Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA324)
Problem Set 08

1. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, with usual assumptions on ε. Show that

$$Cov\left(\widehat{\beta}_0, \widehat{\beta}_1\right) = -\frac{\overline{x}\sigma^2}{S_{xx}} \quad \text{and} \quad Cov\left(\overline{y}, \widehat{\beta}_0\right) = \frac{\sigma^2}{n}.$$

2. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$ with usual assumptions on ε. Show that

$$E\left(MS_R\right) = \sigma^2 + \beta_1^2 S_{xx} \quad \text{and} \quad E\left(MS_{Res}\right) = \sigma^2.$$

3. Suppose that we have fit a simple linear regression model $\widehat{y} = \widehat{\beta}_0 + \widehat{\beta}_1 x_1$, but the response is affected by a second variable x_2 such that the true regression is $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$. Is the least square estimator $\widehat{\beta}_1$ in the simple linear regression model unbiased?

4. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, where ε's are independent and identicall $N(0, \sigma^2)$ random variables. Find the MLEs of β_0, β_1, and σ^2. Is the MLE of σ^2 UE?

5. Suppose that we are fitting a straight line and wish to make standard error of the slope as small as possible. Suppose that the region of interest for x is $-1 \leq x \leq 1$. Where should the observations $x_1, x_2, \ldots, x_n$ be taken?

6. Consider the simple linear regression model $y = \beta_0 + \beta_1 x + \varepsilon$, with usual assumptions on ε. Also assume that β_0 is known. Find the LSE of β_1 and its' variance.