Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA324)
Problem Set 06

1. Let $X_i, \ldots, X_n$ be a RS form an exponential distribution with location parameter θ. That is the PDF of X_i's are given by $f(x) = e^{-(x-\theta)} I_{(\theta, \infty)}(x)$. Find a level α likelihood ratio test for the null hypothesis $H_0 : \theta \leq \theta_0$ against the alternative hypothesis $H_1 : \theta > \theta_0$, where θ_0 is a given real number.

2. Suppose that $X_1, X_2, \ldots, X_n$ are *i.i.d.* random variables form a exponential probability density function
$$f(x;\, \theta) = \theta^{-1} e^{-x/\theta} I_{(0, \infty)}(x),$$
where $\theta > 0$ is assumed unknown. With preassigned $\alpha \in (0,\, 1)$, derive a level α likelihood ratio test for $H_0 : \theta = \theta_0 (> 0)$ against $H_1 : \theta \neq \theta_0$.

3. Let $X_1, X_2, \ldots, X_n$ be a RS from $N(\mu_1, \sigma^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a RS from $N(\mu_2, \sigma^2)$ distribution. Also, assume that X_i's are independent of Y_j's. Derive a level α LRT for $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

4. Let $X_1, X_2, \ldots, X_n$ be a RS from $N(\mu_1, \sigma_1^2)$ distribution and $Y_1, Y_2, \ldots, Y_m$ be a RS from $N(\mu_2, \sigma_2^2)$ distribution. Also, assume that X_i's are independent of Y_j's. Derive a level α LRT for $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$.

5. Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample from a bivaiate normal distribution $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Derive a level α LRT for
   (a) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 > \mu_2$.
   (b) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 < \mu_2$.
   (c) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$.

6. Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be a random sample of size $n \geq 3$ from a bivaiate normal distribution $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Derive a level α LRT for $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$. You may use the fact that
$$\frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim t_{n-2} \quad \text{if} \quad \rho = 0.$$