Q1

```
clear;
format long e
```

Q1 (a)

```
clear;
```

Eg. - 1

```
U = toeplitz([1 linspace(0, 0, 2)], 1:3)
```

```
U =
     1     2     3
     0     1     2
     0     0     1
```

```
b = [6 3 1]'
```

```
b =
     6
     3
     1
```

```
x = colbackward(U, b)
```

```
x =
     1
     1
     1
```

Eg. - 2

```
U = triu(randn(3))
```

```
U =
   -7.341691126967387e-01    4.263875574089450e-01    2.023690886603053e+00
                        0   -3.728087417235042e-01   -2.258353970496191e+00
                        0                        0    2.229445680456899e+00
```

```
b = randn(3, 1)
```

```
b =
    3.375637006131064e-01
    1.000060819589125e+00
   -1.664164474987060e+00
```

```
x = colbackward(U, b)
```

```
x =
   -1.449144160542513e+00
    1.839233107066967e+00
   -7.464476437237122e-01
```

Q1 (b)

```
clear;
```

Eg. - 1

```
L = toeplitz(1:3, [1 linspace(0, 0, 2)])
```

```
L =
     1     0     0
     2     1     0
     3     2     1
```

```
b = [1 3 6]'
```

```
b =
     1
     3
     6
```

```
x = rowforward(L, b)
```

```
x =
     1
     1
     1
```

Eg. - 2

```
U = tril(randn(3))
```

```
U =
   -5.900345642052215e-01                            0                            0
   -2.780641637653093e-01     4.716343264163027e-01                            0
    4.227156912204783e-01    -1.212847199674459e+00     3.270599671770875e-01
```

```
b = randn(3, 1)
```

```
b =
    1.082633504236756e+00
    1.006077110819051e+00
   -6.509077365977526e-01
```

```
x = colbackward(U, b)
```

```
x =
   -1.834864548477879e+00
    2.133171939505960e+00
   -1.990178566382956e+00
```

Q2

```
clear;
A = randn(4)
```

```
A =
    2.570561574339689e-01     4.984907525081331e-05     3.502011738745352e-01    -6.903611031112258e-01
   -9.443778064042190e-01    -5.491891460940670e-02     1.250251228304996e+00    -6.515536417502810e-01
   -1.321788521392564e+00     9.111272656538598e-01     9.297894585577157e-01     1.192101870531270e+00
    9.248259334937059e-01     5.945836974090524e-01     2.397632570585800e-01    -1.611830388677811e+00
```

```
[L, U] = genp(A)
```

```
L =
     1.000000000000000e+00                    0                    0                    0
    -3.673819043400294e+00     1.000000000000000e+00                    0                    0
    -5.142022407038032e+00    -1.665060079338925e+01     1.000000000000000e+00                    0
     3.597758337032911e+00    -1.085952137373259e+01     5.899136778749531e-01     1.000000000000000e+00
U =
     2.570561574339689e-01     4.984907525081331e-05     3.502011738745352e-01    -6.903611031112258e-01
                         0    -5.473577812745437e-02     2.536826969906401e+00    -3.187815409183137e+00
                         0                         0     4.497022489940636e+01    -5.543679217193735e+01
                         0                         0                         0    -1.043305586197569e+00
```

Q3

```
clear;
```

Q3 (a)

```
A = [10^-20, 1; 1, 1];
[L, U] = genp(A)
```

```
L =
     1.000000000000000e+00                    0
     9.999999999999998e+19     1.000000000000000e+00
U =
     1.000000000000000e-20     1.000000000000000e+00
                         0    -9.999999999999998e+19
```

```
decomposition_difference = A - L*U
```

```
decomposition_difference =
                         0                    0
     1.110223024625157e-16     1.000000000000000e+00
```

Q3 (b)

```
% Ax = b => LUx = b => Ly = b, Ux = y
b = [1 0]';
y = rowforward(L, b);
x_c = colbackward(U, y)
```

```
x_c =
     0
     1
```

```
x = [-1 1]' / (1 - 10^-20)
```

```
x =
    -1
     1
```

```
norm_diff = norm(x_c - x) / norm(x)
```

```
norm_diff =
     7.071067811865475e-01
```

- The step where things typically start to go wrong is in the forward substitution due to the very small pivot element 10^-20 in the matrix A. This small value can lead to numerical instability, causing large errors in the solution.

