CSIT113
Problem Solving

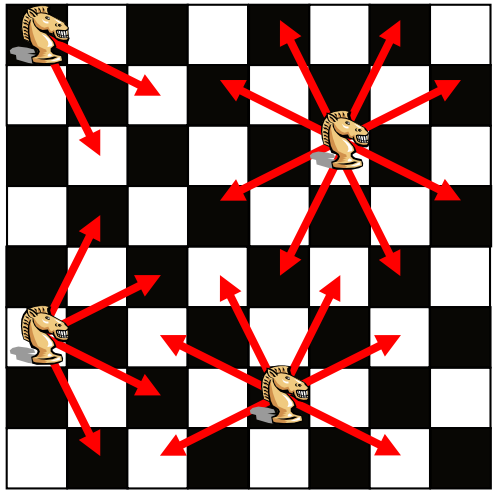
Week 13

The Knight’s Tour

- The problem here is to find a way to have a chess knight travel from square to square of a chess board visiting each square once and returning to its start position.
- The knight moves in the usual way it does in chess.
  - That is two up/down and one left/right, or one up/down and one left/right.

The Moves

- Depending on where the knight is we have between 2 (corner) and 8 (middle) choices.



Where do we Start?

- This problem is going to be hard so let’s look at some easier cases first.
- We could try one of two options:
  - change the allowed moves;
  - change the size of the board.
- Let us consider tours with other sorts of allowed move.

Different Pieces

- Suppose you have a piece that can move to any square.
- The tour is now trivially easy to construct
  - ```
    BEGIN Tour
        WHILE there are free squares
            move to a free square
        END WHILE
        move to the starting square
    END
    ```
- This is too easy.
    ```

Different Pieces

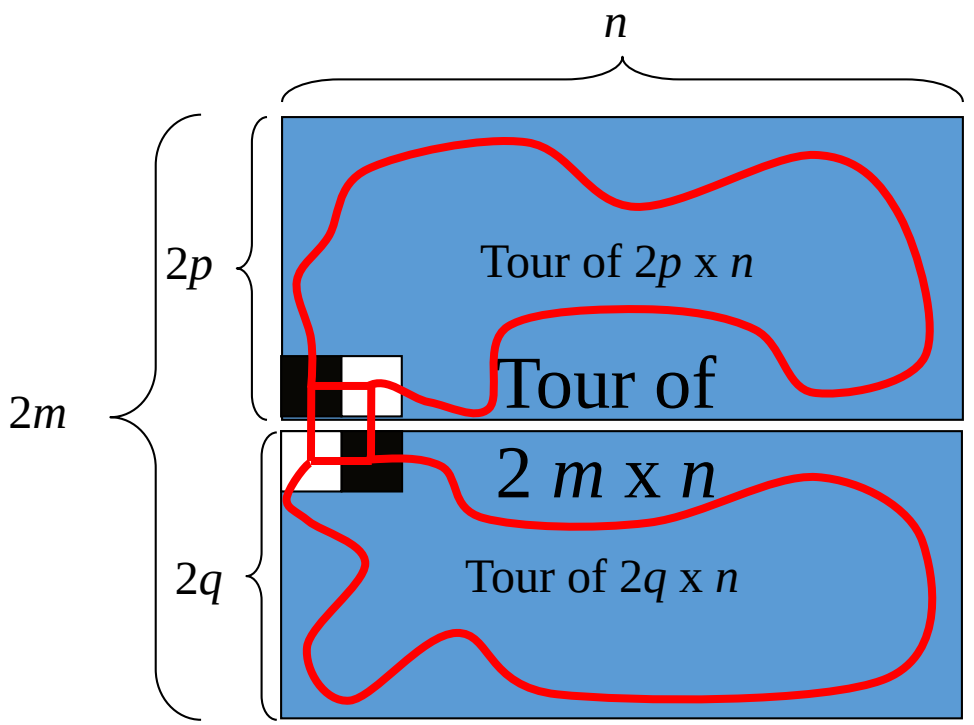
- The chess piece with the simplest move is the king.
