

Q1

```
clear;  
format long e
```

Q1 (a)

```
clear;
```

Eg. - 1

```
U = toeplitz([1 linspace(0, 0, 2)], 1:3)
```

```
U =  
    1    2    3  
    0    1    2  
    0    0    1
```

```
b = [6 3 1]'
```

```
b =  
    6  
    3  
    1
```

```
x = colbackward(U, b)
```

```
x =  
    1  
    1  
    1
```

Eg. - 2

```
U = triu(randn(3))
```

```
U =  
 -7.341691126967387e-01    4.263875574089450e-01    2.023690886603053e+00  
      0    -3.728087417235042e-01   -2.258353970496191e+00  
      0      0      2.229445680456899e+00
```

```
b = randn(3, 1)
```

```
b =  
    3.375637006131064e-01  
    1.000060819589125e+00  
   -1.664164474987060e+00
```

```
x = colbackward(U, b)
```

```
x =  
   -1.449144160542513e+00  
    1.839233107066967e+00  
   -7.464476437237122e-01
```

Q1 (b)

```
clear;
```

Eg. - 1

```
L = toeplitz(1:3, [1 linspace(0, 0, 2)])
```


```
L =  
    1    0    0  
    2    1    0  
    3    2    1
```

```
b = [1 3 6]'
```

```
b =  
    1  
    3  
    6
```

```
x = rowforward(L, b)
```

```
x =  
    1  
    1  
    1
```



Eg. - 2

```
U = tril(randn(3))
```

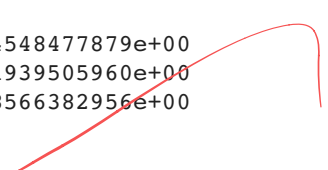
```
U =  
-5.900345642052215e-01    0    0  
-2.780641637653093e-01    4.716343264163027e-01    0  
 4.227156912204783e-01   -1.212847199674459e+00    3.270599671770875e-01
```

```
b = randn(3, 1)
```

```
b =  
 1.082633504236756e+00  
 1.006077110819051e+00  
-6.509077365977526e-01
```

```
x = colbackward(U, b)
```

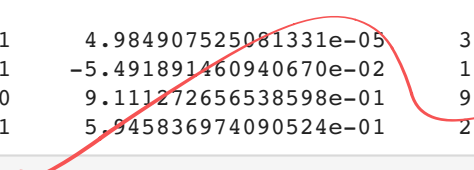
```
x =  
-1.834864548477879e+00  
 2.133171939505960e+00  
-1.990178566382956e+00
```



Q2

```
clear;  
A = randn(4)
```

```
A =  
 2.570561574339689e-01    4.984907525081331e-05    3.502011738745352e-01   -6.903611031112258e-01  
-9.443778064042190e-01   -5.491891460940670e-02    1.250251228304996e+00   -6.515536417502810e-01  
-1.321788521392564e+00    9.111272656538598e-01    9.297894585577157e-01    1.192101870531270e+00  
 9.248259334937059e-01    5.945836974090524e-01    2.397632570585800e-01   -1.611830388677811e+00
```



```
[L, U] = genp(A)
```

```

L =
    1.000000000000000e+00      0      0      0
   -3.673819043400294e+00      1.000000000000000e+00      0      0
   -5.142022407038032e+00     -1.665060079338925e+01      1.000000000000000e+00      0
    3.597758337032911e+00     -1.085952137373259e+01      5.899136778749531e-01      1.000000000000000e+00

U =
    2.570561574339689e-01      4.984907525081331e-05      3.502011738745352e-01     -6.903611031112258e-01
         0      -5.473577812745437e-02      2.536826969906401e+00     -3.187815409183137e+00
         0         0         4.497022489940636e+01     -5.543679217193735e+01
         0         0         0         -1.043305586197569e+00

```

Q3

```
clear;
```

Q3 (a)

```
A = [10^-20, 1; 1, 1];
[L, U] = genp(A)
```

```

L =
    1.000000000000000e+00      0
    9.999999999999998e+19      1.000000000000000e+00

U =
    1.000000000000000e-20      1.000000000000000e+00
         0     -9.999999999999998e+19

```

```
decomposition_difference = A - L*U
```

```

decomposition_difference =
         0      0
    1.110223024625157e-16      1.000000000000000e+00

```

Q3 (b)

```

% Ax = b => LUX = b => Ly = b, Ux = y
b = [1 0]';
y = rowforward(L, b);
x_c = colbackward(U, y)

```

```

x_c =
    0
    1

```

```
x = [-1 1]' / (1 - 10^-20)
```

```

x =
   -1
    1

```

```
norm_diff = norm(x_c - x) / norm(x)
```

```

norm_diff =
    7.071067811865475e-01

```

- The step where things typically start to go wrong is in the forward substitution due to the very small pivot element 10^{-20} in the matrix A. This small value can lead to numerical instability, causing large errors in the solution.

