Indian Institute of Technology Guwahati

Mid Semester Exam, January-May 2022

MA 211M: REAL ANALYSIS

Time: 2 Hours (Including submission time) Maximum Marks: 25

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(1) Prove or disprove (with counterexample) the following statements. No mark without proper justification. [$3 \times 4 = 12$]

(a) If A is a bounded subset of $\mathbb{R}$ (with usual metric) such that $A \subset A'$ then A is compact. (A' denotes the the set of limit points of A.)

(b) If f is a continuous mapping of a metric space X into a metric space Y, then $f(\bar{A}) = \overline{f(A)}$ for every $A \subset X$ ($\bar{A}$ denotes the closure of A).

(c) If A is a compact subsets of $\mathbb{R}^3$, then the set $\{(x^2, y, z) : (x, y, z) \in A\}$ is also compact.

(d) Let $f : (X, d) \to (Y, \rho)$ be a continuous bijection. If (Y, ρ) is compact then f^{-1} is also continuous.

(2) Consider the subset $A = \{(x, y) : xy = 0, y \in \mathbb{Q}\}$ of $\mathbb{R}^2$ with usual metric. Find the following sets: [3+3]

(a) Interior of A.

(b) Boundary of A.

(3) If (X, d) is a discrete metric space then show that $A' = \emptyset$ for every $A \subseteq X$. (A' denotes the set of all limit points of A.) [2]

(4) Suppose K and F are disjoint closed sets in $\mathbb{R}^n$. If one of them is bounded then show that $d(K, F) > 0$ where $d(K, F) = \inf\{\|x - y\| : x \in K, y \in F\}$. What will happen if we remove the boundedness condition from the assumtion. [4+1]

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