MA 374 Financial Engineering – II
Lab Assignment – 3

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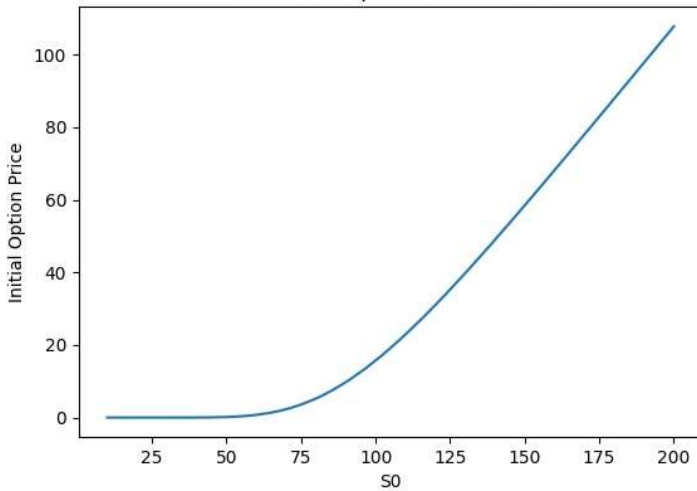## Q1)

The Initial American Option Prices are:
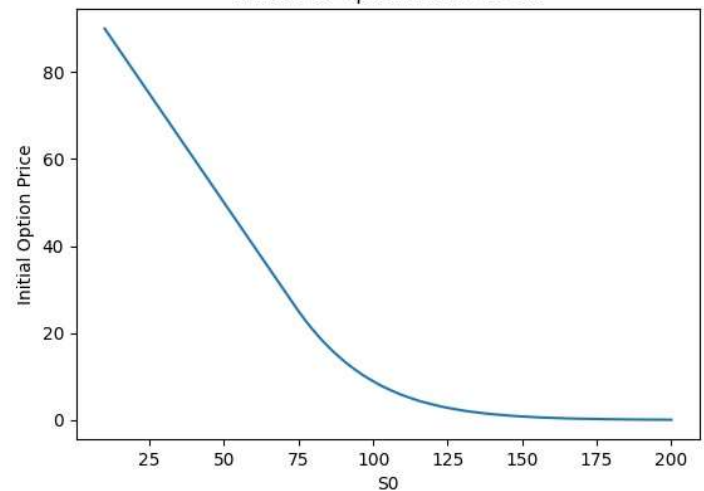- American Call Option: 15.7368
- American Put Option: 8.9231
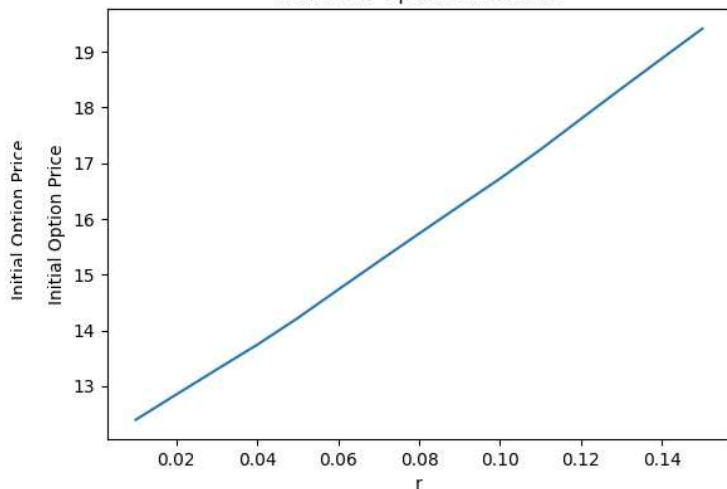
Sensitivity Analysis: i)
Varying S(0):

ii) Varying K:

iii) Varying r:iv) Varying σ:

v) Varying M:

