

$$\text{var}(X|Y) = (1-\rho^2) \sigma_X^2 = (1-\rho^2) \cdot 1$$

$$\begin{aligned} \text{So } E(X^2|Y) &= (1-\rho^2) - (\rho Y)^2 \\ &= 1-\rho^2 - \rho^2 Y^2 \end{aligned}$$

$$\begin{aligned} \text{So, } E(X^2 Y^2) &= E(Y^2 (1-\rho^2 - \rho^2 Y^2)) \\ &= E(Y^2) - E(Y^2 \rho^2) - E(\rho^2 Y^4) \\ &= 1 - \rho^2 \cdot 1 - \rho^2 E(Y^4) \\ &= 1 - \rho^2 - \rho^2 \times 3 \quad (\text{see page 7 also}) \\ &= 1 - \rho^2 - 3\rho^2 \quad \underline{\text{Ans.}} \\ &= 1 - 4\rho^2 \end{aligned}$$