PHYSICS-I

Department of Physics, IIT Guwahati.

Course No: PH 101 **Mid-Semester Examination** **Date:16 Sept., 2018**

Time: 2-4 pm **Total Marks:40**

General Instructions

a) Make sure that there are **six sheets** (including this) in this **Question-cum-Answer** Booklet.

b) Write your **Name** and **Roll Numbers** on **every sheet** in the space provided.

c) You must write the answers **ONLY IN THE SPACE PROVIDED for the given question**. Answers written **elsewhere WILL NOT be evaluated**!

d) **NO** extra **answer-sheets** will be provided!

e) Supplementary sheets provided are **ONLY** for rough work.

f) It is advised that you first solve the problems on the supplementary sheet, and then copy the key steps in the space provided for that problem in **this** Question-cum-Answer **booklet**.

g) Be **legible!** Also, make sure that your answers are systematic, logically as well as mathematically connected.

h) Question 1 is on the rear side of **this sheet**. The answers to **each** of the remaining questions (Q2-Q6) should be contained in a single sheet.

i) Marks for each question is indicated below the question number.

Student's Name :	Rahul. D
Roll No:	180102054
Signature:	Rahul

Signature of the Invigilator:	Jshop 16/9/18

Q1. Report the degree of freedom (**DOF)** of the following systems in the space provided. ***No justification*** **required!**			
a) [3×1]	Consider a ***thin*** uniform rigid rod of mass m and length l, under the following situations.		
	The rod is sliding down such a way that one end of it always maintains contact with a vertical wall while the other end slips on a horizontal floor. **The whole motion is on the** xy **−plane.**	DOF=	1
	The rod is free to move any fashion but constrained ***only to the*** xy ***−plane.***	DOF=	3
	The rod is constrained to move ***inside*** a larger spherical shell of radius R $(2R > l)$ such a way that both ends of the rod always maintain contact with the inner surface of the shell.	DOF=	3
b) [4×1]	Water molecule (H_2O) is composed of two hydrogen atoms bonded to an oxygen atom, with its average O−H bonds measuring 1 Å, and with an equilibrium H−O−H angle of 104.5°. Obtain the DOF of a water molecule that is free to move in ***three dimensions*** under the following models.		
	Model#1: The O−H bonds, or bond lengths, are flexible. But the H−O−H bond angle is fixed at 104.5°	DOF=	8
	Model#2: The O−H bonds are rigid at 1 Å, and the H−O−H bond angle is fixed at 104.5°.	DOF=	6
	Model#3: The O−H bonds are rigid at 1 Å, but the H−O−H bond angle is free to change.	DOF=	7
	Model#4: The O−H bonds as well as the H−O−H bond angle is free to change.	DOF=	9

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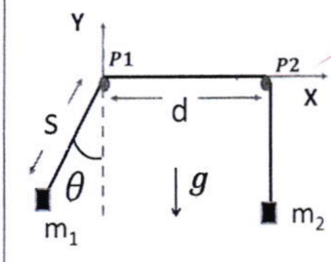
Q2 **[7]**	Two point masses $m1$ and $m2$ are connected by a light inextensible string (of length l), which runs over two massless pulleys of ***negligible radius***, as shown in the figure. The mass $m1$ can oscillate on the xy –plane, while $m2$ can move only in the vertical direction. The first pulley P1 is at the origin, and the 2nd pulley P2 is on the x-axis, at distance d from the first. Gravity acts downwards as shown. Obtain the Lagrangian and the Euler-Lagrange equations for the system, employing S and θ (marked in the figure) as the generalized coordinates.	

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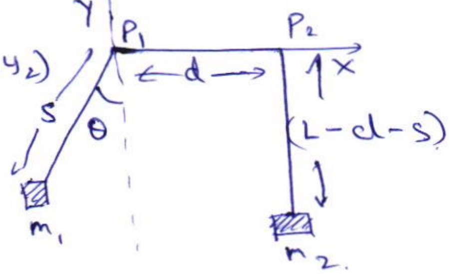
It is given that S and θ should be used as generalised coordinates

Let cartesian coordinates of m_1 & m_2 be (x_1, y_1) & (x_2, y_2)



$x_1 = -s\sin\theta$ $\qquad x_2 = d$

$\dot{x}_1 = -[\dot{s}\sin\theta + \cos\theta\,\dot{\theta}\,s]$ $\qquad \dot{x}_2 = 0$

