CSIT113
Problem Solving

Workshop – Week 5

Proofs by induction

Prove the following formulae by induction

1) $1 + 2 + \ldots + n = \frac{1}{2}n(n + 1)$

2) $1^2 + 2^2 \ldots + n^2 = \frac{1}{6}n(n + 1)(2n + 1)$

Steps

- Check the initial case (either n=0 or n=1)
- Assume it holds for n
- Prove for the case n+1

Notes

- We can prove the above formulae because we know the formula of the left hand side and the right hand side.
- What if we are given only the left hand side and asked to find the formula of the right hand side?
- i.e., find the formulae of the following

$$1^3 + 2^3 + \cdots + n^3$$

- How do we proceed?

Problem in Lecture

Consider the sequence produced by adding successive powers of 2.

- 1 + 2 + 4 + 8...

- 1, 3, 7, 15, ...

- Prove that this sequence is of the form $2^n - 1$

Problem in Lecture

Similarly the sequence produced by adding successive powers of 5...

- 1 + 5 + 25 + 125...

- 1, 6, 31, 156, ...

Prove the sequence to be of the form $(5^n − 1)/4$.