Initial Call Option Prices vs M

K = 95
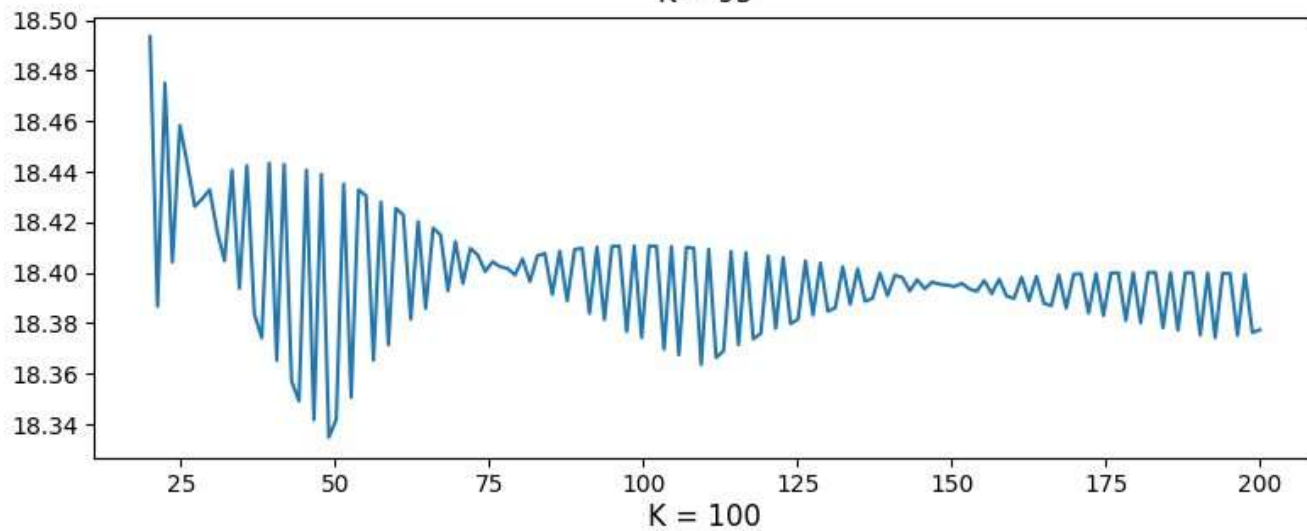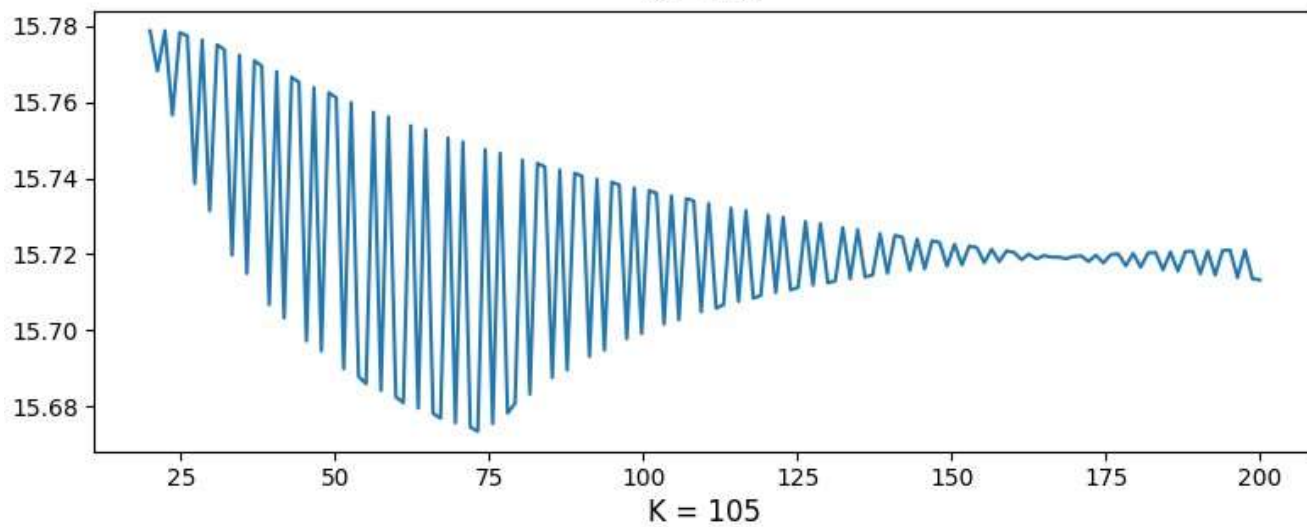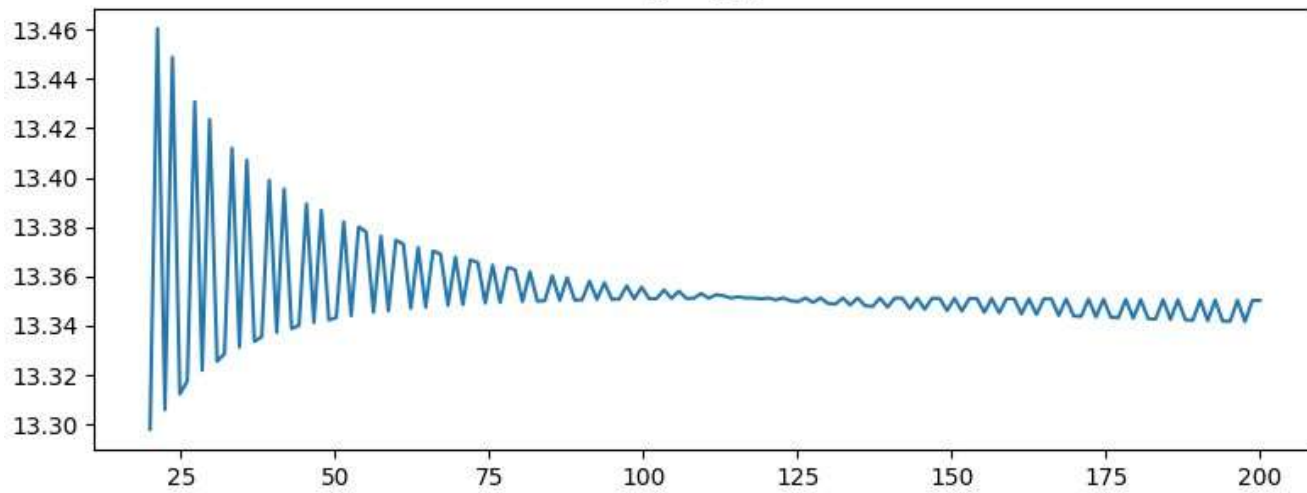
K = 100