$y_1 = -s\cos\theta$ $\qquad y_2 = -[L-d-s]$

$\dot{y}_1 = -[\dot{s}\cos\theta - \sin\theta\,\dot{\theta}\,s]$ $\qquad \dot{y}_2 = \dot{s}$

Kinetic energy for system $(T) = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2)$

$T = \frac{1}{2}m_1((-\dot{s}\sin\theta - s\cos\theta\,\dot{\theta})^2 + (s\dot{\theta}\sin\theta - \dot{s}\cos\theta)^2) + \frac{1}{2}m_2(0^2 + \dot{s}^2)$

$T = \frac{1}{2}m_1(\dot{s}^2 + s^2\dot{\theta}^2) + \frac{1}{2}m_2\dot{s}^2$

$V = [-m_1 g s\cos\theta] + [-m_2 g(L-d-s)]$ [considering gravitational potential energy zero at origin]

$L = T - V$

$L = \frac{1}{2}m_1(\dot{s}^2 + s^2\dot{\theta}^2) + \frac{1}{2}m_2\dot{s}^2 + m_1 g s\cos\theta + m_2 g(L-d-s)$

4

$\frac{\partial L}{\partial s} = m\dot{\theta}^2 s + m_1 g\cos\theta - m_2 g$ $\qquad \frac{\partial L}{\partial \dot{s}} = m_1\dot{s} + m_2\dot{s}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{s}}\right) - \frac{\partial L}{\partial s} = 0$ [1st E-L eq]

$\Rightarrow m_1\ddot{s} + m_2\ddot{s} - m\dot{\theta}^2 s - m_1 g\cos\theta + m_2 g = 0 \longrightarrow (1)$

$\frac{\partial L}{\partial \theta} = m_1 g s(-\sin\theta)$ $\qquad \frac{\partial L}{\partial \dot{\theta}} = m_1 s^2\dot{\theta}$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$ [2nd E-L eq]

3

$\Rightarrow m_1[2s\dot{s}\dot{\theta} + s^2\ddot{\theta}] + m_1 g s\sin\theta = 0 \longrightarrow (2)$

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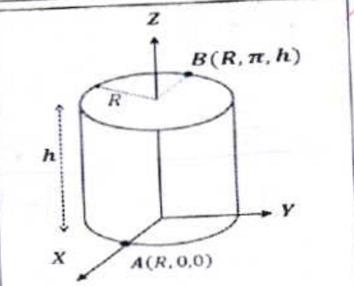
Q3	Find the *geodesic* on the surface of a cylinder of radius R and height h, employing **cylindrical coordinate** system. Find the total length of this path between points $A(R,0,0)$ and $B(R,\pi,h)$. Coordinate of A and B are given in cylindrical coordinate system.	
[6]		

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On considering cylindrical coordinate system, the ~~point~~ any point on the measured path can be shown as (R, θ, z) where R is constant.

$$dr = R\, d\theta\ \hat{\theta} + dz\ \hat{z}$$

thus $ds = \sqrt{(R d\theta)^2 + (dz)^2}$

geodesic means the path length should be minimum so we are considering a ~~path~~ variable which denotes path length (S).

$$S = \int_A^B \sqrt{(R\, d\theta)^2 + (dz)^2}$$

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$$S = \int_A^B \sqrt{R^2 + \left(\frac{dz}{d\theta}\right)^2}\, d\theta$$

$$F(\dot{z}, \dot{\theta}, z) = \sqrt{R^2 + \dot{z}^2} \qquad \text{where } \dot{z} = \frac{dz}{d\theta}$$

For the path to be geodesic the function should obey Euler-lagrange equation

ie $$\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{z}}\right) - \frac{\partial F}{\partial z} = 0$$

$$\frac{d}{dt}\left(\frac{\dot{z}}{\sqrt{R^2+\dot{z}^2}}\right) - 0 = 0$$

$$\Rightarrow \frac{\dot{z}}{\sqrt{R^2+\dot{z}^2}} = K.$$

$$\frac{\dot{z}^2}{R^2+\dot{z}^2} = k^2$$

$$\frac{\dot{z}^2}{R^2} = \frac{k^2}{1-k^2}$$

$$\dot{z}^2 = \frac{R^2k^2}{(1-k^2)}$$

$$\dot{z} = \frac{Rk}{\sqrt{1-k^2}}$$

$$\frac{dz}{d\theta} = \frac{Rk}{\sqrt{1-k^2}}$$

$$\int_0^{z} dz = \frac{Rk}{\sqrt{1-k^2}}\int_0^{\theta} d\theta$$

$$z = \frac{Rk}{\sqrt{1-k^2}}\,\theta$$

On substituting ~~to~~ coordinate of B

$$h = \frac{Rk}{\sqrt{1-k^2}}\,\pi$$

Thus $\frac{Rk}{\sqrt{1-k^2}} = \frac{h}{\pi}$

On substituting the value of constant.

