MA 324 | Quiz I
Statistical Inference and Multivariate Analysis | 14:05–14:55 IST
IIT GUWAHATI | February 11, 2022

Instructions

1. Attempt **all** the questions.
2. There is **no credit** for a solution if the appropriate work is not shown, even if the answer is correct.
3. Notations are standard and same as used during the lectures.
4. No question requires any clarification from the instructor. Even if a question has an error or incomplete data, the students are advised to write answer according to their understanding or write reasons for why it is not possible to solve partially or completely that question by citing errors/insufficient data.
5. The question paper has **1** page. This examination has **3** questions, for a total of **15** points.

Questions

1. (3 points) Let $X_1,\, X_2,\, X_3,\, X_4 \overset{i.i.d.}{\sim} \mathit{Bernoulli}(p)$, where $0 < p < 1$ is unknown parameter. Denote
$$U = X_1(X_3 + X_4) + X_2.$$
Show that U is not a sufficient statistic for p.

2. Let $X_1,\, X_2,\, X_3,\, X_4\, X_5 \overset{i.i.d.}{\sim} N(2\theta,\, 25\theta^2)$, where $\theta > 0$ is an unknown parameter.
   (a) (3 points) Derive the minimal sufficient statistics.
   (b) (4 points) Is the minimal sufficient statistic, that you obtained in part (a), complete? Justify your answer.

3. (5 points) Let $X_1,\, X_2,\, \ldots,\, X_n$ be a random sample of size $n\,(\geq 2)$ from a population having probability density function
$$f(x,\, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$
where $\theta > 0$ is unknown. Find the conditional expectation of $X_{(1)} = \min\{X_1,\, X_2,\, \ldots,\, X_n\}$ given $T = \sum_{i=1}^{n} X_i$. You may assume that the conditional expectation exists.