MA 225 | End-semester Examination
Probability Theory and Random Processes | 9:00–10:00 IST
IIT GUWAHATI | November 25, 2022

PART A

Instructions:

1. ***Be very careful while bubbling the Roll No. and answers. Once bubbled, it cannot be changed. Bubble properly, otherwise computer will not be able to detect it. Use black or blue ball pen only for bubbling.***

Name and Roll Number

Please bubble your Roll No:

○0 ○0 ○0 ○0 ○0 ○0 ○0 ○0 ○0
○1 ○1 ○1 ○1 ○1 ○1 ○1 ○1 ○1
○2 ○2 ○2 ○2 ○2 ○2 ○2 ○2 ○2
○3 ○3 ○3 ○3 ○3 ○3 ○3 ○3 ○3
○4 ○4 ○4 ○4 ○4 ○4 ○4 ○4 ○4
○5 ○5 ○5 ○5 ○5 ○5 ○5 ○5 ○5
○6 ○6 ○6 ○6 ○6 ○6 ○6 ○6 ○6
○7 ○7 ○7 ○7 ○7 ○7 ○7 ○7 ○7
○8 ○8 ○8 ○8 ○8 ○8 ○8 ○8 ○8
○9 ○9 ○9 ○9 ○9 ○9 ○9 ○9 ○9

Signature of Invigilator: ________________

Name:

....

Roll No.:

....

Signature of Student: ________________

Questions and Responses

Question [q1]**:** (3 points) Let X and Y be independent random variables and both of them are uniformly distributed in $[0, 1]$. If the smaller (of the two) is less than $1/4$, then what is the conditional probability that the larger is greater than $3/4$?

○ 1/3 ● 2/7 ○ 1/4 ○ 1/7

Question [q2]**:** (3 points) Let X be a random variable with cumulative distribution function

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x^2}{2} & \text{if } 0 \le x < 1 \\ \frac{x+2}{6} & \text{if } 1 \le x < 2 \\ 1 & \text{if } x \ge 2. \end{cases}$$

Then the value of $P(1 < X < 2)$ equals

○ 1/2 ○ 1/4 ● 1/6 ○ 1/8

Question [q3]**:** (3 points) Let X be random variable with probability mass fucntion, $P(X = x) = e^{-\lambda}\frac{\lambda^x}{x!},\ x = 0, 1, 2, \cdots,$. Then which of the following statements is/are **TRUE**?

- ○ $E(X^3) = \lambda + 2\lambda^2 + \lambda^3$
- ○ $E(X^4) = \lambda^4$
- ● $E(X^2) = \lambda + \lambda^2$
- ● $E(X^3) = \lambda + 3\lambda^2 + \lambda^3$

Question [q4]**:** (3 points) Let $X_1, X_2, \cdots$ are iid random variables having uniform distribution in $[0, \theta]$. Let $X_{(1)} = \min\{X_1, X_2, \cdots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \cdots, X_n\}$.
Then which of the following statements is/are **TRUE**?

- ○ $X_{(n)}$ does NOT converge to θ in probability.
- ○ $nX_{(1)}$ does converge in distribution to a radom variable Y, the CDF of Y is: $F(y) = 0$ if $y < 0$; $F(y) = 1 - e^{-\theta y}$ if $y \geq 0$
- ● $nX_{(1)}$ does converge in distribution to a radom variable Y, the CDF of Y is: $F(y) = 0$ if $y < 0$; $F(y) = 1 - e^{-\frac{y}{\theta}}$ if $y \geq 0$
- ● $X_{(n)}$ converges to θ in probability.

Question [q5]**:** (3 points) Which of the following statements is/are true?

- ○ Suppose that i is a recurrent state for a Markov chain $\{X_n : n \geq 0\}$. Define another Markov chain $\{Y_n : n \geq 0\}$ by $Y_n = X_{2n-1}$ for $n \geq 1$ and $Y_0 = X_0$. Then i is recurrent for $\{Y_n : n \geq 0\}$ as well.
- ● There is a Markov chain with no recurrent states.
- ● There is a Markov chain with countably infinite state space having exactly one recurrent state.
- ● A Markov chain can have more than one stationary distribution.

Question [q6]**:** (3 points) A fair die is rolled independently and repeatedly. Which of the following stochastic processes is/are Markov chain/chains?

- ● $X_n = \begin{cases} \text{number shown at } n\text{th roll} & \text{if } n \geq 1 \\ 0 & \text{if } n = 0. \end{cases}$
- ● $X_n = \begin{cases} \text{largest number shown up to } n\text{th roll} & \text{if } n \geq 1 \\ 0 & \text{if } n = 0. \end{cases}$
- ● $X_n = \begin{cases} \text{number of fives up to } n\text{th roll} & \text{if } n \geq 1 \\ 0 & \text{if } n = 0. \end{cases}$
- ● $X_n = \begin{cases} \text{smallest number shown up to } n\text{th roll} & \text{if } n \geq 1 \\ 0 & \text{if } n = 0. \end{cases}$