INSTRUCTIONS
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1. Attempt **all** the questions.

2. There is **no credit** for a solution if the appropriate work is not shown, even if the answer is correct.

3. Notations are standard and same as used during the lectures.

4. No question requires any clarification from the instructor. Even if a question has an error or incomplete data, the students are advised to write answer according to their understanding or write reasons for why it is not possible to solve the question partially or completely by citing errors/insufficient data.

5. Write the answers on blank papers (preferably white). You must write your name and roll number on the first page. Every page (both sides) must be self-attested and numbered. Please scan all the pages and make a single PDF file. It is your responsibility to check quality of the PDF file so that it is easily readable. Upload the file through Microsoft Teams against the assignment. The portal will remain active till 16:05 hours and you need to complete the submission procedure by 16:05 hours. If you submit through any other means, **a penalty of 10 marks will be imposed**.

6. The question paper has **2** page. This examination has **3** questions, for a total of **20** points.

QUESTIONS
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1. (a) (5 points) Let $X_{i1}, X_{i2}, \ldots, X_{in_i}$ be random samples from $N(\mu_i, \sigma^2)$ distribution, $i = 1, 2$. Also, assume that the X_{1j}'s are independent of the X_{2j}'s. Here, we assume that $\mu_1 \in \mathbb{R}, \mu_2 \in \mathbb{R}, \sigma > 0$ are all unknown parameters. With preassigned $\alpha \in (0, 1)$ and real number D, find (in an implementable form) likelihood ratio level α test for $H_0 : \mu_1 - \mu_2 = D$ against $H_1 : \mu_1 - \mu_2 \neq D$.

   (b) (2 points) Two types of cars were compared for the braking distances. Test runs were made for each car in a driving range. Once a car reached the stable speed of 95 kilometers per hour, the brakes were applied. The distance (in feet) each car traveled from the moment the brakes were applied to the moment the car came to a complete stop are recorded. The summary statistics are shown below:

Car	Sample size	Sample mean	Sample standard deviation
Make A	12	37.1	3.1
Make B	10	39.6	4.3

   Assume that the elapsed times are distributed as $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, respectively, for the Make A and Make B with all parameters unknown. Test at 5% level whether the average braking distance of the two makes are significantly different. You may use the fact that $\tau_{0.1} = 1.28, \tau_{0.05} = 1.64, \tau_{0.025} = 1.96$. Please clearly write the hypotheses and your final decision with justification.

2. (7 points) Let

$$(1.25,\ 1.14,\ 0.62,\ 0.99,\ 0.067,\ 0.03,\ 1.16,\ 0.46,\ 0.61,\ 0.60)$$

and

$$(1.07,\ 0.21,\ 1.47,\ 1.82,\ 2.44,\ 0.28,\ 0.94,\ 1.91,\ 2.72,\ 2.22)$$

be realizations of independent random samples of size 10 from $U(0, \theta_1)$ and $U(0, \theta_2)$ distribution, respectively. Assuming that both $\theta_1 \in \mathbb{R}$ and $\theta_2 \in \mathbb{R}$ are unknown, construct a 95% confidence interval for $\dfrac{\theta_1}{\theta_1 + \theta_2}$. Note that you need to provide numerical limits of the confidence interval.

3. It is difficult and time consuming to measure directly the amount of protein in a liver sample. As a result, medical laboratories often make use of the fact that the amount of protein is related to the amount of light that would be absorbed by the sample. A spectrometer that emits light is shined on a solution that contains the liver sample and the amount of light absorbed is then used to estimate the amount of protein. The above procedure was tried on five sampled having known amounts of protein, with the following data.

Light Absorbed	Amount of Protein (mg)
0.44	2
0.82	16
1.20	30
1.61	46
1.83	55

For all the parts below, please write the steps clearly mentioning statistical modeling and expressions. No need to derive the least square estimators.

(a) (2 points) Compute the coefficients of a linear regression using the above data.

(b) (2 points) Determine the coefficient of determination. Interpret the result.

(c) (2 points) Test, at 5% level, the significance of the linear regression, by stating null and alternative hypotheses clearly. You do not need to derive the test.