DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 473 Computational Finance Lab-I Date: 30.07.2024

1. Solve the following parabolic initial-boundary-value problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} - 4\dfrac{\partial^2 u}{\partial x^2} = 0, \quad (x,t) \in (-1,1)\times(0,1), \\ u(x,0) = \cos\left(\frac{\pi x}{2}\right), \quad x \in (-1,1), \\ u(-1,t) = u(1,t) = 0, \quad t \in (0,1]. \end{cases}$$

by

i) forward–time and central space (FTCS) discretization scheme,
ii) backward–time and central space (BTCS) discretization scheme,
iii) Crank–Nicolson scheme.

with the spatial step-size $h = \triangle x = 1e-02, 1e-03, 1e-04$ and the time steps $k = \triangle t = 5e-04, 1e-03, 1e-02$. The exact solution is given by

$$u(x,t) = e^{-\pi^2 t}\cos\left(\frac{\pi x}{2}\right).$$

2. Determine the numerical solution of the following one-dimensional parabolic IBVP:

$$\begin{cases} u_t - u_{xx} + u_x - u = (2x^2 - 4x + 3)e^{-t}, \quad (x,t) \in (0,1)\times(0,T),\ T = 1 \\ u(0,t) = 0, \quad u_x(1,t) = -xe^{-t}, \quad \forall t \in (0,T] \\ u(x,0) = x(1-x), \quad x \in (0,1), \end{cases}$$

by

i) forward–time and central space (FTCS) discretization scheme,
ii) backward–time and central space (BTCS) discretization scheme,
iii) Crank–Nicolson scheme.

with the spatial step-size $h = \triangle x = 1e-02, 1e-03, 1e-04$ and the time steps $k = \triangle t = 5e-04, 1e-03, 1e-02$. The exact solution is given by

$$u(x,t) = e^{-t}x(1-x).$$

The output files for all of the above problems should include the following:

a) Plot the exact and numerical solutions in different colors at the final time level with some symbols.

b) Draw the surface plot of the exact and numerical solution.