Q4

```
clear;

for iter = 1:5
    A = randn(iter*5, iter*5);

    [L_gepp, U_gepp, p_gepp, ~] = gepp(A);
    [L_matlab, U_matlab, P_matlab] = lu(A); lu(A, 'vector')

    P_gepp = eye(size(A));
    P_gepp = P_gepp(p_gepp, :);

    norm_L = norm(L_gepp - L_matlab);
    norm_U = norm(U_gepp - U_matlab);
    norm_P = norm(P_gepp - P_matlab);

    fprintf('Iteration %d', iter);
    % disp('L from custom gepp and matlab:');
    % L_gepp
    % L_matlab
    % disp('U from custom gepp and matlab:');
    % U_gepp
    % U_matlab
    % disp('P from custom gepp and matlab:');
    % P_gepp
    % P_matlab
    fprintf(' - Norm Difference in L: %e\n', norm_L);
    fprintf(' - Norm Difference in U: %e\n', norm_U);
    fprintf(' - Norm Difference in P: %e\n', norm_P);
end
```

```
Iteration 1
- Norm Difference in L: 1.387793e-16
- Norm Difference in U: 4.440892e-16
- Norm Difference in P: 0.000000e+00
Iteration 2
- Norm Difference in L: 5.077390e-16
- Norm Difference in U: 7.906692e-16
- Norm Difference in P: 0.000000e+00
Iteration 3
- Norm Difference in L: 4.070732e-15
- Norm Difference in U: 4.239090e-15
- Norm Difference in P: 0.000000e+00
Iteration 4
- Norm Difference in L: 3.066433e-15
- Norm Difference in U: 7.747275e-15
```

```

- Norm Difference in P: 0.000000e+00
Iteration 5
- Norm Difference in L: 5.182451e-15
- Norm Difference in U: 1.067127e-14
- Norm Difference in P: 0.000000e+00

```

Q5

```
clear;
```

Q5 Part (a)

```

for iter = 1:5
    A = randn(iter*2, iter*2);
    b = randn(iter*2, 1);

    x_gepp = geppsolve(A, b);
    x_matlab = A \ b;
    relative_difference = norm(x_gepp - x_matlab) / norm(x_matlab);

    fprintf('Iteration %d\n', iter);
    % x_gepp
    % x_matlab
    fprintf(' - Relative difference between geppsolve and A\b: %e\n',
relative_difference);
end

```

```

Iteration 1
- Relative difference between geppsolve and A\b: 5.340211e-17
Iteration 2
- Relative difference between geppsolve and A\b: 2.844027e-16
Iteration 3
- Relative difference between geppsolve and A\b: 2.846213e-16
Iteration 4
- Relative difference between geppsolve and A\b: 9.441142e-16
Iteration 5
- Relative difference between geppsolve and A\b: 8.308429e-16

```

Q5 Part (b)

```

A = [10^-20, 1; 1, 1];
[L, U, p, ~] = gepp(A)

```

```

L =
    1.0000000000000000e+00    0
    1.0000000000000000e-20    1.0000000000000000e+00
U =
    1    1
    0    1
p =
    2
    1

```

```
decomposition_difference = A(p,:) - L*U
```

```
decomposition_difference =
```

```
0 0
0 0
```

```
b = [1 0]';
x_c = geppsolve(A, b)
```

```
x_c =
-1
1
```

```
x = [-1 1]' / (1 - 10^-20)
```

```
x =
-1
1
```

```
norm_diff = norm(x_c - x) / norm(x)
```

```
norm_diff =
0
```

- gepp is able to produce much better results in the case of Q3, with the errors being insignificant.

Q6

```
clear;
for iter = 1:5
    fprintf('Iteration %d\n', iter);
    A = randn(5, 5)
    det_A = mydet(A);
    actual_det_A = det(A);
    fprintf(' - mydet(A): %e', det_A);
    fprintf(' - matlab det(A): %e', actual_det_A);
    fprintf(' - Error: %e', abs(actual_det_A - det_A));
end
```

Iteration 1

```
A =
-1.195411812608376e+00 -6.857045336988162e-01 -2.974925334908478e-01 -1.398802093251064e+00
-1.334688387232651e-02 -9.415243349555418e-01 -2.714974866653881e-01 7.886623415581502e-01
-1.808717136297500e+00 8.433147225737069e-01 6.310417162625513e-01 1.095374445695478e+00
-4.784692698351494e-01 -1.692577689976957e+00 4.917817629800340e-01 6.840426290518494e-01
-5.135679079202427e-01 7.256787280929793e-01 3.122440315664763e+00 2.065446432873658e-01
- mydet(A): 6.750984e+01
- matlab det(A): 6.750984e+01
- Error: 0.000000e+00
```

Iteration 2

```
A =
-3.337149145275614e-01 -2.049825403046277e+00 -2.171863685987873e+00 6.041078532827356e-01
6.380393646156524e-02 4.336478235909531e-02 1.376923291073182e+00 -3.528233742647947e-01
-8.043663802464812e-02 -7.490150894064554e-01 -7.540362453283391e-01 -2.785748361329278e+00
-1.472826588011795e+00 7.411353260974530e-01 -1.443598198376165e+00 1.446397323976541e+00
1.624437188042158e+00 7.434300922580746e-02 -7.727278784050305e-01 -3.370780002527350e-01
- mydet(A): 5.747260e+01
- matlab det(A): 5.747260e+01
- Error: 7.105427e-15
```

