

Week 1 Problem Set

Numbers, Sets, Words, Logic

1. (Numbers)

How many numbers in the interval $[1431, 9758]$ are

- divisible by 3?
- divisible by 5?
- divisible by 3 and 5?
- divisible by 3 or 5?

2. (Sets)

Prove that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

- using Venn diagrams,
- without Venn diagrams.

3. (Alphabets and Words)

Let $\Sigma = \{a, b, c\}$ and $\Psi = \{a, c, e\}$.

- How many words are in the set Σ^2 ?
- What are the elements of $\Sigma^2 \setminus \Psi^*$?
- Is it true that $\Sigma^* \setminus \Psi^* = (\Sigma \setminus \Psi)^*$? Why or why not?

4. (Propositional Logic)

For each of the following formulae, determine all the truth assignments to A , B and C under which the formula is true.

- $A \wedge (\neg C \Rightarrow (B \vee \neg A))$
- $(A \wedge \neg C) \Rightarrow (B \vee \neg A)$
- $(\neg C \Rightarrow \neg A) \wedge (B \Rightarrow (A \wedge \neg C))$
- $\neg(C \Rightarrow A) \wedge (A \vee (B \wedge \neg C))$

5. (Proving properties of algorithms)

Recall the algorithm for computing the greatest common divisor (gcd) of two positive numbers:

$$\gcd(m, n) = \begin{cases} m & \text{if } m = n \\ \gcd(m - n, n) & \text{if } m > n \\ \gcd(m, n - m) & \text{if } m < n \end{cases}$$

Recall the correctness proof given in class. What needs to be changed to adapt it to the faster version below?

$$\gcd(m, n) = \begin{cases} m & \text{if } n = 0 \\ \gcd(n, m \bmod n) & \text{if } n > 0 \end{cases}$$

6. Challenge Exercise

A **multiplication magic square** has the product of the numbers in each row, column and diagonal the same. If the diagram below is filled with positive integers to form a multiplicative magic square, then give the value of y .

5		y
4		
	1	

Assessment

This first problem set is meant to give you your first practice and will not count towards your mark for the weekly homework.

However, in order to familiarise yourself with the environment and structure of the weekly assessments, go to **COMP9020 20T1 Quiz Week 1** to answer 4 quiz questions on this week's problem set (Exercises 1-5 only) and lecture.

There is no time limit on the quiz once you have started it, but the deadline for submitting your quiz answers is **Thursday, 27 February 10:00:00am**.

Please always be mindful of the following **quiz rules**:

Do ...

- use your own best judgement to understand & solve a question
- discuss quizzes on the forum only **after** the deadline on Thursday

Do not ...

- post specific questions about the quiz **before** the Thursday deadline
- agonise too much about a question that you find too difficult