MA 101 (Mathematics - I)

Quiz - IV (Answer Key)

Maximum Marks : 22

Date: February 27, 2021 **Time:** 11 am - 12 pm

Instructions:

- The answers of this Quiz question paper are to be filled in the Quiz - IV response form. You get exactly one hour time (from 11 am to 12 pm) for doing this.
- You should submit the response form at 12 pm (or before). Although you get extra 5 minutes for submission only (the portal will close at 12:05 pm), it is advised not to take any risk of submitting after 12 pm. No request will be entertained if you fail to submit your responses through the portal due to any reason.
- The response form permits only one submission. It does not allow revision.

Type of Questions and Marking scheme:

- The first question is writing your Roll number. It is compulsory. It has no marks.
- Q.2 to Q.7 are of single correct option type questions, where exactly one option is correct. Each of these questions carries 2 marks for correct answer, -1 mark for incorrect answer, and 0 mark for no answer. No answer will be considered if you do not mark any option or if you mark option (E) given in the response form.
- Q.8 to Q.12 are of multiple correct option type questions, where one or more of the options is (are) correct. In each of these questions, you get 2 marks if you choose all the correct options and choose no wrong option. In all other cases, you get 0 mark.

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1. Write your Roll number.

2. Let S_1 and S_2 be two disjoint bounded smooth surfaces in $\mathbb{R}^3$ such that $S_1 \cup S_2$ is a closed orientable surface which enclosed the solid D in $\mathbb{R}^3$ of volume 10. Let $\vec{F}(x, y, x) = (x, y, -2z)$. If $\iint\limits_{S_1} \vec{F} \cdot \hat{n}_1 d\sigma_1 = -5$, then

   (A) $\iint\limits_{S_2} \vec{F} \cdot \hat{n}_2 d\sigma_2 = -5$ (B) $\iint\limits_{S_2} \vec{F} \cdot \hat{n}_2 d\sigma_2 = 5$

   (C) $\iint\limits_{S_1 \cup S_2} \vec{F} \cdot \hat{n} d\sigma = 10$ (D) $\iint\limits_{S_2} \vec{F} \cdot \hat{n}_2 d\sigma_2$ need not exist.

   Answer: (B)

3. Let C be a closed simple and piecewise smooth curve in $\mathbb{R}^2$. Consider a circle C_1 with center at the origin such that C_1 lies in the interior of the domain D enclosed by C. Let $F = (g, h)$ be a continuous vector field on $\mathbb{R}^2$ such that $g_y = h_x$ on D except origin. If F is satisfying $F(R(t)) \cdot R'(t) = 100$ for each point $R(t)$ on C_1, then $\oint\limits_C F \cdot dR$ is equal to

   (A) 200π (B) 100π (C) 50π (D) -100π

   Answer: (A)

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $I = \int_0^x \int_0^y \int_0^z f(t)dtdzdy$ and $\alpha = \int_0^x (x-t)^2 f(t)dt$. Then I is equal to

(A) α (B) 2α (C) -3α (D) $0.5\,\alpha$

Answer: (D)

5. Let $\ln r$ denote natural logarithm of r. The value of the double integral $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dydx$ is equal to

(A) $\frac{21}{2}\ln 2$ (B) $\frac{23}{2}\ln 3$ (C) $\frac{19}{3}\ln 2$ (D) $\frac{25}{2}\ln 2$

Answer: (A)

6. Let $S = \{\left(x, \sin\frac{1}{x}\right) : x \in (0,1]\} \times [0, 0.001]$. Then which of the following statements is true ?

(A) S is a set of content zero in $\mathbb{R}^2$

(B) S cannot be a set of content zero in $\mathbb{R}^2$

(C) S is an open subset of $\mathbb{R}^2$

(D) S is a closed subset of $\mathbb{R}^2$

Answer: (A) if typo $\mathbb{R}^2$ will be corrected to $\mathbb{R}^3$. Hence Q6 has been withdrawn.

7. Consider the following two statements **P** and **Q**.

P : Suppose $\int_a^b f(x)dx = \oint_C F \cdot dR$ if the curve C is oriented counterclockwise and $-\int_a^b f(x)dx = \oint_C F \cdot dR$ if the curve C is oriented clockwise.

Q : Outward unit normal $\hat{n}$ to the surface $z = \sqrt{x^2 + y^2}$ is continuous at each point of the surface.

Then

(A) both **P** and **Q** are true (B) **P** is true but **Q** is false

(C) **Q** is true but **P** is false (D) both **P** and **Q** are false

Answer: (B)

8. Let f and g be continuous functions on a closed and bounded domain D in $\mathbb{R}^2$. Suppose $f \neq g$ on a set of content zero in D. Then which of the following statements is (are) true ?

(A) f agree to g on D

(B) f need not be agree to g on D

(C) If $\iint_T f(x,y)dxdy = 0$ for each triangular disc in D, then $f = 0$

(D) Even if condition in (C) holds, f need not be identically zero

Answer: (A), (C)

9. Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be vector field whose second derivative F'' is continuous. Then
   (A) there exists $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $F = \frac{\nabla\phi}{\|\nabla\phi\|}$
   (B) the statement (A) is not necessarily true for every F with F'' is continuous
   (C) there exists a unique $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $F = \frac{\nabla\phi}{\|\nabla\phi\|}$
   (D) there cannot exist $\phi : \mathbb{R}^3 \to \mathbb{R}$ such that $F = \frac{\nabla\phi}{\|\nabla\phi\|}$ unless $\nabla \times (\|\nabla\phi\| F) = 0$

   Answer: (B), (D)

10. Let f be a continuous function on a bounded solid D in $\mathbb{R}^3$. Suppose the surface S is given by $\{(x, y, z) \in D : f(x, y, z) = 0\}$. Then which of the following statements is (are) true ?

   (A) S is orientable if f' is continuous and $\|\nabla f\| > 0$ on D
   (B) S is orientable even if f' is continuous and $\nabla f \neq 0$ except on a finite set in D
   (C) S is orientable even if f' is continuous and the partial derivative $f_x \neq 0$ on D
   (D) none of the above is true

   Answer: (A), (C)

11. Let $f : D = [0, 1] \times [01] \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0, 1], \\ y & \text{if } x \in \mathbb{Q}^c \cap [0, 1]. \end{cases}$$

   Then which of the following statements is (are) true ?

   (A) $\inf\limits_{P} L(P, f) = 0.5$ and $\sup\limits_{P} L(P, f) = 1$
   (B) $\inf\limits_{P} L(P, f)$ does not exist and $\sup\limits_{P} L(P, f) = 1$
   (C) f is not Riemann integrable on D
   (D) f is discontinuous on a set of content zero in D

   Answer: (A), (C)

12. Let $f : [0, \pi] \to \mathbb{R}$ be defined by

$$f(y) = \begin{cases} \frac{\sin y}{\sqrt{y}} & \text{if } y \in (0, \pi], \\ 0 & \text{if } y = 0. \end{cases}$$

   Denote $J = \int\limits_{0}^{\pi} \int\limits_{x^2}^{\pi} f(y)dydx$. Then which of the following statements is (are) true ?

   (A) $J = 2$
   (B) f is bounded and continuous on $[0, \pi]$
   (C) f is bounded but not continuous on $[0, \pi]$
   (D) f is bounded and uniformly continuous on $[0, \pi]$

   Answer: (A), (B), (D)

——— **END** ———