

Statistical Inference and Multivariate Analysis (MA324)

LECTURE SLIDES Lecture 07

BVN and Sampling Distribution



Indian Institute of Technology Guwahati

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Theorem: Let $\mathbf{X} \sim N_2(\boldsymbol{\mu}, \Sigma)$ be such that Σ is invertible, then

- 1 for all $y \in \mathbb{R}$, the conditional PDF of X given $Y = y$ is given by

$$f_{X|Y}(x|y) = \frac{1}{\sigma_{x|y}\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - \mu_{x|y}}{\sigma_{x|y}} \right)^2 \right] \quad \text{for } x \in \mathbb{R},$$

where $\mu_{x|y} = \mu_x + \rho \frac{\sigma_x}{\sigma_y}(y - \mu_y)$ and $\sigma_{x|y} = \sigma_x^2(1 - \rho^2)$.

- 2 $E(X|Y = y) = \mu_{x|y} = \mu_x + \rho \frac{\sigma_x}{\sigma_y}(y - \mu_y)$ for all $y \in \mathbb{R}$.

Theorem: Let X_1, X_2, \dots, X_n be i.i.d. $N(0, 1)$ random variables. Then $\sum_{i=1}^n X_i^2 \sim \text{Gamma}(n/2, 1/2) \equiv \chi_n^2$.

Theorem: Let X_1, X_2, \dots, X_n be i.i.d. $N(\mu, \sigma^2)$ random variables. Then $\bar{X} \sim N(\mu, \sigma^2/n)$, $\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$, and \bar{X} and S^2 are independently distributed. Here $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ and $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$.