Special Theory of Relativity (PH101)
Course Instructors: Pankaj Mishra and Tapan Mishra
Practice Problems
due on Monday, 28th of October, 2019 (11:00Hrs IST)

1. Electrons in projection television sets are accelerated through a potential difference of 50 kV.

   Calculate the speed of the electrons using the relativistic form of kinetic energy assuming the electrons start from rest.**Answer:** $v = 0.413c$

   Calculate the speed of the electrons using the classical form of kinetic energy.**Answer: v = 0.442c**

   Solution

   A particle with rest mass m moving with speed v has the kinetic energy K given by $K = (\Gamma_u - 1)mc^2$. Therefore, $v = c\sqrt{1 - [1 + (K/mc^2)^{-2}]}$. Also for $v \ll c$ we have $K \simeq \frac{1}{2}mv^2$.

2. As seen from Earth, two spaceships A and B are approaching along perpendicular directions. If A is observed by a stationary Earth observer to have velocity $u_y = -0.90c$ and B to have velocity $u_x = 0.90c$, determine the speed of ship A as measured by the pilot of ship B.**Answer:** $u'_x = -0.9c$**,** $u'_y = -0.39c$**,** $u' = 0.98c$

   Solution:

   We take the S frame to be attached to the Earth and the S' frame to be attached to spaceship B moving with $\beta = 0.90$ along the x-axis. Spaceship A has velocity components $u_x = 0, u_y = 0.90c$ in S.
   $u'_y = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$ give the velocity components of spaceship A in S', from which we have $u'_x = -v = -0.90c, u'_y = u_y/\gamma = -0.39c$ So we have,

   $$u' = \sqrt{(u'_x)^2 + (u'_y)^2} = 0.98c$$

3. A body quadruples its momentum when its speed doubles. What was the initial speed in units of c, i.e., what was u/c? **Answer:** $u/c = \frac{1}{\sqrt{5}}$

4. A body of rest mass m_0 moving at speed v collides with and sticks to an identical body at rest. What is the mass and momentum of the final clump?**Answer:** $M = m_0\sqrt{2(1 + \Gamma_v)}$

5. Two β particles move in opposite direction with velocity $0.6c$ in the laboratory frame. Calculate the velocity of one β particle in the moving frame attached to the other β particle by applying relativistic transformation. Repeat the calculations taking speed of particles as 0.06c

. **Answer:** $u_2' = -0.88c$

6. A 2m long stick, when it is at rest, moves past an observer on the ground with a speed of 0.5c.

   (a) What is the length measured by the observer ?**Answer:** $L = 1.732m$

   (b) If the same stick moves with the velocity of 0.05c what would be its length measured by the observer ?**Answer:** $L = 1.997m$

7. A proton of mass $m_p = 1.67 \times 10^{-27}kg$ moves with a speed of $u = 0.6c$. Compute its relativistic and non-relativistic momentum.**Answer:** $P_{relativistic} = 3.75 \times 10^{-19}kg - m/sec$ **and** $P_{nonrelativistic} = 3.0 \times 10^{-19}kg - m/sec$

8. The change in frequency of wave that happens due to relative motion between the source and observer is known as the "Doppler effect." In Galilean relativity the modified frequency is given by $\nu = \nu_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$, where, ν_0 is the original emitted frequency, ν is the observed(detected) frequency, v, v_s, and v_o is the velocity of wave (e.g., sound wave),observer, and source relative to the medium respectively. Here $+(-)$ stands for the situation when observer is approaching (receding) towards (from) source. This relation gets modified in STR as $\nu = \nu_0 \sqrt{\frac{c \pm v}{c \mp v}}$ where, c is the velocity of light and v is the velocity of the light source. Note that here implicitly it is assumed that the observer is in the frame S and light source is in the moving frame S'. Using this information solve the following problem.

   A driver is caught violating the traffic rule by going through a red light signal. The driver claims to the judge that the color she actually saw was green ($\nu = 5.60 \times 10^{14}Hz$) and not red ($\nu = 4.80 \times 10^{14}Hz$) because of the Doppler effect. The judge accepts this explanation and instead fines her for speeding at the rate of 100 (INR) for each km/h that she exceeded the speed limit of 80 km/h. Compute the total fine amount?

   Solution:
   Using the relation $\nu = \nu_0 \sqrt{\frac{c+v}{c-v}}$, as the observer (driver) is approaching towards the source (signal), we have
   $v = c\left(\frac{\nu^2 - \nu_0^2}{\nu^2 + \nu_0^2} \right) = 3 \times 10^8 m/s \left[\frac{(5.60)^2 - (4.80)^2}{(5.60)^2 + (4.80)^2} \right]$
   $= 4.59 \times 10^7 m/sec = 1.65 \times 10^8 km/h$, (since, $1m/sec = 3.6km/h$).

   The fine will be $(1.65 \times 10^8 - 80) \times 100(INR) = 16499992000(INR)$.

9. The light wave equation is given by $\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$, where, E is the electric field and c is the velocity of light. Show that under Galilean transformation the above equation will have the form as $\frac{\partial^2 E'}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E'}{\partial t'^2} - \frac{2v_x}{c^2}\frac{\partial^2 E'}{\partial x' \partial t'} - \frac{v_x}{c^2}\frac{\partial}{\partial x'}\left[v_x \frac{\partial E'}{\partial x'}\right] = 0$, where v_x is the speed of S' frame w.r.t. the S frame. That shows that light wave equation is not compatible with the Galilean relativity. Hint: Use the transformation formula discussed during the first lecture of STR.

