

```
format long e
```

### Question 3 (a)

```
A = [1e-20, 1; 1, 1];  
[L, U] = genp(A);  
q3a_ans = A - L * U
```

```
q3a_ans =  
    0    0  
    0    1
```

### Question 3 (b)

```
b = [1; 0];  
xc = solve(A, b);  
x = A \ b;  
q3b_ans = norm(xc - x) / norm(x)
```

*hand calculation*

```
q3b_ans =  
7.071067811865475e-01
```

We can conclude that GENP may not give a correct solution when the pivot value is very small even though the function does not exit.

In the first step itself, dividing by  $1e-20$  makes the last row quite large.

### Question 4

```
L_norm = zeros(1, 10);  
U_norm = zeros(1, 10);  
for i = 1:10  
    A = randn(10);  
    [L_hat, U_hat, ~] = gepp(A);  
    [L, U, ~] = lu(A);  
    L_norm(i) = norm(L_hat - L) / norm(L);  
    U_norm(i) = norm(U_hat - U) / norm(U);  
end  
L_norm
```

```
L_norm =  
1.948829867736452e-16    6.768878265393872e-16    1.371431919900664e-16    2.214909043237717e-16
```

```
U_norm
```

```
U_norm =  
2.734446865268963e-16    3.136616222272667e-16    2.045353789745362e-16    2.152617336200437e-16
```

### Question 5 (a)

```
x_norm = zeros(1, 10);  
for i = 1:10  
    A = randn(10);  
    b = randn(10, 1);
```

```

x = geppsolve(A, b);
x_hat = A \ b;
x_norm(i) = norm(x_hat - x) / norm(x);
end
x_norm

```

```

x_norm =
    4.179645902363428e-16    3.321549456635863e-14    4.902262520206317e-16    7.687834629317253e-16

```

Question 5 (b)

```

A = [1e-20, 1; 1, 1];
[L, U, p, ~] = gepp(A);
q5b_ans_1 = A(p, :) - L * U

```

```

q5b_ans_1 =
    0    0
    0    0

```

```

b = [1; 0];
xc = geppsolve(A, b);
x = A \ b;
q5b_ans_2 = norm(xc - x) / norm(x)

```

```

q5b_ans_2 =
    0

```

As we can see, the answer obtained by GEPP is far more accurate than the answer obtained by GENP.

Question 6

```

A = randn(10);
determinant = mydet(A)

```

```

determinant =
   -8.621549577366590e+00

```

```

determinant_matlab = det(A)

```

```

determinant_matlab =
   -8.621549577367128e+00

```

## Functions

Question 1 (a)

```

function x = colbackward(U, b)
[n, ~] = size(U);
x = zeros(n, 1);
for i = n:-1:1
    x(i) = b(i) / U(i,i);
    b(1:i-1) = b(1:i-1) - U(1:i-1,i) * x(i);
end
end

```

what if  $U(i,i) = 0$ ?

Question 1 (b)

```
function x = rowforward(L, b)
    x = zeros(size(b));
    n = length(x);
    x(1) = b(1) / L(1, 1);
    for i = 2:n
        sum = 0;
        for j = 1:(i - 1)
            sum = sum + L(i, j) * x(j);
        end
        x(i) = (b(i) - sum) / L(i, i);
    end
end
```

what if  $L(i,i) == 0$ ?

Question 2

```
function [L, U] = genp(A)
    sz = size(A);
    n = sz(1);
    for k = 1:(n - 1)
        if A(k, k) ~= 0
            A((k + 1):n, k) = A((k + 1):n, k) / A(k, k);
        else
            exit('zero pivot encountered');
        end
        A((k + 1):n, (k + 1):n) = A((k + 1):n, (k + 1):n) - A((k + 1):n, k)
        * A(k, (k + 1):n);
    end
    L = tril(A, -1) + eye(size(A));
    U = triu(A);
end
```

Question 3 (b)

```
function x = solve(A, b)
    [L, U] = genp(A);
    y = rowforward(L, b);
    x = colbackward(U, y);
end
```

Question 4

```
function [L, U, p, sign] = gepp(A)
    [n, ~] = size(A);
    p = (1: n);
    sign = 1;
    for k = 1: n - 1
        [~, idx] = max(abs(A(k: end, k)));
        idx = idx + k - 1;
```

```

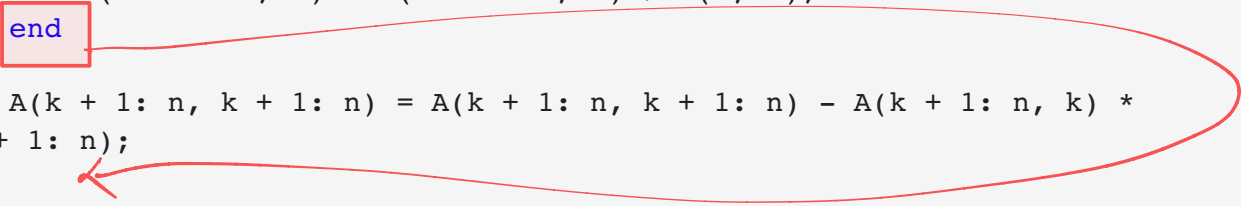
    if idx ~= k
        sign = -1 * sign;
        permutation = [idx, k];
        A(permutation, :) = A(permutation([2, 1]), :);
        p(permutation) = p(permutation([2, 1]));
    end

    if A(k, k) ~= 0
        A(k + 1: n, k) = A(k + 1: n, k) / A(k, k);
    end

    A(k + 1: n, k + 1: n) = A(k + 1: n, k + 1: n) - A(k + 1: n, k) *
A(k, k + 1: n);
end

L = eye(n) + tril(A, -1);
U = triu(A);
end

```




Question 5 (a)

```

function x = geppsolve(A, b)
    [L, U, p, ~] = gepp(A);
    y = rowforward(L, b(p, :));
    x = colbackward(U, y);
end

```



Question 6

```

function d = mydet(A)
    [~, U, ~, sign] = gepp(A);
    [n, ~] = size(A);
    d = sign;

    for i = 1: n
        d = d * U(i, i);
    end
end

```