- Can we find a king’s tour?
- What about on an arbitrary rectangular board?
- Yes, provided at least one side of the rectangle is even.
  - We can generally but there is an advantage for use in looking at the even sided board.

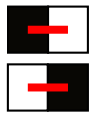
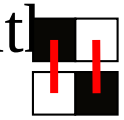
The King’s Tour

- Any board of size $2m$ x n can be broken up into 2 boards of size $2p$ x n and $2q$ x n.
- If we can construct tours for the smaller boards we can always combine these to give a tour for the whole board.
- How?

The King's Tour

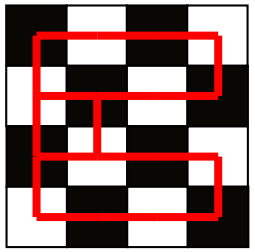
- Consider:



- Replace  with  and we have a full tour.

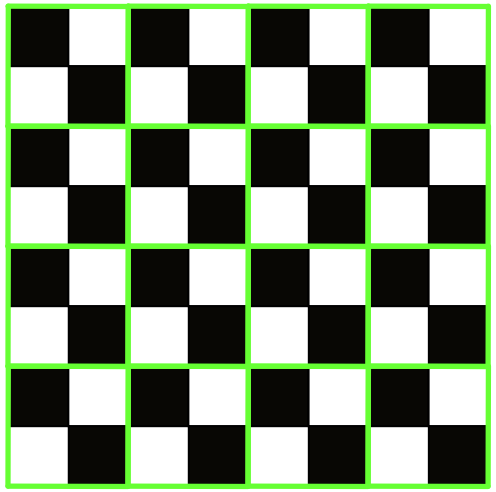
The King’s Tour

- Given that a 2 x *n* tour is a simple loop, we can construct a King’s tour on any 2*m* x *n* board.
- Using 4 x 4 as an example:



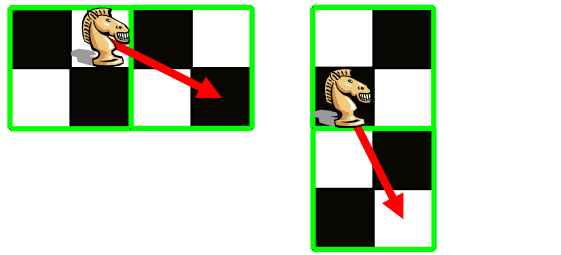
The Knight’s Tour

- Can we find a way of subdividing the board to make the knight’s tour easier to find?
- Let us break the board into 2 x 2 supersquares.

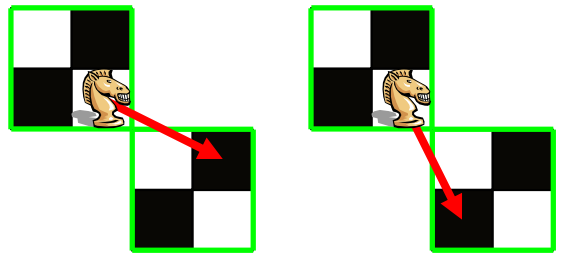


Supersquares

- Now, every knight move must be between a pair of supersquares.
- Either adjacent, horizontal or vertical:
  - *straight* moves;

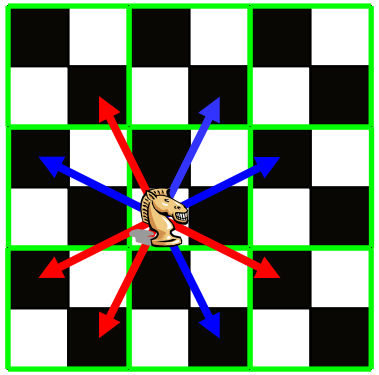


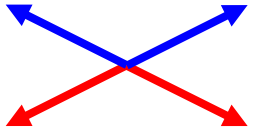
- or not:
  - *diagonal* moves.

