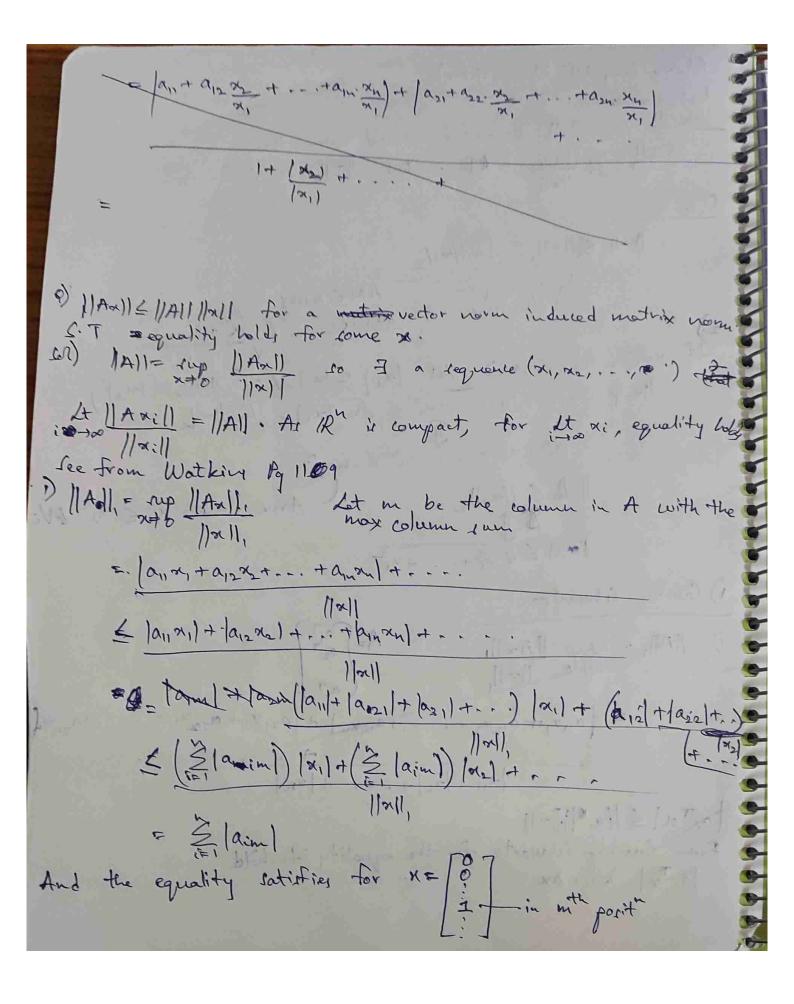
Mathia Computation (P in slider) Believenter Clide 11th II Ilp is vector induced norm, then A Ille say (Jos) = 01 but III/ by def = In Clide 121-North sides

