MA 322 Scientific Computing

Lab-11

Pavan Kumar A

210123043

Q1)

(a)

Implicit Euler:

```
Implicit Euler Method:
    t         approximation              actual                   error

    1                     0                     0                     0
   1.2    0.250454583026496    0.221242772757631    0.0292118102688649
   1.4    0.563305747476104    0.489681663750943    0.0736240837251615
   1.6     0.95352968025127    0.812752740561542     0.140776939689727
   1.8      1.4421515146776     1.19943864032594     0.242712874351662
    2     2.06047770329629     1.66128175572157      0.39919594757472
   2.2     2.85773744096705     2.21350181348063     0.644235627486413
   2.4     3.91683450001299     2.87655141999484      1.04028308001815
   2.6     5.39162138733281     3.67847533085185      1.71314605648097
   2.8     7.61470635189775     4.65866505823951      2.95604129365824
    3     11.5481105505512     5.87409997818417      5.67401057236705
```
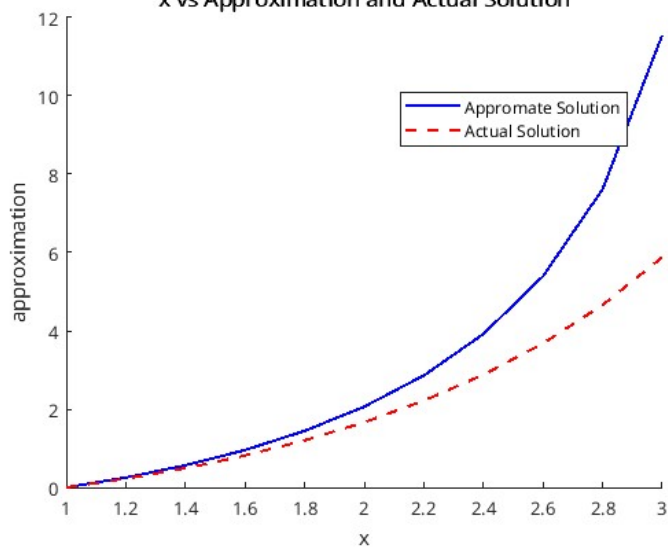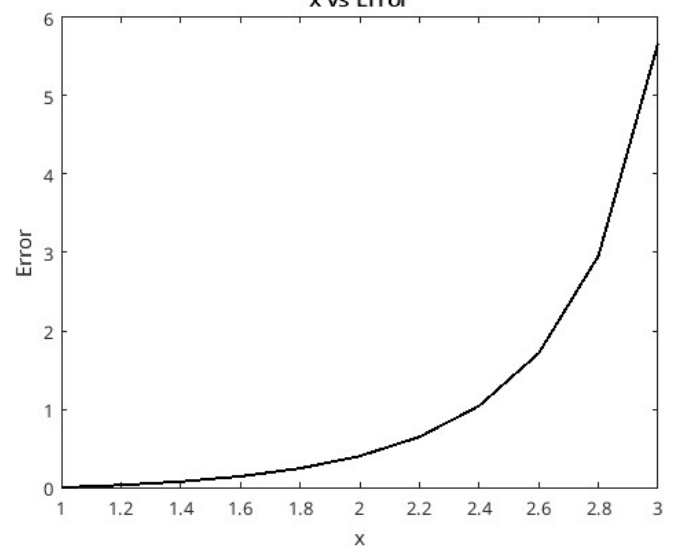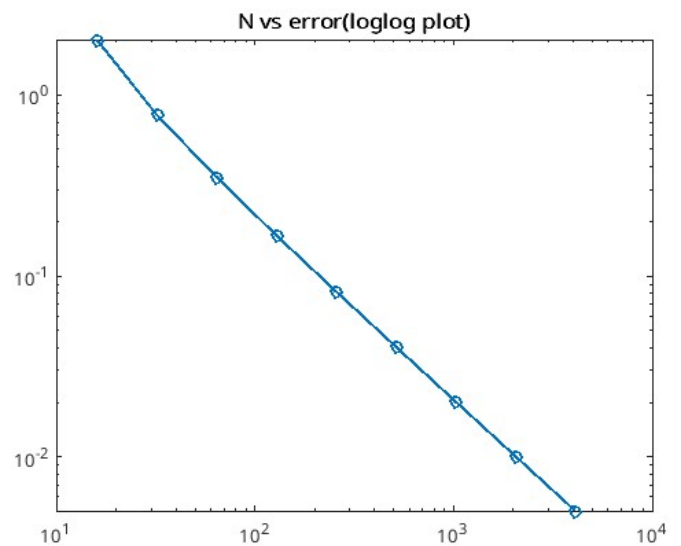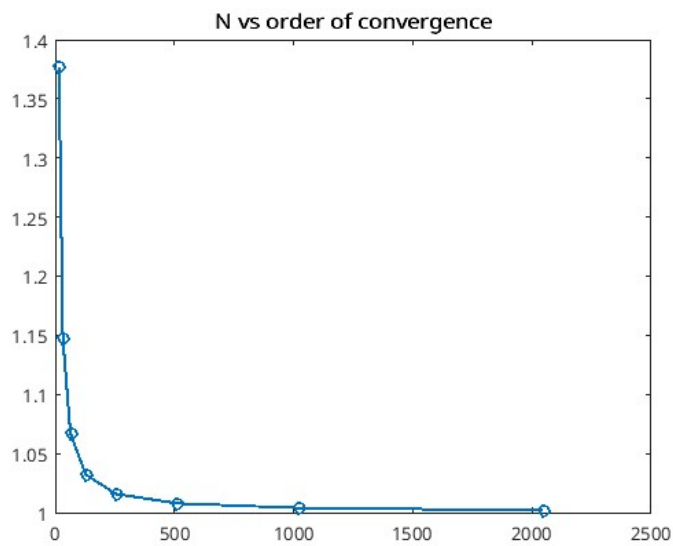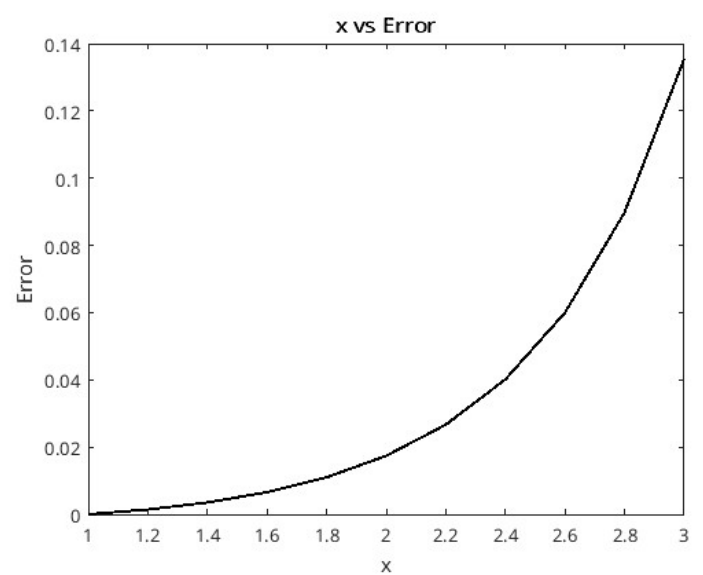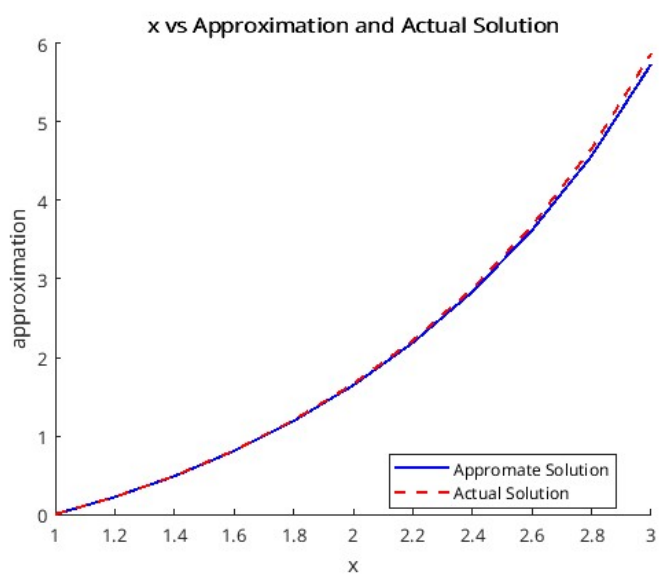

