COMP9020

Foundations of Computer Science

COMP9020 18s2 Staff

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Consults:	Wednesdays 1-2 (Room 204, K17)
Research:	Theoretical CS: Algorithms, Formal verification

Course Aims

"Computer science no more about computers than astronomy is about telescopes"

– E. Dijkstra

The course aims to increase your level of mathematical maturity to assist with the fundamental problem of **finding, formulating, and proving** properties of programs.

Course Aims

The actual content is taken from a list of subjects that constitute the basis of the tool box of every serious practitioner of computing:

- numbers, sets, words — week 2
- functions and relations — weeks 3–4
- logic — week 5

- graph theory — week 6
- induction and recursion — weeks 7–8
- program analysis — week 9

- combinatorics — week 10
- probability and expectation — weeks 11–12

Course Material

All course information is placed on the course website

www.cse.unsw.edu.au/$\sim$cs9020/

Lecture slides and Problem Sets (from last semester) are publicly readable.

Textbook:

- KA Ross and CR Wright: Discrete Mathematics

Supplementary textbook:

- E Lehman, FT Leighton, A Meyer:
  Mathematics for Computer Science

Assessment Summary

60% exam, 30% assignments, 10% quizzes:

- 10 weekly quizzes, worth up to 2 marks each
- 3 assignments, worth up to 10 marks each
- final exam (2 hours) worth up to 60 marks

Quizzes available after lecture, due before next lecture.
Assignments due at the end of weeks 5, 8 (after stuvac), 12.

You must achieve 40% on the final exam to pass

Your final score will be taken from your 5 best results quiz, 3 assignments and final exam.

More information

View the course outline at:

www.cse.unsw.edu.au/~cs9020/outline.html

Particularly the sections on **Student conduct** and **Plagiarism**.

COMP9020 Week 2
Session 2, 2018
Numbers, Sets, Alphabets

- Textbook (R & W) - Ch. 1, Sec. 1.1-1.5, 1.7
- Supplementary Exercises Ch. 1 (R & W)

Notation for Numbers

Definition

Integers $\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

Reals $\mathbb{R}$

$\lfloor . \rfloor : \mathbb{R} \longrightarrow \mathbb{Z}$ — **floor** of x, the greatest integer $\leq x$

$\lceil . \rceil : \mathbb{R} \longrightarrow \mathbb{Z}$ — **ceiling** of x, the least integer $\geq x$

Example

$\lfloor \pi \rfloor = 3 = \lceil e \rceil \qquad \pi, e \in \mathbb{R};\ \lfloor \pi \rfloor, \lceil e \rceil \in \mathbb{Z}$

Simple properties

- $\lfloor -x \rfloor = -\lceil x \rceil$, hence $\lceil x \rceil = -\lfloor -x \rfloor$
- $\lfloor x + t \rfloor = \lfloor x \rfloor + t$ and $\lceil x + t \rceil = \lceil x \rceil + t$, for all $t \in \mathbb{Z}$

Fact

Let $k, m, n \in \mathbb{Z}$ such that $k > 0$ and $m \geq n$. The number of multiples of k in the interval $[n, m]$ is

$$\left\lfloor \frac{m}{k} \right\rfloor - \left\lfloor \frac{n-1}{k} \right\rfloor$$

Exercise

Examples

$\boxed{1.1.4}$

(b) $2\lfloor 0.6\rfloor - \lfloor 1.2\rfloor = -1$

$2\lceil 0.6\rceil - \lceil 1.2\rceil = 0$

(d) $\lceil\sqrt{3}\rceil - \lfloor\sqrt{3}\rfloor = 1$; the same for every non-integer

$\boxed{1.1.19(a)}$

Give x, y s.t. $\lfloor x\rfloor + \lfloor y\rfloor < \lfloor x+y\rfloor$

$\lfloor 3\pi\rfloor + \lfloor e\rfloor = 9 + 2 = 11 < 12 = \lfloor 9.42\ldots + 2.71\ldots\rfloor = \lfloor 3\pi + e\rfloor$

