```
for i=5:8
    A=rand(i);
    hess(A);
    reflecthess(A);
    norm(hess(A)-reflecthess(A))
end
```

```
ans =
    0.7167   -0.5846    0.6063    0.3117    0.4601
   -1.3110    1.8499   -0.7510   -0.5271    0.3204
         0   -0.6846    0.3502   -0.0235   -0.0349
         0         0    0.2004   -0.0867   -0.1171
         0         0         0    0.3812    0.1086
ans =
    0.7167   -0.5846    0.6063    0.3117    0.4601
   -1.3110    1.8499   -0.7510   -0.5271    0.3204
         0   -0.6846    0.3502   -0.0235   -0.0349
         0         0    0.2004   -0.0867   -0.1171
         0         0         0    0.3812    0.1086
ans = 1.3585e-15
ans =
    0.5466   -1.0116    0.3089    0.6621   -0.4044   -0.4458
   -1.1528    2.2453   -0.4052    0.2547    0.1712   -0.3883
         0   -0.6170    0.1829    0.4380    0.1760    0.2882
         0         0   -0.7116    0.2826   -0.0635    0.0350
         0         0         0    0.3348    0.1285   -0.5622
         0         0         0         0    0.2334   -0.1089
ans =
    0.5466   -1.0116    0.3089    0.6621   -0.4044   -0.4458
   -1.1528    2.2453   -0.4052    0.2547    0.1712   -0.3883
         0   -0.6170    0.1829    0.4380    0.1760    0.2882
         0         0   -0.7116    0.2826   -0.0635    0.0350
         0         0         0    0.3348    0.1285   -0.5622
         0         0         0         0    0.2334   -0.1089
ans = 1.4226e-15
ans =
    0.6135   -0.9688   -0.5189    0.2834   -0.0380   -0.0639   -0.7596
   -1.7978    2.7306    1.2182   -0.1143   -0.1549   -0.0460    0.4556
         0    1.2318    0.5658   -0.0250    0.2391    0.1179   -0.0927
         0         0   -0.2911   -0.2168    0.2684    0.0704   -0.2981
         0         0         0   -0.1584    0.0906   -0.7639   -0.0865
         0         0         0         0    0.5701    0.1368    0.2044
         0         0         0         0         0   -0.2388    0.5058
ans =
    0.6135   -0.9688   -0.5189    0.2834   -0.0380   -0.0639   -0.7596
   -1.7978    2.7306    1.2182   -0.1143   -0.1549   -0.0460    0.4556
         0    1.2318    0.5658   -0.0250    0.2391    0.1179   -0.0927
         0         0   -0.2911   -0.2168    0.2684    0.0704   -0.2981
         0         0         0   -0.1584    0.0906   -0.7639   -0.0865
         0         0         0         0    0.5701    0.1368    0.2044
         0         0         0         0         0   -0.2388    0.5058
ans = 3.3212e-15
ans =
    0.5002   -1.0642    0.7504   -0.1982    0.5259   -0.5202   -0.5833    0.0344
   -1.0979    2.3318   -1.0247   -0.3100   -0.5569    0.2659   -0.4485   -0.0580
         0   -1.7214    1.2367   -0.0243    0.6270    0.0573   -0.2582    0.3832
         0         0    0.4387   -0.4898    0.5301   -0.2065    0.2729    0.0217
         0         0         0    0.8429   -0.2501   -0.4724    0.0753    0.0985
         0         0         0         0   -0.3670    0.2327   -0.2825   -0.0589
         0         0         0         0         0    0.3942    0.1644    0.0754
         0         0         0         0         0         0    0.1086   -0.1402
ans =
```