K = 105

Initial Put Option Prices vs M
K = 95
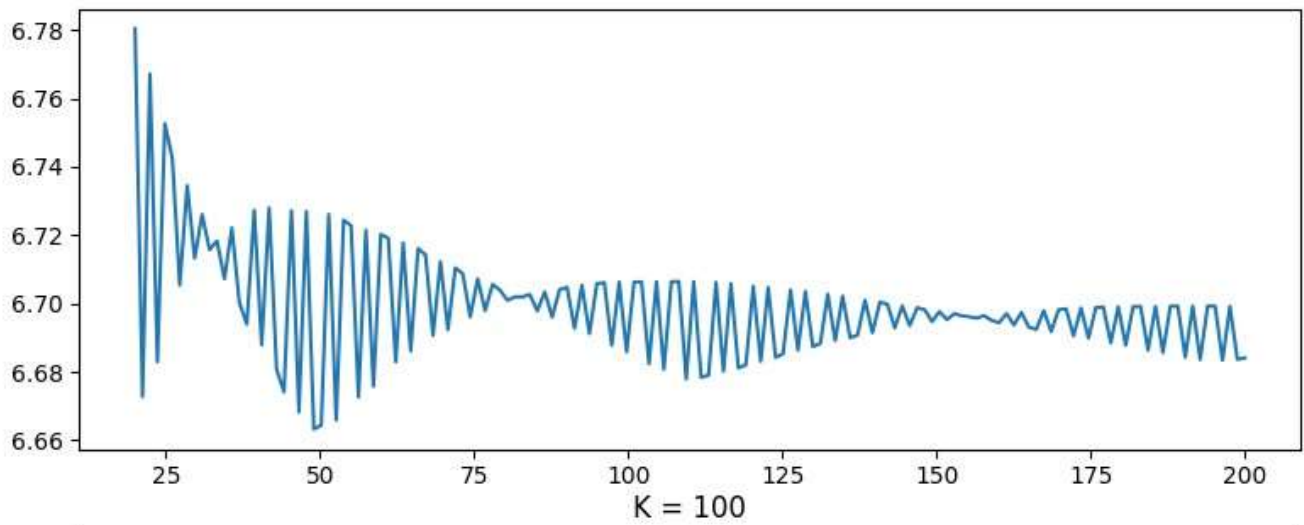
K = 100
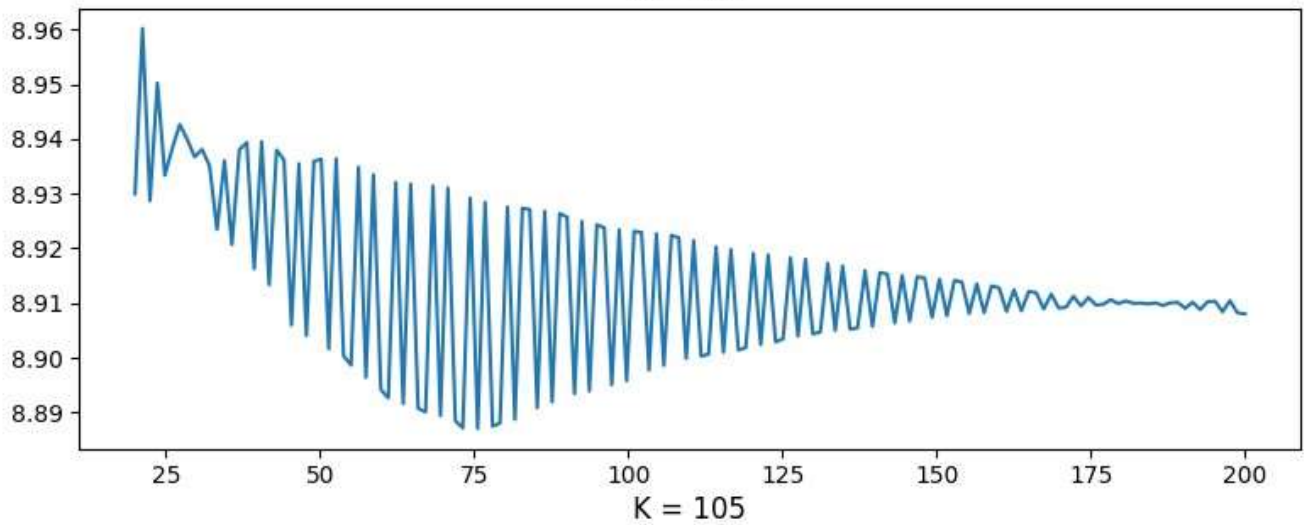
K = 105
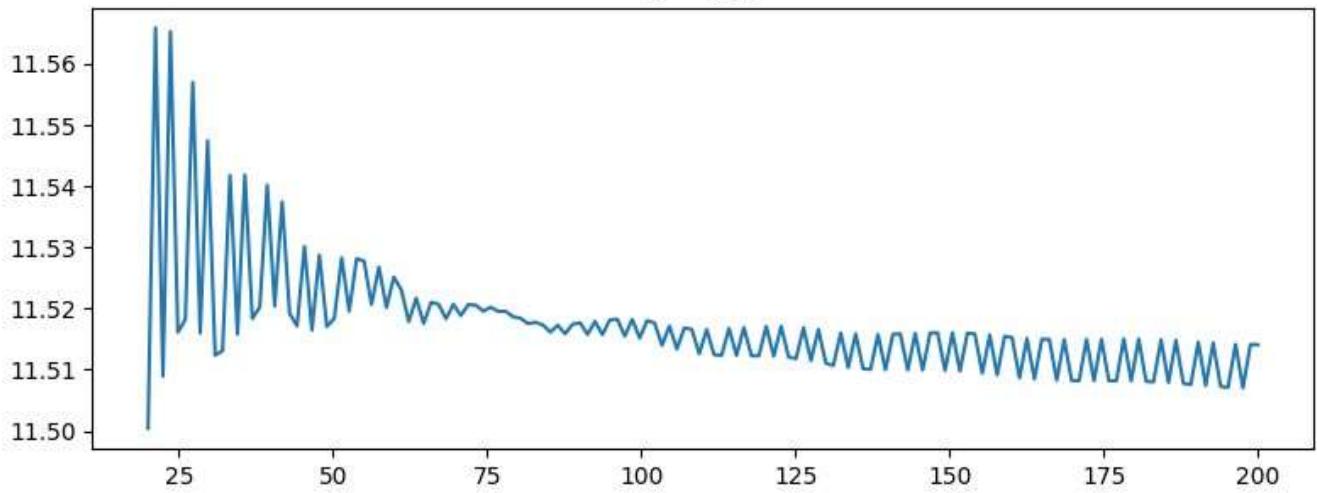
Q2)

(a) The Initial Option Prices for Lookback Option are:

Si.No.	M	Initial Option Price	Execution Time(s)
1.	5	15.372952	0.000305
2.	10	16.950340	0.010537
3.	25	18.533782	347.2713721
4.	50	Not feasible	Not feasible

For M = 50, it is unfeasible to calculate the Lookback Initial Option price since the algorithm has an exponential time complexity.