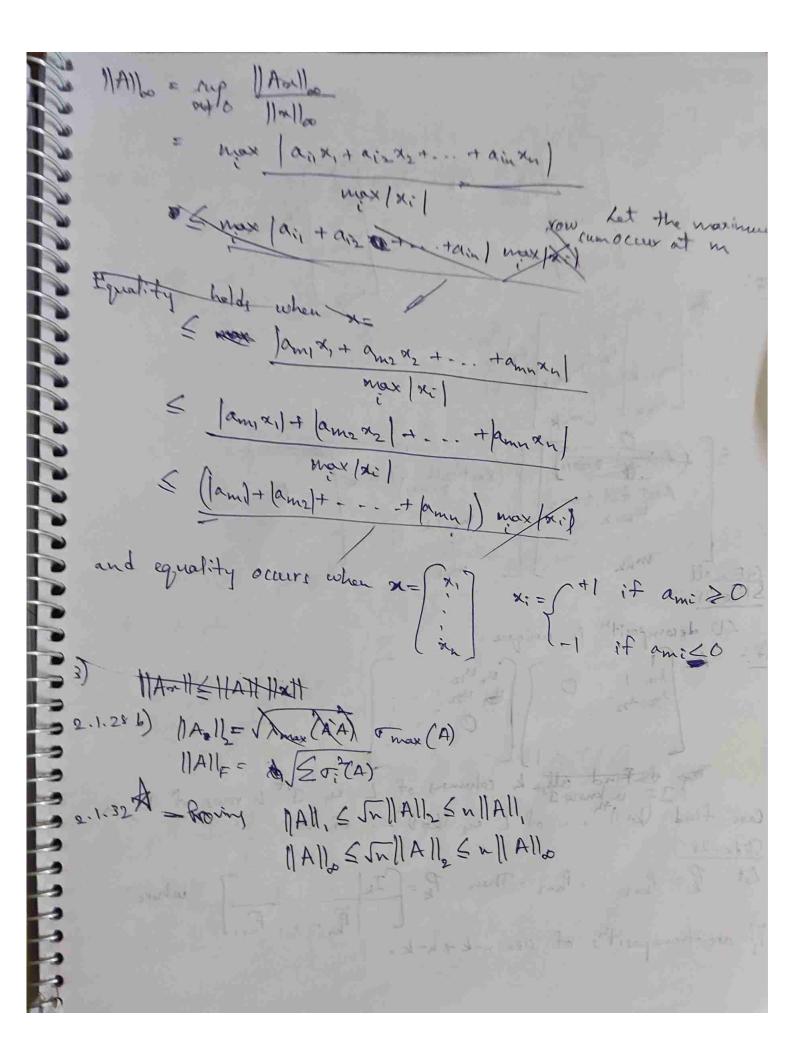
AM < RMS

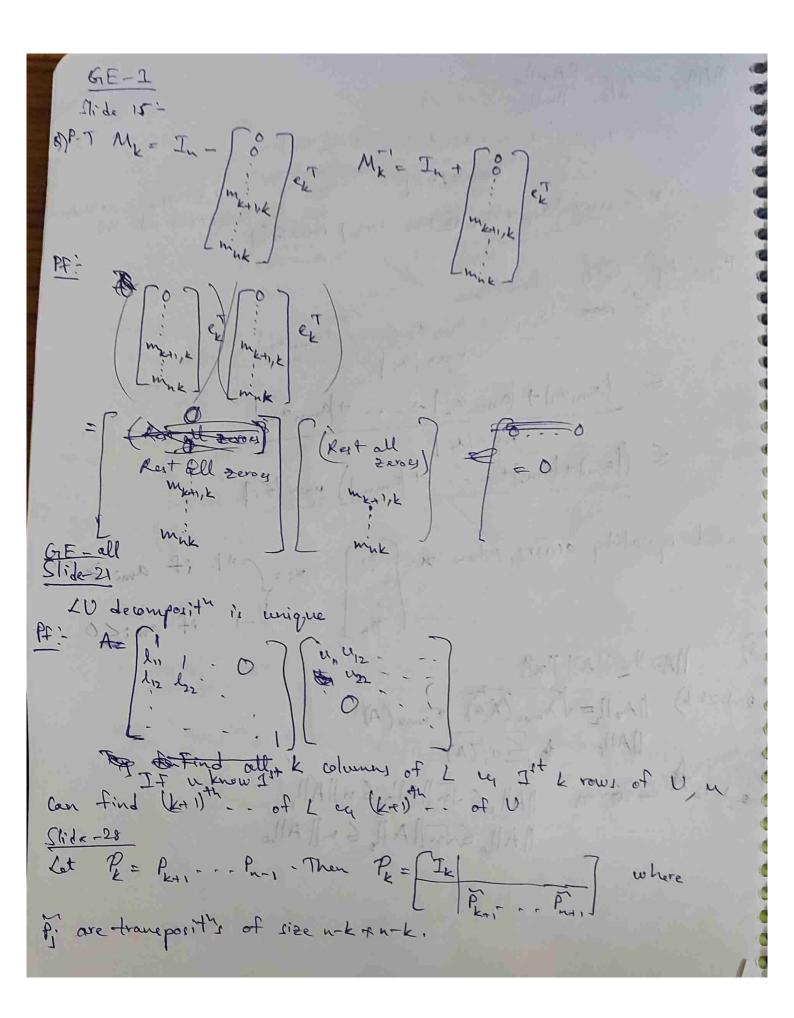
both sides

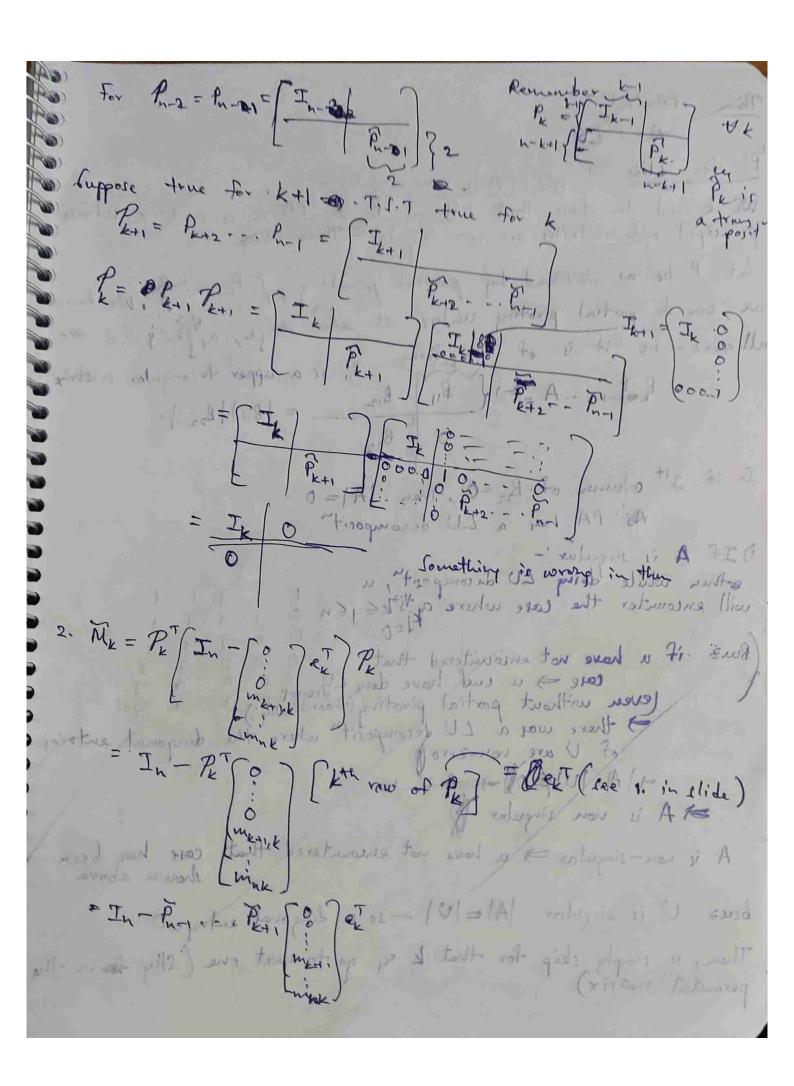
i.e.

arb+c+, -. < \ar2+b^2+c^2+... 212+x22+...+xn & nx2 I alo & 1/x 1/2 C true since x2 < x2 ov. [mx] 5 / m,2+... + x,2 1) Cauchy Schwartz 1) HAHR = sup !! Ax!! 1x1)+ (x2)+---+ |xn| atx 5 1 a: 1 m1 From Couchy Schwarts, For the equality to hold tain hair Ax

