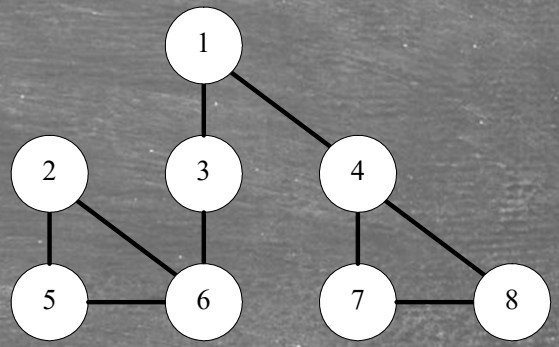
CSCI203

Week 8 – Lecture B

Articulation Points

Articulation Points

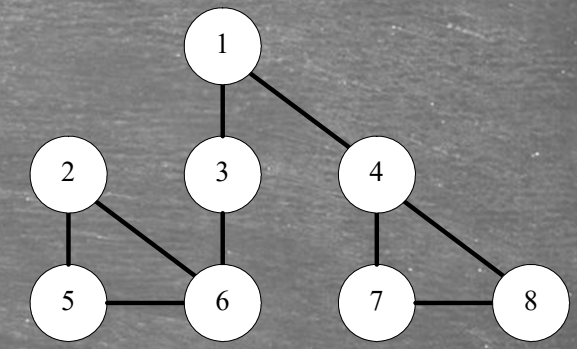
- A vertex, *v*, of a connected graph, *G*, is an articulation point if the graph obtained by deleting node *v* and all its connected edges is no longer connected.
- In the graph shown to the right:
- Node 1 and node 3 (among others) are both articulation points.
- How can we find articulation points in a systematic manner?

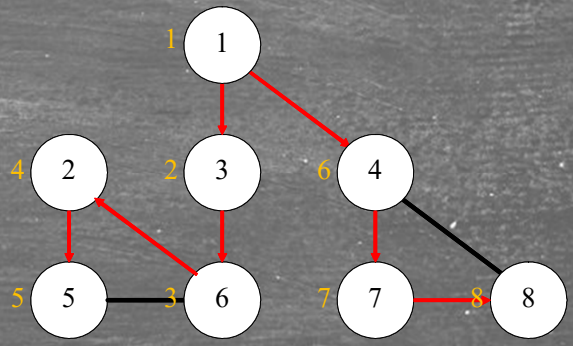


Articulation Points

- Traversing a graph
  - Using the example graph:
  - We perform a DFS in numerical order and list the vertices in the order in which they are pushed onto the stack.

  - This gives us the spanning tree, T.
  - It also gives us the preorder numbering shown on the right.



