DEPARTMENT OF MATHEMATICS
IIT GUWAHATI

MA 473 | Computational Finance | Lab – II | Date: 06.08.2024

1. Consider the following Black-Scholes PDE for European call:

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 S^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta)S\dfrac{\partial V}{\partial S} - rV = 0, \quad (0,\infty)\times(0,T],\ T>0 \\ V(S,t) = 0, \quad \text{for } S = 0, \\ V(S,t) = S - Ke^{-r(T-t)}, \quad \text{for } S \to \infty \\ \text{with suitable initial/terminal condition } V(S,0)/V(S,T). \end{cases}$$

With the following transformation

$$\begin{cases} S = Ke^x, \quad t = T - \dfrac{2\tau}{\sigma^2}, \quad q := \dfrac{2r}{\sigma^2}, \quad q_\delta := \dfrac{2(r-\delta)}{\sigma^2}, \\ V(s,t) = V\left(Ke^x,\, T - \dfrac{2\tau}{\sigma^2}\right) =: v(x,\tau), \text{ and} \\ v(x,\tau) =: K\exp\left\{-\dfrac{1}{2}(q_\delta - 1)x - \left[\dfrac{1}{4}(q_\delta-1)^2 + q\right]\tau\right\} y(x,\tau) \end{cases}$$

the above Black-Scholes PDE becomes the following 1-D heat conduction parabolic PDE:

$$\begin{cases} \dfrac{\partial y}{\partial \tau} = \dfrac{\partial^2 y}{\partial x^2},\ x\in\mathbb{R},\ \tau \ge 0, \\ y(x,0) = \max\left\{\exp(\frac{x}{2}(q_\delta+1)) - \exp(\frac{x}{2}(q_\delta-1)), 0\right\},\ x\in\mathbb{R}, \\ y(x,\tau) = 0,\ \text{for}\, x\to-\infty, \\ y(x,\tau) = \exp\left(\frac{1}{2}(q_\delta+1)x + \frac{1}{4}(q_\delta+1)^2\tau\right)\ \text{for}\, x\to\infty. \end{cases}$$

Solve the transformed PDE by the following schemes:

(i) Forward-Euler for time & central difference for space (FTCS) scheme.
(ii) Backward-Euler for time & central difference for space (BTCS) scheme.
(iii) Crank-Nicolson finite difference scheme

The values of the parameters are $T = 1$, $K = 10$, $r = 0.06$, $\sigma = 0.3$ and $\delta = 0$.

The output files should contain the following for above problem:

a) Plot the numerical solutions at the final time level in different colors with some symbols.

b) Draw the surface plot of the numerical solutions.