Q4

```
clear;

for iter = 1:5
    A = randn(iter*5, iter*5);

    [L_gepp, U_gepp, p_gepp, ~] = gepp(A);
    [L_matlab, U_matlab, P_matlab] = lu(A);

    P_gepp = eye(size(A));
    P_gepp = P_gepp(p_gepp, :);

    norm_L = norm(L_gepp - L_matlab);
    norm_U = norm(U_gepp - U_matlab);
    norm_P = norm(P_gepp - P_matlab);

    fprintf('Iteration %d', iter);
    % disp('L from custom gepp and matlab:');
    % L_gepp
    % L_matlab
    % disp('U from custom gepp and matlab:');
    % U_gepp
    % U_matlab
    % disp('P from custom gepp and matlab:');
    % P_gepp
    % P_matlab
    fprintf(' - Norm Difference in L: %e\n', norm_L);
    fprintf(' - Norm Difference in U: %e\n', norm_U);
    fprintf(' - Norm Difference in P: %e\n', norm_P);
end
```

```
Iteration 1
 - Norm Difference in L: 1.387793e-16
 - Norm Difference in U: 4.440892e-16
 - Norm Difference in P: 0.000000e+00
Iteration 2
 - Norm Difference in L: 5.077390e-16
 - Norm Difference in U: 7.906692e-16
 - Norm Difference in P: 0.000000e+00
Iteration 3
 - Norm Difference in L: 4.070732e-15
 - Norm Difference in U: 4.239090e-15
 - Norm Difference in P: 0.000000e+00
Iteration 4
 - Norm Difference in L: 3.066433e-15
 - Norm Difference in U: 7.747275e-15
```

```
 - Norm Difference in P: 0.000000e+00
Iteration 5
 - Norm Difference in L: 5.182451e-15
 - Norm Difference in U: 1.067127e-14
 - Norm Difference in P: 0.000000e+00
```

Q5

```
clear;
```

Q5 Part (a)

```
for iter = 1:5
    A = randn(iter*2, iter*2);
    b = randn(iter*2, 1);

    x_gepp = geppsolve(A, b);
    x_matlab = A \ b;
    relative_difference = norm(x_gepp - x_matlab) / norm(x_matlab);

    fprintf('Iteration %d\n', iter);
    % x_gepp
    % x_matlab
    fprintf(' - Relative difference between geppsolve and A\\b: %e\n',
relative_difference);
end
```

```
Iteration 1
 - Relative difference between geppsolve and A\b: 5.340211e-17
Iteration 2
 - Relative difference between geppsolve and A\b: 2.844027e-16
Iteration 3
 - Relative difference between geppsolve and A\b: 2.846213e-16
Iteration 4
 - Relative difference between geppsolve and A\b: 9.441142e-16
Iteration 5
 - Relative difference between geppsolve and A\b: 8.308429e-16
```

Q5 Part (b)

```
A = [10^-20, 1; 1, 1];
[L, U, p, ~] = gepp(A)
```

```
L =
     1.000000000000000e+00                         0
     1.000000000000000e-20     1.000000000000000e+00
U =
     1     1
     0     1
p =
     2
     1
```

```
decomposition_difference = A(p,:) - L*U
```

```
decomposition_difference =
```

```
     0     0
     0     0
```

```
b = [1 0]';
x_c = geppsolve(A, b)
```

```
x_c =
    -1
     1
```

```
x = [-1 1]' / (1 - 10^-20)
```

```
x =
    -1
     1
```

```
norm_diff = norm(x_c - x) / norm(x)
```

```
norm_diff =
     0
```

- gepp is able to produce much better results in the case of Q3, with the errors being insignificant.