Exercise

Examples

$\boxed{1.1.4}$

(b) $2\lfloor 0.6 \rfloor - \lfloor 1.2 \rfloor = -1$

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Divisibility

Let $m, n \in \mathbb{Z}$.
'$m|n$' — m is a **divisor** of n, defined by $n = k \cdot m$ for some $k \in \mathbb{Z}$
Also stated as: 'n is divisible by m', 'm divides n', 'm multiple of n'

$m \nmid n$ — negation of $m|n$

Notion of divisibility applies to all integers — positive, negative and zero.
$1|m$, $-1|m$, $m|m$, $m|-m$, for every m
$n|0$ for every n; $0 \nmid n$ except $n = 0$

Numbers > 1 divisible only by 1 and itself are called **prime**.
Greatest common divisor $\gcd(m, n)$
Numbers m, n s.t. $\gcd(m, n) = 1$ are said to be **relatively prime**.
Least common multiple $\operatorname{lcm}(m, n)$

NB

$\gcd(m, n)$ *and* $\operatorname{lcm}(m, n)$ *are always taken as positive, even if* m *or* n *is negative.*

$$\begin{aligned} \gcd(-4, 6) = \gcd(4, -6) = \gcd(-4, -6) = \gcd(4, 6) &= 2 \\ \operatorname{lcm}(-5, -5) = \ldots &= 5 \end{aligned}$$

NB

Number theory (the study of prime numbers, divisibility etc.) is important in cryptography, for example.

Absolute Value

$$|x| = \begin{cases} x & , \text{if } x \geq 0 \\ -x & , \text{if } x < 0 \end{cases}$$

Fact

$\gcd(m, n) \cdot \operatorname{lcm}(m, n) = |m| \cdot |n|$

Examples

$\boxed{1.2.2}$ *True* or *False*. Explain briefly.
(a) $n|1$
(b) $n|n$
(c) $n|n^2$

$\boxed{1.2.7(b)}$ $\gcd(0, n) \stackrel{?}{=}$

$\boxed{1.2.12}$ Can two even integers be relatively prime?

$\boxed{1.2.9}$ Let m, n be positive integers.
(a) What can you say about m and n if $\operatorname{lcm}(m, n) = m \cdot n$?
(b) What if $\operatorname{lcm}(m, n) = n$?

Examples

$\boxed{1.2.2}$ *True* or *False*. Explain briefly.
(a) $n|1$ — only if $n = 1$ (for $n \in \mathbb{Z}$ also $n = -1$)
(b) $n|n$ — always
(c) $n|n^2$ — always

$\boxed{1.2.7(b)}$ $\gcd(0, n) = |n|$

$\boxed{1.2.12}$ Can two even integers be relatively prime? No. (why?)

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(a) What can you say about m and n if $\operatorname{lcm}(m, n) = m \cdot n$?
They must be relatively prime since always $\operatorname{lcm}(m, n) = \frac{mn}{\gcd(m,n)}$
(b) What if $\operatorname{lcm}(m, n) = n$?
m must be a divisor of n

Examples

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Euclid's gcd Algorithm

$$f(m,n) = \begin{cases} m & \text{if } m = n \\ f(m-n,n) & \text{if } m > n \\ f(m,n-m) & \text{if } m < n \end{cases}$$

Fact

For $m > 0, n > 0$ the algorithm always terminates. (Proof?)

Fact

For $m, n \in \mathbb{Z}$, if $m > n$ then $\gcd(m,n) = \gcd(m-n,n)$

Proof.
For all $d \in \mathbb{Z}$, ($d|m$ and $d|n$) if, and only if, ($d|m-n$ and $d|n$):
"$\rightarrow$": if $d|m$ and $d|n$ then $m = a \cdot d$ and $n = b \cdot d$, for some a, b
then $m - n = (a-b) \cdot d$, hence $d|m-n$
"$\leftarrow$": if $d|m-n$ and $d|n$ then ... $d|m$ (why?)