(b)
Plots comparing M vs Initial Option Price:

(c)
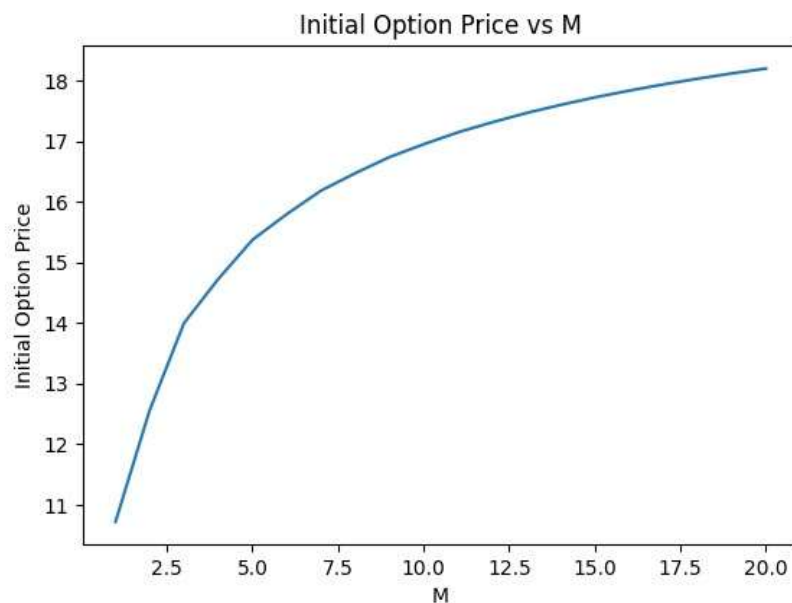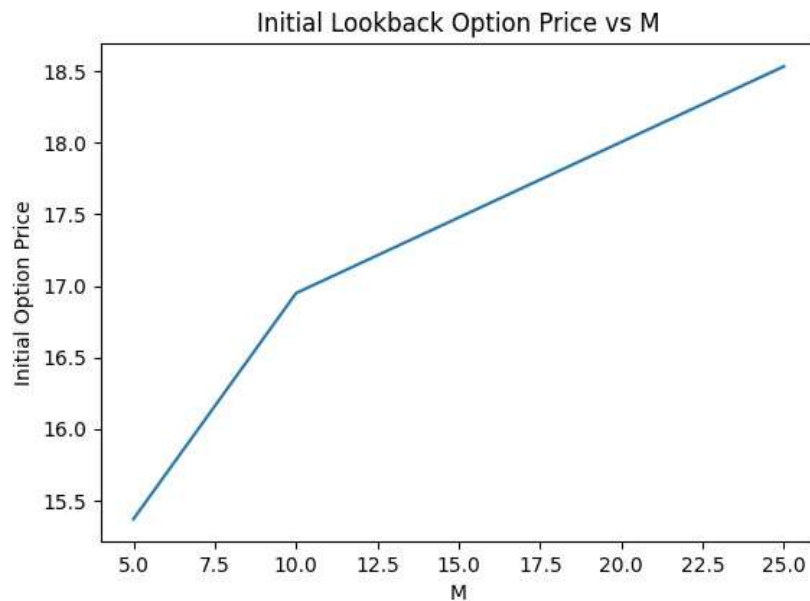
Values of the option at all intermediate time points for M = 5:

	t=0	t=1	t=2	t=3	t=4	t=5
1.	15.373	15.53	15.1998	13.3862	10.3325	0.0
2.		15.7097	16.3658	17.5045	16.873	21.0025
3.			11.6226	10.2358	7.9008	0.0
4.			20.3053	23.0262	27.6768	34.2971
5.				10.2358	7.9008	0.0
6.				13.3849	12.902	16.0597
7.				12.7023	12.1033	14.1899
8.				28.5665	34.6965	42.062
9.					7.9008	0.0
10.					12.902	16.0597
11.					6.0414	0.0
12.					21.1632	26.2255
13.					6.0414	0.0
14.					19.7758	24.6019
15.					19.7758	24.6019
16.					38.2824	45.9145
17.						0.0
18.						16.0597
19.						0.0
20.						26.2255
21.						0.0
22.						12.2802
23.						10.8504
24.						32.163
25.						0.0
26.						12.2802
27.						9.4406

28.	30.7531
29.	9.4406
30.	30.7531
31.	30.7531
32.	47.0499

Q3)