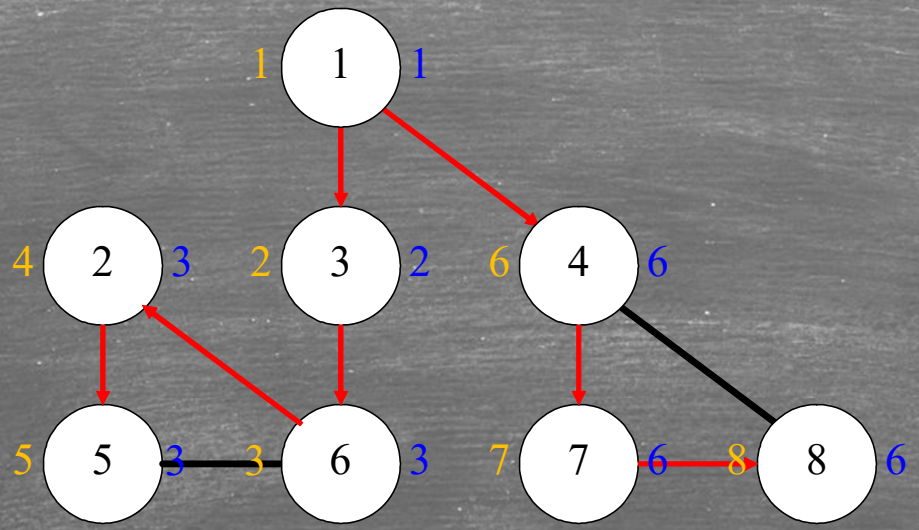


Articulation points

- Let us now number the graph again.
- In this instance the number is determined as follows:
  - Let w be the highest node that can be reached by following down zero or more arrows and then going along at most one non-arrow edge.
- Define highest[v] = prenum[w]
- E.g. from node 7 we can reach node 4
  - Highest[7] = prenum[4] = 6

Articulation Points

- The highest numbering is shown below:

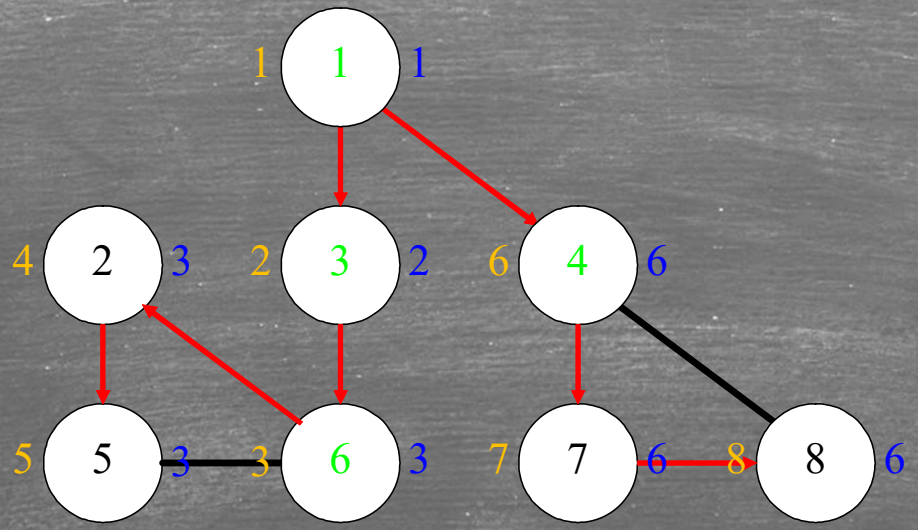


Articulation Points

- Consider each node *v* of the graph:
- If *v* is not the root of *T* then:
  - If *v* has no children in *T*, it is not an articulation point;
  - Otherwise, let *x* be a child of *v:*
    - If highest[*x*] < prenum[*v*] then we can reach a node higher up the tree than node *v* from node *x* which means that node *v* need not be used to reach node *x;*
    - If highest[*x*] ≥ prenum[*v*] then node *x* can only be reached via node *v.*
  - Node *v* is an articulation point if at least one of its children *x* satisfies highest[*x*] ≥ prenum[*v*].
- The root is an articulation point if it has more than one child.

Articulation points

- Nodes 1, 3, 4 and 6 are articulation points.



How: in Detail

- The complete algorithm for finding the articulation points of a graph *G* is:
  1. Conduct a depth first traversal of *G*, producing the spanning tree *T*.
  2. Traverse *T* in preorder and record the preorder sequence number prenum[*v*] (this can be done at the same time as the DFS).
  3. Traverse *T* in postorder. For each node, *v*, calculate highest[*v*] as the minimum of:
     1. prenum[*v*];
     2. prenum[*w*] for each node *w* such that *G* contains {*v*, *w*} and *T* does not;
     3. highest[*x*] for every child *x* of *v*.
  4. Determine the articulation points of *G* as follows:
     1. The root is an articulation point if it has more than one child;
     2. Any other node *v* is an articulation point if it has at least one child *x* such that highest[*x*] $\geq$ prenum[*v*].

Why?

- The identification of articulation points is important in determining the critical components of networks.
- A component which corresponds to an articulation point is critical.
- If such a component fails, the network is compromised.