Indian Institute of Technology Guwahati
Statistical Inference and Multivariate Analysis (MA 324)
Problem Set 02

1. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} Bernoilli(p)$, where $p \in (0,\, 1)$ is an unknown parameter. Evaluate $\mathcal{I}_{\boldsymbol{X}}\left(p\right)$, $\mathcal{I}_{\overline{X}}\left(p\right)$, and compare. Can we use these Fisher information to claim the sufficiency of $\overline{X}$?

2. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} N(\mu,\, \sigma^2)$, where $\sigma > 0$ is unknown, but $\mu \in \mathbb{R}$ is known parameter. Assume that $n \geq 2$. Let
$$U^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \mu\right)^2$$
$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2.$$
Evaluate $\mathcal{I}_{U^2}\left(\sigma^2\right)$ and $\mathcal{I}_{S^2}\left(\sigma^2\right)$. Show that $\mathcal{I}_{U^2}\left(\sigma^2\right) > \mathcal{I}_{S^2}\left(\sigma^2\right)$.

3. Suppose that $X_1,\, X_2,\, \ldots,\, X_n$ are *i.i.d.* RVs with common Rayleigh PDF
$$f(x,\, \sigma) = \begin{cases} \frac{2}{\sigma} x e^{-\frac{x^2}{\sigma}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$
where $\sigma > 0$ is unknown parameter. Denote a statistic $T = \sum_{i=1}^{n} X_i^2$. Evaluate $\mathcal{I}_{\boldsymbol{X}}\left(\sigma\right)$ and $\mathcal{I}_{T}\left(\sigma\right)$. Are they same? If so, what conclusion can one draw from this?

4. Let $X_1,\, X_2,\, X_3 \overset{i.i.d.}{\sim} N(\theta,\, 1)$, where $\theta \in \mathbb{R}$ is an unknown parameter. Denote $T_1 = X_1 - X_2$, $T_2 = X_1 + X_2 - 2X_3$, and $\boldsymbol{T} = (T_1,\, T_2)$. Is $\boldsymbol{T}$ ancillary for θ? Are T_1 and T_2 independent?

5. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} U(\theta - \frac{1}{2},\, \theta + \frac{1}{2})$, where $\theta \in \mathbb{R}$ is unknown parameter. Assume that $n \geq 2$. Show that $X_{(n)} - X_{(1)}$ is ancillary for θ. Is $\frac{X_{(n)} - X_{(1)}}{\overline{X} - X_{(1)}}$ ancillary for θ?

6. Let $X_1,\, X_2,\, \ldots,\, X_n \overset{i.i.d.}{\sim} N(2\theta,\, 5\theta^2)$, where $\theta > 0$ is an unknown parameter. Is the minimal sufficient statistic complete?