Using Markov property, the algorithm for finding the option price at t = 0 is given by:

$$v_n(s,m)=R^{-1}[\ p.v_{n+1}(s.u,max(s.u,m))+(1+p).v_{n+1}(s.d,m)]$$

(a)

Initial Option Prices for Lookback Option:

Si.No.	M	Initial Option Price	Execution Time(s)
1.	5	15.372952	0.000178
2.	10	16.950340	0.001149
3.	25	18.533782	0.08640
4.	50	19.390465	5.71798

(b)

Plot comparing M vs Initial Option Price:

Initial Lookback Option Price vs M
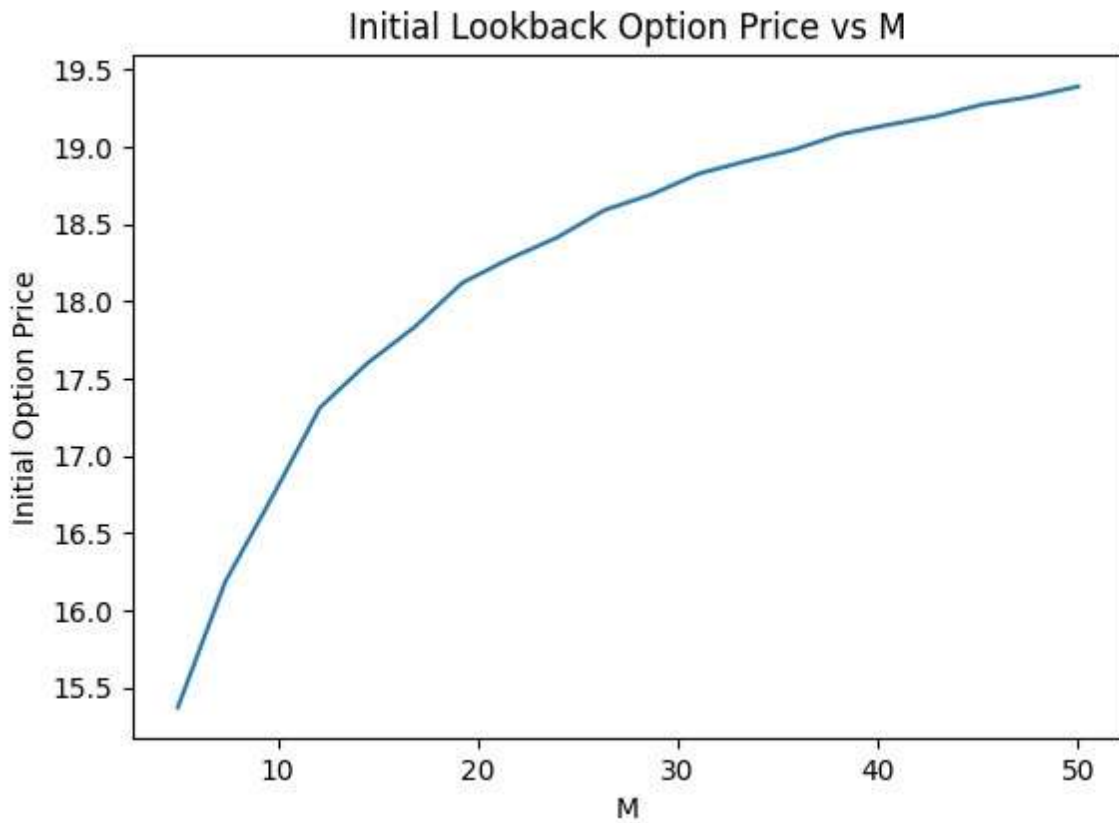
The prices increase as M increases initially and then they tend to converge.

(c)

Values of the option at all intermediate time points for M = 5 (here each state is characterized by a tuple denoting – (stock price at that instant, maximum stock price among all states in the current path):

At_timestamp:_0

Index	Price
Index:(100, 100)	Price:15.373

At_timestamp:_1

Index	Price
Index:(115.16135876866093, 115.16135876866093)	Price:15.5321
Index:(88.05891748599798, 100)	Price:15.7097

At_timestamp:_2

Index	Price
Index:(132.6213855344424, 132.6213855344424)	Price:15.1998
Index:(101.40984589384922, 115.16135876866093)	Price:16.3658
Index:(101.40984589384924, 101.40984589384924)	Price:11.6226
Index:(77.543729488058, 100)	Price:20.3053

