COMP9020 20T1 Foundations of Computer Science

Week 9 Problem Set
Probability

[Show with no answers] [Show with all answers]

1. (General probability)

   Consider two events A and B such that $0 < P(B) < 1$. For each of the following statements, decide whether they are always true for any such A and B, always false or could be either.

   a. $P(A \cup B) \geq P(A) + P(B)$

   b. $P(A \mid B) \cdot P(A) = P(A \cap B)$

   c. $P(A \cup B \mid B) \geq P(A \mid B)$

   d. $P(A \cap B \mid B) + P(A \cap B^c \mid B) = 1$

   e. $P(A \mid B) + P(A^c \mid B) < 1$

   f. $A \perp B \Rightarrow P(A \mid B) = P(A \mid B^c)$

   g. $A \perp B \Rightarrow P(A \cup B) = P(A) + P(B)$

   [show answer]

2. (Events and probability)

   a. Let E_1, E_2 be two events. Prove that $P(E_1 \setminus E_2) = P(E_1) - P(E_2)$ implies $P(E_2 \setminus E_1) = 0$.

   b. A red, a blue and a green die are rolled simultaneously.

      i. What is the probability that the sum of the 3 values is a prime number?

      ii. What is the probability of a doublet (2 of the 3 values are equal but the third value is different)?

      iii. What is the probability that all 3 values are different and the value of the red die is higher than the other two?

   [show answer]

3. (Counting and probability)

   Let $\Sigma = \{a, c, e, n, s\}$. Suppose we choose a 4-letter word at random from Σ^4. What is the probability that the letters of the word are in alphabetical order (e.g. as in $aces$ or $cees$ but not in $sees$)?

   [show answer]

4. (Probability of sequences)

   Alice and Bob repeatedly toss a coin (outcome H – head, or T – tails) until either Alice's or Bob's winning sequence is observed. What is the probability for Alice to win if

   a. Alice's winning sequence is HTH and Bob's is HHH?

   b. Alice's winning sequence is HTH and Bob's is THT?

   [show answer]

5. (Conditional Probability)

   a. Prove that

   $$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2|A_1) \cdot P(A_3|A_1 \cap A_2) \cdot \ldots \cdot P(A_n|A_1 \cap A_2 \cap \ldots \cap A_{n-1})$$

   b. Consider three urns: Urn 1 contains one red and two black marbles, Urn 2 contains three red and four black marbles, Urn 3 contains two red and two black marbles. One urn is selected at random and then two

marbles are randomly drawn from that urn without replacement. Given that these two marbles are red, what is the probability that Urn 2 was chosen?

[show answer]

6. **Challenge Exercise**

   a. Jamie and Charlie have two kids, one of which is a girl. Assume that the probability of each gender is ½. What is the probability that the other kid is also a girl?

   b. There are three cards in a hat. One card is red on both sides, one card is blue on both sides, and one card is red on one side and blue on the other. You draw one card at random, put it down on the table and then observe that the side of the card facing up is red. What is the probability that the other side is red too?

[show answer]

Assessment

After you have solved the exercises, go to COMP9020 20T1 Quiz Week 9 to answer 4 quiz questions on this week's problem set (Exercises 1-5 only) and lecture.

The quiz is worth 2.5 marks.

There is no time limit on the quiz once you have started it, but the deadline for submitting your quiz answers is **Thursday, 23 April 10:00:00am**.

As always please respect the **quiz rules**:

Do …

- use your own best judgement to understand & solve a question
- discuss quizzes on the forum only **after** the deadline on Thursday

Do not …

- post specific questions about the quiz **before** the Thursday deadline
- agonise too much about a question that you find too difficult