Solution:

Similar problem has been done in the class where incompatibility of the Ampere's law was shown with the Galilean relativity (GR). Use the relation $E/B = c$ which will give the relation $E' = E$ if it is assumed that for $E = E(x,t)\hat{j}$ and $B = B(x,t)\hat{k}$ the Lorentz force remains invariant which will give the relation $E = E' + v_x B$ (See the lecture note for details).
hint:

$$x' = x - v_x t \quad t' = t \rightarrow \frac{\partial x'}{\partial x} = 1 \quad \frac{\partial x'}{\partial t} = -v_x \quad \frac{\partial t'}{\partial t} = 1 \quad \frac{\partial t'}{\partial x} = 0$$

using the chain rule,Electrons in projection television sets are accelerated through a potential difference of 50 kV.

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using the chain rule, $\dfrac{\partial E}{\partial x} = \dfrac{\partial E}{\partial x'}\dfrac{\partial x'}{\partial x} + \dfrac{\partial E}{\partial t'}\dfrac{\partial t'}{\partial x} = \dfrac{\partial E}{\partial x'}$.

In the similar way it can be shown that $\dfrac{\partial^2 E}{\partial x^2} = \dfrac{\partial^2 E}{\partial x'^2}$.

A ball moves at speed v 1 with respect to a train. The train moves at speed v 2 with respect to the ground. What is the speed of the ball with respect to the ground? Solve this problem (that is, derive the velocity addition formula, eq. (11.31)) in the following way (dont use time dilation, length contraction, etc; use only the fact that the speed of light is the same in any inertial frame): Let the ball be thrown from the back of the train. At the same instant, a photon is released next to it (see Fig. 11.44). The photon heads to the front of the train, bounces off a mirror, heads back, and eventually runs into the ball. In both the frame of the train and the frame of the ground, calculate the fraction of the way along the train where the meeting occurs, and then equate these fractions.

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial t} = \frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right) = \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}\right) = \frac{\partial}{\partial t'}\left(\frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}\right)\frac{\partial t'}{\partial t} + \frac{\partial}{\partial x'}\left(\frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}\right)\frac{\partial x'}{\partial t}$$
$$= \frac{\partial^2 E}{\partial t'^2} - 2v_x\frac{\partial^2 E}{\partial x'\partial t'} - v_x^2\frac{\partial^2 E}{\partial x'^2}.$$

$$\frac{\partial E}{\partial x} = \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial x} = \frac{\partial E}{\partial x'}.$$

In the similar way it can be shown that $\dfrac{\partial^2 E}{\partial x^2} = \dfrac{\partial^2 E}{\partial x'^2}$.

$$\frac{\partial E}{\partial t} = \frac{\partial E}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial E}{\partial t'}\frac{\partial t'}{\partial t} = \frac{\partial E}{\partial t'} - v_x\frac{\partial E}{\partial x'}$$

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$$= \frac{\partial^2 E}{\partial t'^2} - 2v_x\frac{\partial^2 E}{\partial x'\partial t'} - v_x^2\frac{\partial^2 E}{\partial x'^2}.$$

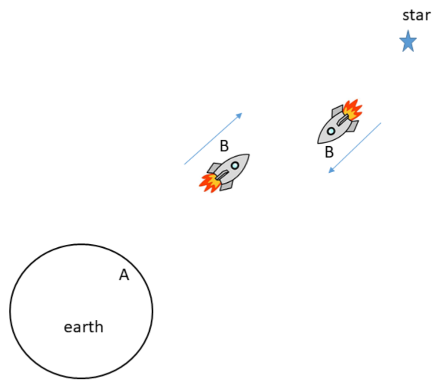
Indian Institute of Technology Guwahati

Tutorial-6 PH101: Physics-I Due on : 22 Jan, 2021

1. It was shown in the class that Newton's second law is invariant under Galilean transformation. Using similar arguments show that Newton's first and third laws are also invariant under Galilean transformation.

2. An earth's satellite is travelling at a speed of 7800 m/s. For a moving frame at this speed how much the corresponding γ exceeds 1. Let us assume that it takes 90 minutes to complete one orbit around the earth as per the clock attached to the satellite. What is the corresponding time as per the clock attached to ground (i.e. earth). What is the percentage difference?

3. A and B are twins and each of the two are 20 years old. B sets off on a rocket at a constant velocity of 0.95c (c being the velocity of light in vacuum) to a distant star. After exploring the star for a very short duration B then gets back home on earth.



As measured by A the total period of absence of B is 40 years. What is the time of absence as per B's clock? Is this entire experiment symmetrical with respect to the frames of reference of the two twins? If not why.

4. A π^+ meson is created in the atmosphere at a height 200km from the ground when primary cosmic ray is incident. It descends vertically at a speed 0.99c. It then disintegrates after 2.5×10^{-8} sec of its creation as measured from the frame of reference of the π^+ meson. At what height from the ground the π^+ meson is observed to disintegrate from earth?

5. A bullet train has a total length (as measured from the frame of reference of the train itself) of 450 metres. An observer standing on a railway platform is measuring the length of the bullet train when the train is traveling with a constant velocity at speed equal to 400 km/hour. Obtain the amount by which the length of the train appears to get shortened as per the observer on the platform. (Hint: As in this case $\beta^2 \ll 1$, hence expand γ binomially and retain upto the second term).

6. A rod of proper length L_0 moves with speed v along the positive x-direction and is attached to the S$'$ frame. The rod makes an angle θ_0 with respect to the x$'$-axis of the S$'$ frame. Find the length of the rod as measured in frame S and the angle made by the rod with the x axis.