At_timestamp:_3

Index	Price
Index:(152.7285895992882, 152.7285895992882)	Price:13.3862
Index:(116.78495645656189, 132.6213855344424)	Price:17.5045
Index:(116.78495645656187, 116.78495645656187)	Price:10.2358
Index:(89.3004125183424, 115.16135876866093)	Price:23.0262
Index:(116.78495645656189, 116.78495645656189)	Price:10.2358
Index:(89.30041251834241, 101.40984589384924)	Price:13.3849
Index:(89.3004125183424, 100)	Price:12.7023
Index:(68.28416876545448, 100)	Price:28.5665

Index	Price
Index:(175.88431901075205, 175.88431901075205)	Price:10.3325
Index:(134.4911426927657, 152.7285895992882)	Price:16.873
Index:(134.4911426927657, 134.4911426927657)	Price:7.9008
Index:(102.8395684421425, 132.6213855344424)	Price:27.6768
Index:(134.49114269276566, 134.49114269276566)	Price:7.9008
Index:(102.8395684421425, 116.78495645656187)	Price:12.902
Index:(102.8395684421425, 115.16135876866093)	Price:12.1033
Index:(78.63697657418294, 115.16135876866093)	Price:34.6965
Index:(102.8395684421425, 116.78495645656189)	Price:12.902
Index:(102.83956844214251, 102.83956844214251)	Price:6.0414
Index:(78.63697657418295, 101.40984589384924)	Price:21.1632
Index:(102.8395684421425, 102.8395684421425)	Price:6.0414
Index:(78.63697657418294, 100)	Price:19.7758
Index:(78.63697657418295, 100)	Price:19.7758
Index:(60.130299829171165, 100)	Price:38.2824

Index	Price
Index:(202.5507716337883, 202.5507716337883)	Price:0.0
Index:(154.8818273484876, 175.88431901075205)	Price:21.0025
Index:(154.8818273484876, 154.8818273484876)	Price:0.0
Index:(118.43144436979834, 152.7285895992882)	Price:34.2971
Index:(118.43144436979834, 134.4911426927657)	Price:16.0597
Index:(118.43144436979833, 132.6213855344424)	Price:14.1899
Index:(90.55941071742268, 132.6213855344424)	Price:42.062
Index:(154.88182734848758, 154.88182734848758)	Price:0.0
Index:(118.43144436979831, 134.49114269276566)	Price:16.0597
Index:(118.43144436979833, 118.43144436979833)	Price:0.0
Index:(90.55941071742268, 116.78495645656187)	Price:26.2255
Index:(90.55941071742268, 115.16135876866093)	Price:24.6019
Index:(90.55941071742267, 115.16135876866093)	Price:24.6019
Index:(69.24687031494331, 115.16135876866093)	Price:45.9145
Index:(90.55941071742268, 116.78495645656189)	Price:26.2255
Index:(118.43144436979834, 118.43144436979834)	Price:0.0
Index:(90.5594107174227, 102.83956844214251)	Price:12.2802
Index:(90.5594107174227, 101.40984589384924)	Price:10.8504
Index:(69.24687031494332, 101.40984589384924)	Price:32.163
Index:(90.55941071742268, 102.8395684421425)	Price:12.2802
Index:(90.55941071742267, 100)	Price:9.4406
Index:(69.24687031494331, 100)	Price:30.7531
Index:(90.5594107174227, 100)	Price:9.4406
Index:(69.24687031494332, 100)	Price:30.7531
Index:(52.95009111065302, 100)	Price:47.0499

Comparative Analysis:

- Table comparing execution times(s) of the algorithms:

M	Unoptimized Algorithm	Markov Based Algorithm
5	0.000305	0.000178
10	0.010537	0.001149
25	347.2713721	0.08640
50	Infeasible	5.71798