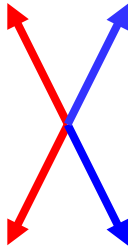


Supersquares

- For a single piece:
  - straight
  - diagonal.
- Note the symmetries:
  - horizontal moves;
  - vertical moves.

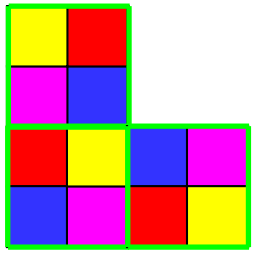






Straight Moves

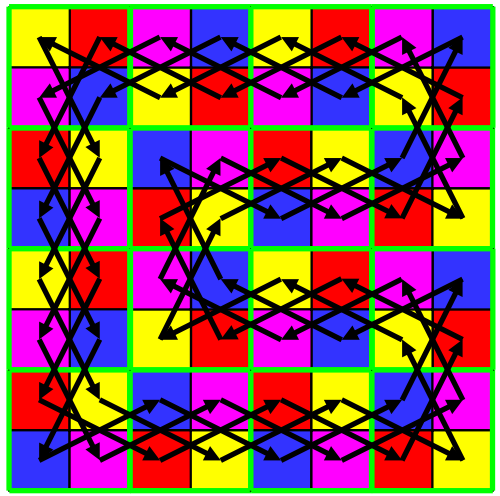
- If we consider only straight moves we get the following pattern:
  - vertical;
  - horizontal.



- In each case moves are between squares of the same colour.

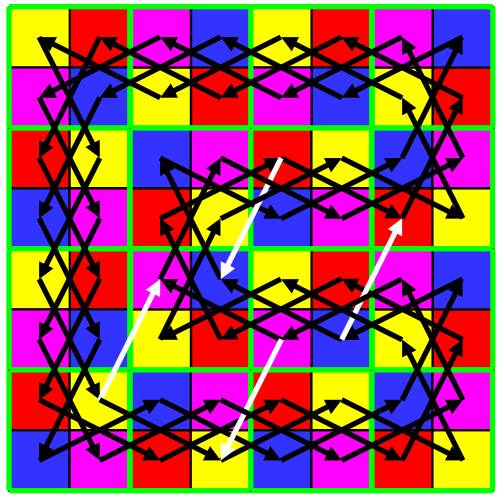
Straight Moves

- If we continue this pattern over the whole board we get.
- We can now create:
  - a red tour;
  - a yellow tour;
  - a blue tour;
  - and a magenta tour.



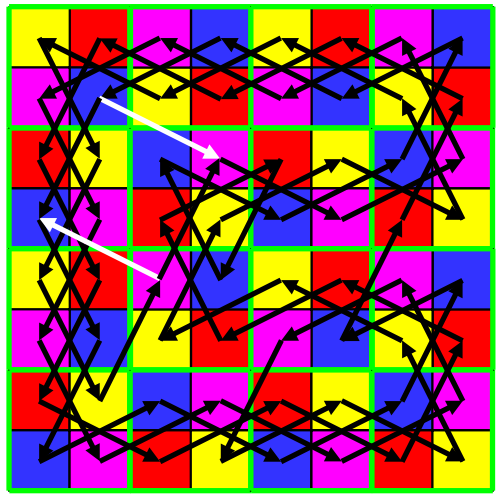
Combining Tours

- We can now combine the tours in pairs:
- Red
  - with
- Blue.
- Yellow
  - with
- Magenta



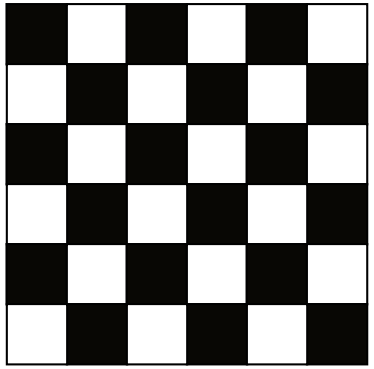
Combining Tours

- Finally, we can combine these tours:
- Red / Blue.