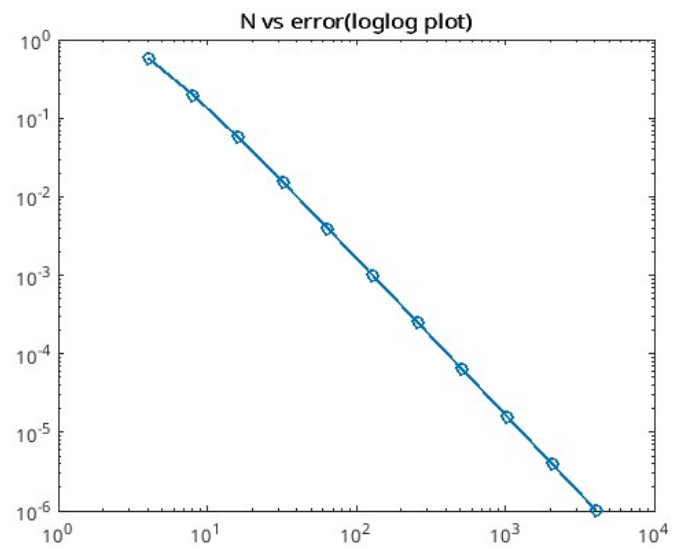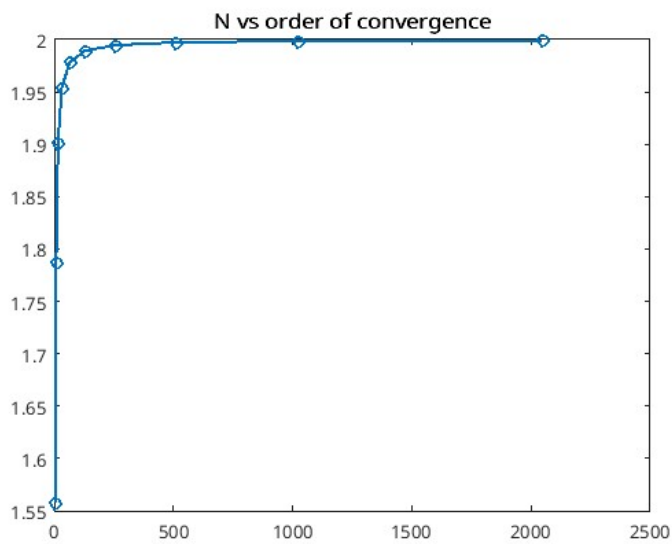x vs Approximation and Actual Solution

