# MA 322: Scientific Computing


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# Chapter 4: Numerical Integrations or Quadratures

# Trapezoidal rule

$$I = \int_0^{\pi} e^x \cos x \mathrm{d}x.$$

| $n$ | $I_n$ | $E_n$ | $\tilde{E}_n$ |
|---|---|---|---|
| 2 | -17.389259 | 5.32 | 4.96 |
| 4 | -13.336023 | 1.27 | 1.24 |
| 8 | -12.382162 | 3.12E-1 | 3.10E-1 |
| 16 | -12.148004 | 7.77E-2 | 7.76E-2 |
| 32 | -12.089742 | 1.94E-2 | 1.94E-2 |
| 64 | -12.075194 | 4.85E-3 | 4.85E-3 |
| 128 | -12.071558 | 1.21E-3 | 1.21E-3 |
| 256 | -12.070649 | 3.03E-4 | 3.03E-4 |
| 516 | -12.070422 | 7.57E-5 | 7.57E-5 |

# Corrected trapezoidal rule

$$CT_n(f) = \frac{h}{2}\left[f(x_0) + f(x_n) + 2\sum_{j=1}^{n-1} f(x_j)\right] - \frac{h^2}{12}\left[f'(b) - f'(a)\right].$$

| $n$ | $CT_n(f)$ | Error Trap | Error |
|---|---|---|---|
| 2 | -12.425528367 | 5.32 | 3.55E-1 |
| 4 | -12.05090106 | 1.27 | 2.47E-2 |
| 8 | -12.071929245 | 3.12E-1 | 1.58E-3 |
| 16 | -12.070445804 | 7.77E-2 | 9.95E-5 |
| 32 | -12.070352543 | 1.94E-2 | 6.23E-6 |
| 64 | -12.070346706 | 4.85E-3 | 3.89E-7 |
| 128 | -12.070346341 | 1.21E-3 | 2.43E-8 |

# Simpson's rule

$$I_n(f) = \frac{3h}{8}\left[f(x_0) + f(x_n) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + 2\sum_{j=1}^{n/2-1} f(x_{2j})\right].$$

| $n$ | $I_n$ | $E_n$ | $\tilde{E}_n$ |
|---|---|---|---|
| 2 | -11.5928395534 | -4.78E-1 | -1.63 |
| 4 | -11.9849440198 | -8.54E-2 | -1.02E-1 |
| 8 | -12.0642089572 | -6.14-3 | -6.38E-3 |
| 16 | -12.0699513233 | -3.95E-4 | -3.99E-4 |
| 32 | -12.0703214561 | -2.49E-5 | -2.49E-5 |
| 64 | -12.0703447599 | -1.56E-6 | -1.56E-6 |
| 128 | -12.0703462191 | -9.73E-8 | -9.73E-8 |