Iteration 3

```

A =
-1.435127816125753e+00    1.322267749225890e+00    -6.585141903681243e-01    2.796553966921837e-01
-6.577116192785718e-01    1.707785191460827e-01    -1.388991550629430e+00    2.234843118537121e-01
-1.286463762790578e+00    5.365315018855938e-01    -1.116604948273796e+00    -2.977428140639953e-01
-4.715015425553275e-01    6.013374578678989e-01    5.489961711477550e-01    -1.720468700822851e+00
-1.484967735796905e+00    1.685771031293984e+00    1.729276969412223e-01    1.987548404507014e-01
- mydet(A): 4.301823e+00
- matlab det(A): 4.301823e+00
- Error: 8.881784e-16
Iteration 4
A =
-2.661048167405558e-02    1.360596268499852e-01    2.303922941012403e+00    1.268163704069089e+00
2.622252047475515e-01    -5.429699608628925e-01    1.125960109483281e+00    5.561766298182860e-02
1.099483089910364e+00    1.898335299918788e-01    -1.491095706258468e+00    -1.491649807834865e+00
1.095342266854256e+00    -1.768144219531031e+00    1.299364011637519e+00    -1.098669499383252e+00
1.285337839163083e-01    5.260763010757622e-01    -1.664134805582986e+00    2.000561678867744e-01
- mydet(A): -3.888453e+00
- matlab det(A): -3.888453e+00
- Error: 0.000000e+00
Iteration 5
A =
8.943811495704282e-01    -9.463707012173982e-01    6.613019182999430e-01    6.057509333100209e-01
7.821033892312837e-02    2.811939762783778e-01    9.055677538507401e-01    2.028369299015762e+00
1.450818828240354e+00    2.343432279205171e-01    -1.170451590152022e+00    -3.522273390901558e-01
-1.196558034844856e-01    -4.568070294812646e-01    -5.391688062756526e-01    -1.765604517202779e-01
6.259096830391265e-01    -3.595816710475149e-01    1.376459133783378e+00    -2.042281427116826e-01
- mydet(A): -3.002746e+01
- matlab det(A): -3.002746e+01
- Error: 7.105427e-15

```

Functions defined :-

Q1

```

function x = colbackward(U,b)
    n = size(b, 1);
    x = zeros(n, 1);
    for i = n:-1:1
        if U(i, i) == 0
            error('Provided U is singular');
        end
        x(i) = b(i) / U(i, i);
        b(1 : i - 1) = b(1 : i - 1) - U(1 : i-1, i)*x(i);
    end
end
function x = rowforward(L,b)
    n = size(b, 1);
    x = zeros(n, 1);
    for i = 1:n
        if L(i, i) == 0
            error('Provided L is singular');
        end
        x(i) = (b(i) - L(i, 1:i-1)*x(1:i-1)) / L(i, i);
    end
end

```

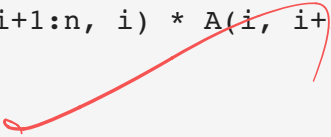
Q2

```
function [L, U] = genp(A)
    n = size(A, 1);

    for i = 1:n-1
        if A(i, i) == 0
            error('Zero pivot encountered');
        end

        A(i+1:n, i) = A(i+1:n, i) / A(i, i);
        A(i+1:n, i+1:n) = A(i+1:n, i+1:n) - A(i+1:n, i) * A(i, i+1:n);
    end

    L = eye(n) + tril(A, -1);
    U = triu(A);
end
```



Q4

```
function [L, U, p, detP] = gepp(A)
    detP = 1;
    n = size(A, 1);
    p = (1:n)';

    for i = 1:n-1
        [~, max_row] = max(abs(A(i:n, i)));
        max_row = max_row + i - 1;

        if max_row ~= i
            detP = -detP;
            A([i, max_row], :) = A([max_row, i], :);
            p([i, max_row]) = p([max_row, i]);
        end


        if A(i, i) == 0
            warning('Matrix is singular or nearly singular. Pivoting failed.');
```

```
            break; % If no pivot is found, exit the loop gracefully
        end
```

```
        A(i+1:n, i) = A(i+1:n, i) / A(i, i);
        A(i+1:n, i+1:n) = A(i+1:n, i+1:n) - A(i+1:n, i) * A(i, i+1:n);
    end

    L = eye(n) + tril(A, -1);
    U = triu(A);
end
```

read 4(b)



Q5

```
function x = geppsolve(A, b)
    [L, U, p, ~] = gepp(A);

    % PA = LU, Ax = b => LUX = Pb => Ly = Pb, Ux = y
    b = b(p);
    y = rowforward(L, b);
    x = colbackward(U, y);
end
```

Q6

```
function d = mydet(A)
    [~, U, ~, detP] = gepp(A);
    detU = prod(diag(U));
    d = detP * detU;
end
```