$$z = \frac{h}{\pi}\,\theta$$

$$\dot{z} = h/\pi$$

$$S = \int_0^{\pi} \sqrt{R^2+\left(\frac{h}{\pi}\right)^2}\,d\theta$$

$$S = \sqrt{R^2+\left(\frac{h}{\pi}\right)^2}\,[\theta]_0^{\pi}$$

$$S = \sqrt{R^2+\left(\frac{h}{\pi}\right)^2}\,(\pi)$$

$$\boxed{S = \sqrt{R^2\pi^2+h^2}}$$ → path length

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Q4	A point particle of mass m is constrained to move on the surface of a rigid-fixed sphere of
[6]	radius R. Obtain the Hamiltonian and Hamilton's equations of motion of the mass m in ***spherical polar*** coordinate system. The particle is acted upon **only** by the forces of constraint.

The particle is constrained to move on rigid fixed sphere of radius R,
so on considering the spherical polar coordinate system,
the coordinate of particle ~~at~~ is (R, θ, ϕ)

$R \to$ constant θ & ϕ variables

velocity of a particle in spherical polar coordinate system

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\,\dot{\phi}\hat{\phi}$$

in this case $r = R$ so $\dot{r} = 0$.

thus $\vec{v} = R\dot{\theta}\hat{\theta} + R\sin\theta\,\dot{\phi}\hat{\phi}$.

Kinetic energy$(T) = \frac{1}{2}mv^2 = \frac{m}{2}\left[(R\dot{\theta})^2 + (R\sin\theta\,\dot{\phi})^2\right]$

Since the particle is acted upon only by forces of constraint,
the potential at all points on the surface of sphere
remains constant and let us assume it as C

thus $V = C$

$$L = T - V$$

$$L = \frac{m}{2}R^2\dot{\theta}^2 + \frac{m}{2}R^2\sin^2\theta\,\dot{\phi}^2 - C.$$

$$P_\theta = \frac{\partial L}{\partial\dot{\theta}} = mR^2\dot{\theta} \quad \Rightarrow \quad \dot{\theta} = \frac{P_\theta}{mR^2}.$$

$$P_\phi = \frac{\partial L}{\partial\dot{\phi}} = mR^2\sin^2\theta\,\dot{\phi} \quad \Rightarrow \quad \dot{\phi} = \frac{P_\phi}{mR^2\sin^2\theta}$$

~~$H = \sum p_j$~~ $H = \sum p_j\dot{q}_j - L$

$$H = mR^2\dot{\theta}^2 + mR^2\sin^2\theta\,\dot{\phi}^2 - \frac{mR^2\dot{\theta}^2}{2} - \frac{m}{2}R^2\sin^2\theta\,\dot{\phi}^2 + C$$

$$H = \frac{mR^2\dot{\theta}^2}{2} + \frac{mR^2\sin^2\theta\,\dot{\phi}^2}{2} + C.$$

$$H = \frac{mR^2}{2}\left(\frac{P_\theta}{mR^2}\right)^2 + \frac{mR^2\sin^2\theta}{2}\left(\frac{P_\phi}{mR^2\sin^2\theta}\right)^2 + C$$

$$H = \frac{P_\theta^2}{2mR^2} + \frac{P_\phi^2}{2mR^2\sin^2\theta} + C$$

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The required equation are $\dot{P}_j = -\dfrac{\partial H}{\partial q_j}$ $\quad \dot{q}_j = \dfrac{\partial H}{\partial p_j}$

$$\dot{P}_\phi = 0 \qquad \dot{\phi} = \frac{2P_\phi}{2mR^2 \sin^2\theta} = \frac{P_\phi}{mR^2 \sin^2\theta}$$

$$\dot{P}_\theta = -\frac{\partial H}{\partial \theta} = \frac{P_\phi^2 \cot\theta \operatorname{cosec}^2\theta}{mR^2} \qquad \dot{\theta} = \frac{\partial H}{\partial p_\theta} = \frac{P_\theta}{mR^2}$$

