x_n)

x x b

Q1.

$$Q = (\phi(x) - \alpha)\left(L(\theta_1) - k\, L(\theta_0)\right)$$

$$\phi(x) = \begin{cases} 1 & L(\theta_1) > k\, L(\theta_0) \\ \gamma & L(\theta_1) = k\, L(\theta_0) \\ 0 & L(\theta_1) < k\, L(\theta_0) \end{cases}$$

$$\alpha = \underline{\underline{\beta(\theta_0)}} = \cancel{P_{\theta_0}(x} \; E_{\theta_0}[\phi(x)]$$

$$\underline{0 \leq \alpha \leq 1}$$

$$Q \geq 0 \quad \forall \; \underline{x \in \mathcal{X}^n}$$

$$\int_{x \in \mathcal{X}^n} Q \, dx \geq 0$$

$$\Rightarrow \int_{x \in \mathcal{X}^n} \phi(x)\left(L(\theta_1) - k\, L(\theta_0)\right) dx - \alpha \int_{x \in \mathcal{X}^n} \left(L(\theta_1) - k L(\theta_0)\right) dx \geq 0$$

$$\Rightarrow \beta(\theta_1) - \cancel{k\beta(\theta_0)} - \alpha - \cancel{\alpha k} \geq 0$$

$$\Rightarrow \beta(\theta_1) \geq \alpha$$

$$\frac{L(\mu_1)}{L(\mu_0)} = \frac{\cancel{(2\pi\sigma^2)^{-\frac{n}{2}}}\, e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu_1)^2}}{\cancel{(2\pi\sigma^2)^{-\frac{n}{2}}}\, e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu_0)^2}}$$

$$= e^{\frac{1}{2\sigma^2}\Sigma\left[(x_i-\mu_0)^2 - (x_i-\mu_1)^2\right]}$$

$$= e^{\frac{1}{2\sigma^2}\Sigma(\mu_1-\mu_0)\left(2x_i-(\mu_1+\mu_0)\right)}$$

$$= e^{\frac{(\mu_1-\mu_0)}{2\sigma^2}\left[2n\bar{x} - n(\mu_1+\mu_0)\right]}.$$

Dec $\mu_0 > \mu_1$

~~Inc~~ in $\bar{x}$ ($\cancel{\mu_0 < \mu_1}$ $\cancel{\mu_1 > \mu_0}$)

$$\therefore \quad \frac{L(\mu_1)}{L(\mu_0)} > k \Rightarrow \bar{x} < k'$$

$$\therefore \quad \psi(x) = \begin{cases} 1 & \bar{x} < k' \\ \gamma & \bar{x} = k' \\ 0 & \bar{x} > k' \end{cases}$$

where k' is s.t.

$$P_{\theta_0}(\bar{x} < k') + \cancel{\gamma P_{\theta_0}(\bar{x} = k')} \rightarrow 0 \quad = \alpha.$$

$$\Rightarrow \quad P_\theta\left(\frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} < \frac{\sqrt{n}(k'-\mu)}{\sigma}\right) = \alpha.$$

$$\Rightarrow \quad P_\theta\left(\frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} < \lambda\right) = \alpha$$

$$\Rightarrow \quad \cancel{\frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} > z_\alpha}. \qquad \frac{\sqrt{n}(\bar{x}-\mu)}{\sigma} > z_{1-\alpha}.$$

hypothesis ... "meaningful

b)

An $\cancel{\mu_1 \in R}$ $\mu_1 \in R$ s.t. $\mu_1 < \mu_0$.

Then the MP level α test is given by

$$\phi(x) = \begin{cases} 1 & \frac{\sqrt{n}\,(\bar{x} - \mu_0)}{\sigma} < z_{1-\alpha} \\ 0 & \text{otherwise.} \end{cases}$$

---

c).

