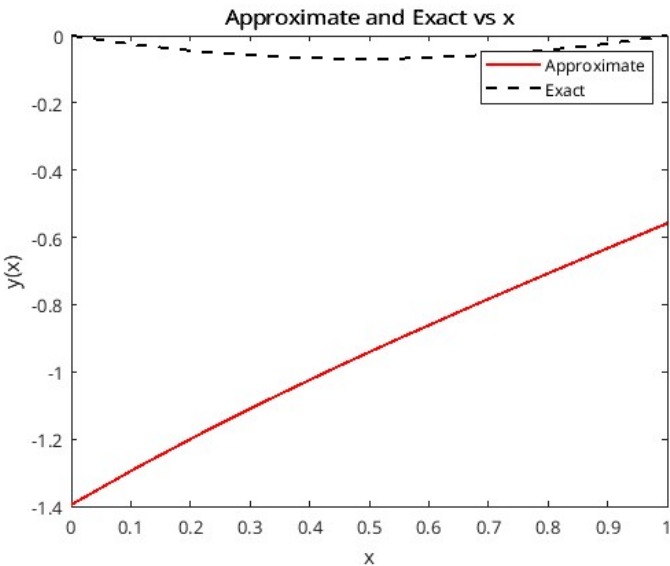
Using Backward Difference method with N: 10

t	approx	exact	error
0	-11.497	0	11.497
0.11111	-10.375	-0.027794	10.347
0.22222	9.373	0.048491	9.3245

Using Central Difference method with N: 10

t	approx	exact	error
0	4.0643	1	3.0643
0.087266	3.8562	1.0323	2.8239
0.17453	3.6579	1.0567	2.6012
0.2618	3.4687	1.0731	2.3955
0.34907	3.2876	1.0814	2.2063
0.43633	3.114	1.0814	2.0327
0.5236	2.9473	1.0731	1.8741
0.61087	2.7868	1.0567	1.7301
0.69813	2.6322	1.0323	1.5999
0.7854	2.4832	1	1.4832
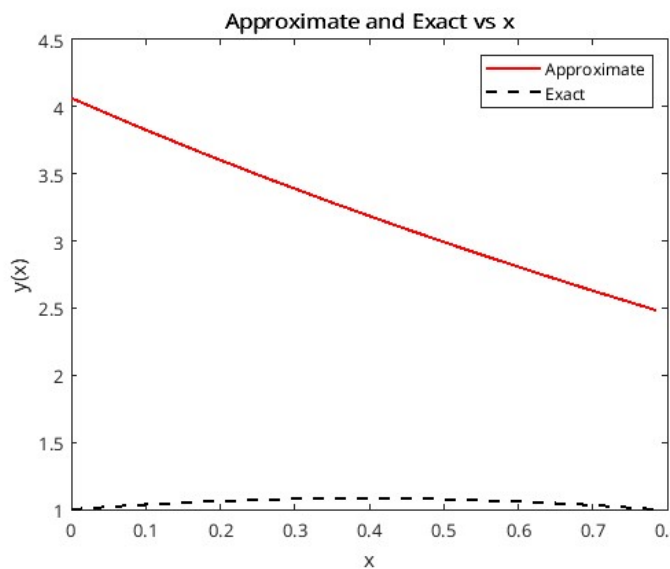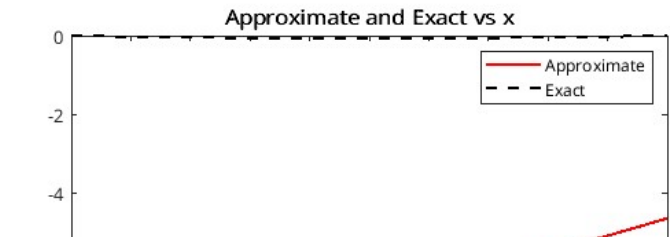
(f)

```
Using Forward Difference method with N: 10

      t        approx     exact      error
   _____    _____    _____    _____

         0     4.1232          1     3.1232
  0.087266     3.9104     1.0323     2.8781
   0.17453     3.7079     1.0567     2.6512
    0.2618     3.5148     1.0731     2.4417
   0.34907     3.3302     1.0814     2.2488
   0.43633     3.1534     1.0814      2.072
    0.5236     2.9836     1.0731     1.9105
   0.61087     2.8204     1.0567     1.7637
   0.69813     2.6634     1.0323     1.6311
    0.7854     2.5121          1     1.5121
```


Approximate and Exact vs x
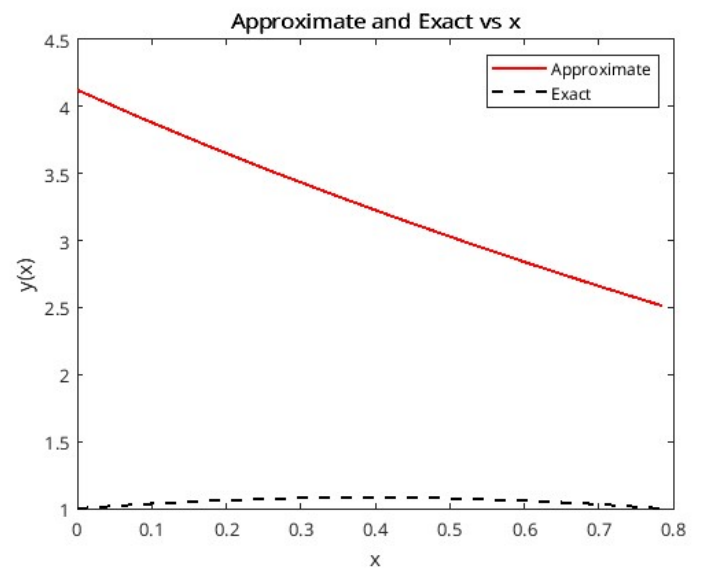
```
Using Backward Difference method with N: 10

      t        approx     exact      error
   _____    _____    _____    _____

         0     4.0098          1     3.0098
  0.087266     3.8059     1.0323     2.7736
   0.17453     3.6116     1.0567     2.5548
    0.2618     3.4259     1.0731     2.3528
   0.34907     3.2482     1.0814     2.1668
   0.43633     3.0776     1.0814     1.9962
    0.5236     2.9136     1.0731     1.8405
   0.61087     2.7556     1.0567     1.6989
   0.69813     2.6034     1.0323     1.5711
    0.7854     2.4564          1     1.4564
```


Approximate and Exact vs x