  - with
- Yellow / Magenta



A problem

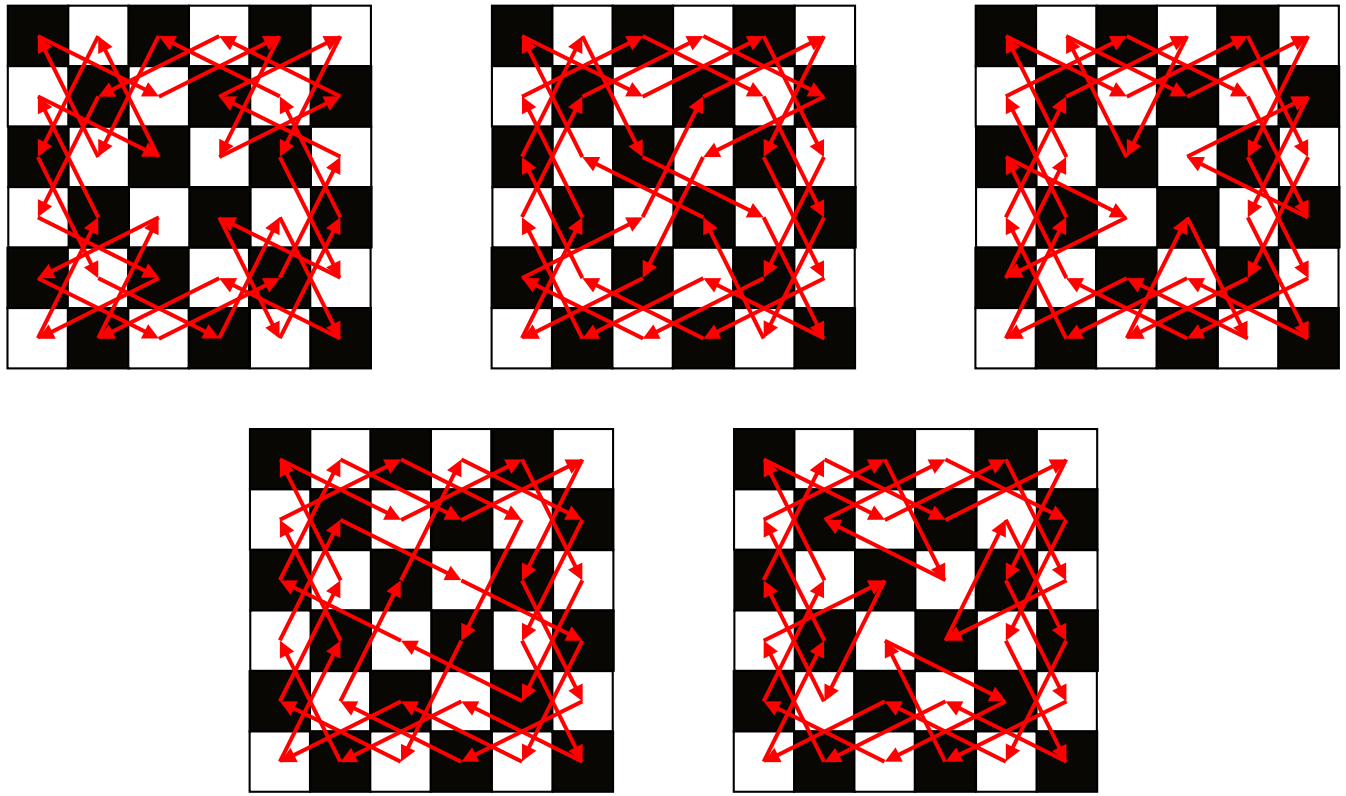
- Can we construct a Knight's tour for a 6 by 6 board?



