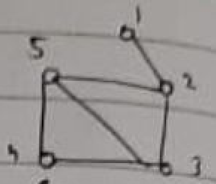
MA-221

GRAPHS

$G = (V, E)$, $E = \{ \{x, y\} \mid x, y \in V \}$.



$V = \{1, 2, 3, 4, 5\}$.

→ Incidence function

$E = \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{\ldots$
$\{5,3\}\}$.

$\phi : E \to V \times V$

(Edges from a vertex).

Degree of vertex → # edges incident on it.

$d(1) = 1 \quad d(2) = 3 \quad d(3) = 3 \quad d(4) = 2 \quad d(5) = 3$.

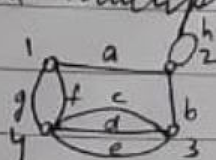
$\boxed{\sum_{v \in V} d(v) = 2|E|.}$ → Handshake lemma.

Proof 1:- Every edge contributes 2 to the degree sum.

Proof 2:- $S = \{(v, e) \mid v \in V, e \in E\}$.

$|S| = \boxed{\sum_{v \in V} d(v) = \sum_{e \in E} 2 = 2|E|.}$

Multigraphs:- Simple graph with multiple edges and loops.



Adjacency matrix

$n \times n$ matrix, $A = (a_{ij})$

$n = |V|$

no edge-repitition walk $\rightarrow$ trail.

no edge-and-vertex-repitition walk $\rightarrow$ path.

cycle $\rightarrow$ closed path.

no odd cycle

Bipartite graphs :- $G(V, E)$

$V = V_1 \cup V_2, \quad V_1 \cap V_2 = \phi$.

There are no edges between any two vertices in the same partition.

$K_{m,n} \rightarrow$ complete bipartite graph.

Isomorphic graphs

$G \cong H$

if $\exists$a bijection $\quad \nu : V(G) \longrightarrow V(H)$

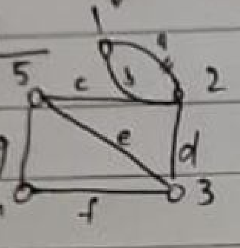
$\epsilon : E(G) \longrightarrow E(H)$.

undirected: Every edge $e = \{x, y\}$ in G has $\epsilon(e) = \{\nu(x), \nu(y)\}$ in H

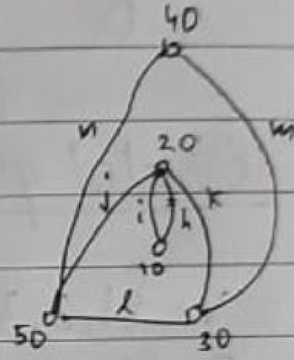
directed: Every edge $e = (x, y)$ in G has $\epsilon(e) = (\nu(x), \nu(y))$ in H

no ... cycle

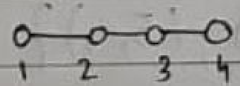
eg:- G



H

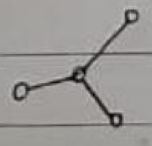


Path graph P_4.



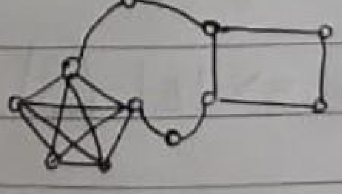
$\# = n!/2$

Star graph S_4



$\# = n$

Sub graphs

$G =$ 

K_5

$V(K_5) \subseteq V(G)$

$E(K_5) \subseteq E(G)$

Walk:-

$e_1, e_2, e_3, \ldots, e_k$.

$(u,v), (v,w) \ldots$

$$a_{ij} = \begin{cases} 1 & , \text{ if } \{i,j\} \in E \\ 0 & \text{o.w.} \end{cases}$$

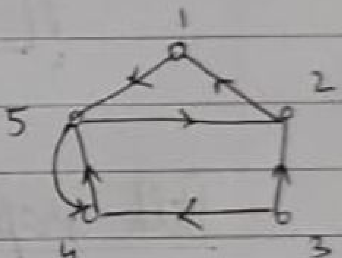
$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 2 \\ 2 & 1 & 2 & 1 & 0 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 2 & 0 & 1 & 0 \end{array}$$ → For hand shaking lemma

Incidence Matrix

$|V| \times |E|$ matrix , $B = (b_{ij})$.

$$b_{ij} = \begin{cases} 1 & \text{edge } j \text{ has one end at } i. \\ 0 & \text{o.w.} \end{cases}$$

Directed graph



$V = \{1, 2, 3, 4, 5\}$

$E = \{(2,1), (1,5), (3,2), (3,4), (4,5), (5,2), (5,4)\}$

In-degree →

Out-degree →

(K) Complete graph → Edge between all pairs of vertices.

$|V| = n \Rightarrow |E| = \binom{n}{2}$.

simple graphs on n vertices $= 2^{\binom{n}{2}}$

simple directed graph on n vertices $= 4^{\binom{n}{2}} = 2^{2\binom{n}{2}} = 2^{n(n-1)}$