x vs Error

Runge Kutta 2nd Order


```
Runge Kutta Method 2 order:
     t       approximation           actual                 error
    ___    _____      _____      _____

     1                  0                    0                      0
    1.2    0.219834710743802    0.221242772757631    0.00140806201382948
    1.4    0.486177046768206    0.489681663750943    0.00350461698273657
    1.6    0.806184891605961    0.812752740561542    0.00656784895558093
    1.8     1.18843925837304     1.19943864032594     0.0109993819529002
     2     1.64388892124877      1.66128175572157      0.0173928344727998
    2.2    2.18686089286093      2.21350181348063       0.0266409206197
    2.4    2.83643571306701      2.87655141999484      0.0401157069278355
    2.6    3.61849255493937      3.67847533085185      0.0599827759124798
    2.8    4.56889438371891      4.65866505823951      0.0897706745206071
     3     5.73864746521413      5.87409997818417       0.135452512970037
```
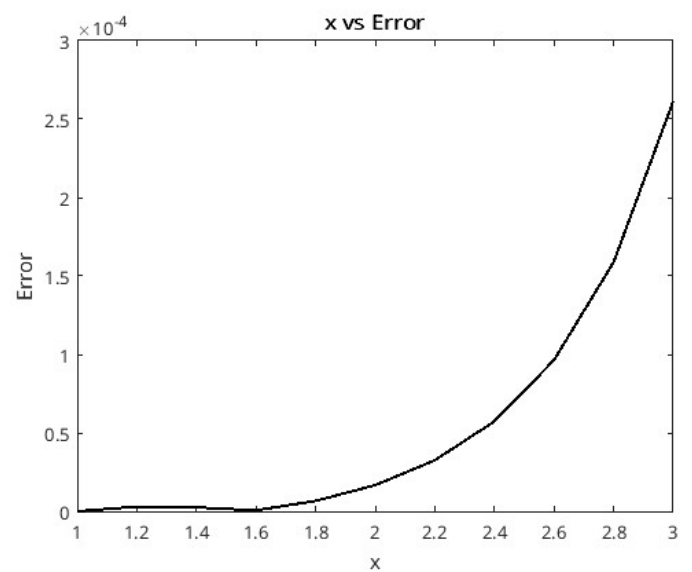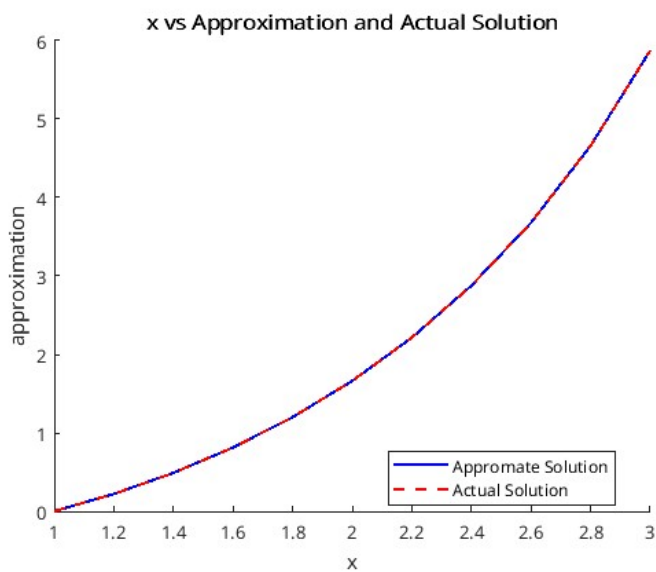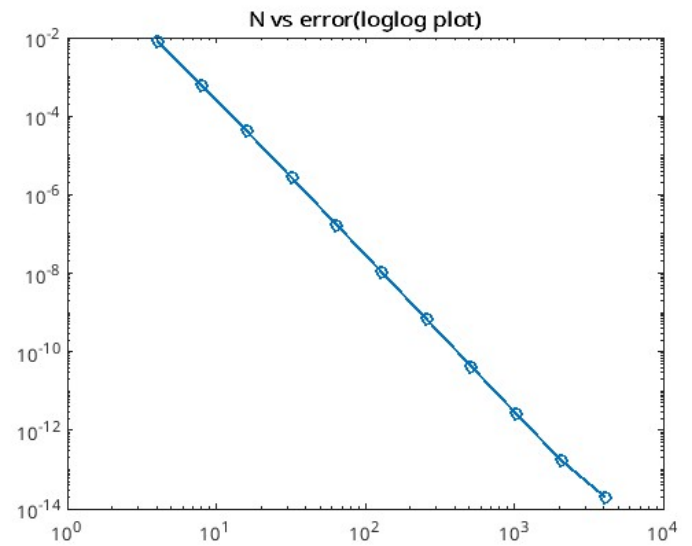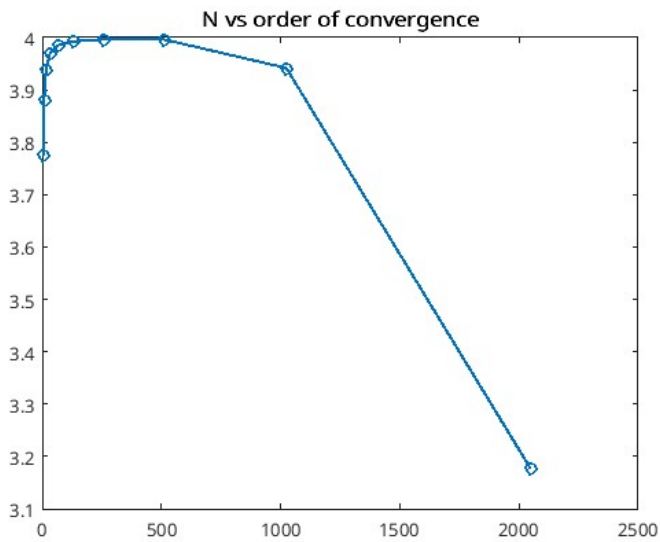
