# Statistical Inference and Multivariate Analysis (MA324)

## LECTURE SLIDES
## Lecture 12

Method of Moment Estimator

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# Method of Moment Estimator (MME):

MME was first introduced by Karl Pearson in the year 1902. The basic method can be summarized in following algorithm:

1. Suppose that we have a RS of size $n$ from a population with PMF/PDF $f(x; \boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_k)$ is the unknown parameter vector. We want to find estimator of $\boldsymbol{\theta}$.
2. Calculate first $k$ (number of unknown parameters) **moments** $\mu'_1, \ldots, \mu'_k$ of $f(x; \boldsymbol{\theta})$, where $\mu'_r = E_{\boldsymbol{\theta}}(X^r)$.
3. Calculate first $k$ **sample moments** $m'_1, \ldots, m'_k$, where $m'_r = \frac{1}{n}\sum_{i=1}^{n} X_i^r$.
4. Equate $\mu'_r = m'_r$ for $r = 1, 2, \ldots, k$.
5. **Solve** the system of $k$ equations (if they are consistent) for $\theta_i$'s. The solutions are the MMEs of the unknown parameters.

# Examples:

Example 1: Let $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} Bernoulli(\theta)$, $\theta \in (0, 1) = \Theta$. Here, we have one parameter $\theta$. Thus, $k = 1$. $E(X_1) = \theta$. Hence, we get the MME of $\theta$ is $\widehat{\theta} = \bar{X}$.

Example 2: Let $X_1, X_2, \ldots, X_n \overset{i.i.d.}{\sim} N(\mu, \sigma^2)$, $\boldsymbol{\theta} = (\mu, \sigma^2) \in \mathbb{R} \times \mathbb{R}^+ = \Theta$. Here $k = 2$, $E(X) = \mu$, and $E(X^2) = \sigma^2 + \mu^2$. Hence, we get the MMEs of $\mu$ and $\sigma^2$ are $\widehat{\mu} = \bar{X}$ and $\widehat{\sigma^2} = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2$, respectively.

# Examples:

Example 3: Let $X_1, X_2, \ldots, X_n \stackrel{i.i.d.}{\sim} N(0, \sigma^2)$, $\sigma > 0$. Here $k = 1$. However, as $E(X) = 0$, equating $E(X) = \bar{X}$ does not provide any solution (inconsistent). Alternatively, we can find $E(X^2) = \sigma^2$ and equate to $m_2'$ to obtain MME of $\sigma^2$ as $\widehat{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n} X_i^2$.

Example 4: Let $X_1, X_2, \ldots, X_n \stackrel{i.i.d.}{\sim} N(\theta, \theta^2)$, $\theta > 0$. Here $k = 1$. $E(X) = \theta$. Equating $E(X) = \bar{X}$, we get MME of $\theta$ is $\widehat{\theta} = \bar{X}$. However, this may not be a meaningful estimator as $\bar{X}$ can be negative with positive probability, while $\theta > 0$.

Remark: Previous two examples show that there are some degrees of arbitrariness in this method.