$$\text{var}(X|Y) = (1-\rho^2)\,\sigma_X^2 = (1-\rho^2)\cdot 1$$

$$\text{So} \quad E(X^2|Y) = (1-\rho^2) - (\rho Y)^2$$

$$= 1 - \rho^2 - \rho^2 Y^2$$

$$\text{So,} \quad E(X^2Y^2) = E\left(Y^2(1-\rho^2-\rho^2Y^2)\right)$$

$$= E(Y^2) - E(Y^2\rho^2) - E(\rho^2 Y^4)$$

$$= 1 - \rho^2 \cdot 1 - \rho^2 E(Y^4)$$

$$= 1 - \rho^2 - \rho^2 \times 3 \quad \text{(see page 7 also)}$$

$$= 1 - \rho^2 - 3\rho^2 \quad \text{Ans.}$$

$$= 1 - 4\rho^2$$