QUIZ 1 for MA224 (Real Analysis)

Total marks 20 — Date and Time: 15/02/21, 9:00 - 9:50 AM

Problem 1: Let (X, d) be a compact metric space. Let A and B be two closed subsets of X such that $A \cap B = \phi$. Show that there exists two open sets U and V such that $A \subset U$, $B \subset V$ and $U \cap V = \phi$. **(Marks 6)**

Problem 2: Let (X, d) be a complete metric space. Let $T : X \to X$ be a continuous function such that $\sum_{n=1}^{\infty} d(T^n(x), T^n(y)) < \infty$ for all $x, y \in X$. Then prove that T has a unique fixed point. **(Marks 4)**

Problem 3: Let (X, d) be a metric space. Suppose that $f : X \to X$ is a function with the property that $\{f(x_n)\}_{n\geq 1}$ is Cauchy whenever $\{x_n\}_{n\geq 1}$ is Cauchy. Show that f is continuous on X. **(Marks 3)**

Problem 4a: Prove or disprove: If $f : \mathbb{R} \to \mathbb{R}$ is bounded and continuous, then f is also uniformly continuous.

Problem 4b: Let A be a subset of a metric space (X, d). Then prove that A^0 (interior of A) is an open set and $\bar{A}$ (closure of A) is a closed set.

(Marks 3+4=7)

Important : Step marking is there, so write the steps clearly, that will fetch you some marks even if the solution is not completely correct. Solution without proper justification will fetch no marks.