Special Theory of Relativity (PH101)

Course Instructors: Pankaj Mishra and Tapan Mishra

Tutorial-6

due on Wednesday, 16th of October, 2019 (8:00Hrs IST)

1. A helicopter is flying with a constant speed of 180 km/hr along the horizontal x-direction with respect to an observer on the ground at a height of 500 m. Consider the y-direction vertically upwards. At certain time, the pilot releases a packet which drops down with zero initial vertical speed. Set the coordinates of the observer and the pilot so that at the time of release, the packet is at the origin in both the reference frames. Set this time to be t=0. Consider acceleration due to gravity $g = 9.8m/sec^2$. Find the position and velocity of the packet in the two frames, 5 seconds after it is dropped, according to Galilean Relativity.

2. Compare the speeds of the following with that of light (express as fraction of c). In each of the cases find the prefactor γ that appear in the Lorentz transformation.
(i) Hima Das running at a speed of 100m in 10 sec.
(ii) Maglev train running at a speed of 500 km/hr.
(iii)Concord aircraft flying with ground speed 1800 km/hr.
(iv) Space-shuttle moving with a speed of 27000km/hr.
(v)Earth orbiting around Sun with a speed of 30km per second.
(vi)Proton making one round of 27km circumference of the Large Hadron Collider (LHC) tunnel in 100 micro second.

3. Two events occur at $\left(t = \frac{X}{2c}, X, 0, 0\right)$ and $\left(t = \frac{X}{c}, 3X, 0, 0\right)$ in a frame S. What is the speed of frame S' (moving along x-axis with constant speed), so that the above two events occur at the same time in this frame? What is the value of this time, and what are the values of x-coordinate in S'?

4. The earth and Sun are 8.3 light-minutes (the distance traveled by light in one minute) apart. Ignore their relative motion for this problem and assume they live in a single inertial frame, the Earth-Sun frame. Events A and B occur at t = 0 on the earth and at t=2 minutes on the Sun respectively. Find the time difference between the events according to an observer moving at $u = 0.8c$ from Earth to Sun. Repeat if observer is moving in the opposite direction at $u = 0.8c$.

5. An observer in frame S who lives on the x-axis sees a flash of red light at $x = 1210m$. After $4.96\mu s$, he sees flash of blue light at $x = 480m$. Use subscripts R and B to label the coordinates of the events related to the red and blue light respectively.
(i) Now suppose there is an observer in S' which is moving with a velocity 'v' with respect to the S frame watches these events. Compute the velocity v for the situation when the observer in S' records both the events occurring at the same place?
(ii) Which event occurs first according to S' and what is the measured time interval between these flashes?

6. The average lifetime of a π meson in its own frame of reference is 26.0 ns. (This is its proper lifetime.) What will be the lifetime of π meson as measured by an observer at rest on earth if it moves with respect to the Earth with a speed $0.95c$. Also compute the average distance it will travel before decaying as measured by an observer at rest on Earth.

7. The change in frequency of wave that happens due to relative motion between the source and observer is known as the "Doppler effect." In Galilean relativity the modified frequency is given by $\nu = \nu_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$, where, ν_0 is the original emitted frequency, ν is the observed(detected) frequency, v, v_s, and v_o is the velocity of wave (e.g., sound wave),observer, and source relative to the medium respectively. Here $+(-)$ stands for the situation when observer is approaching (receding) towards (from) source. This relation gets modified in STR as $\nu = \nu_0 \sqrt{\frac{c \pm v}{c \mp v}}$ where, c is the velocity of light and v is the velocity of the light source. Note that here implicitly it is assumed that the observer is in the frame S and light source is in the moving frame S'. Using this information solve the following problem.

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Special Theory of Relativity (PH101)
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Tutorial-7
due on Wednesday, 23rd of October, 2019 (8:00Hrs IST)

1. Two rockets of rest length L_0 are approaching the earth from opposite directions at velocities $\pm c/2$. What will be size of each rocket measured from the other rocket frame of reference?

Solution:

Let's pick one rocket (call it rocket 1) and consider how fast the other rocket (rocket 2) looks in this frame. In the Earth frame, rocket 1 has velocity $c/2$ and rocket 2 has velocity $c/2$. Applying the velocity addition law gives

$$v_2' = \frac{v_2 - v_1}{1 - v_1 v_2/c^2} = \frac{(-c/2) - (c/2)}{1 - (c/2)(-c/2)/c^2} = -\frac{4}{5}c.$$

As Rocket 2 looks like it is approaching at $\frac{4}{5}c$. Applying the Lorentz contraction relation we have

$$L' = \frac{L_0}{\gamma} = L_0\sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}L_0.$$

2. A body of rest mass m_0 moving at speed u in positive x-direction approaches an identical body of same mass which is at rest. Find the speed V of a frame which is also moving in positive x-direction from which the total momentum of the system would be zero. Further repeat the calculation for non-relativistic range, i.e., $u \ll c$ and see whether your relativistic result reduces to that value or not.

Solution:

Two identical particles with equal and opposite velocities will have equal and opposite momenta as the relativistic momentum is given by $p = \Gamma_u m u$ where $\Gamma_u = \frac{1}{\sqrt{1 - u^2/c^2}}$. So we will look for a frame where the velocities are equal and opposite.

