INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

DEPARTMENT OF MATHEMATICS

MA 322: SCIENTIFIC COMPUTING

Semester–II, Academic Year 2022-23

Assignment

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1. Derive the **Adams-Bashforth and Adams-Multon methods of order 5** based on equally spaced points $x_i = x_0 + ih, \quad 0 \leq i \leq h$. Solve the following IVP using your Adams-Bashforth-Multon methods.

$$\begin{aligned} &x'' = e^t + x\cos t - (t+1)x' \\ &x(0) = 1 \qquad x'(0) = 1.85. \end{aligned}$$

2. Consider the following partial differential equation (PDE)

$$u_t + au_{xxx} = f.$$

Show that the scheme

$$\frac{u_m^{k+1} - u_m^k}{\Delta t} + a\frac{u_{m+2}^k - 3v_{m+1}^k + 3v_m^k - v_{m-1}^k}{\Delta x^3} = f_m^k$$

is consistent with the above and, if $\nu = \Delta t(\Delta x)^{-3}$ is constant, then it is stable when $0 \leq a\nu \leq 1/4$.

3. Consider the nodes $x_0 < x_1 < \cdots < x_n$ such that $x_{i+1} - x_i = h_i, \quad 0 \leq i \leq n-1$. Solve the following BVP using second-order finite difference method on non-equidistant points with $0.01 \leq h_i \leq 0.1$.

$$\begin{aligned} &\frac{\mathrm{d}^2C}{\mathrm{d}R^2} + \frac{2}{R}\frac{\mathrm{d}C}{\mathrm{d}R} = \Phi^2 C \quad 0 < R < 1 \\ &\frac{\mathrm{d}C}{\mathrm{d}R} = 0 \quad \text{at} \quad R = 0 \\ &C = 1 \quad \text{at} \quad R = 1 \\ &\Phi = 2.236. \end{aligned} \tag{1}$$

Determine $C(R=0)$. Show details of your calculation and derivation of the system of linear algebraic equations.

4. Solve the following **initial boundary value problem (IBVP)** using **backward time central space (BTCS)** scheme.

$$\begin{aligned} &u_t = \nu u_{xx} + u^2 \qquad (x,t) \in (-1,1) \times (0,T] \\ &u_x(-1,0) = u_x(1,0) = 0 \quad t > 0 \\ &u(x,0) = \begin{cases} 1 & x \leq 0 \\ 0 & x > 0 \end{cases} \quad t > 0. \end{aligned}$$

Show your result graphically for different values of T and $\nu = 1/4, \ 4$. Discuss the stability of the scheme for the corresponding linear equation in terms of ν.

5. Consider the IVP
$$x' = -kx, \ x(0) = 1$$
on the interval $[0, 1]$ with various values for the decay constant $k \in \mathbb{R}$. For Euler's method, the **region of stability** for this problem is $0 \le hk \le 2$. Determine the **region of stability** for **fourth-order RK method**. Verify your theoretical prediction with numerical solutions for at least two different values of k. Determine the order of the method numerically.

6. Solve the following **initial boundary value problem (IBVP)** using **Crank-Nicolson (CN)** method.
$$\begin{aligned} &u_t = \nu u_{xx} \qquad (x,t) \in (0,1) \times (0,T] \\ &u(0,t) = u(1,t) = 0 \quad t > 0 \\ &u(x,0) = u_0 \quad 0 < x < 1 \end{aligned}$$
Show your result graphically for $T = 10$ and $\nu = 1/4, \ 4$. Discuss the stability of the method.

7. Discuss shooting method for nonlinear BVP. Solve the following BVP using nonlinear shooting method with a tolerance 10^{-6} and $h = 0.1$.
$$y'' = y' + 2(y - \ln x)^3 - \frac{1}{x}, \quad 1 \le x \le 2, \qquad y(1) = 1, \ \ y(2) = \frac{1 + \ln 4}{2}.$$
Is it possible to provide your best approximation of $y(1.15)$ using the numerically computed result? – Explain.

8. Solve the following boundary value problem (BVP)
$$y'' + 4y = 0 \qquad y(\pi) = 0, \ \ y(49\pi/4) = 1$$
on a non-uniform mesh given by $x_{i+1} = r x_i, \ r > 1, \ i = 0, 1, 2, \ldots, n$. Determine the relative error of your numerical results.

(**Hint: Use a suitable transformation so that the non-uniform mesh is mapped into a uniform mesh. Solve your problem in this transformed domain.**)