Sets

A set is defined by the collection of its elements.
Sets are typically described by:
(a) Explicit enumeration of their elements

$$
\begin{aligned}
S_1 &= \{a, b, c\} = \{a, a, b, b, b, c\} \\
&= \{b, c, a\} = \ldots \quad \text{three elements} \\
S_2 &= \{a, \{a\}\} \quad \text{two elements} \\
S_3 &= \{a, b, \{a, b\}\} \quad \text{three elements} \\
S_4 &= \{\} \quad \text{zero elements} \\
S_5 &= \{\{\{\}\}\} \quad \text{one element} \\
S_6 &= \{\,\{\}, \{\{\}\}\,\} \quad \text{two elements}
\end{aligned}
$$

(b) Specifying the properties their elements must satisfy; the elements are taken from some 'universal' domain. A typical description involves a **logical** property $P(x)$

$$S = \{\, x : x \in X \text{ and } P(x) \,\} = \{\, x \in X : P(x) \,\}$$

We distinguish between an element and the set comprising this single element. Thus always $a \neq \{a\}$.
Set $\{\}$ is empty (no elements);
set $\{\{\}\}$ is nonempty — it has one element.
There is only one empty set; only one set consisting of a single a; only one set of all natural numbers.

(c) Constructions from other sets (already defined)

- Union, intersection, set difference, symmetric difference, complement
- **Power set** $\mathrm{Pow}(X) = \{ A : A \subseteq X \}$
- Cartesian product (below)
- Empty set $\emptyset$
  $\emptyset \subseteq X$ for all sets X.

$S \subseteq T$ — S is a **subset** of T; includes the case of $T \subseteq T$
$S \subset T$ — a **proper** subset: $S \subseteq T$ and $S \neq T$

NB

An element of a set and a subset of that set are two different concepts

$$a \in \{a, b\}, \quad a \not\subseteq \{a, b\}; \qquad \{a\} \subseteq \{a, b\}, \quad \{a\} \notin \{a, b\}$$

Cardinality

Number of elements in a set X (various notations):

$$|X| = \#(X) = \text{card}(X)$$

Fact

Always $|Pow(X)| = 2^{|X|}$

$|\emptyset| = 0 \qquad \text{Pow}(\emptyset) = \{\emptyset\} \qquad |\text{Pow}(\emptyset)| = 1$

$\text{Pow}(\text{Pow}(\emptyset)) = \{\emptyset, \{\emptyset\}\} \qquad |\text{Pow}(\text{Pow}(\emptyset))| = 2 \quad \ldots$

$|\{a\}| = 1 \qquad \text{Pow}(\{a\}) = \{\emptyset, \{a\}\} \qquad |\text{Pow}(\{a\})| = 2 \quad \ldots$

$[m, n]$ — interval of integers; it is empty if $n < m$

$|[m, n]| = n - m + 1$, for $n \geq m$

Examples

$\boxed{1.3.2}$ Find the cardinalities of sets

1. $|\{\ \frac{1}{n} : n \in [1,4]\ \}| \stackrel{?}{=}$
2. $|\{\ n^2 - n : n \in [0,4]\ \}| \stackrel{?}{=}$
3. $|\{\ \frac{1}{n^2} : n \in \mathbb{P} \text{ and } 2|n \text{ and } n < 11\ \}| \stackrel{?}{=}$
4. $|\{\ 2 + (-1)^n : n \in \mathbb{N}\ \}| \stackrel{?}{=}$

Examples

$\boxed{1.3.2}$ Find the cardinalities of sets

1. $|\{ \frac{1}{n} : n \in [1,4] \}| = 4$ — four 'indices', no repetitions of values
2. $|\{ n^2 - n : n \in [0,4] \}| = 4$ — one 'repetition' of value
3. $|\{ \frac{1}{n^2} : n \in \mathbb{P} \text{ and } 2|n \text{ and } n < 11 \}| = 5$
4. $|\{ 2 + (-1)^n : n \in \mathbb{N} \}| = 2$ — what are the two elements?