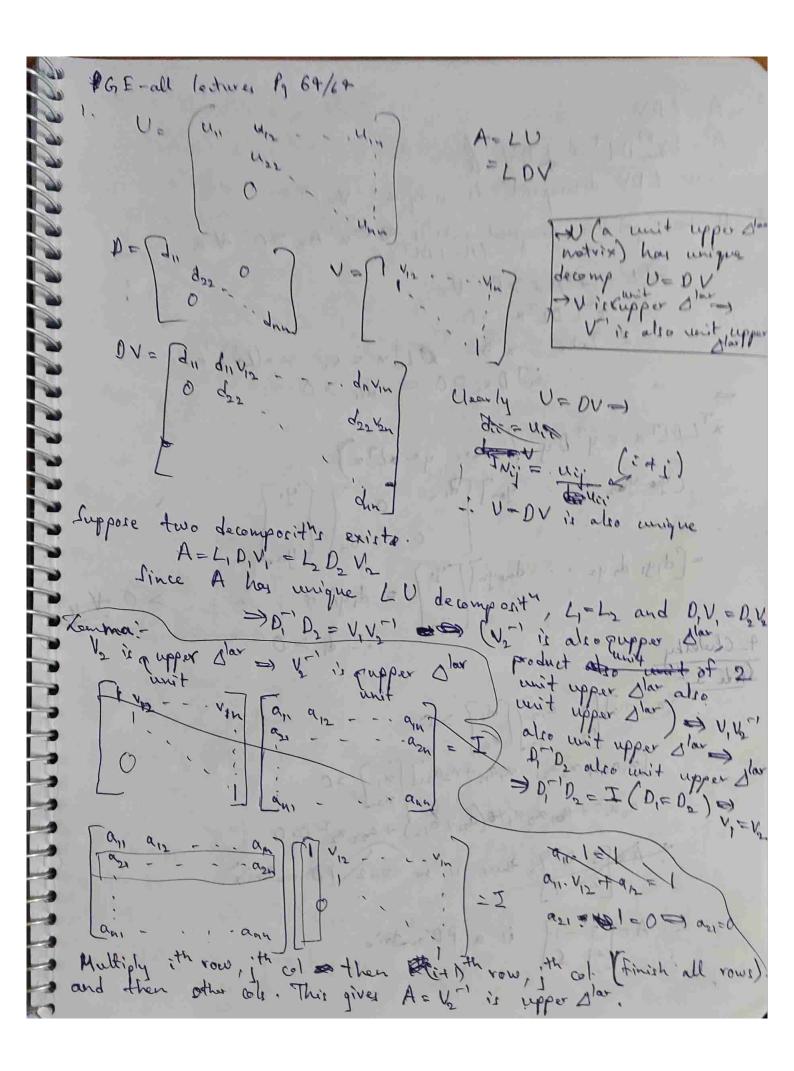


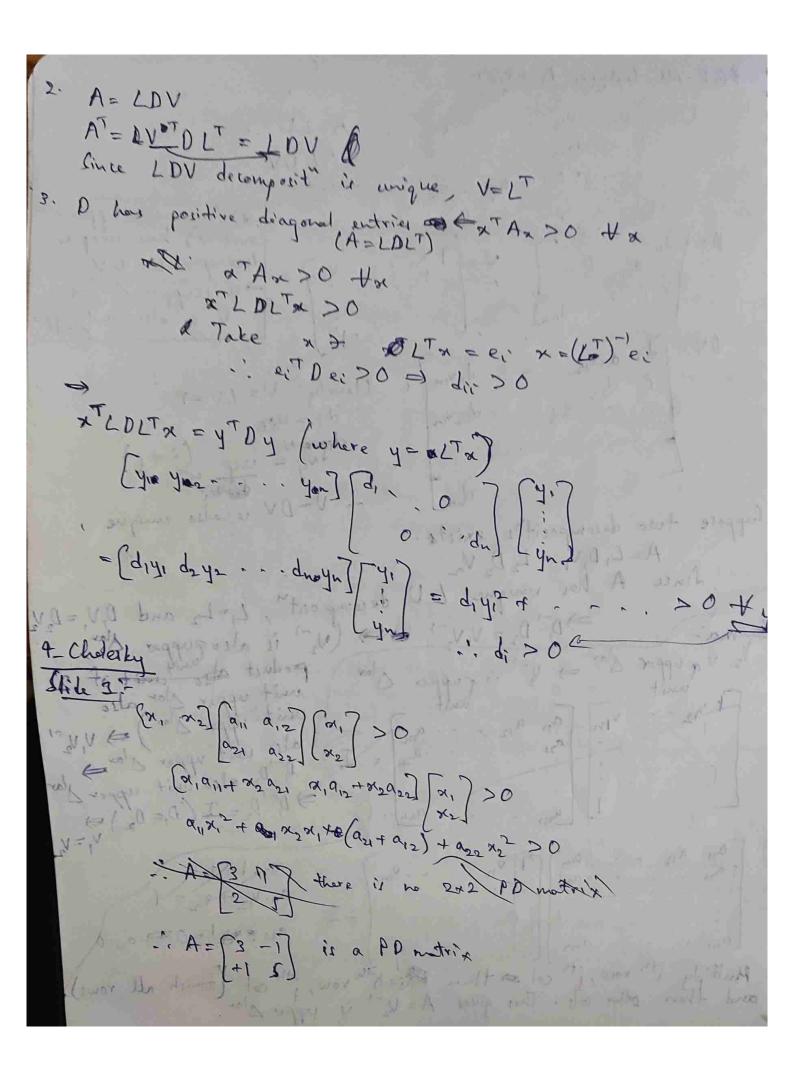






This - PAEXU Pt for Alide 29:- DIF A is non-singular: We kneed to show that I a principal sub-matrices are con-singular than Let P be as defined by partial piroting Potis we can do partial pivoting unless at some stage, axiking in all zero a i.e it is of the form In is a upper triangular matrix - Robert - A = 1 { B11 | B12 | - | B11 | B22 | So if 1st column of Biz =0, then |A|=0. Ai PA has a LU de composit) If A is singular !when while drings EV de comport, u will encounter the case where attaking Burz if a have not encountered that (even without portial pivoting (theoretically)) Here was a LU decomposit where the diagonal entries of U are non-zero (abile of AdVIED A Air no A is non singular A is non-singular > & have not encountered that cose boux U is singular A = 101 - so a diagonal entry =0 Then, a simply skip for that k of go to next one (Illy in the permutat matrix)