Runge Kutta 4th order

```
Runge Kutta Method 4 order:
    t        approximation            actual                  error
   ___      _____        _____        _____

    1                  0                      0                       0
   1.2    0.221245707288813      0.221242772757631     2.9345311814144e-06
   1.4    0.489684166443365      0.489681663750943    2.50269242257906e-06
   1.6     0.81275216202401      0.812752740561542     5.7853753210324e-07
   1.8     1.19943202163924       1.19943864032594    6.61868670004395e-06
    2      1.66126511544869       1.66128175572157    1.66402728811743e-05
   2.2     2.21346931657636       2.21350181348063    3.24969042724277e-05
   2.4     2.87649411454154       2.87655141999484    5.73054533026252e-05
   2.6     3.67837899609925       3.67847533085185    9.63347525999581e-05
   2.8     4.65850628368228       4.65866505823951    0.000158774557231567
    3      5.87383856983279       5.87409997818417    0.000261408351379444
```
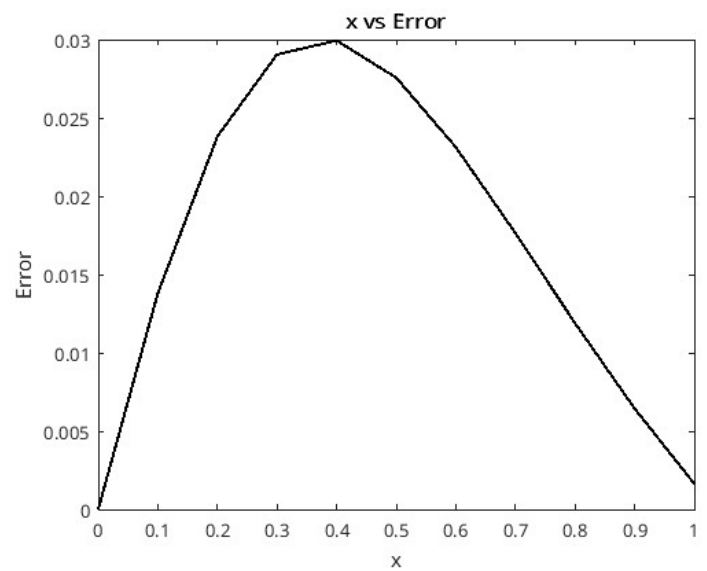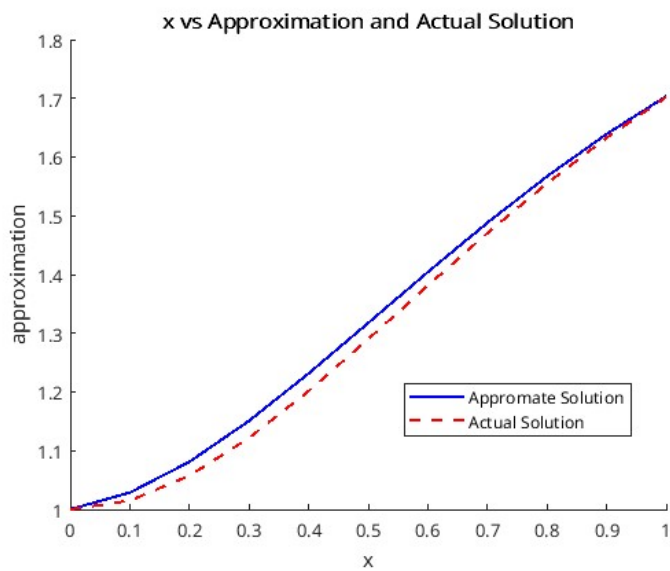
N vs order of convergence

