1 a) $(x, y) \longleftrightarrow z \in [0,1]$

$0 < x, y \leq 1$ bijection

$x = 0.3 \quad 01 \quad 2 \quad 007 \quad 08 \ldots$

$y = 0.009 \quad 2 \quad 05 \quad 1 \quad 0008 \ldots$

$z = 0.3\ 009\ 01\ 2\ 2\ 05\ 007\ 1\ 08\ 0008 \ldots$

b) $\{x + b \,/\, b \in \mathbb{R}\}$ and $\{ax \,/\, a \in \mathbb{R}\}$

c) T: 0 2 4 6 8 ....

B: 1 3 5 7 9 ....

form a set by either top or bottom element in every column.

- Any two sets formed this way are not comparable.
- The set of such sets is uncountable. ∴ the set of all infinite binary functions are uncountable.

2 a)

1.	$\neg p \to q$	Premise
2.	$r \to p$	Premise
3.	$\neg q \vee r$	Assumption
4.	$\neg q$	Assumption
5.	$\neg\neg p$	MT 1, 4
6.	p	$\neg\neg e$ 5
7.	r	Assumption
8.	p	$\to e$ 2, 7
9.	p	$\vee e$ 3, 4–6, 7–8
10.	$\neg q \vee r \to p$	$\to i$ 3–9

2. b)

1. $c_1 = c_2 \lor d_1 = d_2$ Premise
2. $c_1 = c_2$ Assumption
3. $f(c_1) = f(c_1)$ $=i$
4. $f(c_1) = f(c_2)$ $=e$ 2, 3
5. $f(c_1) = f(c_2) \lor f(d_1) = f(d_2)$ $\lor i_1$ 4
6. $d_1 = d_2$ Assumption
7. $f(d_1) = f(d_1)$ $=i$
8. $f(d_1) = f(d_2)$ $=e$ 6, 7
9. $f(c_1) = f(c_2) \lor f(d_1) = f(d_2)$ $\lor i_2$ 8
10. $f(c_1) = f(c_2) \lor f(d_1) = f(d_2)$ $\lor e$ 1, 2–5, 6–9

2. (c)

1. x_0
2. $P(x_0) \land \neg P(f(x_0))$ Assumption
3. $x_0 = f(x_0)$ Assumption
4. $P(x_0)$ $\land e_1$ 2
5. $\neg P(f(x_0))$ $\land e_2$ 2
6. $P(f(x_0))$ $=e$ 3, 4
7. $\perp$ $\neg e$ 5, 6
8. $x_0 \neq f(x_0)$ $\neg i$ 3–7
9. $P(x_0) \land \neg P(f(x_0)) \to x_0 \neq f(x_0)$ $\to i$ 2–8
10. $\forall x\, P(x) \land \neg P(f(x)) \to x \neq f(x)$ $\forall i$ 1–9

4 a) A – nonempty set

$P^M \subseteq A^2$

$g^M : A^2 \to A$

$c^M \in A$.

b) $\phi = P(g(c,c), c)$

$A = \{0,1\}$ $\quad P^M = \{(0,0)\}$ $\quad g^M((x,y)) = x$

$c^M = 1$

$M \not\models_l \phi$ for arbitrary looking table.

$\because (1,1) \notin P^M$

(3) $(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r)$

(5) a) Not valid. Take the model M with $A = \{0,1\}$ and $R^M = \{(0,1),(1,0)\}$

b) Valid.

1.	$\forall x \forall y\ (R(x,y) \to \neg R(y,x))$	Premise
2.	x_0	Assumption
3.	$R(x_0,x_0)$	Assumption
4.	$\forall y\ R(x_0,y) \to \neg R(y,x_0)$	$\forall e$ $(1,x_0)$
5.	$R(x_0,x_0) \to \neg R(x_0,x_0)$	$\forall e$ $(4,x_0)$
6.	$\neg R(x_0,x_0)$	$\to e$ 5,3
7.	$\perp$	$\neg e$ 6,3
8.	$\neg R(x_0,x_0)$	$\neg i$ 3-7
9.	$\forall x \neg R(x,x)$	$\forall i$ 2-8