From a frame S' moving with a velocity V with respect to the S-frame, the body which is at rest rest will have the velocity V , while the mass that is moving with a velocity u w.r.t. S-frame will have the velocity

$$u' = \frac{u - V}{1 - \frac{uV}{c^2}}$$

If the total momentum is zero from S', then we have

$$\frac{u - V}{1 - \frac{uV}{c^2}} = V \Rightarrow uV^2 - 2c^2V + c^2u = 0$$

$$\Rightarrow V = \frac{2c^2 \pm \sqrt{4c^4 - 4u^2c^2}}{2u} = \frac{c^2 \pm c^2/\Gamma_u}{u}$$

If we consider + sign $V > c$ which is not accepted in STR. Considering the $-$ sign we have,

$$V = \frac{c^2(1 - 1/\Gamma_u)}{u}$$

We can further simplify this by

$$V = u\frac{1 - 1/\Gamma_u}{u^2/c^2} = u\frac{1 - 1/\Gamma_u}{1 - 1/\Gamma_u^2} = u\frac{\Gamma_u}{1 + \Gamma_u}$$

For non-relativistic range ($\Gamma_u \simeq 1$), we have $V = u/2$, which we can easily get using the Galilean relativity also.

3. A spaceship moves away from Earth with speed v and fires a shuttle craft in the forward direction at a speed v relative to the spaceship. The pilot of the shuttle craft launches a probe in the forward direction at speed v relative to the shuttle craft.

   (i)Determine the speed of the shuttle craft relative to the Earth.
   (ii) Determine the speed of the probe relative to the Earth.

   Analyze your results for the limiting cases $v \ll c$ and $v \to c$.
   Solution:

   We consider the S frame to be attached to the Earth and the S' frame to be attached to the spaceship moving with v along the $x-$axis. The shuttle craft has speed $u'_x = v$. Therefore,

$$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}. \tag{1}$$

   Gives the speed $u_x = \frac{2v}{1+\beta^2}$, where $\beta = v/c$.

   Now consider S' frame is attached to the shuttle craft moving with speed,

   $v' = \frac{2v}{1+\beta^2}$ with respect to the S-frame (Earth)

   along the $x-$ axis. The probe has a speed $u'_x = v$ in S'. Its speed u_x in the S frame would be given by Eq. (1) with v replaced by v' from which we get

$$u_x = \left(\frac{3+\beta^2}{1+3\beta^2}v\right) \tag{2}$$

   It follows from Eq.(2) that $u_x \to 3v$ when $\beta \ll 1$ and $u_x \to c$ when $\beta \to 1$

4. An electron e^- ($m_0 = 0.511 MeV/c^2$) with kinetic energy 1 MeV undergoes a head-on collision with a positron e^+ at rest (A positron is an antimatter particle that has the same mass as the electron but opposite charge). In the process of collision the two particles annihilate resulting in creation of two photons of equal energy. Consider one photon makes an angle θ and another makes and $-\theta$ with the electrons direction of motion (A photon γ is a massless particle of electromagnetic radiation having energy $E = pc$). The process is denoted as

   $e^- + e^+ \to 2\gamma$

   Determine the energy E, momentum p and angle of emission θ of each photon.

   Solution:

   Using the Kinetic Energy (KE) and momentum relation we can compute the momentum using the relation

   $p = \sqrt{K(K + 2m_0c^2)}/c = 1.422 Mev/c$, when $K = 1 Mev$.
   (The above relation can be obtained from
   $E^2 = p^2c^2 + m_0^2c^4$. Substitute here $E = K + m_0c^2$.)
   The total energy E of the electron and the stationary positron before the collision is

   $E = K + 2m_0c^2 = 2.022 Mev.$

   Using energy conservation we compute the energy of the two photons emerge from the collision as

   $E_\gamma = (E/2) = 1.011 Mev.$

   The magnitude of the momentum of each photon will be $p_\gamma = E_\gamma/c = 1.011 Mev/c$.

   The momentum vectors of the photons make angles $\pm\theta$ with the x-axis. Conservation of momentum yields,

   $p = 2p_\gamma \cos\theta$ which gives $\theta = 45.3^o$

5. A bullet of rest mass $m_0 = 100gm$ fired in the x-direction at a speed of $0.5c$ hits and get stuck on to a ball of rest mass $M_0 = 200gm$ sitting at rest with respect to the earth. What is the speed of the combined system after the collision as per special theory of relativity (momentum defined as $\vec{p} = \Gamma_u m_0 \vec{u}$ is conserved)? (Neglect all other effects including that of air drag and gravity.)

   Solution:

   Consider the bullet is fired along the $x-$ direction.

Initial momentum of the bullet is $\vec{p}_1 = \frac{1}{\sqrt{1-0.5^2}} 0.1 \times 0.5c\hat{x} = 0.057735c\hat{x}$ (in SI units).

Initial momentum of the ball $= 0$.
Hence, the total initial momentum $\vec{p}_i = 0.057735c\hat{x}$.

The final momentum is given as $\vec{p}_f = \Gamma_u(M_0 + m_0)\vec{u}_f$.

According to momentum conservation; $\vec{p}_f = \vec{p}_i = 0.057735c\hat{x}$.

Therefore, $\Gamma_u(M_0 + m_0)\vec{u}_f = 0.057735c\hat{x} \Rightarrow \vec{u}_f = \frac{c\vec{p}_f}{\sqrt{|\vec{p}_f|^2 + (M_0 + m_0)^2 c^2}}$
$= \frac{0.057735c\vec{x}}{\sqrt{0.057735^2 + 0.3^2}} = 0.189c\vec{x}$.