- No. It doesn't have sides a multiple of 4 in length.
- Really?

The Knights Tour

- So, explain this:



The Knight’s Tour

- That's five unique tours.
- That's slightly more than “none.”
- There are in fact no fewer than 1245 distinct 6x6 knight’s tours!
- Not counting rotations and reflections!

The Knight’s Tour

- What question did you *actually* answer?
- **Question**: can you find a Knight's Tour for a 6x6 board **?**

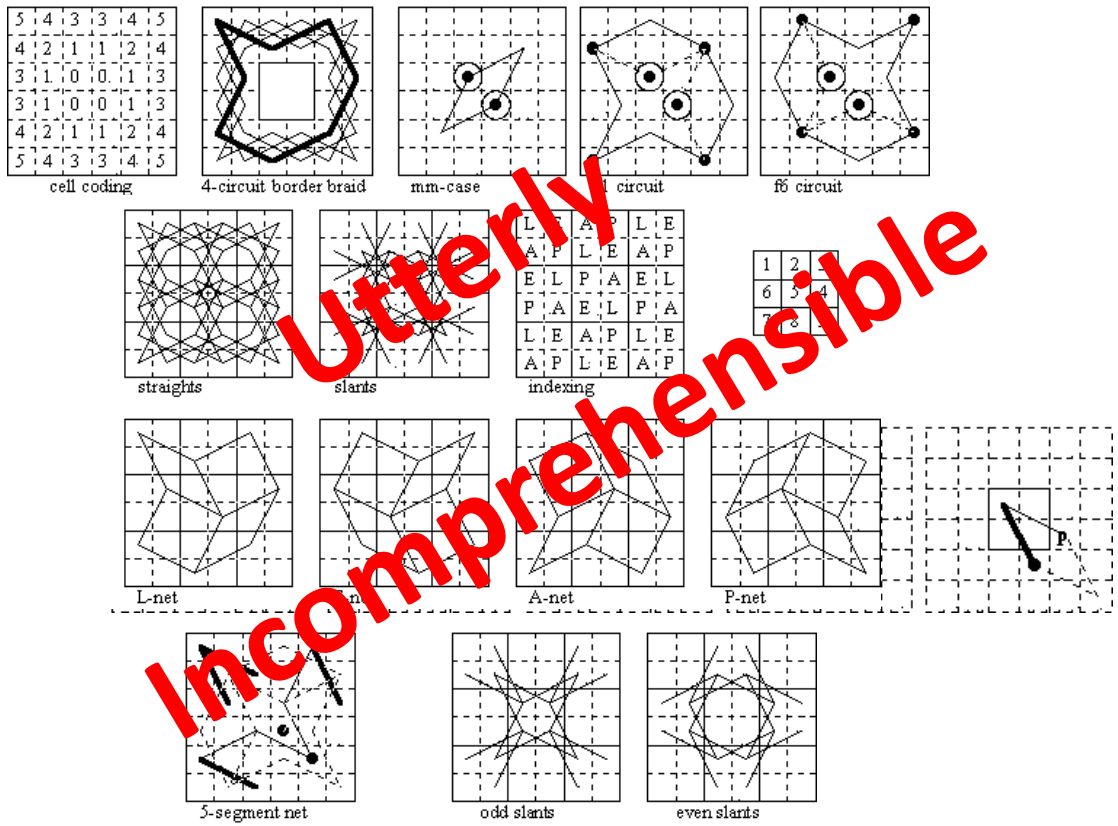
using a king’s tour?

- **Answer**: no, because it violates the rules for a King's Tour.

Knight’s Tour

- Ok, so the King's Tour technique won't work.
- So how do we find Knight's Tours?

Knight’s Tour



Knight’s Tour

- It's an interesting problem.
- So of course, there's a lot of people who spend a lot of time trying to solve it.
  - The Chinese were solving a similar puzzle as far back as 2200 BC.
