$\cancel{L(\theta_1)}$

$$\Lambda(x) = \frac{\sup\limits_{\theta \leq \theta_0} \cancel{e^{-n\theta}}\ e^{-\sum x_i + n\theta}\ \mathbb{I}_{x_{(1)} > \theta}}{\sup\limits_{\substack{\cancel{\theta > \theta_0} \\ \theta \in \mathbb{R}}} e^{-\sum x_i + n\theta} \cdot \mathbb{I}_{x_{(1)} > \theta}}$$

$$\Lambda(x) = \begin{cases} 1 & \theta_0 \geq x_{(1)} \\ \dfrac{e^{-\sum x_i + n\theta_0}}{e^{-\sum x_i + n x_{(1)}}} & \theta_0 < x_{(1)} \end{cases}$$

$$e^{n\theta_0 - n x_{(1)}}$$

$\Lambda(x) \leq K \Rightarrow x_{(1)} \geq K'$

$$\psi(x) = \begin{cases} 1 & \Lambda(x) < K \\ \gamma & \Lambda(x) = K \\ 0 & \Lambda(x) > K \end{cases}$$

$$\sup_{\theta \in \Theta_0} E_{\theta}[\psi(X)] = \alpha.$$

$$\sup_{\theta \le \theta_0} E_\theta[\varphi(x)'] = \alpha$$

$$\Rightarrow \cancel{\beta} \quad \underline{P_\theta(\cancel{x}(x) < K) \cdot t}$$

$$\sup_{\theta \le \theta_0} \left[P_\theta(\underline{x_{(1)} > K'})\right] = \alpha$$

$\cancel{\beta}$

$$P_\theta(x_1 > K')\, P(x_2 > K') \cdots P(x_n > K')$$

$$\cancel{\beta}\, P_\theta(x_i > K) \qquad \underline{K > \theta}$$

$$= \int_K^\infty e^{-(x-\theta)} I_{(\theta,\infty)}(x)\, dx$$

$$= \int_K^\infty e^{-(x-\theta)}\, dx. \qquad e^{-(K-\theta_0)n} = \alpha$$

$$= \left.\frac{e^{-(x-\theta)}}{-1}\right]_K^\infty \qquad \Rightarrow -(K-\theta_0)n = \ln\alpha$$

$$\Rightarrow K = \theta_0 - \frac{1}{n}\ln\alpha$$

$$= \left[e^{-(K-\theta)n}\right]$$

2.

Ans
$$\frac{\sup_{\theta=\theta_0} \theta^{-n} e^{-\frac{\Sigma x_i}{\theta}} \, \mathbb{I}_{x_{(1)}>0}}{\sup_{\theta \in \mathbb{R}} \theta^{-n} e^{-\frac{\Sigma x_i}{\theta}} \, \mathbb{I}_{x_{(1)}>0}}$$

$$f(x) = \theta^{-n} e^{-\frac{n\bar{x}}{\theta}}.$$

$$\frac{e^{-\frac{\cancel{\Sigma x} \, n\bar{x}}{\theta}}}{\theta^{n}}$$

$$\frac{df}{d\theta} = -n\theta^{-n-1} e^{-\frac{n\bar{x}}{\theta}}.$$
$$+ \theta^{-n} e^{-\frac{n\bar{x}}{\theta}} \times \left(\frac{-n\bar{x}}{-\theta^2}\right).$$

$$= \theta^{-n} e^{-\frac{n\bar{x}}{\theta}} \left[\frac{n\bar{x}}{\theta^2} - \frac{n}{\theta}\right].$$

$$\theta = \bar{x}$$

$$\frac{\theta_0^{-n} e^{-\frac{n\bar{x}}{\theta_0}} \, \mathbb{I}_{x_{(1)}>0}}{(\bar{x})^{-n} e^{-n} \, \mathbb{I}_{x_{(1)}>0}}$$

$$\frac{1}{\theta} = t$$

$$\frac{e^{-n\bar{x}t} \times t^{n}}{}$$

$$\frac{t^n}{e^{\alpha t}} < 1$$

$$\Rightarrow t^n < e^{\alpha t}$$

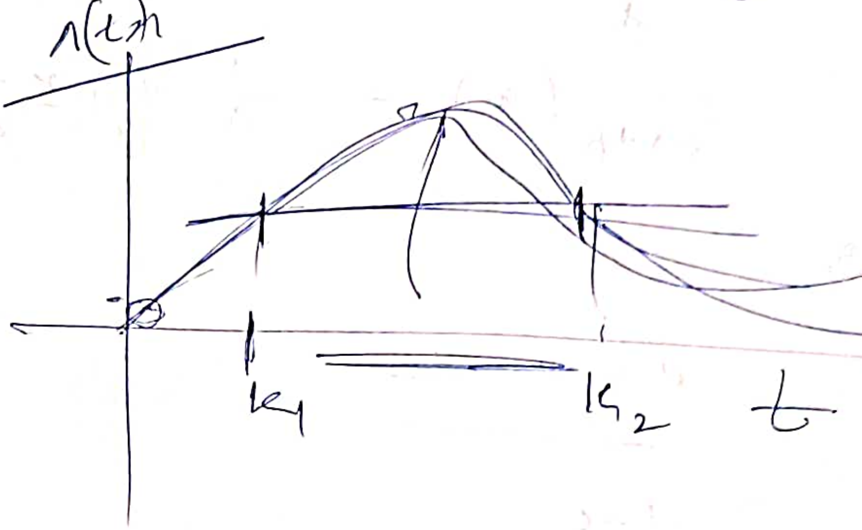
$$\Rightarrow \left(\frac{1}{\theta}\right)^n < e^{\alpha}$$