6. Consider a shell of total mass $M_0 = 1$ kg at rest with respect to an observer on earth. At time $t = 0$ it explodes into three equal parts, each of which flies away with the same speed $u = 0.2c$, making an angle 120^o between each other. The velocities may be considered as $\vec{u}_1 = 0.2c\hat{x}$, $\vec{u}_2 = 0.2c\Big(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\Big)$, $\vec{u}_3 = 0.2c\Big(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\Big)$. For the observer on the earth, what is the energy and what is momentum of each of the pieces, (let us denote these by $E_1,\ E_2,\ E_3$ and $\vec{p}_1,\ \vec{p}_2,\ \vec{p}_3$ respectively)? Denoting the energy-momentum four-vectors as $p_1,\ p_2,\ p_3$ with $p_1 \equiv \left(\frac{E_1}{c},\ \vec{p}_1\right)$ etc., find $(p_1+p_2+p_3)\cdot(p_1+p_2+p_3)$. (In general, $p \cdot p = \frac{E^2}{c^2} - \vec{p}\cdot\vec{p} = m_0^2c^2$.) Consider another observer moving with velocity $\vec{v} = 0.5c\hat{x}$ with respect to the observer on the Earth. Find $E_1',\ E_2',\ E_3'$ and $\vec{p}\,'_1,\ \vec{p}\,'_2, \vec{p}\,'_3$. What are the masses of each of these parts as seen from the moving frame? Find the magnitude of the sum of the energy-momentum four-vector in this frame $((p_1' + p_2' + p_3')\cdot(p_1' + p_2' + p_3'))$.

Solution:

The energy of an object moving with velocity $\vec{u}$ is given as $E = \Gamma_u m_0 c^2$ and the momentum as $\vec{p} = \Gamma_u m_0 \vec{u}$.
Situation before the explosion: Total energy available $E_i = M_0c^2 = 9\times10^{16}$J. Initial momentum, $\vec{p}_i = 0$.
As all the pieces have the same mass and move with same speed after the expolsion, they have to same energy.

i.e. $E_1 = E_2 = E_3 = \frac{E_i}{3} = 3 \times 10^{16}$J

Magnitude of momentum of each piece , $|\vec{p}_1| = E_1\frac{|\vec{u}_1|}{c^2}\ kg\ m/s = 3\times10^{16}\ \frac{0.2}{c} = 2\times10^7\ kg\ m/s = |\vec{p}_2| = |\vec{p}_3|$.

Therefore, $\vec{p}_1 = 2\times10^7\ \hat{x},\quad \vec{p}_2 = 2\times10^7\Big(-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}\Big),\quad \vec{p}_3 = 2\times10^7\Big(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\Big)$.

The rest mass of each piece is $m_{0j} = \dfrac{2\sqrt{\dfrac{2}{3}}}{5}\ kg,\ \ j = 1, 2, 3.$

The relativistic mass is E_j/c^2.

Here $\gamma_v = \dfrac{1}{\sqrt{1 - v^2/c^2}} = \dfrac{2}{\sqrt{3}}$.

Hence, from $p'_x = \gamma_v\left(p_x - \dfrac{v}{c^2}E\right);\ p'_y = p_y;\ p'_z = p_z;\ E' = \gamma_v\left(E - vp_x\right)$ we have

For mass m_1,

$$p'_{1,x} = \frac{2}{\sqrt{3}}\left(2\times 10^7 - \frac{0.5}{3\times 10^8}\times 3\times 10^{16}\right);\ p'_{1,y} = 0;\ p'_{1,z} = 0;\ E'_1 = \frac{2}{\sqrt{3}}\left(3\times 10^{16} - 0.5\times 3\times 10^8\times 2\times 10^7\right)$$

For mass m_2,

$$p'_{2,x} = \frac{2}{\sqrt{3}}\left(-\frac{1}{2}\times 2\times 10^7 - \frac{0.5}{3\times 10^8}\times 3\times 10^{16}\right);\ p'_{2,y} = \frac{\sqrt{3}}{2}\times 2\times 10^7;\ p'_{2,z} = 0;\ E'_2 = \frac{2}{\sqrt{3}}\left(3\times 10^{16} + \frac{1}{2}\times 0.5\times 3\times 10^8\times 2\times 10^7\right)$$

For mass m_3,

$$p'_{3,x} = \frac{2}{\sqrt{3}}\left(-\frac{1}{2}\times 2\times 10^7 - \frac{0.5}{3\times 10^8}\times 3\times 10^{16}\right);\ p'_{3,y} = -\frac{\sqrt{3}}{2}\times 2\times 10^7;\ p'_{2,z} = 0;\ E'_3 = \frac{2}{\sqrt{3}}\left(3\times 10^{16} + \frac{1}{2}\times 0.5\times 3\times 10^8\times 2\times 10^7\right)$$

Further simplification leads to :

For mass m_1,

$$p'_{1,x} = -2\sqrt{3}\times 10^7\ kg\ m/s;\ p'_{1,y} = 0;\ p'_{1,z} = 0;\ E'_1 = 18\sqrt{3}\times 10^{15}\ J$$

For mass m_2,

$$p'_{2,x} = -4\sqrt{3}\times 10^7\ kg\ m/s;\ p'_{2,y} = \sqrt{3}\times 10^7\ kg\ m/s;\ p'_{2,z} = 0;\ E'_2 = 21\sqrt{3}\times 10^{15}\ J$$

For mass m_3,

$$p'_{3,x} = -4\sqrt{3}\times 10^7\ kg\ m/s;\ p'_{3,y} = -\sqrt{3}\times 10^7;\ p'_{2,z} = 0;\ E'_3 = 21\sqrt{3}\times 10^{15}\ J$$

Define the 4-vector $p' = \sum_{j=1,2,3} \left(\frac{E'_j}{c},\ p'_{j,x},\ p'_{j,y},\ p'_{j,z} \right)$.