Q6

```
clear;
for iter = 1:5
    fprintf('Iteration %d\n', iter);
    A = randn(5, 5)
    det_A = mydet(A);
    actual_det_A = det(A);
    fprintf(' - mydet(A): %e', det_A);
    fprintf(' - matlab det(A): %e', actual_det_A);
    fprintf(' - Error: %e', abs(actual_det_A - det_A));
end
```

```
Iteration 1
A =
    -1.195411812608376e+00    -6.857045336988162e-01    -2.974925334908478e-01    -1.398802093251064e+00
    -1.334688387232651e-02    -9.415243349555418e-01    -2.714974866653881e-01     7.886623415581502e-01
    -1.808717136297500e+00     8.433147225737069e-01     6.310417162625513e-01     1.095374445695478e+00
    -4.784692698351494e-01    -1.692577689976957e+00     4.917817629800340e-01     6.840426290518494e-01
    -5.135679079202427e-01     7.256787280929793e-01     3.122440315664763e+00     2.065446432873658e-01
 - mydet(A): 6.750984e+01
 - matlab det(A): 6.750984e+01
 - Error: 0.000000e+00
Iteration 2
A =
    -3.337149145275614e-01    -2.049825403046277e+00    -2.171863685987873e+00     6.041078532827356e-01
     6.380393646156524e-02     4.336478235909531e-02     1.376923291073182e+00    -3.528233742647947e-01
    -8.043663802464812e-02    -7.490150894064554e-01    -7.540362453283391e-01    -2.785748361329278e+00
    -1.472826588011795e+00     7.411353260974530e-01    -1.443598198376165e+00     1.446397323976541e+00
     1.624437188042158e+00     7.434300922580746e-02    -7.727278784050305e-01    -3.370780002527350e-01
 - mydet(A): 5.747260e+01
 - matlab det(A): 5.747260e+01
 - Error: 7.105427e-15
Iteration 3
```

```
A =
    -1.435127816125753e+00     1.322267749225890e+00    -6.585141903681243e-01     2.796553966921837e-01
    -6.577116192785718e-01     1.707785191460827e-01    -1.388991550629430e+00     2.234843118537121e-01
    -1.286463762790578e+00     5.365315018855938e-01    -1.116604948273796e+00    -2.977428140639953e-01
    -4.715015425553275e-01     6.013374578678989e-01     5.489961711477550e-01    -1.720468700822851e+00
    -1.484967735796905e+00     1.685771031293984e+00     1.729276969412223e-01     1.987548404507014e-01
 - mydet(A): 4.301823e+00
 - matlab det(A): 4.301823e+00
 - Error: 8.881784e-16
Iteration 4
A =
    -2.661048167405558e-02     1.360596268499852e-01     2.303922941012403e+00     1.268163704069089e+00
     2.622252047475515e-01    -5.429699608628925e-01     1.125960109483281e+00     5.561766298182860e-02
     1.099483089910364e+00     1.898335299918788e-01    -1.491095706258468e+00    -1.491649807834865e+00
     1.095342266854256e+00    -1.768144219531031e+00     1.299364011637519e+00    -1.098669499383252e+00
     1.285337839163083e-01     5.260763010757622e-01    -1.664134805582986e+00     2.000561678867744e-01
 - mydet(A): -3.888453e+00
 - matlab det(A): -3.888453e+00
 - Error: 0.000000e+00
Iteration 5
A =
     8.943811495704282e-01    -9.463707012173982e-01     6.613019182999430e-01     6.057509333100209e-01
     7.821033892312837e-02     2.811939762783778e-01     9.055677538507401e-01     2.028369299015762e+00
     1.450818828240354e+00     2.343432279205171e-01    -1.170451590152022e+00    -3.522273390901558e-01
    -1.196558034844856e-01    -4.568070294812646e-01    -5.391688062756526e-01    -1.765604517202779e-01
     6.259096830391265e-01    -3.595816710475149e-01     1.376459133783378e+00    -2.042281427116826e-01
 - mydet(A): -3.002746e+01
 - matlab det(A): -3.002746e+01
 - Error: 7.105427e-15
```

Functions defined :-

Q1

```
function x = colbackward(U,b)
    n = size(b, 1);
    x = zeros(n, 1);
    for i = n:-1:1
        if U(i, i) == 0
            error('Provided U is singular');
        end
        x(i) = b(i) / U(i, i);
        b(1 : i - 1) = b(1 : i - 1) - U(1 : i-1, i)*x(i);
    end
end
function x = rowforward(L,b)
    n = size(b, 1);
    x = zeros(n, 1);
    for i = 1:n
        if L(i, i) == 0
            error('Provided L is singular');
        end
        x(i) = (b(i) - L(i, 1:i-1)*x(1:i-1)) / L(i, i);
    end
end
```

Q2

```
function [L, U] = genp(A)
    n = size(A, 1);

    for i = 1:n-1
        if A(i, i) == 0
            error('Zero pivot encountered');
        end

        A(i+1:n, i) = A(i+1:n, i) / A(i, i);
        A(i+1:n, i+1:n) = A(i+1:n, i+1:n) - A(i+1:n, i) * A(i, i+1:n);
    end

    L = eye(n) + tril(A, -1);
    U = triu(A);
end
```

Q4

```
function [L, U, p, detP] = gepp(A)
    detP = 1;
    n = size(A, 1);
    p = (1:n)';

    for i = 1:n-1
        [~, max_row] = max(abs(A(i:n, i)));
        max_row = max_row + i - 1;

        if max_row ~= i
            detP = -detP;
            A([i, max_row], :) = A([max_row, i], :);
            p([i, max_row]) = p([max_row, i]);
        end

        if A(i, i) == 0
            warning('Matrix is singular or nearly singular. Pivoting
failed.');
            break;  % If no pivot is found, exit the loop gracefully
        end

        A(i+1:n, i) = A(i+1:n, i) / A(i, i);
        A(i+1:n, i+1:n) = A(i+1:n, i+1:n) - A(i+1:n, i) * A(i, i+1:n);
    end

    L = eye(n) + tril(A, -1);
    U = triu(A);
end
```

Q5

```
function x = geppsolve(A, b)
    [L, U, p, ~] = gepp(A);

    % PA = LU, Ax = b => LUX = Pb => Ly = Pb, Ux = y
    b = b(p);
    y = rowforward(L, b);
    x = colbackward(U, y);
end
```

Q6

```
function d = mydet(A)
    [~, U, ~, detP] = gepp(A);
    detU = prod(diag(U));
    d = detP * detU;
end
```