x' An = (1")" x = x' (M')"

-(1" x)" x)" Scince n" An e R ~ (A-A") x = 0 + x -10 (xin) (A-A") (any) = of that any in place of min & (xing) (A-A") (any) = 0 Put nainy in place of a in O Pf: Put atky in place of or in x Bx (a+ky)"B(x+ky) = a*Bx+ ky"Bx+ x*Bky+ky"Bky Put kel ug kei = y Bn = 0 tyte C Vige I If A is PD, its leading principal submatrices are PD. $A = \begin{cases} A_{11} & A_{12} \\ A_{21} & A_{32} \end{cases}$ at $A_{31} = \begin{cases} x_1 & \dots & x_k \\ x_1 & A_{22} \end{cases}$ where $x_1^2 = [a_1, \dots, a_k]$ n-k{ | A22 | AA 6-Finite Previor systemy 35 (2/3,-1, 1) Emax ATA , person in 6) E = (1.01) 2 - 1 & Prin = (1.00) x 2 Nmax = [1.11] 2+2 41(x+4+5)= 41(41(x+4)+5)=+1(6.11)+3)=+1(2+4+ fl(fl(2+x)+y)=fl((0.11)2+1)=(1.01)2

Slide 10 PF: xest yest then (m, 4)=0 (y, e)=0 + ces (an+by, s) = a(2,s) + b(y,s) = 0 - antby ect Lone properties: 1) If at N(ATA) then Ax is in both R(A) up N(AT) e) N(ATA) = N(A) (N(A) CN(ATA) NOLV, XEN(ATA) => ATAX=0 3) A cy ATA have some rank

