Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA 324)
Problem Set 01

1. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} Geo(p)$ with common PMF
$$f(x;\, p) = \begin{cases} p(1-p)^x & \text{if } x - 0,\, 1,\, 2,\, \ldots \\ 0 & \text{otherwise,} \end{cases}$$
where $p \in (0,\, 1)$ is an unknown parameter. Find a sufficient statistic for p.

2. Let $X_1,\, X_2,\, \ldots,\, X_m \overset{i.i.d.}{\sim} Poi(\lambda)$, $Y_1,\, Y_2,\, \ldots,\, Y_n \overset{i.i.d.}{\sim} Poi(2\lambda)$, where $\lambda > 0$ is unknown parameter. Also, assume that X's and Y's are independent. Show that $\sum_{i=1}^m X_i + \sum_{i=1}^n Y_i$ is a sufficient statistic for λ.

3. Let $X_1,\, X_2,\, X_3,\, X_4 \overset{i.i.d.}{\sim} Bernoulli(p)$, where $0 < p < 1$ is unknown parameter. Denote
$$U = X_1(X_3 + X_4) + X_2.$$
Show that U is not a sufficient statistic for p.

4. Suppose that $X_1,\, X_2,\, \ldots,\, X_n$ are *i.i.d.* RVs with common PDF
$$f(x,\, \sigma,\, \mu) = \begin{cases} \frac{1}{\sigma} e^{-\frac{x-\mu}{\sigma}} & \text{if } x > \mu \\ 0 & \text{otherwise,} \end{cases}$$
where $\mu \in \mathbb{R}$ and $\sigma > 0$. Show that
   (a) $X_{(1)} = \min\{X_1,\, X_2,\, \ldots,\, X_n\}$ is minimal sufficient for μ if σ is known.
   (b) $\frac{1}{n}\sum_{i=1}^n (X_i - \mu)$ is minimal sufficient statistic for σ if μ is known.
   (c) $\left(X_{(1)},\, \sum_{i=1}^n \left(X_i - X_{(1)}\right)\right)$ is minimal sufficient statistic for $(\mu,\, \sigma)$ if both parameters are unknown.

5. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} N(\theta,\, \theta^2)$, where $\theta > 0$ is unknown parameter. Derive a minimal sufficient statistic for θ.

6. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} N(\theta,\, \theta)$, where $\theta > 0$ is unknown parameter. Derive a minimal sufficient statistic for θ.

7. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} U(-\theta,\, \theta)$, where $\theta > 0$ is unknown parameter. Derive a minimal sufficient statistic for θ.