An
$$\frac{L(b_1, \delta^*)}{L(b_0, \delta^*)} = \frac{\cancel{(b_1 \Gamma(\delta^*))^{-n}} \prod_{i=1} \left(\cancel{x_i^{\delta^*-1}}\, e^{-\frac{x_i}{b_1}} \right)}{\cancel{(b_0 \Gamma(\delta^*))^{-n}} \prod_{i=1} \left(\cancel{x_i^{\delta^*-1}}\, e^{-\frac{x_i}{b_0}} \right)}$$

$$= e^{\left(\frac{1}{b_0} - \frac{1}{b_1}\right) \Sigma x_i}$$

inc in Σx_i $\longrightarrow$

$$\phi(x) = \begin{cases} 1 & \Sigma x_i > \\ \gamma & \Sigma x_i = \\ 0 & \Sigma x_i < \end{cases}$$

Given $x_i \sim Gamma(\delta, b)$

$\Sigma x_i \sim Gamma(n\delta, b)$

$P_{\theta_0}(\Sigma x_i > k) = \alpha$.

$k = Gamma_{\alpha}(n\delta, b)$.

4.

AN

$$\frac{L(\lambda_1)}{L(\lambda_0)} = \frac{\prod_{i=1}^{n} \frac{e^{-\lambda_1}\lambda_1^{x_i}}{\cancel{x_i!}}}{\prod_{i=1}^{n} \frac{e^{-\lambda_0}\lambda_0^{x_i}}{\cancel{x_i!}}}$$

$$= \frac{e^{-n\lambda_1}\lambda_1^{\Sigma x_i}}{e^{-n\lambda_0}\lambda_0^{\Sigma x_i}}$$

$$= e^{n(\lambda_0-\lambda_1)}\left(\frac{\lambda_1}{\lambda_0}\right)^{\Sigma x_i}$$

Inc in Σx_i as $\lambda_1 > \lambda_0$

$$\psi(x) = \begin{cases} 1 & \Sigma x_i > K \\ \gamma & \Sigma x_i = K \\ 0 & \Sigma x_i < K. \end{cases}$$

$\Sigma x_i \sim P(n\lambda)$

Let, $\tilde{K}$ be such that,

$$P_{\theta_0}(\Sigma X_i \geq \tilde{K}) > \alpha \geq P(\Sigma X_i > \tilde{K})$$

$$P_\theta(\Sigma X_i > \tilde{K}) + \gamma P_\theta(\Sigma X_i = K) = \alpha.$$

Let, $K = \tilde{K}$, $\gamma = \dfrac{\alpha - P(\Sigma X_i > \tilde{K})}{P(\Sigma X_i = \tilde{K})}$

$0 \leq \gamma \leq 1$

$\gamma \geq 0 \Rightarrow \alpha \geq P(\Sigma X_i > \tilde{K})$.

$\gamma \leq 1 \Rightarrow \alpha \leq P(\Sigma X_i > \tilde{K})$ } Already true from assumption

$$\psi(x) = \begin{cases} 1 & , \Sigma x_i > \tilde{K} \\ \dfrac{\alpha - P(\Sigma X_i > \tilde{K})}{P(\Sigma X_i = \tilde{K})} & , \Sigma x_i = \tilde{K} \\ 0 & , \Sigma x_i < \tilde{K} \end{cases}$$

5.

Ans

$$\frac{L(\theta_1)}{L(\theta_0)} = \frac{3}{16 \cdot 2^{\frac{4}{3}(\lambda - \frac{1}{2})}} x^{\lambda} \, I_{(0,4)}(x) \left(\frac{x}{8}\right)^{x} \quad \text{———} \quad x^{\lambda}$$

~~64x~~ 4×16.