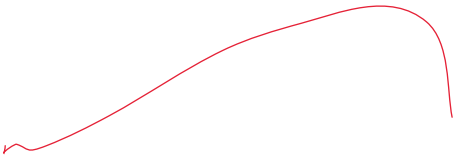
```
    0.5002   -1.0642    0.7504   -0.1982    0.5259   -0.5202   -0.5833    0.0344
   -1.0979    2.3318   -1.0247   -0.3100   -0.5569    0.2659   -0.4485   -0.0580
         0   -1.7214    1.2367   -0.0243    0.6270    0.0573   -0.2582    0.3832
         0         0    0.4387   -0.4898    0.5301   -0.2065    0.2729    0.0217
         0         0         0    0.8429   -0.2501   -0.4724    0.0753    0.0985
         0         0         0         0   -0.3670    0.2327   -0.2825   -0.0589
         0         0         0         0         0    0.3942    0.1644    0.0754
         0         0         0         0         0         0    0.1086   -0.1402
ans = 2.4521e-15
```

```
d0=zeros(3,1);
d1=zeros(3,1);
d2=zeros(3,1);
d3=zeros(3,1);
c0=zeros(3,1);
c1=zeros(3,1);
c2=zeros(3,1);
c3=zeros(3,1);
for i=[8,10,12]
    A=rand(i);
    [Q0,R0]=cgs(A);
    d0(i/2-3)=norm(eye(i)-Q0'*Q0);
    c0(i/2-3)=norm(A-Q0*R0);
    [Q1,R1]=mgs(A);
    d1(i/2-3)=norm(eye(i)-Q1'*Q1);
    c1(i/2-3)=norm(A-Q1*R1);
    [Q2,R2]=mgsrep(A);
    d2(i/2-3)=norm(eye(i)-Q2'*Q2);
    c2(i/2-3)=norm(A-Q2*R2);
    [Q3,R3]=qr(A);
    d3(i/2-3)=norm(eye(i)-Q3'*Q3);
    c3(i/2-3)=norm(A-Q3*R3);
end

deviation_from_orthonormality=table(d0,d1,d2,d3)
```

```
deviation_from_orthonormality =
        d0            d1            d2            d3
    __________    __________    __________    __________

    1.6542e-14    2.0398e-15    1.9531e-16    5.5793e-16
    3.3499e-15    1.0535e-15    2.5322e-16     7.361e-16
    2.8462e-14    1.9922e-15    2.7486e-16    9.8988e-16
```



```
deviation_from_A_to_qr=table(c0,c1,c2,c3)
```

```
deviation_from_A_to_qr =
        c0            c1            c2            c3
    __________    __________    __________    __________

    7.2046e-17    3.6403e-16    3.5139e-16    6.4987e-16
    3.8851e-16     3.273e-16    3.6105e-16    7.5232e-16
     5.422e-16    3.1539e-16    4.2369e-16    1.1917e-15
```

```
d0=zeros(3,1);
d1=zeros(3,1);
d2=zeros(3,1);
d3=zeros(3,1);
for i=[8,10,12]
    A=hilb(i);
    [Q0,R0]=cgs(A);
    d0(i/2-3)=norm(eye(i)-Q0'*Q0);
    [Q1,R1]=mgs(A);
    d1(i/2-3)=norm(eye(i)-Q1'*Q1);
    [Q2,R2]=mgsrep(A);
    d2(i/2-3)=norm(eye(i)-Q2'*Q2);
    [Q3,R3]=qr(A);
    d3(i/2-3)=norm(eye(i)-Q3'*Q3);
end

deviation_from_orthonormality=table(d0,d1,d2,d3)
```

```
deviation_from_orthonormality =
      d0          d1             d2             d3
    _____    _________    __________    __________

    1.0025    1.4734e-07    2.4549e-16    8.0837e-16
    2.9975    7.2373e-05    2.6025e-16    7.3514e-16
    4.9993       0.16674    3.4952e-16    7.9867e-16
```

```
disp('cgs,mgs as they are not backward stable')
```

```
cgs,mgs as they are not backward stable
```

```
function B=applreflect(u,gamma,A)
    B=A-(gamma*u)*(u.'*A);
end

function B=applreflectr(u,gamma,A)
    B=(applreflect(u,gamma,A'))';
end

function [u,gamma,tau]=reflect(x)
    a=norm(x,2);
    if(a==0)
        disp('x has two norm 0');
    end
    tau=sign(x(1))*a;
    gamma=(x(1)+tau)/tau;
    u=[1;x(2:end)/(x(1)+tau)];
end
```

reflector does not works when x(1) is zero.

```
function H=reflecthess(A)
    [~,n]=size(A);

    for k=1:n-2
        [u,gamma,tau]=reflect(A(k+1:n,k));
        A(k+1,k)=-tau;
        A(k+1:n,k+1:n)=applreflect(u,gamma,A(k+1:n,k+1:n));
        A(:,k+1:n)=applreflectr(u,gamma,A(:,k+1:n));
        A(k+2:n,k)=u(2:end);
    end

    H=triu(A,-1);
end

function [V,R]=cgs(V)
    [n,m]=size(V);
    R=zeros(m,m);

    for k=1:m
        R(1:k-1,k)=V(:,1:k-1)'*V(:,k);
        V(:,k)=V(:,k)-V(:,1:k-1)*R(1:k-1,k);
        R(k,k)=norm(V(:,k),2);
        if(R(k,k)==0)
            fprintf('V1 to V%d are l.d',k)
        end
        V(:,k)=V(:,k)/R(k,k);
    end
end

function [V,R]=mgs(V)
    [n,m]=size(V);
    R=zeros(m,m);

    for k=1:m
        for i=1:k-1
            R(i,k)=V(:,i)'*V(:,k);
            V(:,k)=V(:,k)-R(i,k)*V(:,i);
        end
        R(k,k)=norm(V(:,k),2);
        if(R(k,k)==0)
            fprintf('V1 to V%d are l.d',k)
        end
        V(:,k)=V(:,k)/R(k,k);
    end
end

function [V,R]=mgsrep(V)
    [n,m]=size(V);
    R=zeros(m,m);
```

skip the reflect() applreflect() and applreflectr() if max(abs(A(k+2:n,k)) is zero

```
    for k=1:m
        for i=1:k-1
            R(i,k)=V(:,i)'*V(:,k);
            V(:,k)=V(:,k)-R(i,k)*V(:,i);
        end

        for i=1:k-1
            alpha=V(:,i)'*V(:,k);
            R(i,k)=R(i,k)+alpha;
            V(:,k)=V(:,k)-alpha*V(:,i);
        end

        R(k,k)=norm(V(:,k),2);
        if(R(k,k)==0)
            fprintf('V1 to V%d are l.d',k)
        end
        V(:,k)=V(:,k)/R(k,k);
    end
end
```