Hence, $p' = 10^8(2\sqrt{3}, -\sqrt{3}, 0, 0)$.

The length of the 4-vector is ,

$|p'|^2 = 10^{16}(12 - 3) = 9 \times 10^{16}(kg\ m/s)^2$

PH 101: Physics I

Module 2: Special Theory of Relativity-Basics

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Review of Important formula in STR

$$x' = \frac{1}{\sqrt{1-v^2/c^2}}(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{1}{\sqrt{1-v^2/c^2}}(t - vx/c^2)$$

$$p_x = \frac{1}{\sqrt{1-v^2/c^2}}\left(p'_x + \frac{E'v}{c^2}\right)$$

$$p_y = p'_y$$

$$p_z = p'_z$$

$$E = \frac{1}{\sqrt{1-v^2/c^2}}(E' + vp'_x)$$

$$p \equiv \left(p_0 = \frac{E}{c},\ p_x,\ p_y,\ p_z\right)$$

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} \qquad u'_y = \frac{u_y}{\gamma_v(1 - \frac{u_x v}{c^2})} \qquad u'_z = \frac{u_z}{\gamma_v(1 - \frac{u_x v}{c^2})}$$

Review of Important formula in STR

$$p^2c^2 = \Gamma_u^2 m_0^2 c^4 \left(1 - \frac{1}{\Gamma_u^2}\right) = E^2 - m_0^2 c^4$$

Relativistic Momentum $\vec{p} = \Gamma_u m_0 \vec{u}$

Total energy of an object of mass m moving with speed u: $E = \Gamma_u m_0 c^2$

Kinetic energy of the object: $K.E. = E - m_0 c^2 = (\Gamma_u - 1) m_0 c^2$

Practice Problem Set

As seen from Earth, two spaceships A and B are approaching along perpendicular directions. If A is observed by a stationary Earth observer to have velocity $u_y = -0.90c$ and B to have velocity $u_x = 0.90c$, determine the speed of ship A as measured by the pilot of ship B. **Answer:** $u'_x = -0.9c$**,** $u'_y = -0.39c$**,** $u' = 0.98c$

Solution:

We take the S frame to be attached to the Earth and the S' frame to be attached to spaceship B moving with $\beta = 0.90$ along the x-axis. Spaceship A has velocity components $u_x = 0, u_y = 0.90c$ in S.
$u'_y = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$ give the velocity components of spaceship A in S', from which we have $u'_x = -v = -0.90c, u'_y = u_y/\gamma = -0.39c$ So we have,

$$u' = \sqrt{(u'_x)^2 + (u'_y)^2} = 0.98c$$

Example 1

A free electron is moving with velocity $\vec{u} = \frac{1}{2\sqrt{2}} c(\hat{x} + \hat{y})$ as seen by an observer on earth (frame: S). What is its momentum, total energy and kinetic energy? Mass of electron may be taken to be $m_e = 10^{-30}\ kg$

(a) As seen by the observer in S.
(b) As seen by an observer in S', which is moving with velocity $\vec{v} = 0.2c\ \hat{x}$

Example 1

Solution: $\vec{u}=\frac{1}{2\sqrt{2}}c(\hat{x}+\hat{y})$ $|\vec{u}|=0.5c$ $\gamma_u=\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}=\frac{1}{\sqrt{1-0.5^2}}=1.1547$

$$\vec{p}=\gamma_u m\vec{u}=1.1547\times10^{-30}\times\frac{1}{2\sqrt{2}}c(\hat{x}+\hat{y})=1.2247\times10^{-22}(\hat{x}+\hat{y})\ kg\,m/s$$

$$E=\gamma_u mc^2=1.1547\times10^{-30}\times9\times10^{16}=1.0392\times10^{-13}\ J,\qquad K.E.=(\gamma_u-1)mc^2=0.1392\times10^{-13}\ J$$

As seen by S': $\gamma_v=\left(1-0.2^2\right)^{-\frac{1}{2}}=1.0206$

$$E'=\gamma_v(E-\beta c p_x)=1.0206\left(1.0392\times10^{-13}-0.2\times3\times10^8\times1.2247\times10^{-22}\right)J=0.98566\times10^{-13}\,J$$

$$p_x'=\gamma_v\left(p_x-\beta\frac{E}{c}\right)=1.0206\left(1.2247\times10^{-22}-0.2\times\frac{1.0392\times10^{-13}}{3\times10^8}\right)=0.5429\times10^{-22}\quad kgm/s$$

$$p_y'=p_y=1.2247\times10^{-22}\quad kgm/s,\qquad p'_z=p_z=0$$

$$K.E.=E'-mc^2=0.98566\times10^{-13}-10^{-30}\times9\times10^{-16}=0.08566\times10^{-13}\,J$$

Example 1

Alternatively (in frame S'):

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{(0.3536 - 0.2)c}{1 - 0.3536 \times 0.2} = 0.1652c, \quad u'_y = \frac{u_y}{\gamma_v\left(1 - \frac{u_x v}{c^2}\right)} = \frac{0.3536c}{1.0206 \times 0.9293} = 0.3728c, \quad u'_z = 0$$

$$|\vec{u}'| = \sqrt{0.1652^2 + 0.3728^2}\, c = 0.40775c, \qquad \gamma_{u'} = \frac{1}{\sqrt{1 - \frac{u'^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.40775^2}} = 1.095178$$

$$E' = \gamma_{u'} mc^2 = 1.095178 \times 10^{-30} \times 9 \times 10^{16} = 0.98566 \times 10^{-13}\ J,$$

$$p'_x = \gamma_{u'} mu'_x = 1.095178 \times 10^{-30} \times 0.1652 \times 3 \times 10^{8} = 0.5429 \times 10^{-22} \quad kgm/s,$$

$$p'_y = \gamma_{u'} mu'_y = 1.095178 \times 10^{-30} \times 0.3728 \times 3 \times 10^{8} = 1.2247 \times 10^{-22} \quad kgm/s,$$

Example 2

An object moving with velocity $\vec{u} = 0.2c\,\hat{x}$ has momentum $|\vec{p}| = 3\times10^{-20}\ kg\,m/s$

What is the total energy of the object?

