# Statistical Inference and Multivariate Analysis (MA324)

## LECTURE SLIDES
## Lecture 21

p-value

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# $p$-value

We have discussed hypothesis testing at a **fixed level** $\alpha$. Note that this approach is one of the two standard approaches to the evaluation of hypotheses. To explain the other, first we need to define nested test.

Def: [**Nested Test**] For varying level $\alpha$, assume that the test is a non-randomized test with critical region $R_\alpha$. The test is called nested if

$$R_\alpha \subset R_{\alpha'} \quad \text{for all } \alpha < \alpha'.$$

When a test is nested, it is good practice to determine not only whether the null hypothesis is accepted or rejected at a given level $\alpha$, but also to **determine the smallest level at which the null hypothesis would be rejected** for the given observation. This smallest level is called $p$-value.

Def: [$p$**-value**] The $p$-value of a nested test is defined by

$$\widehat{p} = \widehat{p}(\boldsymbol{X}) = \inf \{\alpha \in [0,\, 1] : \boldsymbol{X} \in R_\alpha\}\,.$$

The $p$-value provides an idea of how strong the data contradict the null hypothesis. It also enables other to reach a verdict based on the level of their choice. **If $p$-value is smaller than $\alpha$, we reject the null hypothesis**. Otherwise, we accept the null hypothesis.

Example 1: Let $X_1, X_2, \ldots, X_n$ be a RS form a population having normal distribution with unknown mean $\mu$ and known variance $\sigma^2$. Consider $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. We have seen that the critical region of likelihood ratio level $\alpha$ test is given by

$$R_\alpha = \left\{ \boldsymbol{x} \in \mathbb{R}^n : \sqrt{n}\frac{|\overline{x} - \mu_0|}{\sigma} > z_{\frac{\alpha}{2}} \right\}.$$

As for $\alpha < \alpha'$, $z_{\frac{\alpha}{2}} > z_{\frac{\alpha'}{2}}$, $R_\alpha \subset R_{\alpha'}$. Therefore, the test is a nested test and we can talk about $p$-value.