$$\Rightarrow \frac{1}{e^{\frac{\alpha}{\theta}}} \quad \frac{1}{\theta}$$

$$\theta^{n} \quad \theta^{-n} < e^{\alpha}$$

$(\bar{x})^{n} e^{-n\frac{x}{\theta_0}}$ .. $\qquad \bar{x} = \frac{1}{t}$

$t^{-n} e^{-\frac{n\left(\frac{1}{\theta_0}\right)}{t}}$



$\Lambda(x) > k$

$\cancel{\frac{1}{t}}\ t > k_1 \quad \& \quad t < k_2$

1 $\quad \bar{x} < k_1 \quad \& \quad \bar{x} > k_2$

γ $\quad \bar{x} = k_1 \quad , \quad \bar{x} = k_2$

0 $\quad k_2 \geq \bar{x} > k_1$

$P(\bar{x} < k_1) \quad P(\bar{x} > k_2) = \alpha$

$\bar{x}$

$$f_{ax+b}(x) = \frac{1}{a} f_X\left(\frac{x-b}{a}\right).$$

$\underline{\underline{8.}}$

$\mu = \mu_1 - \mu_2$

$\mu = 0$

$$\frac{1}{(2\pi\sigma^2)^{\frac{n}{2}}} e^{-\frac{1}{2\sigma^2}\sum(x_i - \mu_1)^2} \times \frac{1}{(2\pi\sigma^2)^{\frac{m}{2}}} e^{-\frac{1}{2\sigma^2}\sum(y_i - \mu_2)^2}$$

$$(2\pi\sigma^2)^{-\frac{n+m}{2}} e^{-\frac{1}{2\sigma^2}\left[\sum(x_i - \mu_1)^2 + \sum(y_i - \mu_2)^2\right]}$$

$$\{\mu_1 = \mu_2\}^c = \{(\mu, \mu) : \mu \in R\}^c$$

$$e^{-\frac{1}{2\sigma^2}\left[\sum(x_i - \mu)^2 + \sum(y_i - \mu - k)^2\right]}$$

$$\frac{n\bar{x} + m\bar{y}}{m+n}$$

$$\frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\sum\left(x_i - \left(\frac{n\bar{x}+m\bar{y}}{n+m}\right)\right)^2}$$

$$\times \frac{1}{(2\pi\sigma^2)^{m/2}} e^{-\frac{1}{2\sigma^2}\sum\left(y_i - \left(\frac{n\bar{x}+m\bar{y}}{n+m}\right)\right)^2}$$

$$> 1/k$$

$$e^{-\frac{1}{2\sigma^2}\sum(x_i-\bar{x})^2} \quad e^{-\frac{1}{2\sigma^2}\sum(y_i-\bar{y})^2}$$

$\downarrow$

$$SS^x_{total} + SS^y_{total} + SS^{xy}_{total} -$$

$$(x_i-\bar{x})^2 - \left(x_i - \frac{n\bar{x}+m\bar{y}}{n+m}\right)^2$$

$$\Lambda(x,y) = \frac{1}{\sigma^2}\sum\left(\frac{n\bar{x}+m\bar{y}}{n+m} - \bar{x}\right)\left(2x_i - \frac{2n\bar{x}+m\bar{y}}{n+m}\right)$$

$$+ \sum\left(\frac{n\bar{x}}{n+m}\right)\left(2y_i - \frac{2m\bar{y}+n\bar{x}}{n+m}\right)$$

$$\left(\frac{m\bar{y}}{n+m}\right)\left(2n\bar{x} - \frac{2n^2\bar{x}+mn\bar{y}}{n+m}\right) + \left(\frac{n\bar{x}}{n+m}\right)\left(2m\bar{y} - \frac{2m^2\bar{y}+mn\bar{x}}{n+m}\right)$$

$$\frac{4mn\bar{x}\bar{y}}{n+m} - 2n^2 m\bar{x}\bar{y} \quad \frac{m\bar{y}}{n+m}\left(\frac{2mn\bar{x}+mn\bar{y}}{n+m}\right)$$

$$\frac{mn(2\bar{x}+\bar{y})}{n+m} + \frac{n^2 m}{n+m}(2\bar{y}+\bar{x}) \quad + \frac{n\bar{x}}{n+m}\left(\frac{2mn\bar{y}+mn\bar{x}}{n+m}\right)$$

$$\frac{(n+m)\,2m\bar{x} + m\bar{y} + 2m\bar{y} + 2n\bar{x}}{(n+m)^2} \quad \frac{mn}{(n+m)^2}(2m$$

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\Sigma(x_i-\mu_1)}{2\sigma^2}} \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{\Sigma(y_i-\mu_2)}{2\sigma^2}}$$

n

$$\frac{1}{2\pi\sigma^2} e^{\frac{-1}{2\sigma^2}(n\bar{x}-n\mu_1+m\bar{y}-m\mu_2)}$$

$\frac{1}{\sigma^2} = t$

$$\frac{t}{2\pi} e^{-t\alpha} = 1$$

$$e^{-b\alpha} - \alpha b e^{-}$$

$$\frac{t}{2\pi} e$$

$t = \frac{1}{\alpha}$ $\frac{1}{\sigma^2} = \frac{1}{\alpha}$

$\sigma^2 = n\bar{x}$

$$\sigma^2 = n\bar{x} - n\mu_1 + m\bar{y} - m\mu_2$$