N vs error(loglog plot)

(b)

Implicit Euler:

```
Implicit Euler Method:
    t         approximation              actual                    error

    0                       1                    1                        0
  0.1     1.02860172615079     1.01481549985822       0.0137862262925679
  0.2     1.08098831062131      1.0571810074642       0.0238073031571016
  0.3     1.15074609870334      1.12169801826798      0.0290480804353561
  0.4      1.2314205216256      1.2014860103644       0.0299345112611962
  0.5     1.31736970673967      1.28980527630561      0.0275644304340572
  0.6     1.40405913082249      1.38093121544373      0.0231279153787602
  0.7     1.48805948728035      1.47041518539722      0.0176443018831272
  0.8      1.5669267611553      1.55503142343518      0.0118953377201196
  0.9     1.63905137779954      1.63261318178287     0.00643819601667794
    1      1.7035097441446      1.70187005276128     0.00163969138332076
```
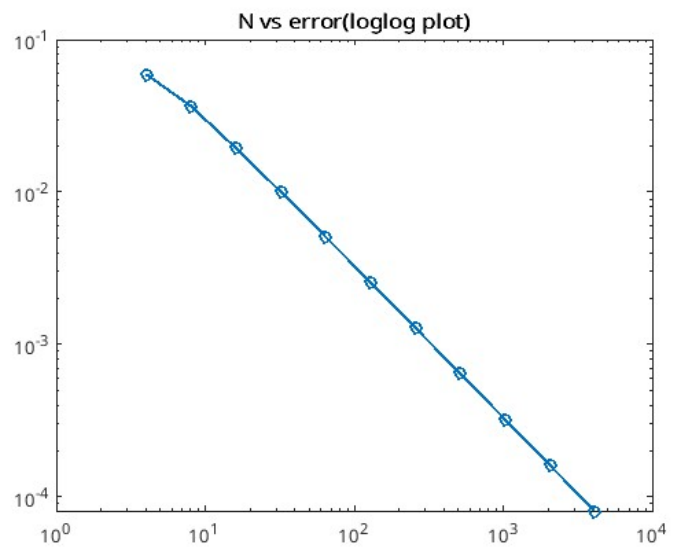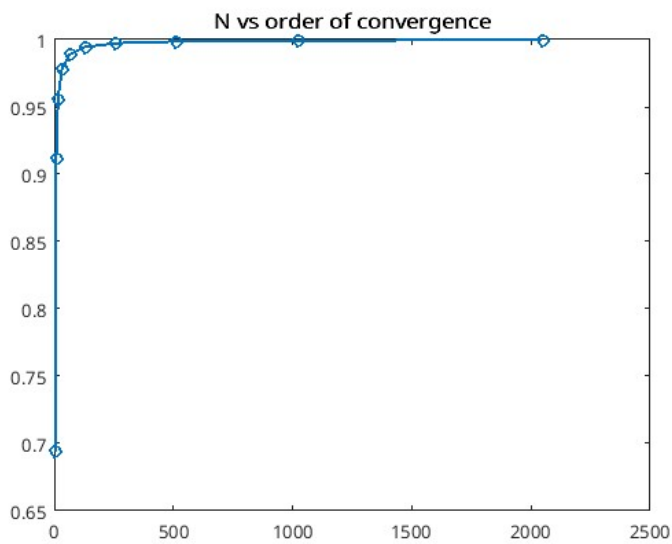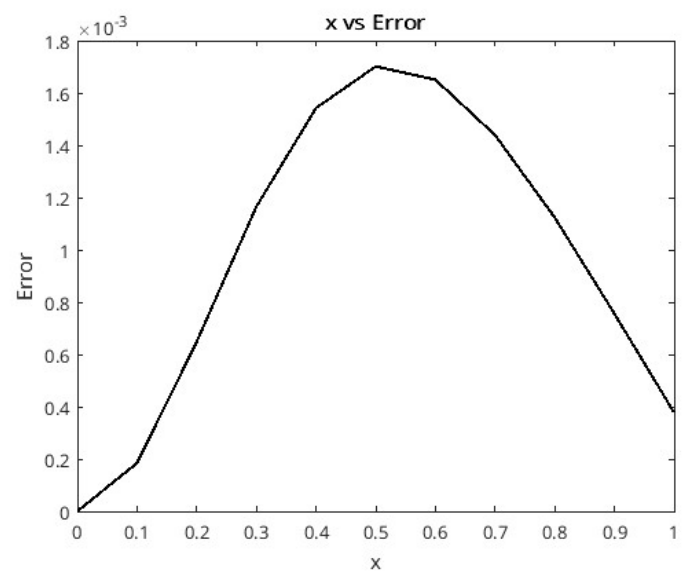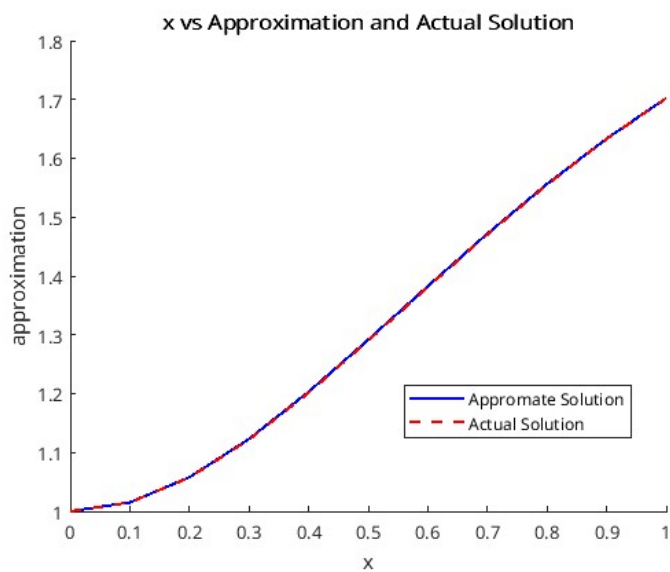

x vs Approximation and Actual Solution

x vs Error

Runge Kutta 2nd Order


```
Runge Kutta Method 2 order:
   t         approximation              actual                      error
  ___      _____        _____        _____

   0                     1                     1                            0
  0.1                 1.015      1.01481549985822        0.0001845001417764
  0.2      1.05782895089288       1.0571810074642       0.000647943428678577
  0.3      1.12286281792299      1.12169801826798        0.00116479965500993
  0.4      1.20302831702477       1.2014860103644        0.00154230666036415
  0.5      1.29150646848125      1.28980527630561        0.00170119217564579
  0.6      1.38258169760259      1.38093121544373        0.00165048215885255
  0.7       1.4718540869499      1.47041518539722        0.00143890155267368
  0.8      1.55615483380428      1.55503142343518        0.00112341036909891
  0.9      1.63336880578335      1.63261318178287       0.000755624000484012
    1      1.70224778342493      1.70187005276128       0.000377730663653653
```