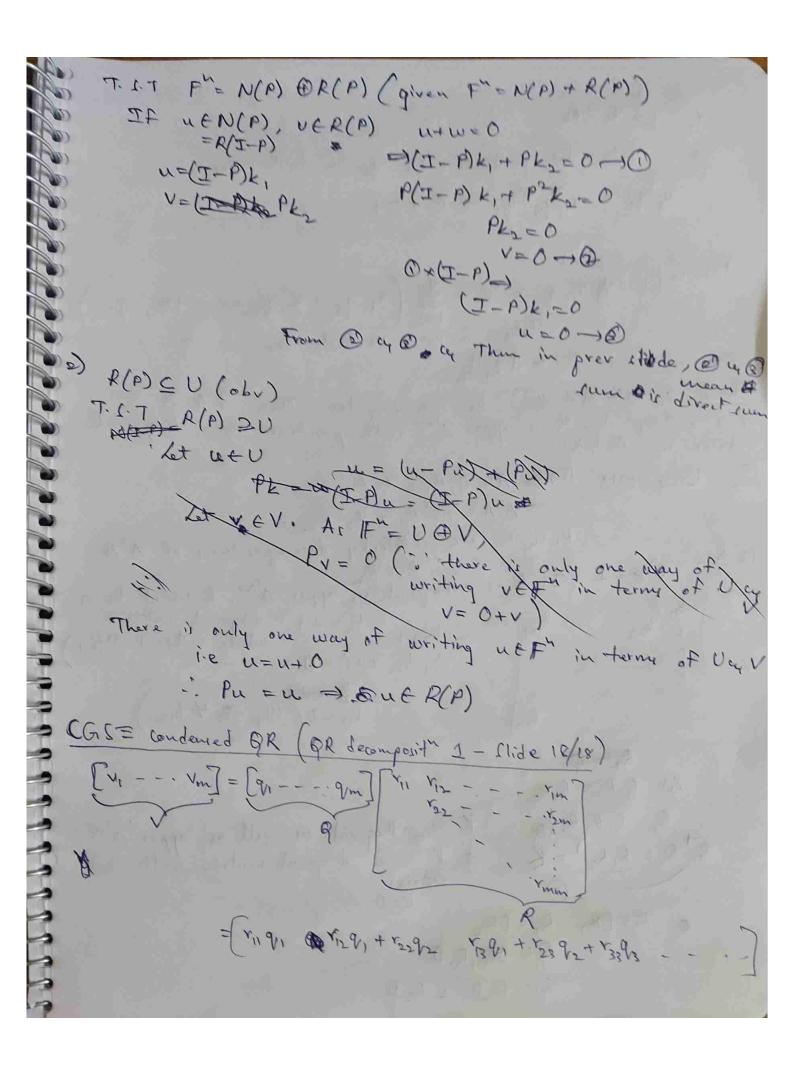
Use rank-Nullity Thu,

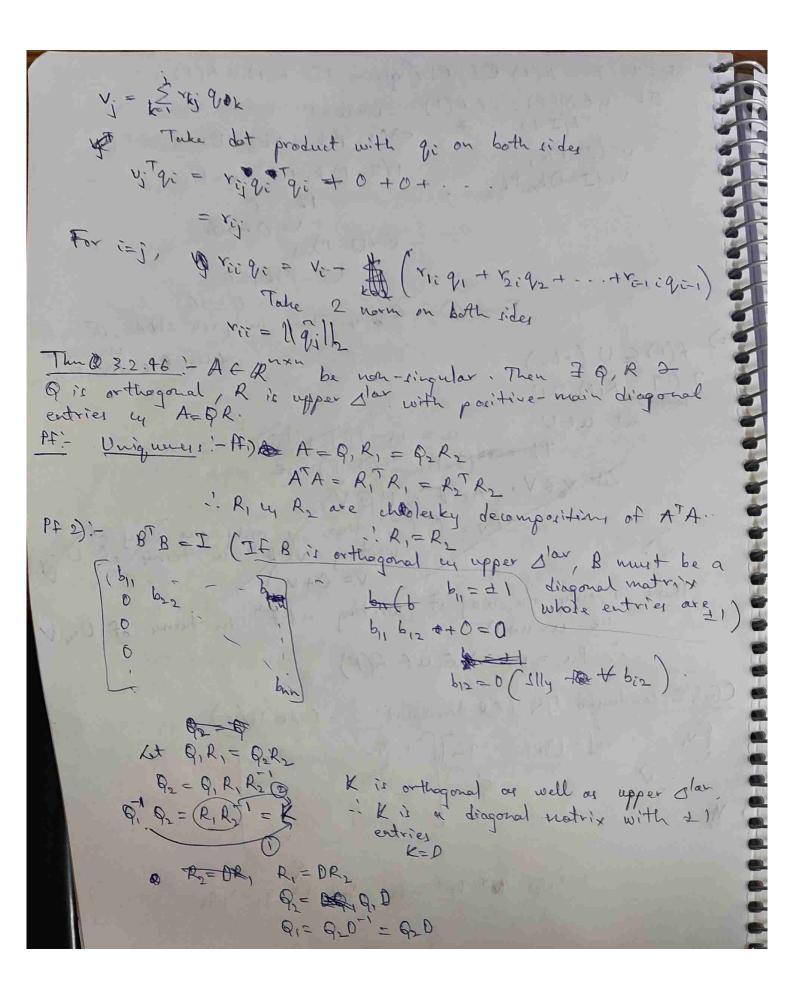
(ANTANCO (An) Anol) American rank (A) = dim (R(A)) (definith) vank(A) + mullity(A) - dim(A) (rank-mullity thu) rank (ATA) + mullity (ATA) = dim (ATA)=n From 2), as mult spaces are same, multities are range rank (A) = rank (ATA) Extract Charles afterns The Carpent : 17 4) IF A hay lie columns, ATA is non-singular