Sets of Numbers

Natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$
Positive integers $\mathbb{P} = \{1, 2, \ldots\}$
Common notation $\mathbb{N}_{>0} = \mathbb{Z}_{>0} = \mathbb{N} \setminus \{0\}$

Integers $\mathbb{Z} = \{\ldots, -n, -(n-1), \ldots, -1, 0, 1, 2, \ldots\}$
Rational numbers (fractions) $\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0 \right\}$
Real numbers (decimal or binary expansions) $\mathbb{R}$
$r = a_1 a_2 \ldots a_k \,.\, b_1 b_2 \ldots$

In $\mathbb{P} \subset \mathbb{N} \subset \mathbb{Z}$ different symbols denote different numbers.
In $\mathbb{Q}$ and $\mathbb{R}$ the standard representation is not necessarily unique.

NB

Proper ways to introduce reals include Dedekind cuts and Cauchy sequences, neither of which will be discussed here. Natural numbers etc. are either axiomatised or constructed from sets $(\,0 \stackrel{def}{=} \{\},\ n+1 \stackrel{def}{=} n \cup \{n\}\,)$

NB

If we need to emphasise that an object (expression, formula) is defined through an equality we use the symbol $\stackrel{def}{=}$*. It denotes that the object on the left is defined by the formula/expression given on the right.*

Number sets and their containments

$$\mathbb{P} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

Derived sets of positive numbers

$$\mathbb{P} = \mathbb{N}_{>0} = \mathbb{Z}_{>0} = \{n : n \geq 1\} \subset \mathbb{Q}_{>0} = \{r : r = \tfrac{k}{l} > 0\} \subset \mathbb{R}_{>0}$$

Derived sets of integers

$$2\mathbb{Z} = \{\ 2x : x \in \mathbb{Z}\ \} \qquad \text{the even numbers}$$

$$3\mathbb{Z} + 1 = \{\ 3x + 1 : x \in \mathbb{Z}\ \}$$

Intervals of numbers (applies to any type)

$$[a, b] = \{x | a \leq x \leq b\}; \quad (a, b) = \{x | a < x < b\}$$

$$[a, b] \supseteq [a, b), \; (a, b] \supseteq (a, b)$$

NB

$(a, a) = (a, a] = [a, a) = \emptyset$; *however* $[a, a] = \{a\}$.

Intervals of $\mathbb{P}, \mathbb{N}, \mathbb{Z}$ are finite: if $m \leq n$

$$[m, n] = \{m, m + 1, \ldots, n\} \qquad |[m, n]| = n - m + 1$$

Examples

$\boxed{1.3.10}$ Number of elements in the sets

1. $\{-1, 1\}$
2. $[-1, 1]$
3. $(-1, 1)$
4. $\{\ n \in \mathbb{Z} :\ -1 \leq n \leq 1\ \}$

Examples

$\boxed{1.3.10}$ Number of elements in the sets

1. $\{-1, 1\}$ — 2
2. $[-1, 1]$ — 3 (if over $\mathbb{Z}$); ∞ (if over $\mathbb{Q}$ or $\mathbb{R}$)
3. $(-1, 1)$ — 1 (if over $\mathbb{Z}$); ∞ (if over $\mathbb{Q}$ or $\mathbb{R}$)
4. $\{\, n \in \mathbb{Z} : -1 \leq n \leq 1 \,\}$ — 3

Set Operations

Union $\quad A \cup B$; $\qquad$ Intersection $\quad A \cap B$

Note that there is a correspondence between set operations and logical operators (to be discussed in Week 6):
One can match set A with that subset of the universal domain, where the property a holds, then match B with the subset where b holds. Then
$A \cup B \leftrightarrow a$ or b; $\qquad A \cap B \leftrightarrow a$ and b

We say that A, B are **disjoint** if $A \cap B = \emptyset$

NB

$A \cup B = B \leftrightarrow A \subseteq B \qquad A \cap B = B \leftrightarrow A \supseteq B$