1+ $\qquad\qquad$ 1+

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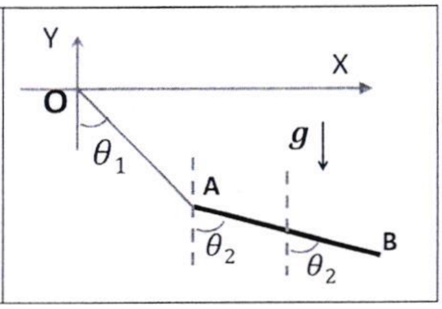
Q5 [6]	A thin uniform rod, AB, of mass m and length l is suspended to the ceiling (at O), by one end, through a light inextensible string of length d (see figure). Assume that the rod can only move on the xy−plane, and without any slackening of the string. Obtain the Lagrangian of the system using $\theta 1$ and $\theta 2$ (marked in the figure) as the generalized coordinates. The system is under gravity. Euler-Lagrange equations are **not required!**



It is given that the generalised coordinates to be used. is θ_1 and θ_2.

Since the ~~mass~~ string is light, ~~its the~~ kinetic and potential energy is neglected.

Kinetic energy (rigid body) $= \frac{1}{2} m (v_c)^2$ + Kinetic energy in COM frame.

where COM → centre of mass

v_c → velocity of COM

~~Since~~ Rod is uniform so COM will be at $l/2$ dist from end.

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$$\vec{r}_{com} = \left(d \sin\theta_1 + \frac{l}{2}\sin\theta_2\right)\hat{i} - \left(d\cos\theta_1 + \frac{l}{2}\cos\theta_2\right)\hat{j}$$

$$\vec{V}_{com} = \left(d\cos\theta_1\,\dot{\theta}_1 + \frac{l}{2}\cos\theta_2\,\dot{\theta}_2\right)\hat{i} - \left(d(-\sin\theta_1)\dot{\theta}_1 + \frac{l}{2}(-\sin\theta_2)\dot{\theta}_2\right)\hat{j}$$

$$\vec{v}_{com} = \left(d\cos\theta_1\,\dot{\theta}_1 + \frac{l}{2}\cos\theta_2\,\dot{\theta}_2\right)\hat{i} + \left(d\sin\theta_1\,\dot{\theta}_1 + \frac{l}{2}\sin\theta_2\,\dot{\theta}_2\right)\hat{j}$$

~~T~~ Kinetic energy (T) of system $= \frac{1}{2}\cancel{m((d\cos\theta_1\dot{\theta}_1)^2 + \frac{1}{2}}$

$$T = \frac{m}{2}\left[\left(d\cos\theta_1\dot{\theta}_1 + \frac{l}{2}\cos\theta_2\dot{\theta}_2\right)^2 + \left(d\sin\theta_1\dot{\theta}_1 + \frac{l}{2}\sin\theta_2\dot{\theta}_2\right)^2\right] + \frac{1}{2}\frac{ml^2}{12}\dot{\theta}_2^2$$

$$T = \frac{m}{2}\left[(d\dot{\theta}_1)^2 + \left(\frac{l}{2}\dot{\theta}_2\right)^2 + dl\cos(\theta_1-\theta_2)\right] + \frac{ml^2\dot{\theta}_2^2}{24}$$

$$V = -mg\left[d\cos\theta_1 + \frac{l}{2}\cos\theta_2\right]$$

$$L = \frac{m}{2}\left[(d\dot{\theta}_1)^2 + \left(\frac{l}{2}\dot{\theta}_2\right)^2 + dl\cos(\theta_1-\theta_2)\right] + \frac{ml^2\dot{\theta}_2^2}{24} + mg\left[d\cos\theta_1 + \frac{l}{2}\cos\theta_2\right]$$

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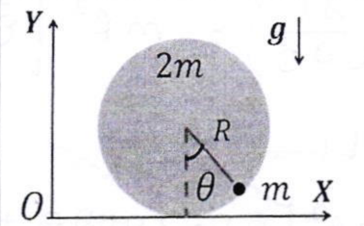
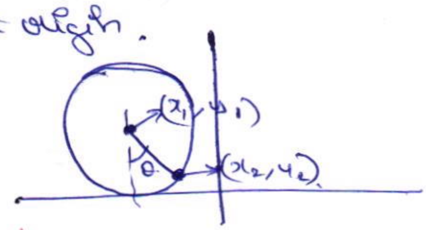
Q6 [8]	A uniform disk of mass $2m$ and radius R has a point mass m ***fixed*** close to its edge. The disk can roll without sliding along the x-axis, under gravity. Obtain the Euler-Lagrange (E-L) equations for the system. Using the E-L equation find the frequency of small angle oscillations in θ (as marked in the figure). `Hint: For small angle oscillations,` $\theta^2 \approx 0$ and $\dot{\theta}^2 \approx 0$.	