As rank(A)=n, rank(ATA)=n eq hence non-singular -> N(A)=R(AT) AT: Axco = at AT=0 = x is orthogonal to the columns of A I = Lpou (st) = R(AT) = N(A) = st where s is set of rows of A (column of AT) (10.1) = (1 + 11.0) /11 - (pa(n-0)

This Let U in W be 2 subspaces of F & Fo U+W. Then Dedin U+ dim W - dim (UNW) = h uy EE- 1000 span (UUW)=U+W+0 Extend this to get unit of unit B= du,, ..., un, u,, -., vor - basis of U B= fu,,..., way of ways of w T.S.T B = {u,,..., ur, uy..., v, wi,..., w+} - is too basis of U+u ∑α; νι: + ξ δινι: + ξ κα (iwi = 0 → Φ) ξα; νι + ξ b; νι = - ξ c; ω; EU EU EW Ecowi = Sdini (ZHIEUNW) (9-D) = 5 C: W: + E L: W: = 0 => die C: = 0 + (Illy bi=0) 10 on B is basic, ai=0 Obve that span(B)=W,+W2. .. dim (U+W) = ++++

Their Suppose U,W are subspaces of F" & F"= U+W. Then F= UDW & utw= O for utilinew implies u= w=0 a) of a mo = x+100 T.S.T u1+w, = 112+w2 = 11+=112 cq w,= w2 (1-12) + (w,-w2) = 0 41-112=0 W1-W2=0 (N/P) = R(PT) R(P) = N(PT) N(P)=N(PT) O) N(P)=R(I-P) As I-P is also a projexu, (N(I-P) = R(I-(I-P)) = R(P) $Re x \in N(P) \Rightarrow Px = 0$ D(I-P)= I+P=2P=I-P) TO TO THE TIME OF @ R(2-P) OR R) (I-P) x (I-P) x (I-P) x (I-P) = (I+P^2-2P) x (I-P) I have to Prove = x $N(P) \rightarrow R(P) = F^{n} + i \cdot i \cdot i + x \in R(I-P) \Rightarrow (I-P) \cdot y = x$ Let $x \in F^{n}$ x = (fx) + (x - Px) x = (fx) + (x - Px)Pa ER(P) N-Pa EN(P) (: P(n-Pa) = 0)





En: - 3.2.68) Porey (I - Kint) or = y n-y = Junta = ((or w) u ". I is a multiple of may 3.2.69) ax 5) Gra = 0 = 1 a i's a multiple of u (I-UVT) WE D Re(VISOU sis a multiple of a x is a multiple of u > Gros=0 $G(\lambda u) = \lambda (u - (v^{T}u)u)$ Lemma - Gr is singular & Vu= 1 Gilgular > Gra= D for a +0 (I - UVT) a = 0 - Ve me MOBBC to the left with va - (vu) (va) = 0 (va) (1-va) =0 Va=0 or va=1 But if va= 0, then O - ' VTU= 1 becomes Vu=1 >> G is singular a\$ -0=0 =>€ Grand with Gu = u - w (v u) u = 0 ey u +0 As vu=1 G(hu) = 0 [-w] (I-Bur) = I OF (I-UV) G-1 = I G-1 (I-UV) = I I - UV (8+1) + P(UV) = 0 G= I+ G- un G= I+ G w Gia= I+ Gulva) u- ku(8+1) + 8 k u= C Gu (1-174) = I 1-k(B+1)+Bk2=0

G=I-uv" is non-singular => G'-I-BINT Suppose G'= I-Burt G'G = I I=(Tu-I) (Tux-I)=I - furt - urt furturt = 0 Let vu= k = utv MOBS with u on right -\$ & ku - ku + \$ k2 u = 0 - B-1+ Bk=0 B(k-1)=1 As we know GT boonow, GT has the form I-Puvt d) G=G= I- vut = I-BurT vut = fart MOBS with u on the right Q(uTu)v = Box (vTu) u => Qv= Pu Now unel $V = \begin{cases} \sqrt{u} & 1 \\ \sqrt{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \\ \frac{1}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{1}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u = \begin{cases} \frac{\rho}{u} & \frac{\rho}{u} \\ \frac{\rho}{u} & 1 \end{cases} u$ T: P=2

