

# Week 10 Problem Set

## Expectation

[Show with no answers] [Show with all answers]

Congratulations on reaching the end of this course!

*Please note:*

- Just 3 (+1 challenge) exercises this week.
- Note the early submission deadline: Tuesday, 28 April, at 10:00:00am.

### 1. (Expectation)

- a. You randomly draw one card at a time from a deck of 52 Poker cards:  $\{2, 3, \dots, 10, J, Q, K, A\} \times \{\spadesuit, \clubsuit, \heartsuit, \diamondsuit\}$ . The cards are **not** put back into the deck after each drawing.
  - i. Is the event of drawing a specific card independent of the previous draw?
  - ii. What is the expected number of drawing attempts until a card **other than** an ace is drawn?
  - iii. What is the expected number of drawing attempts until the sum of the cards drawn is  $\geq 5$ ? (Assume that 2–10 are counted as their numeric value; J, Q, K are counted as 10; A is counted as 11.)
- b. Answer questions (i)–(iii) for the case when the cards **are** put back after each drawing.

[show answer]

### 2. (Variance)

Consider an urn with four balls: one ball is worth 5, two balls are worth 20 each, and one ball is worth 25.

Suppose you randomly draw two balls from the urn at the same time. Let random variable  $X$  denote the sum of the values of these two balls.

- a. What is the variance of  $X$ ?
- b. How does your answer to question a. change if you double the initial value of each ball?

[show answer]

### 3. (Decision making)

An airline is selling tickets for AU\$100 each for a plane with 10 seats. Each ticket holder independently has the probability of  $\frac{1}{8}$  of not turning up to the flight – in which case the airline keeps the AU\$100 for the ticket. Suppose 12 people want tickets. The airline has a choice of three strategies:

- X: sell 10 tickets
- Y: sell 11 tickets, but if everyone turns up the airline has to pay AU\$400 in compensation.
- Z: sell 12 tickets, but if 11 people turn up the airline has to pay AU\$300 in compensation, and if 12 people turn up the compensation will be AU\$600.

Calculate the expected values  $E(X)$ ,  $E(Y)$  and  $E(Z)$  to determine the best strategy.

[\[show answer\]](#)

#### 4. Challenge Exercise

You keep rolling a die until you have observed each of the 6 numbers at least once. Calculate the expected number of times you have to roll the die.

[\[show answer\]](#)

## Assessment

After you have solved the exercises, go to [COMP9020 20T1 Quiz Week 10](#) to answer 4 quiz questions on this week's problem set (Exercises 1-3 only) and lecture.

The quiz is worth 2.5 marks.

There is no time limit on the quiz once you have started it, but the deadline for submitting your quiz answers is **Tuesday, 28 April 10:00:00am**.

Please also for this final quiz respect the **quiz rules**:

Do ...

- use your own best judgement to understand & solve a question
- discuss quizzes on the forum only **after** the deadline on Tuesday

Do not ...

- post specific questions about the quiz **before** the Tuesday deadline
- agonise too much about a question that you find too difficult