As the point mass is rigidly fixed to disk, ~~the~~ it can be considered as a rigid body.

DOF = 6 − 5 = 1 [z = 0, rotationally allowed in one axis, condition of no slipping, move on x axis.]

Consider ~~that~~ that the disk was initially at origin. Now on ~~move~~ rotating it by θ



$x_1 = -R\theta \qquad y_1 = R.$

$\dot{x}_1 = -R\dot{\theta} \qquad \dot{y}_1 = 0$

$x_2 = -R\theta + R\sin\theta \qquad y_2 = R - R\cos\theta.$

$\dot{x}_2 = -R\dot{\theta} + R\cos\theta\,\dot{\theta} \qquad \dot{y}_2 = R\sin\theta\,\dot{\theta}$ 1+

KE of rigid body = KE of COM + KE (in COM frame).

~~Thus T for the sy~~

$$KE_{system}\ (T) = \frac{1}{2}2m(-R\dot{\theta})^2 + \frac{1}{2}\frac{2mR^2}{2}\dot{\theta}^2 + \frac{1}{2}m\left[(-R\dot{\theta} + R\cos\theta\,\dot{\theta})^2 + (R\sin\theta\,\dot{\theta})^2\right]$$

$$T = mR^2\dot{\theta}^2 + \frac{mR^2\dot{\theta}^2}{2} + \frac{m}{2}\left[2R^2\dot{\theta}^2 - 2R^2\dot{\theta}^2\cos\theta\right]$$

$$= \frac{3}{2}mR^2\dot{\theta}^2 + mR^2\dot{\theta}^2(1-\cos\theta)$$

$$T = \frac{5}{2}mR^2\dot{\theta}^2 - mR^2\dot{\theta}^2\cos\theta.$$ 1+

$$V = 2mgR + mg(R - R\cos\theta)$$ 1+

$L = T - V$

$$L = \frac{5}{2}mR^2\dot{\theta}^2 - mR^2\dot{\theta}^2\cos\theta - 3mgR + mgR\cos\theta.$$ 1+

$$\frac{\partial L}{\partial \theta} = mgR(-\sin\theta) + mR^2\dot{\theta}^2\sin\theta. \qquad \frac{\partial L}{\partial \dot{\theta}} = 5mR^2\dot{\theta} - 2mR^2\dot{\theta}\cos\theta.$$

$$\frac{\partial L}{\partial \theta} = mR\sin\theta\left[R\dot{\theta}^2 - g\right] \qquad \frac{\partial L}{\partial \dot{\theta}} = mR^2\dot{\theta}\left[5 - 2\cos\theta\right]$$

E-L equation $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0.$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = mR^2\left[\ddot{\theta}\,[5 - 2\cos\theta] + 2\,\cancel{\cos\theta}\sin\theta\,\dot{\theta}^2\right]$$

Thus ~~EL equation~~

$$mR^2\left[5\ddot{\theta} - 2\cos\theta\,\ddot{\theta} + 2\sin\theta\,\dot{\theta}^2\right] + mR\sin\theta\left[g - R\dot{\theta}^2\right] = 0$$

~~in eq~~ It is given that for small ~~osc~~ oscillation $\dot{\theta}^2 \approx 0$.

so. $mR^2\ddot{\theta}\,[5 - 2\cos\theta] + mR\sin\theta . g = 0.$

$$\ddot{\theta} = -\frac{mR\sin\theta\ g}{mR^2(5 - 2\cos\theta)}$$

$$= -\frac{g\sin\theta}{R(5 - 2\cos\theta)}$$

~~as $\theta^2 \to$~~

as $\theta^2 \approx 0 \quad \theta \to 0 \Rightarrow \sin\theta \to \theta$ & $\cos\theta \to 0$

$$\ddot{\theta} \simeq -\frac{g\,\theta}{5R.}$$

for SHM.

$$\ddot{\theta} = -\omega^2\theta$$

on comparing $\omega^2 = \frac{g}{5R.}$ $\quad \omega = \sqrt{\frac{g}{5R.}}$

frequency $= \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{5R}}$

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