Runge Kutta 4th Order

```
Runge Kutta Method 4 order:
    t        approximation              actual                    error

    0                      1                    1                          0
   0.1    1.01481586700284    1.01481549985822    3.67144615154302e-07
   0.2    1.05718219810417     1.0571810074642    1.19063996151247e-06
   0.3    1.12169988971777    1.12169801826798    1.87144978847087e-06
   0.4    1.20148810362448     1.2014860103644    2.09326007616362e-06
   0.5    1.28980714875618    1.28980527630561    1.87245057259311e-06
   0.6    1.38093254642537    1.38093121544373    1.33098163468404e-06
   0.7    1.47041575933439    1.47041518539722    5.73937165126992e-07
   0.8    1.55503109713822    1.55503142343518    3.26296961894457e-07
   0.9    1.63261186640638    1.63261318178287    1.31537648861979e-06
    1     1.70186770854212    1.70187005276128     2.3442191539047e-06
```
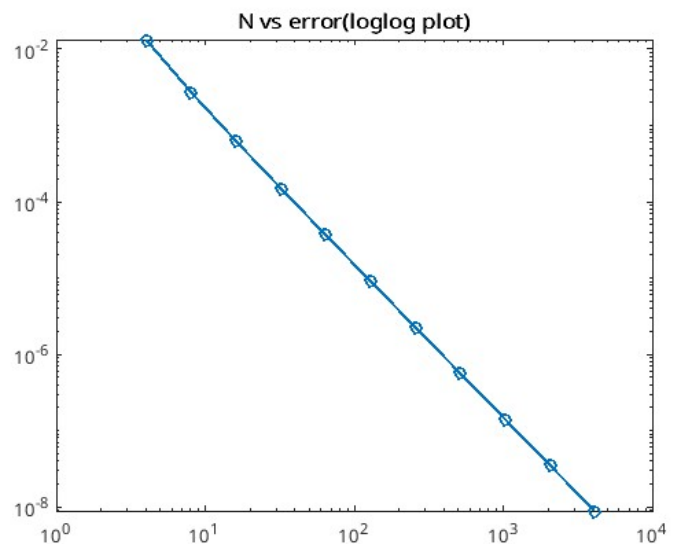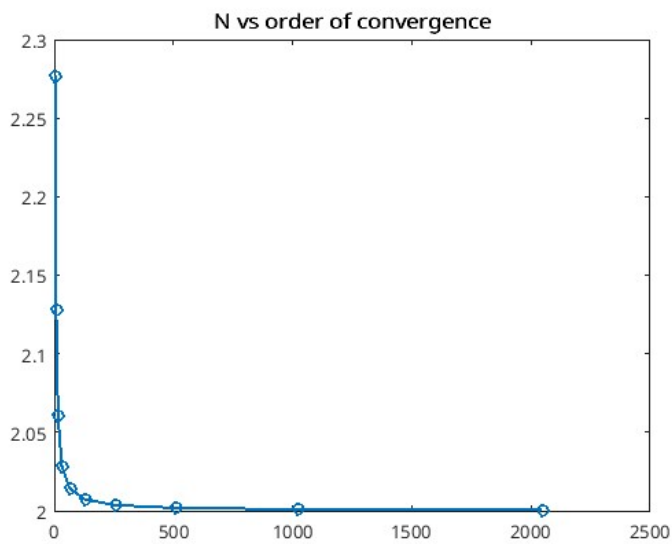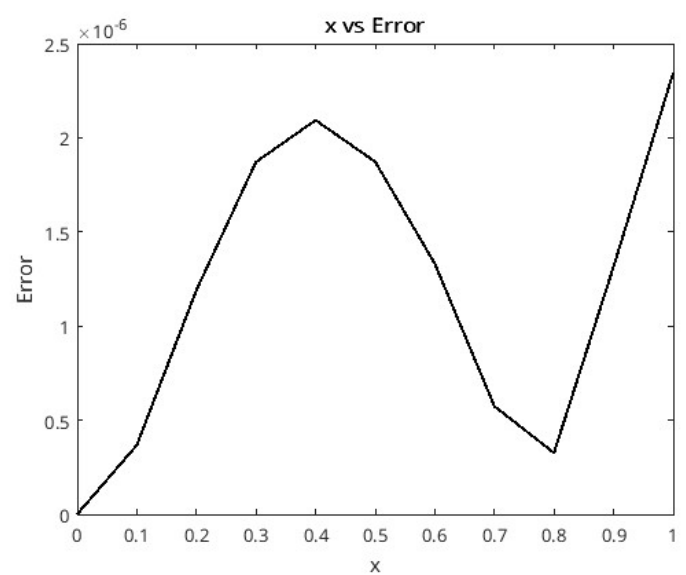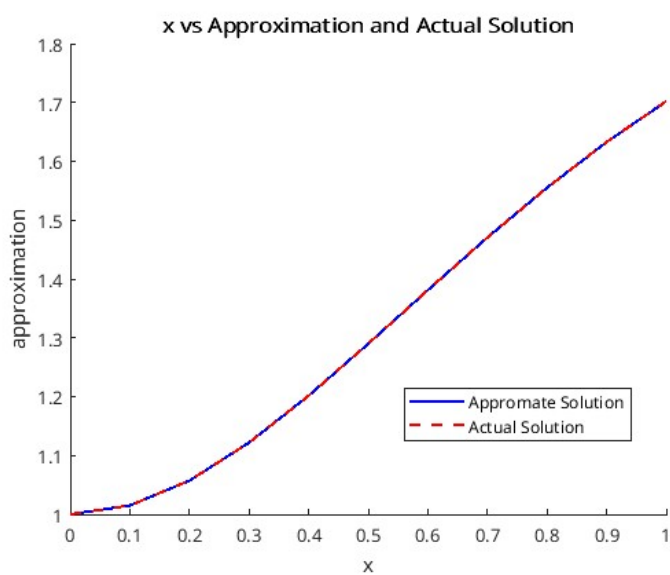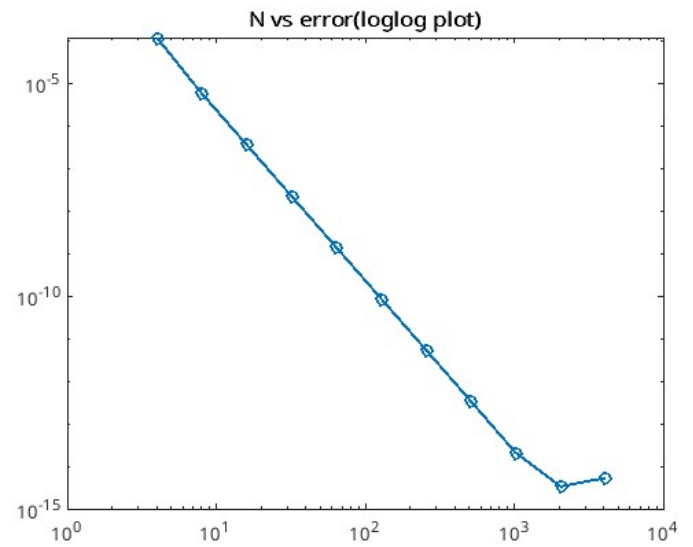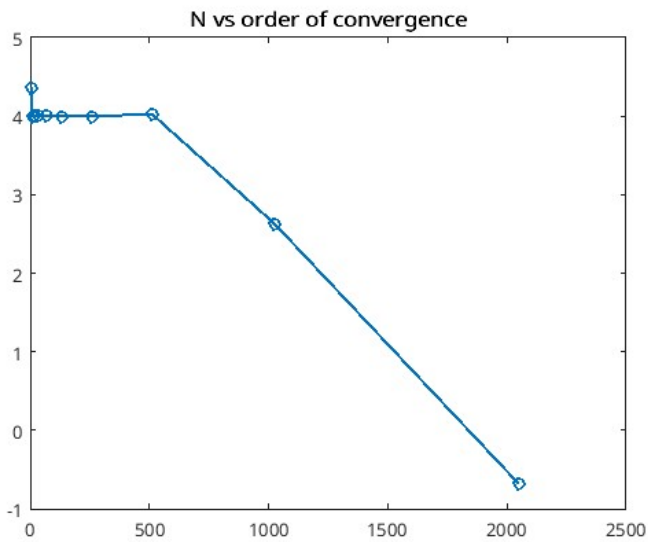
N vs order of convergence and **N vs error(loglog plot)**

Q2)

```
Runge Kutta Method 4 order:
     t        approximation          actual                error

     0                    1                    1                    0
   0.1     1.11422259959182     1.11422250235566     9.7236156237912e-08
   0.2     1.25293090533928      1.2529306328511    2.72488182906372e-07
   0.3     1.40845175999911     1.40845124727396    5.12725151002869e-07
   0.4     1.57122625952187      1.5712254606242    7.98897670017951e-07
   0.5     1.73049086605814     1.73048975851471    1.10754342430575e-06
   0.6     1.87508832068056     1.87508690788131    1.41279924692483e-06
   0.7     1.99434192183752     1.99434023319418    1.68864333693897e-06
   0.8     2.07891995120454      2.0789180400345    1.91117004311181e-06
   0.9     2.12161659014078     2.12161452944478    2.06069599562042e-06
     1     2.11798166912361     2.11797954561299    2.12351061801286e-06
```


x vs Approximation and Actual Solution

x vs Error

N vs order of convergence

N vs error(loglog plot)