Department of Mathematics, Indian Institute of Technology Guwahati

MA 374 Financial Engineering Lab (January - May 2023 Semester)

Lab Examination - I

Marks: 40 | **February 23, 2023** | **Duration: 3 hours**

Instructions

- *Please create a folder with your IITG Roll Number and Email ID in your* **Desktop** *as per the following instructions: If your IITG Roll Number is '200123099' and email address is 'abc@iitg.ac.in' then you have to create a folder* **'99ma374ms.abc'**. *This is very important, since programs outside this folder will not be collected and it will as such be treated as, "The student has not submitted the outputs and programs".*
- *Put down all your observations and outputs (including graphs) of the questions asked in a single word document file. Finally create a pdf file from the word file.*
- *All your programs and output files (in word and pdf format) must be put inside the folder that you created as per the instructions above.*
- *Before leaving after the exam is over, make sure that your files have been received.*

---

1. Write a program to determine the initial price of an American put option in the binomial model with the following data: $S(0) = 80; T = 3; r = 0.06; \sigma = 0.3; K = 100$. Use the following values of u and d for your program:

$$u = e^{\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t}, \quad d = e^{-\sigma\sqrt{\Delta t}+(r-\frac{1}{2}\sigma^2)\Delta t},$$

where $\Delta t = \frac{T}{M}$ (M being the number of subintervals of the time interval $[0, T]$). Use the continuous compounding convention in all your calculations (i.e., both in $\tilde{p}$ and in the pricing formula). Note that $r = 0.06$ is the interest rate per time interval of one unit and here the option matures in three time units.

   (a) Obtain and tabulate the initial price of the option for different values of M starting from $M = 1$ to $M = 20$, in steps of 1.

   (b) Compute and plot the initial price of the option for different values of M starting from $M = 200$ to $M = 400$ in steps of 1.

   (c) Study the sensitivity of the initial option price by varying the parameters K, $S(0)$, r and σ one-at-a-time. Present your results in the form of figures and write down your observations in the report (take $M = 50$ for all cases).

   (d) Repeat part (a) and (b) above, assuming now that the option is an European put option. What are your observations?

2. Consider the data given in the file midsemdata2023.csv/midsemdata2023.xls (available in 'Data' folder in your home directory) showing the yearly prices of ten stocks along with the values of the market index.

   (a) Find the mean return vector and covariance matrix for returns of all the ten stocks. Also find the mean return and variance of the market index return.

   (b) Compute the beta, alpha, the systematic risk and the unsystematic (diversifiable) risk of each of the stocks.

(c) Rank the stocks (in descending order) in terms of the excess return over beta ratio. You may take the risk-free rate to be 4%. Now, rank the stocks (in descending order) in terms of the excess return over risk (i.e. standard deviation) ratio. What do you observe?

(d) Plot the return and risk of the ten stocks in a σ-μ (i.e., risk-return) graph. Mark each of these 10 points clearly in the graph. Identify the dominating asset(s) and the corresponding dominated asset(s).

(e) Determine the minimum variance portfolio (i.e., weights, return and risk). How does this return and risk compare with that of the market (index)?

Note: No computations in Excel sheet. Your program should read the file and do all the calculations.

Department of Mathematics, Indian Institute of Technology Guwahati

MA 374 Financial Engineering Lab (January - May 2023 Semester)

Lab Examination - II

Marks: 40 | **April 27, 2023** | **Duration: 3 hours**

Instructions

- *Please create a folder with your IITG Roll Number and Email ID in your* **Desktop** *as per the following instructions: If your IITG Roll Number is '200123099' and email address is 'abc@iitg.ac.in' then you have to create a folder* **'99ma374es.abc'**. *This is very important, since programs outside this folder will not be collected and it will as such be treated as, "The student has not submitted the outputs and programs".*
- *Put down all your observations and outputs (including graphs) of the questions asked in a single word document file. Finally create a pdf file from the word file.*
- *All your programs and output files (in word and pdf format) must be put inside the folder that you created as per the instructions above.*
- *Before leaving after the exam is over, make sure that your files have been received.*

---

1. **25 marks**

Consider a classical BSM market wherein the price process S of the risky asset follows a geometric Brownian motion (GBM) with the initial price $S(0) = 100$ and with the parameters for the GBM as: drift/risk-free rate $r = 8\%$ and volatility $\sigma = 20\%$.

(a) Generate and plot 10 sample paths of the process S using 10^3 equally spaced time points over $[0, 3]$. Using the same set of random numbers, plot now 10 paths of the process S but with $\sigma = 40\%$. What do you observe? Explain how you are ensuring that you are using the same set of random numbers for both the graphs.

(b) For each of $M \in \{50, 100, 500, 1000\}$, generate M paths for the process S using 10^3 equally spaced time points over $[0, 3]$ and plot the estimated mean and variance against time. Also, in the same graph, plot the theoretical values against time (write down the formula). How do the estimated values match with the theoretical ones? Also, plot the probability distribution of S_3 with $M = 1000$.

(c) Consider a one year European type lookback option on this asset whose payoff at maturity is given by

$$V(T) = S_{\max} - S_{\min},$$

where $S_{\max} = \max\{S(t) : t \in [0, T]\}$ and $S_{\min} = \min\{S(t) : t \in [0, T]\}$.

Let Δt and M be the interval length and the number of sample paths generated, respectively. Now compute the initial price (through simulation) of the lookback option for each of $M = 10^2, 10^3, 10^4$ with each of $\Delta t = 10^{-2}, 10^{-3}, 10^{-4}$. Find the sampling variance for each of the estimates. Tabulate the values in the following format:

M	Δt	Price of Option	Sampling Variance

2. **15 marks**

Consider a dividend paying asset whose price process $\{S_t\}$ follows a GBM with initial price $S_0 = 100$, drift/risk free rate $r = 10\%$ and volatility $\sigma = 25\%$. The dividends are paid in lumps at the end of three, six and nine months at the rate of 2% on each occasion. Consider an one-year European call option on this asset with strike $K = 102$. What is the initial price of this option and how does it vary as a function of model parameters? Present your results in the form of graphs.