MA 322: Scientific Computing

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Chapter 4: Numerical Integrations or Quadratures

Open Newton-Cotes formulas

- For $n = 0$,

$$\int_{x_{-1}}^{x_1} f(x)\mathrm{d}x = 2hf(x_0) + \frac{h^3}{3}f''(\xi), \quad x_{-1} < \xi < x_1.$$

- For $n = 1$,

$$\int_{x_{-1}}^{x_2} f(x)\mathrm{d}x = \frac{3h}{2}\left[f(x_0) + f(x_1)\right] + \frac{3h^3}{4}f''(\xi), \quad x_{-1} < \xi < x_2.$$

- For $n = 2$,

$$\int_{x_{-1}}^{x_3} f(x)\mathrm{d}x = \frac{4h}{3}\left[2f(x_0) - f(x_1) + 2f(x_2)\right] + \frac{14h^5}{45}f^{(iv)}(\xi), \quad x_{-1} < \xi < x_3.$$

Open Newton-Cotes formulas

- For $n = 3$,

$$\int_{x_{-1}}^{x_4} f(x)\mathrm{d}x = \frac{5h}{24}\left[11f(x_0) + f(x_1) + f(x_2) + 11f(x_3)\right] + \frac{95h^5}{144} f^{(iv)}(\xi), \quad x_{-1} < \xi < x_4.$$

- For $n = 4$,

$$\int_{x_{-1}}^{x_5} f(x)\mathrm{d}x = \frac{7h}{20}\left[11f(x_0) - 14f(x_1) + 26f(x_2) - 14f(x_3) + 11f(x_4)\right]$$
$$- \frac{41h^7}{140} f^{(vi)}(\xi), \quad x_{-1} < \xi < x_5.$$

- For $n = 5$,

$$\int_{x_{-1}}^{x_6} f(x)\mathrm{d}x = \frac{6h}{1440}\left[611(f(x_0) + f(x_5)) - 453(f(x_1) + f(x_4)) + 562(f(x_2) + f(x_3))\right]$$
$$- \frac{5257h^7}{8640} f^{(vi)}(\xi), \quad x_{-1} < \xi < x_6.$$

Newton-Cotes integration: Convergence

Theorem

Let $I_n(f) = \sum_{j=0}^{n} w_j f(x_j), \quad n \geq 1$ *be a sequence of numerical integration formula that approximate* $I(f) = \int_a^b f(x)\mathrm{d}x$*. Let* $\mathcal{F}$ *be a family dense in* $C[a,b]$*.*
Then $I_n(f) \to I(f)$ *for all* $f \in C[a,b]$ *if and only if*

1. $I_n(f) \to I(f), \ \forall f \in \mathcal{F}$, *and*
2. $B \equiv \sup_{n\geq 1} \sum_{j=0}^{n} |w_j| < \infty.$