Lecture - 9

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1 Multinomial Distribution:

The multinomial distribution is a generalization of the binomial distribution : each trial can now produce more than just two outcomes. For example, one may imagine rolling a "die" with k faces, with p_i being the probability for the die to land on face E_i.

A multinomial distribution is entirely characterized by :

1. n, total number of trials.
2. The set of probabilities $\{p_1, p_2, \cdots, p_k\}$ with $p_1 + p_2 + \cdots + p_k = 1$ are probabilities that $E_1, E_2, \cdots, E_k$ will occur.

After making n trials, suppose x_i is the number of times the event E_i will appear. Therefore, $x_1 + x_2 + \cdots + x_n = n$.

Multinomial Probability Distribution:

The distribution $Mult(n, p_1, p_2, \cdots, p_k)$ is determined by the values of probabilities of each possible k-tuples. We denote these probabilities

$$P(X_1 = x_1, X_2 = x_2, \cdots, X_k = x_k) = \frac{n!}{n_1!n_2! \cdots n_k!} p_1^{n_1} p_2^{n_2} \cdots p_k^{n_k}$$

2 Generating random number from Multinomial distribution:

We follow the steps to generate random number from multinomial distribution.

1. Generate $u \sim U(0,1)$. Set $j = 1$;
2. Calculate $P_1 = p_1,\ P_2 = p_1 + p_2,\ \cdots,\ P_k = p_1 + p_2 + \cdots + p_k$. Surely $P_k = 1$ and set $P_0 = 0$.
3. If $P_{i-1} < u < P_i$, $X_i = 1$, $j = j + 1$, Go to step 1.
4. repeat until $j > n$.

The above generation will give you a random vector which is one random sample from $Mult(n, p_1, p_2, \cdots, p_k)$.