Statistical Inference and Multivariate Analysis (MA324)

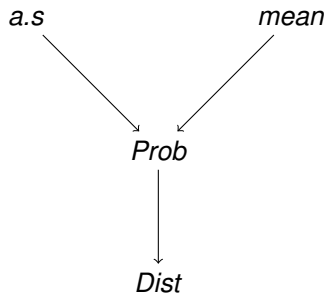## Lecture Slides
Lecture 04

Relation among different Convergences

Indian Institute of Technology Guwahati

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Relation between Modes of Convergence

Counter Examples

Example 1: Let $\mathcal{S} = [0,1], \mathcal{F} = \mathcal{B}([0,1])$ and P be the uniform measure. Define $X_n = n1_{[0,\frac{1}{n}]}$. X_n converges to 0 in probability and almost surely but not in rth mean for any $r \geq 1$.

Example 2: Let X be a $N(0,1)$ RV defined on some probability space $(\mathcal{S}, \mathcal{F}, P)$. Define $X_n = X$ for all n. Then X_n converges in distribution to $-X$ but not in probability.

Example 3: Let $X_{1,1} = 1_{[0,1/2]}, X_{2,1} = 1_{[1/2,1]}$
$X_{1,2} = 1_{[0,1/4]}, X_{2,2} = 1_{[1/4,1/2]}, X_{3,2} = 1_{[1/2,3/4]}, X_{4,2} = 1_{[3/4,1]} \ldots$.
Then $X_{m,n}$ converges (as $n \to \infty$) in rth mean and in probability but not almost surely.