What is the mass of the object?

Solution:

$$|\vec{p}| = p_x = \gamma_u m u_x = \gamma_u m\ 0.2c = 3\times10^{-20} \quad kg\,m/s,$$

$$E = \gamma_u mc^2 = \frac{\vec{p}\cdot\vec{u}}{|\vec{u}|^2}c^2 = \frac{3\times10^{-20}}{0.2}c = 0.45\times10^{-10} \quad J,$$

$$\gamma_u = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{1}{\sqrt{1-0.2^2}} = 1.0206$$

$$E = \gamma_u mc^2 \quad \Rightarrow \quad m = \frac{E}{\gamma_u c^2} = \frac{0.45\times10^{-10}}{1.0206\times9\times10^{16}} = 0.4899\times10^{-27}\ kg$$

Example 3

A body of rest mass m_0 moving at speed u in the x-direction collides with and sticks to an identical body at rest. What would be the mass and momentum of the final clump?

Solution:

The energy-momentum vector of the moving mass is $\mathbf{P_1} = (\Gamma_u m_0 c \equiv E_1/c,\ \gamma m_0 u,\ 0,\ 0)$, while for the mass at rest it is $\mathbf{P_2} = (m_0 c,\ 0,\ ,0\ ,0)$, where, $\Gamma_u = \dfrac{1}{\sqrt{1 - u^2/c^2}}$.

Therefore, the total energy-momentum vector of the final clump is $\mathbf{P} = \mathbf{P_1} + \mathbf{P_2}$.
Therefore, the momentum of the clump is $\gamma m_0 v$.

For the mass we can write ;

$$P^2 = M^2c^2 = (P_1 + P_2)^2 = P_1^2 + 2P_1.P_2 + P_2^2 = m_0^2c^2 + 2\gamma m_0^2c^2 + m_0^2c^2 = 2m_0^2c^2(1+\gamma),$$

$$\Rightarrow M = m_0\sqrt{2(1+\gamma)}.$$

Tutorial-7 (Q3)

A spaceship moves away from Earth with speed v and fires a shuttle craft in the forward direction at a speed v relative to the spaceship. The pilot of the shuttle craft launches a probe in the forward direction at speed v relative to the shuttle craft.

(i)Determine the speed of the shuttle craft relative to the Earth.
(ii) Determine the speed of the probe relative to the Earth.

Tutorial-7 (Q3)

Solution:

We consider the S frame to be attached to the Earth and the S' frame to be attached to the spaceship moving with v along the $x-$axis. The shuttle craft has speed $u'_x = v$. Therefore,

$$u_x = \frac{u'_x + v}{1 + u'_x v/c^2}. \tag{1}$$

Gives the speed $u_x = \frac{2v}{1+\beta^2}$, where $\beta = v/c$.

Now consider S' frame is attached to the shuttle craft moving with speed,

$v' = \frac{2v}{1+\beta^2}$ with respect to the S-frame (Earth)

along the $x-$ axis. The probe has a speed $u'_x = v$ in S'. Its speed u_x in the S frame would be given by Eq. (1) with v replaced by v' from which we get

$$u_x = \left(\frac{3+\beta^2}{1+3\beta^2} v\right) \tag{2}$$

It follows from Eq.(2) that $u_x \to 3v$ when $\beta \ll 1$ and $u_x \to c$ when $\beta \to 1$

Tutorial-6 (Q5)

An observer in frame S who lives on the x-axis sees a flash of red light at $x = 1210m$. After $4.96\mu s$, he sees flash of blue light at $x = 480m$. Use subscripts R and B to label the coordinates of the events related to the red and blue light respectively.
(i) Now suppose there is an observer in S' which is moving with a velocity 'v' with respect to the S frame watches these events. Compute the velocity v for the situation when the observer in S' records both the events occurring at the same place?
(ii) Which event occurs first according to S' and what is the measured time interval between these flashes?

Tutorial-6 (Q5)

Solution:

(i)As the observer from S' finds $x'_R = x'_B$, where x'_R and x'_B are the x-coordinate of the R and B events, respectively.

Therefore we have, $x_R - vt_R = x_B - vt_B$

$$\Rightarrow v = \frac{x_R - x_B}{t_R - t_B} = \frac{1210m - 480m}{0 - 4.96 \times 10^{-6}s} = -1.47 \times 10^8 m/s$$

(ii) Note that the order of the events must be the same in every frame for light-like event, i.e. $\Delta x = c\Delta t$. This can be explicitly shown as follows.

$$(t'_R - t'_B) = \gamma\left[(t_R - t_B) - \frac{v}{c^2}(x_R - x_B)\right]$$

$$= \gamma\left[(t_R - t_B) - \frac{v}{c^2}c(t_R - t_B)\right] = \gamma(t_R - t_B)\left[1 - \frac{v}{c}\right]$$

Since $v \leq c$, $t'_R - t'_B$ and $t_R - t_B$ will have same sign.