  - The problem in its present form also shows up as far back as 840 AD

Knight’s Tour

- No closed tours for 5x5 or smaller square boards.
- 5x5 is, incidentally, the smallest square board that has open tours a.k.a. paths.
  - There are *112* distinct paths for 5x5 boards.
- You can find these open tours by circling around the middle square, and then fiddling with the edges.

Knight’s Tour

- There are lots of techniques involving heuristics, AI, and old fashioned “throw it at a wall and see what sticks.”
- Just because your elegant “all-in-one” technique didn't work, doesn't mean it's not possible.
  - (Just because you couldn't do it, doesn't mean it's not possible.)

Knight’s Tour

- Remember:
  1. Don't jump to conclusions: when you think you have a solution, make sure!
  2. Always double-check you're still answering the question you thought you were.
  3. Sometimes there *is no* really easy generalised solution.
     - (And sometimes there is!)
  4. Just because one strategy doesn’t work it doesn’t mean the problem is impossible.

Revision

- Week 1: What is a problem?
- Week 2: Brute force vs finesse, invariants, representations.
- Week 3: Invariants.
- Week 4: Logic, knights & knaves, calculational logic.
- Week 5: Induction, recursion.
- Week 6: Greediness.
- Week 7: Decomposition: Reduce and conquer, divide and conquer.
- Week 8: Sorting: Selection, insertion, bubble. Quicksort but more later.
- Week 9: Graphs and trees.
- Week 10: Games and graphs, backtracking.
- Week 11: Complexity, transforming problems, branch and bound.
- Week 12: Wicked problems.
- Week 13: Knight tours.

Polya's Problem Solving Method

Polya's method has four steps:

- Understand the problem: Make sure you understand what the problem is asking.
- Plan a strategy for solving the problem.
- Execute your strategy, and revise it if necessary.
- Check and interpret your result.

Learning is a skill*

Research has shown that

- Accuracy and thoroughness are mental habits
- Which can be cultivated through training and exercise.
- Learning skills happens gradually
- Then will become natural to you
  - Idea: to identify/list the least accurate areas and work on
  - Practice and practice

Borrow from slides at here: http://people.cs.vt.edu/prsardar/slides/L4-Problem%20Solving%20Methods.pdf

How to Succeed*

Four Keys to Success

- Learn to network: Take advantage of the resources that are allowed.
- Learn to focus: Multitasking interferes with this; so does procrastination.
- Learn to present: Be clear and complete and engaging.
- Learn to play the game: Don’t change a winning game, but always change a losing one.

Borrow slides from http://people.cs.vt.edu/prsardar/slides/T02_HowToSucceed.pdf

Learn to network

- Lack of interaction with faculty/staff.
  - Not only can they help you academically, they also help you professionally
- Lack of interaction with other students in the discipline.
  - Build a support environment/community.

Learn to focus: Organization

- Make creative use of communication options.
- Keep track of deadlines in-the-large:
  - Degree/semester deadlines
- Keep track of deadlines in-the-small:
  - Assignment due dates
  - Class meeting times
  - Group meeting times
- Good habits trump good memory

Learn to Focus: Environment

- Doing hard work requires a conducive environment.
  - Set yourself up for success by reducing distractors and gathering resources.
- Certain types of work don’t need full attention.
  - Some distractors, such as music, might be OK.
- Other types of work need complete focus.
  - Writing (prose or code), most homework problems, hard debugging all generally require your full attention.

Learn to Present

The primary goal of communication is to invoke the desired response in your audience.