$H_0 : \lambda = 2$

$H_1 : \lambda = \frac{1}{2}$

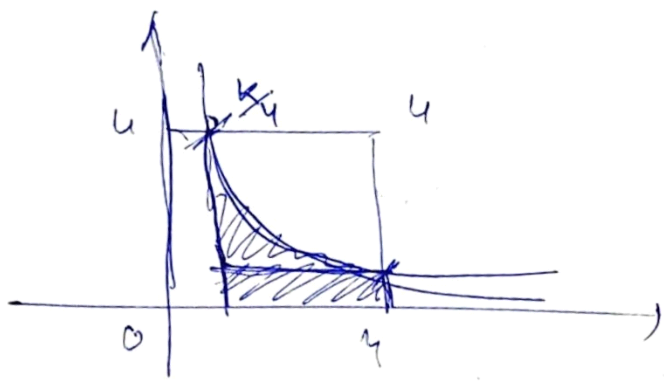
$$\frac{L(\frac{1}{2})}{L(2)} = \frac{\frac{3}{16} \sqrt{x_1 x_2} \, I_{(0,4)}(x)}{\frac{3}{64} (x_1 x_2)^2 \, I_{(0,4)}(x)}$$

$$= 4 (x_1 x_2)^{-\frac{3}{2}}$$

$\frac{L}{L_0} > K \Rightarrow x_1 x_2 < K'$

$P_{\theta_0}(x_1 x_2 < K') = \alpha$

$$\iint_{x_1 x_2 < K'} \left(\frac{3}{64}\right)^2 (x_1 x_2)^2 \, \mathbb{I}_{(0,4)}(x_1) \, \mathbb{I}_{(0,4)}(x_2) \, dx_1 \, dx_2 = \alpha$$



$x_1 x_2$

$$+ \int_{K/4}^{4} \int_0^{K/x_1} \frac{9}{(64)^2} (x_1 x_2)^2 \, dx_2 \, dx_1 = \alpha$$

$$\Rightarrow \frac{9}{(64)^2} \int_{K/4}^{4} x_1^2 \int_0^{K/x_1} x_2^2 \, dx_2 \, dx_1 = \alpha.$$

$$\Rightarrow \frac{9}{(64)^2} \int_{K/4}^{4} \cancel{x_1^2} \times \frac{1}{3} \times \frac{K^3}{x_1^3} \, dx_1 = \alpha$$

$$\Rightarrow \frac{3}{(64)^2} K^3 \times \ln\left(\frac{4}{K/4}\right).$$

c.

Ans

$$\frac{L(\mu_1)}{L(\mu_0)} = \frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x_1-\mu_1)^2} \frac{1}{\sqrt{2\pi 4\sigma^2}} e^{-\frac{1}{8\sigma^2}(x_2-\mu_1)^2}}{e^{-\frac{1}{2\sigma^2}(x_1-\mu_0)^2} \; e^{-\frac{1}{8\sigma^2}(x_2-\mu_0)^2}}$$

$$= e^{\frac{1}{2\sigma^2}(\mu_1-\mu_0)(2x_1-\mu_0-\mu_1)}$$

$$\times e^{\frac{1}{8\sigma^2}(\mu_1-\mu_0)(2x_2-\mu_0-\mu_1)}$$

$$= e^{\frac{(\mu_1-\mu_0)}{2\sigma^2}\left[(2x_1-\mu_0-\mu_1)+\frac{1}{4}(2x_2-\mu_0-\mu_1)\right]}$$

$(\mu_1 > \mu_0)$

$$\Rightarrow \quad x_1 + \frac{x_2}{4} > k.$$

$X_1 \sim N(\mu, \sigma^2)$

$$\frac{X_2}{4} \sim N\left(\frac{\mu}{4}, \frac{\sigma^2}{4}\right)$$

$$T = X_1 + \frac{X_2}{4} \sim N\left(\frac{5\mu}{4}, \frac{5\sigma^2}{4}\right)$$

$$P_{\mu_0}(T > k) = \alpha$$

$$\Rightarrow P_{\mu_0}\left(\frac{T-\frac{5\mu_0}{4}}{\sqrt{\frac{5\sigma^2}{4}}} > \frac{k-\frac{5\mu_0}{4}}{\sqrt{\frac{5\sigma^2}{4}}}\right) = \alpha.$$