Other set operations

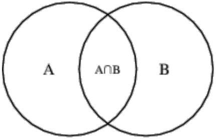
- $A \setminus B$ — **difference**, set difference, relative complement
  It corresponds (logically) to a but not b
- $A \oplus B$ — **symmetric difference**

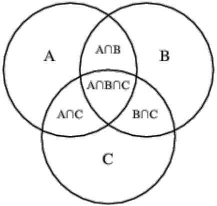
  $$A \oplus B \stackrel{\text{def}}{=} (A \setminus B) \cup (B \setminus A)$$

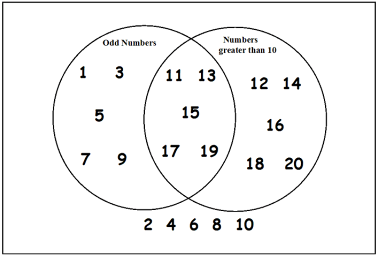
  It corresponds to a and not b or b and not a; also known as **xor (exclusive or)**
- A^c — set **complement** w.r.t. the 'universe'
  It corresponds to 'not a'

Venn Diagrams

p23–26: are a simple graphical tool to reason about the algebraic properties of set operations.







Laws of Set Operations

Commutativity	$A \cup B = B \cup A$
	$A \cap B = B \cap A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$
	$(A \cap B) \cap C = A \cap (B \cap C)$
Distribution	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Idempotence	$A \cup A = A$
	$A \cap A = A$
Identity	$A \cup \emptyset = A$
	$A \cap \emptyset = \emptyset$
Double Complementation	$(A^c)^c = A$
De Morgan laws	$(A \cup B)^c = A^c \cap B^c$
	$(A \cap B)^c = A^c \cup B^c$

Examples

$\boxed{1.4.4}$ $\Sigma = \{a, b\}$
(d) All subsets of Σ: ?
(e) $|\mathrm{Pow}(\Sigma)| \stackrel{?}{=}$
$\boxed{1.4.7}$ $A \oplus A \stackrel{?}{=}$, $\quad A \oplus \emptyset \stackrel{?}{=}$
$\boxed{1.4.8}$ Relate the cardinalities $|A \cup B|$, $|A \cap B|$, $|A \setminus B|$, $|A \oplus B|$, $|A|$, $|B|$

Examples

$\boxed{1.4.4}$ $\Sigma = \{a, b\}$

(d) All subsets of Σ : $\emptyset, \{a\}, \{b\}, \{a, b\}$

(e) $|\mathrm{Pow}(\Sigma)| = 4$

$\boxed{1.4.7}$ $A \oplus A \stackrel{?}{=} \emptyset, \quad A \oplus \emptyset \stackrel{?}{=} A$ for all A

$\boxed{1.4.8}$ Relate the cardinalities

$|A \cup B| = |A| + |B| - |A \cap B|$

hence $|A \cup B| + |A \cap B| = |A| + |B|$

$|A \setminus B| = |A| - |A \cap B|$

$|A \oplus B| = |A| + |B| - 2|A \cap B|$

Cartesian Product

$S \times T \stackrel{\text{def}}{=} \{ (s, t) : s \in S,\ t \in T \}$ where (s, t) is an **ordered** pair

$\bigtimes_{i=1}^{n} S_i \stackrel{\text{def}}{=} \{ (s_1, \ldots, s_n) : s_k \in S_k,$ for $1 \leq k \leq n \}$

$S^2 = S \times S, \quad S^3 = S \times S \times S, \ldots, \quad S^n = \times_1^n S, \ldots$

$\emptyset \times S = \emptyset$, for every S
$|S \times T| = |S| \cdot |T|, \quad | \times_{i=1}^{n} S_i| = \prod_{i=1}^{n} |S_i|$

Formal Languages

Σ — **alphabet**, a finite, nonempty set

Examples (of various alphabets and their intended uses)

$\Sigma = \{a, b, \ldots, z\}$ for single words (in lower case)
$\Sigma = \{\sqcup, -, a, b, \ldots, z\}$ for composite terms
$\Sigma = \{0, 1\}$ for binary integers
$\Sigma = \{0, 1, \ldots, 9\}$ for decimal integers

The above cases all have a natural ordering; this is not required in general, thus the set of all Chinese characters forms a (formal) alphabet.

