MA 324 | Mid-semester Examination
Statistical Inference and Multivariate Analysis | 14:00–16:00 IST
IIT GUWAHATI | March 01, 2022

Instructions

1. Attempt **all** the questions.
2. There is **no credit** for a solution if the appropriate work is not shown, even if the answer is correct.
3. Notations are standard and same as used during the lectures.
4. No question requires any clarification from the instructor. Even if a question has an error or incomplete data, the students are advised to write answer according to their understanding or write reasons for why it is not possible to solve partially or completely that question by citing errors/insufficient data.
5. The question paper has **1** page. This examination has **5** questions, for a total of **30** points.

Questions

1. (2 points) Let $X_1,\, X_2 \overset{i.i.d.}{\sim} Poi(\lambda)$, where $\lambda > 0$ is unknown parameter. Is the family of distributions induced by the statistic $\boldsymbol{T} = (X_1,\, X_2)$ complete?

2. (5 points) Let $X_1,\, X_2,\, \ldots,\, X_9$ be a random sample of size 9 form population having $U(\theta_1,\, \theta_2)$ distribution, where both θ_1 and θ_2 are unknown and $-\infty < \theta_1 < \theta_2 < \infty$. Derive the estimators of θ_1 and θ_2 using method of moment.

3. Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample of size $n\,(\geq 2)$ from a population having probability density function
$$f(x,\, \theta) = \begin{cases} \frac{2}{\theta} x \exp\left[-\frac{x^2}{\theta}\right] & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$
where $\theta > 0$ is a unknown parameter. Consider the problem of estimation of $\tau(\theta) = \dfrac{1}{\sqrt{\theta}}$.
   (a) (5 points) Derive minimum variance unbiased estimator of $\tau(\theta)$.
   (b) (3 points) Show that the estimator that you obtained in (a) is consistent. You may use Stirling's approximation for $\Gamma(n)$: $\Gamma(n) \sim \sqrt{2\pi}\,(n-1)^{n-\frac{1}{2}}\, e^{-n+1}$.
   (c) (3 points) Compute Cramer-Rao lower bound of an unbiased estimator of $\tau(\theta)$.

4. (5 points) Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample from a Bernoulli distribution with success probability $p = \frac{1}{1+e^{\theta}}$, where $\theta \in \mathbb{R}$. Find the maximum likelihood estimator of θ. [Hint: Investigate the existence and non-existence of maximum likelihood estimator.]

5. (7 points) Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample of size $n\,(\geq 2)$ from a population with probability density function
$$f(x;\, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$
where $\theta > 0$ is unknown. With preassigned $\alpha \in (0,\, 1)$, derive a level α likelihood ratio test for $H_0 : \theta = \theta_0 (> 0)$ against $H_1 : \theta \neq \theta_0$.