Therefore the observer in S' sees event R happen before event B. To find the time interval in S' , we can use the invariant spacetime interval and use the fact that the events occur at the same place in S' :

$$(c\Delta t')^2 - (\Delta x')^2 = (c\Delta t')^2 = (c\Delta t)^2 - (\Delta x)^2$$

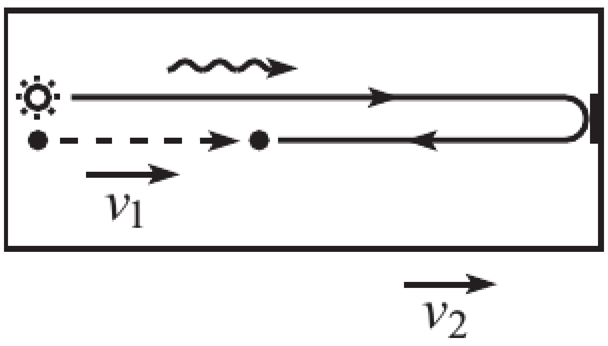
Therefore,

$$\Delta t' = \sqrt{(\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2} = \sqrt{(4.96 \times 10^6 s)^2 - \left(\frac{1210\ m\ -\ 480\ m}{3 \times 10^8\ m/s}\right)^2} = 4.32\mu s.$$

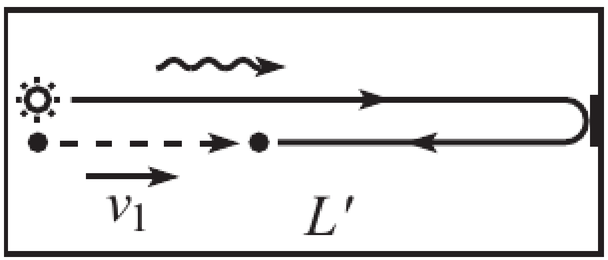
Example 2

A ball moves at speed v_1 with respect to a train. The train moves at speed v_2 with respect to the ground. What is the speed of the ball with respect to the ground?

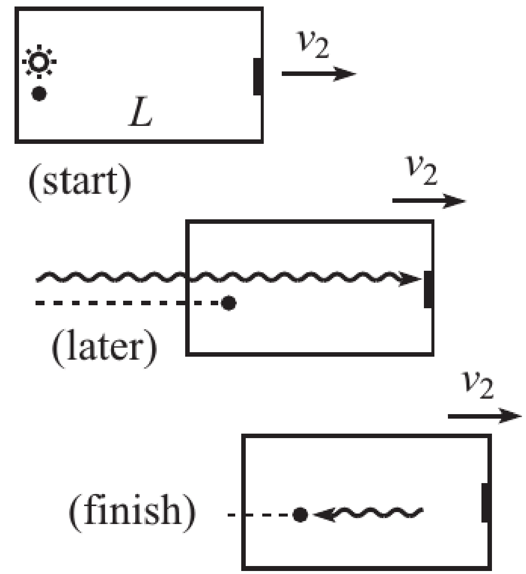
Let the ball be thrown from the back of the train. At the same instant, a photon is released next to it (as shown in the figure). The photon heads to the front of the train, bounces off a mirror, heads back, and eventually runs into the ball. In both the frame of the train and the frame of the ground, calculate the fraction of the way along the train where the meeting occurs, and then equate these fractions.



Example 2



(frame S')



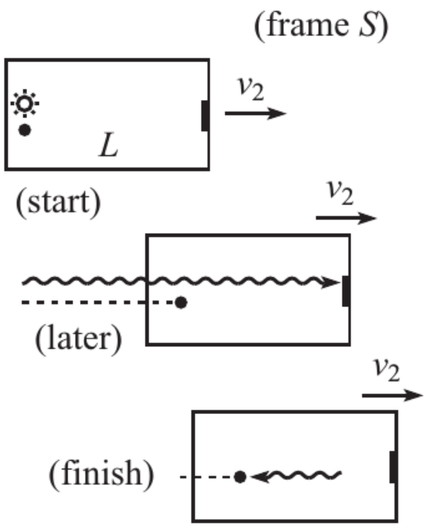
(frame S)

TRAIN FRAME: Let the train have length L' in the train frame. Let's first find the time at which the photon meets the ball (see Figure ’). From the figure, we see that the sum of the distances traveled by the ball and the photon, which is $v_1 t' + ct'$, must equal twice the length of the train, which is $2L'$. The time of the meeting is therefore $t' = \dfrac{2L'}{c + v_1}$.

The distance the ball has traveled is then $v_1 t' = 2v_1 L'/(c + v_1)$, and the desired fraction F' is

$$\boxed{F' = \frac{2v_1}{c + v_1}}$$

Example 2



GROUND FRAME: Let the speed of the ball with respect to the ground be v, and let the train have length L in the ground frame (L equals L'/γ, but we're not going to use this). Again, let's first find the time at which the photon meets the ball Light takes a time $L/(c-v_2)$ to reach the mirror, because the mirror is receding at speed v_2. At this time, the light has traveled a distance $cL/(c-v_2)$. From the figure, we see that we can use the same reasoning as in the train-frame case, but now with the sum of the distances traveled by the ball and the photon, which is $vt + ct$, equalling $2cL/(c-v_2)$. The time of the meeting is therefore

$$\boxed{t = \frac{2cL}{(c-v_2)(c+v)}\,.}$$

The relative speed of the ball and the back of the train (as viewed in the ground frame) is $v - v_2$, so the distance between the ball and the back of the train at this time is $2(v - v_2)cL/[(c-v_2)(c+v)]$. The desired fraction F is therefore

$$\boxed{F = \frac{2(v-v_2)c}{(c-v_2)(c+v)}}$$

We shall continue in the next class