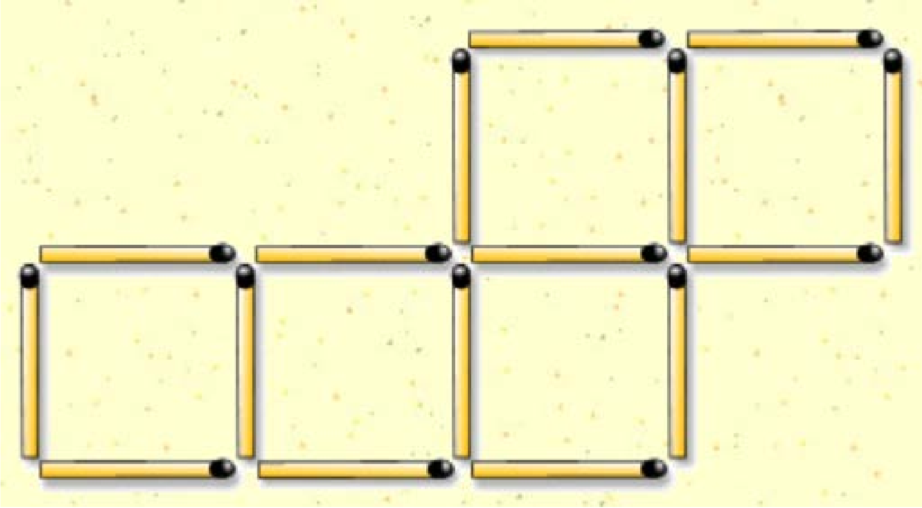
CSIT113: Problem Solving

Week 3B: Games

Matchstick Games

- Play with one or more piles of matches
- Two players take turns to make a move
- Moves: removing one or more matches from one of the piles, following the rule
- The game ends when it is no longer possible to make a move
- The player whose turn it is to move is the loser, and the other is the winner



Matchstick Games

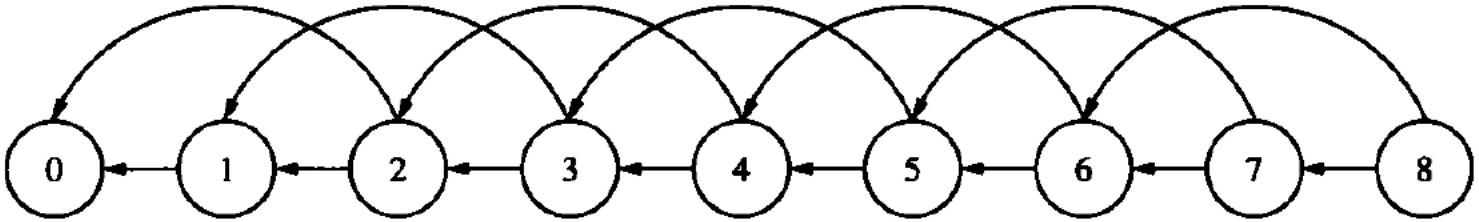
- It is an example of an **impartial**, two-person game with **complete information**:
  - Impartial: Rules of moving apply equally to both players
  - Complete: Both players know the complete state of the game
- A **winning position**: one from which a perfect player is always assured of a win
- A **winning strategy**: an algorithm for choosing moves from winning positions that guarantees a win

Easy Matchstick Games

- One pile of matches
- Allowed move: remove 1 or 2 matches

Labelling positions

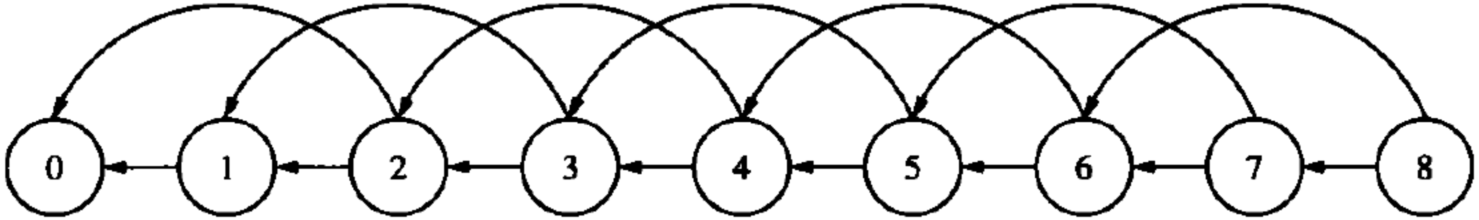
- Using a directed graph
  - Has a set of nodes and a set of edges.
  - Each edge is from one node to another node