- Limits on M:
  - Unoptimized Algorithm: this can handle values of M up-to around 25 to 30. The execution time doubles as M increases by 1. At higher values of M the algorithm incurs a RAM limit and is killed.
  - Markov Based Algorithm: this can handle higher values of M up-to 90. This is because unlike the previous algorithm it does not have exponential space complexity and uses concept of memoization based on Dynamic Programming.
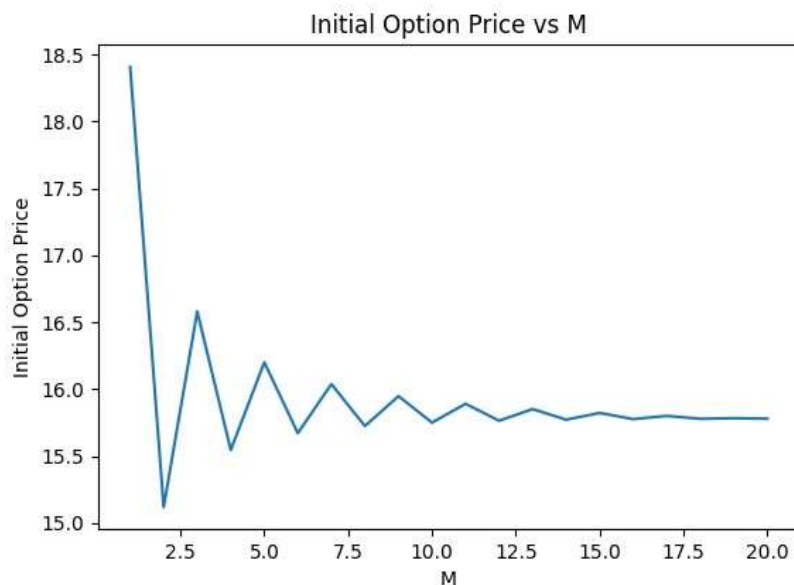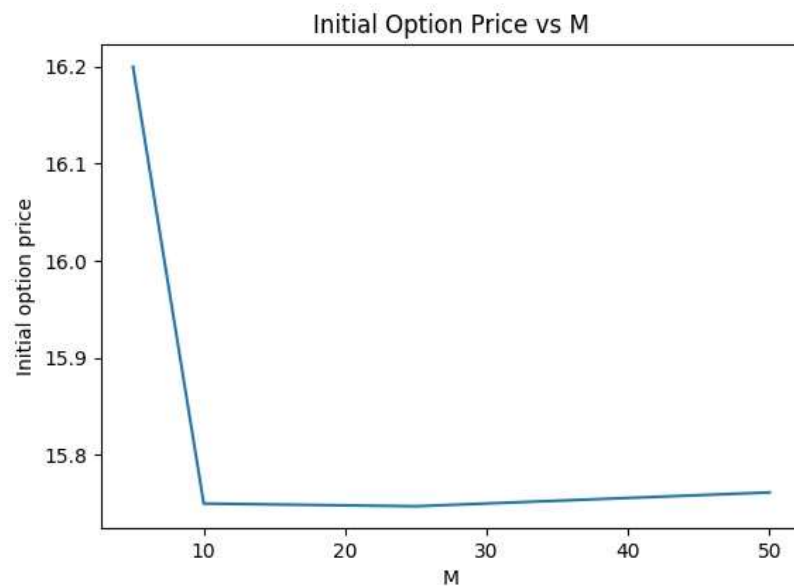
Q4)

(a)

Initial Option Prices for European Option:

Si.No.	M	Initial Option Price	Execution Time(s)
1.	5	16.200136	3.29017639160156e-05
2.	10	15.749707	3.814697265625e-05
3.	25	15.746918	8.7738037109375e-05
4.	50	15.761197	0.0002191066741943

(b)

Plots comparing M vs Initial European Option Price:

(c)

The values of options at all intermediate points for M = 5:

	t=0	t=1	t=2	t=3	t=4	t=5
1.	16.200136	25.375256	38.432095	55.877931	77.471587	102.550772
2.		7.543997	13.131858	22.219195	36.078411	54.881827
3.			2.197282	4.464542	9.071271	18.431444
4.				0.0	0.0	0.0
5.					0.0	0.0
6.						0.0

Comparative Analysis:

- Table comparing Execution times of the algorithms:

M	Unoptimized	Efficient	Most Efficient
5	0.000146865844727	7.939338684082e-05	3.290176391602e-05
10	0.003410816192627	8.916854858398e-05	3.814697265625e-05
25	159.4813978672028	0.000393867492676	8.773803710937e-05
50	Infeasible	0.001422405242919	0.000219106674194

- Limits on M:
  - Unoptimized Algorithm: this can handle values of M up-to around 25 to 30. The execution time doubles as M increases by 1. At higher values of M the algorithm incurs a RAM limit and is killed.
  - Efficient Algorithm: Quadratic space and time complexity (in M), M can be as high as of order 10^4.
  - Most Efficient Algorithm: Works on the same principle as efficient algorithm but summarizes the computation of efficient algorithm to one formula, thus making it linear time and space complexity. However the upper limit of M is lower as we are computing nCr (n choose r) which results in integer overflow for values of M over 1000.