- Email communications
- Tests
- Homework
- Writing
- Proof and argument

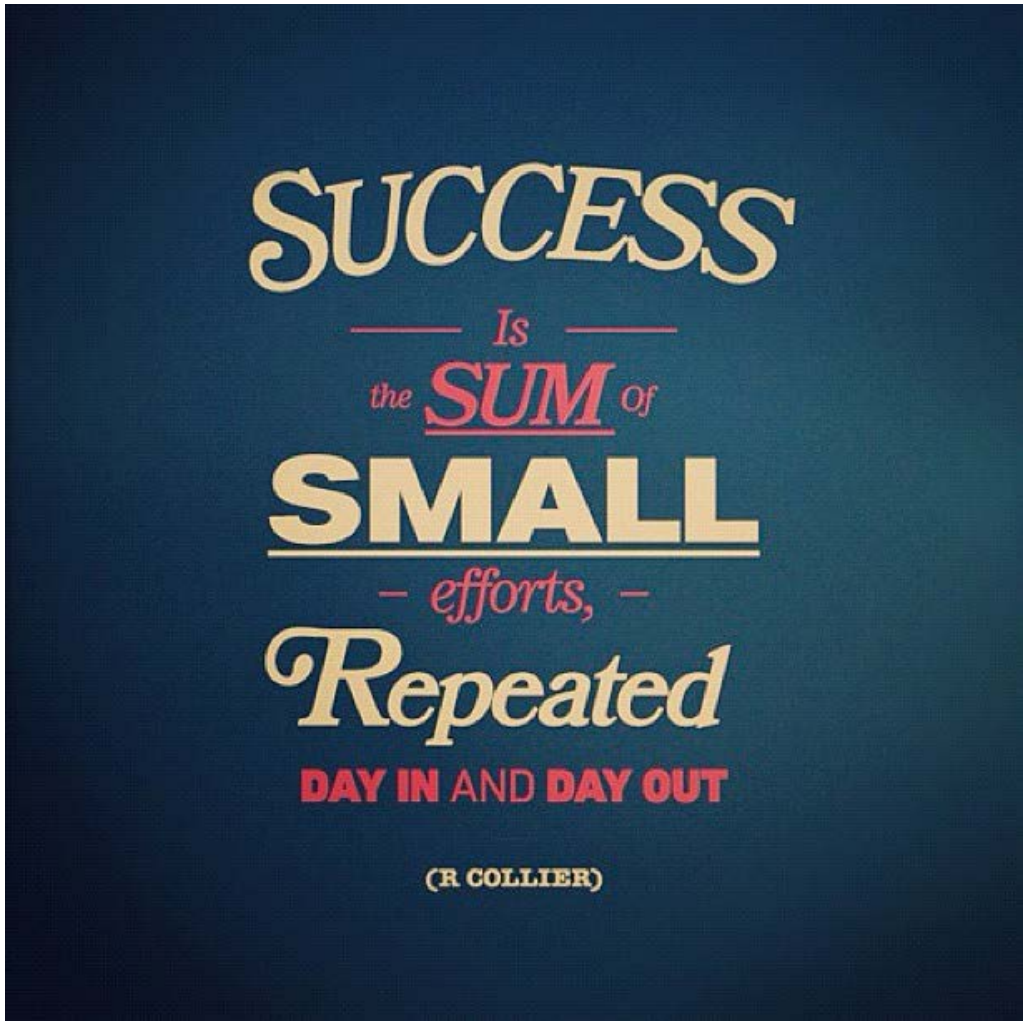
Learn to Play the Game

- Being a good student is a (learned) skill, not an (innate) ability.
- Get the easy points.
  - Never short-change easy assignments or classes
  - A little investment (or reallocation) of time could raise your overall score
- Learn the testing game.
- Learn time management/stress management.

Change a Losing Game

- What is your preferred approach to learning?
- Be prepared to adjust your approach to various styles of delivery.
- Seek adjustment in the course conditions if practical, but in most situations individuals must adapt to the existing reality.

Practice and Practice



THANK YOU
AND
GOOD LUCK

Please enrol in your Spring 2020 subjects as soon as possible.

ADVANTAGES:

- To ensure that you get into the computer lab or workshop of your choice
- To map out your timetable for Spring session
- To get details of your subject

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