- From node 0: there are no edges.
- At node 1: one match remains, there is exactly one move: remove the remaining match
- From all other nodes: there are two edges. One may remove one or two matches

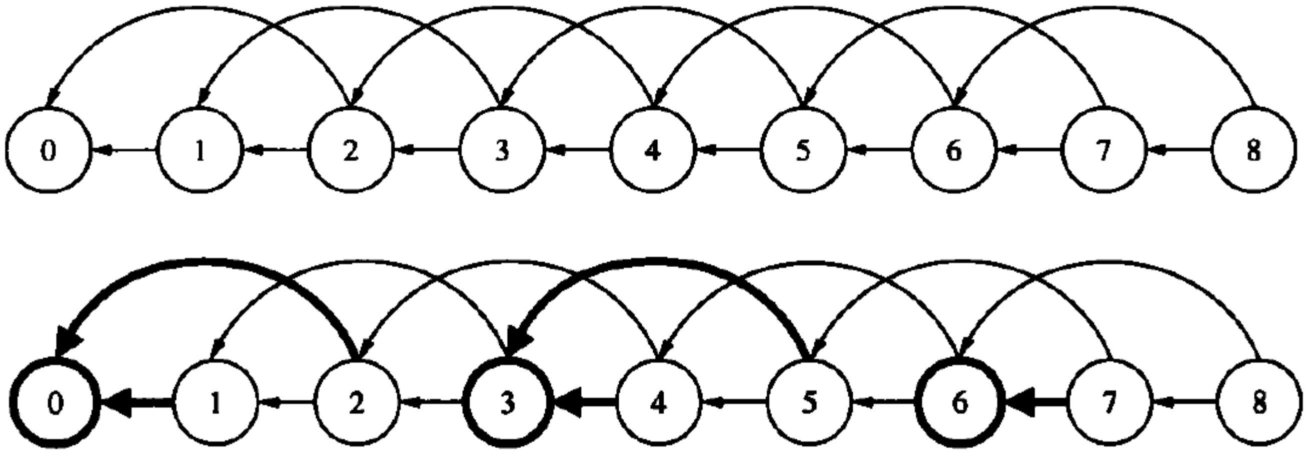
Labelling positions

- **Losing node**: every edge from the node is to a winning position
- **Winning node**: there is an edge from the node to a losing position
- What are losing nodes and winning nodes in the graph?



Labelling positions

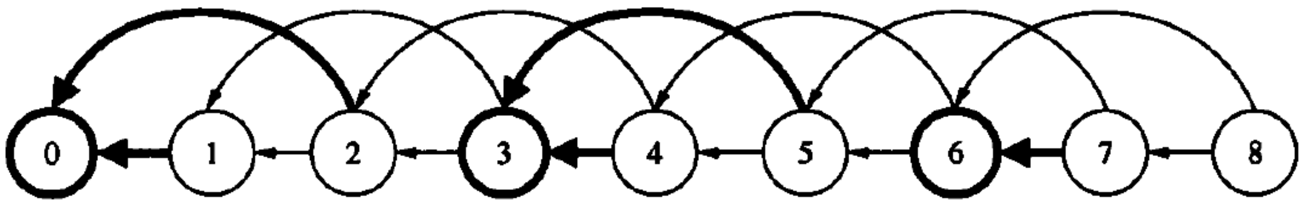
- **Losing nodes**: 0,3,6
- **Winning nodes**: 1,2,4,5,7,8



- **Pattern**:
  - a losing position: the one with the number of matches is a multiple of 3
  - all others are winning positions

Winning strategy

- What is the winning strategy?



- **Winning strategy**: remove one or two matches so as to leave the opponent in a position where the number of matches is a multiple of 3

Winning strategy

- Winning strategy is obtained by characterising the losing positions
- End positions are losing positions
- From a losing position that is not an end position, every move is to a winning position
- From a winning position, it is always possible to apply the winning strategy resulting in a losing position

Winning strategy

$$\{\, n \text{ is a multiple of 3, and } n \neq 0 \}$$

$$\text{If } 1 \leq n \text{ then } n := n-1$$

$$\text{If } 2 \leq n \text{ then } n := n-2$$

$$\{ n \text{ is not a multiple of 3} \}$$

$$n := n - (n \bmod 3)$$

$$\{ n \text{ is a multiple of 3} \}$$

The winner is decided by the starting position.

- If the starting position is a losing position, the second player is guaranteed to win
- If the starting position is a winning position, the first player is guaranteed to win