Definition

word — any finite string of symbols from Σ
empty word — λ

Example

$\omega = aba$, $\omega = 01101\ldots1$, etc.

$\text{length}(\omega)$ — $\#$ of symbols in ω
$\text{length}(aaa) = 3, \text{length}(\lambda) = 0$
The only operation on words (discussed here) is **concatenation**, written as juxtaposition $\nu\omega, \omega\nu\omega, ab\omega, \omega b\nu, \ldots$

NB

$\lambda\omega = \omega = \omega\lambda$
$\text{length}(\nu\omega) = \text{length}(\nu) + \text{length}(\omega)$

Notation: Σ^k — set of all words of length k
We often identify $\Sigma^0 = \{\lambda\}$, $\Sigma^1 = \Sigma$
Σ^* — set of all words (of all lengths)
Σ^+ — set of all nonempty words (of any positive length)

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots; \quad \Sigma^{\leq n} = \bigcup_{i=0}^{n} \Sigma^i$$

$$\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \ldots = \Sigma^* \setminus \{\lambda\}$$

A **language** is a subset of Σ^*. Typically, only the subsets that can be formed (or described) according to certain rules are of interest. Such a collection of 'descriptive/formative' rules is called a **grammar**.

Examples: Programming languages, Database query languages

Examples

$\boxed{1.3.10}$ Number of elements in the sets (cont'd)

(e) Σ^* where $\Sigma = \{a, b, c\}$ — $|\Sigma^*| = \infty$

(f) $\{\ \omega \in \Sigma^* : \text{length}(\omega) \leq 4\ \}$ where $\Sigma = \{a, b, c\}$
$|\Sigma^{\leq 4}| = 3^0 + 3^1 + \ldots + 3^4 = \frac{3^5-1}{3-1} = \frac{243-1}{2} = 121$

Examples

$\boxed{1.3.10}$ Number of elements in the sets (cont'd)

(e) Σ^* where $\Sigma = \{a, b, c\}$ — $|\Sigma^*| = \infty$

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Supplementary Exercises

$\boxed{1.8.2(b)}$ When is $(A \setminus B) \setminus C = A \setminus (B \setminus C)$?

$\boxed{1.8.9}$ How many third powers are $\leq 1,000,000$ and end in 9? (Solve without calculator!)

$\boxed{1.8.2(b)}$ When is $(A \setminus B) \setminus C = A \setminus (B \setminus C)$?
From Venn diagram
$(A \setminus B) \setminus C = A \cap B^c \cap C^c$; $A \setminus (B \setminus C) = (A \cap B^c) \cup (A \cap C)$.
Equality would require that $A \cap C \subseteq A \cap B^c \cap C^c$; however, these two sets are disjoint, thus $A \cap C = \emptyset$ is a necessary condition for the equality.
One verifies that $A \cap C = \emptyset$ is also a sufficient condition and that, in this case, both set expressions simplify to $A \setminus B$.
$\boxed{1.8.9}$ How many third powers are $\leq 1,000,000$ and end in 9? (Solve without calculator!)
$n^3 = 9 \pmod{10}$ only when $n = 9 \pmod{10}$, and $n^3 \leq 1,000,000$ when $n \leq 100$. Hence all such n are $9, 19, \ldots, 99$.
Try the same question for n^4.

Summary

- Notation for numbers
  $\lfloor m \rfloor$, $\lceil m \rceil$, $m|n$, $|a|$, $[a, b]$, (a, b), gcd, lcm
- Sets and set operations
  $|A|$, $\in$, $\cup$, $\cap$, $\setminus$, $\oplus$, A^c, $\text{Pow}(A)$, $\subseteq$, $\subset$, $\times$
- Formal languages: alphabets and words
  λ, Σ^*, Σ^+, Σ^1, $\Sigma^2, \ldots$