CSIT113
Problem Solving

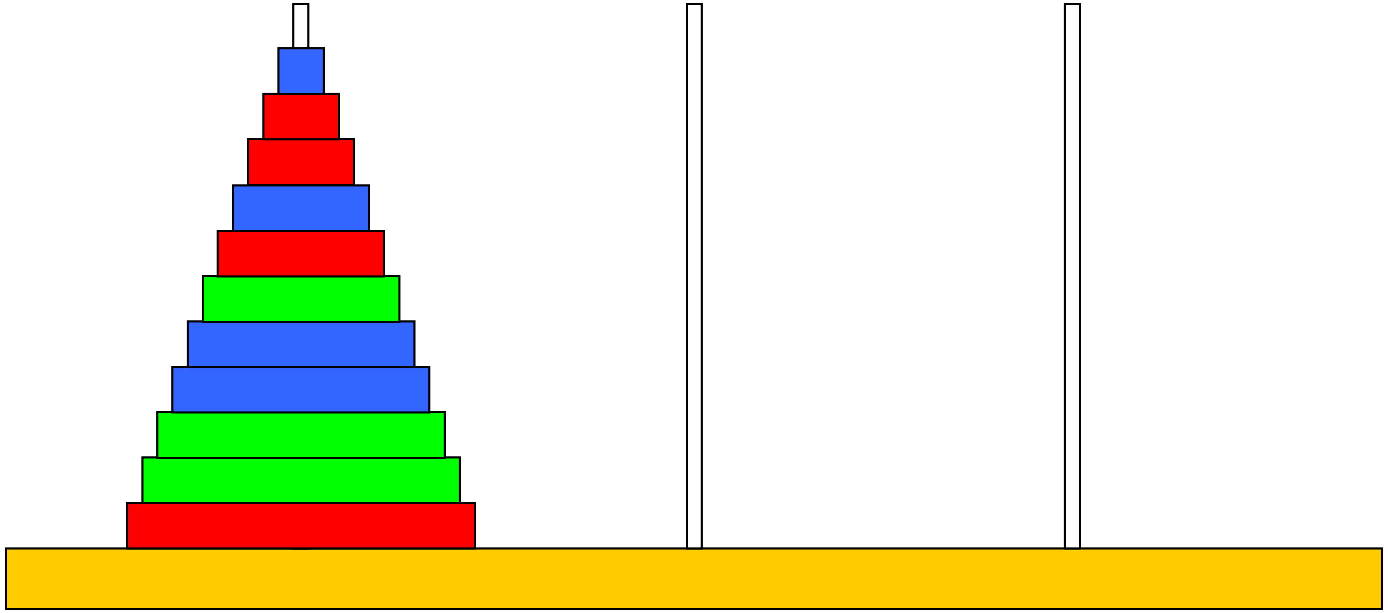
Workshop - Week 7 - Solutions

Induction

- Consider a “Towers of Hanoi” problem with one extra condition:
    - Each of the disks is coloured randomly either red, green or blue.
- Using the normal rules devise an algorithm to put each colour of disc on its own needle.

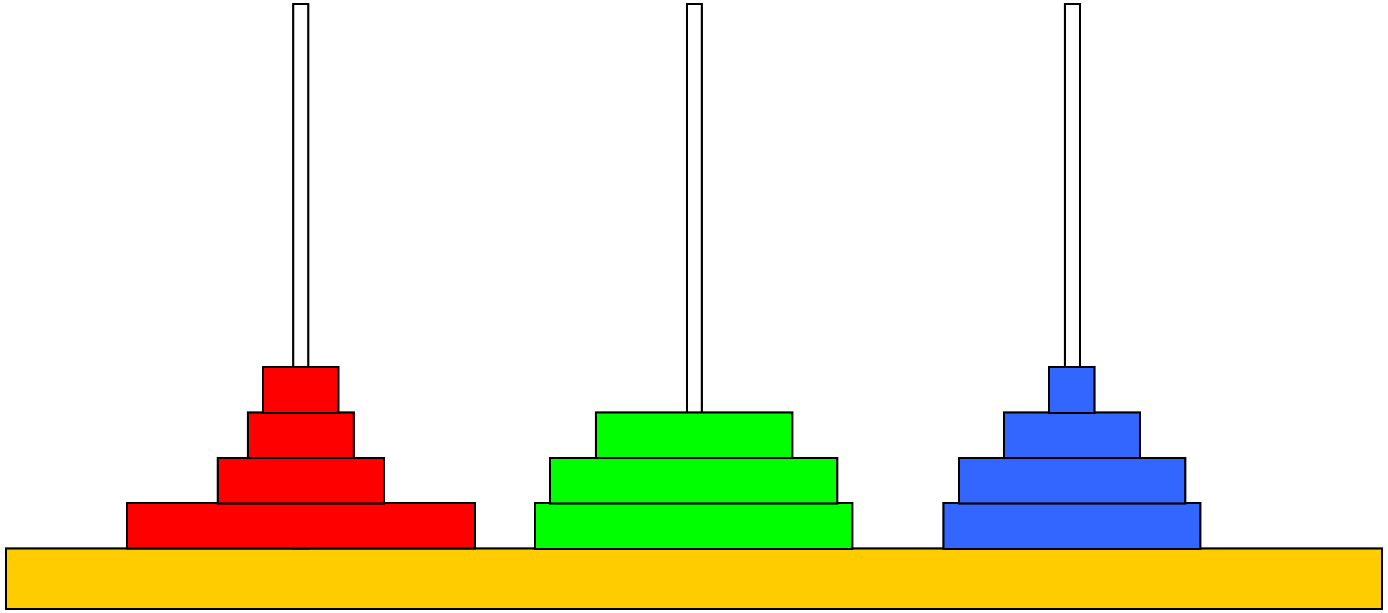
Induction

- Start: there are 11 disks



Induction

- Finish:



Induction

- You must still obey all the standard rules:
  1. Only one disc may be moved at a time
  2. Discs may only be placed on needles
  3. A larger disc may never be placed on a smaller disc
- You may assume that you have $H_{k,d}$ and $\langle k,d \rangle$ already defined.
  - $H_{k.d}$ be the sequence of moves required to move the *k* smallest discs in direction *d*
  - $\langle k,d \rangle$ represents a single move of disk *k* in direction *d*.

Details of small cases

- Let us analyze the notation further:
  - $H_{1, a}$:= <1, a> (1 step)
  - $H_{1, c}$:= <1, c>
  - $H_{2, a}$:= <1, c>; <2, a>; <1, c> (3 steps)
  - $H_{2, c}$:= <1, a>; <2, c>; <1, a>
  - $H_{3, a}$:= <1, a>; <2, c>; <1, a>; <3, a>; <1, a>; <2, c>; <1, a> (7 steps)
  - $H_{3, c}$:= <1, c>; <2, a>; <1, c>; <3, c>; <1, c>; <2, a>; <1, c>

Linking to the smaller problem (recursion)

- Note the possibility of recursion with the given definitions:
- $H_{1,\ a}$ = <1, a> 1 step
- $H_{2,\ c}$ = <1, a>; <2, c>; <1, a> 3 steps
- $H_{3,\ a}$ = <1, a>; <2, c>; <1, a>; <3, a>; <1, a>; <2, c>; <1, a> 7 steps

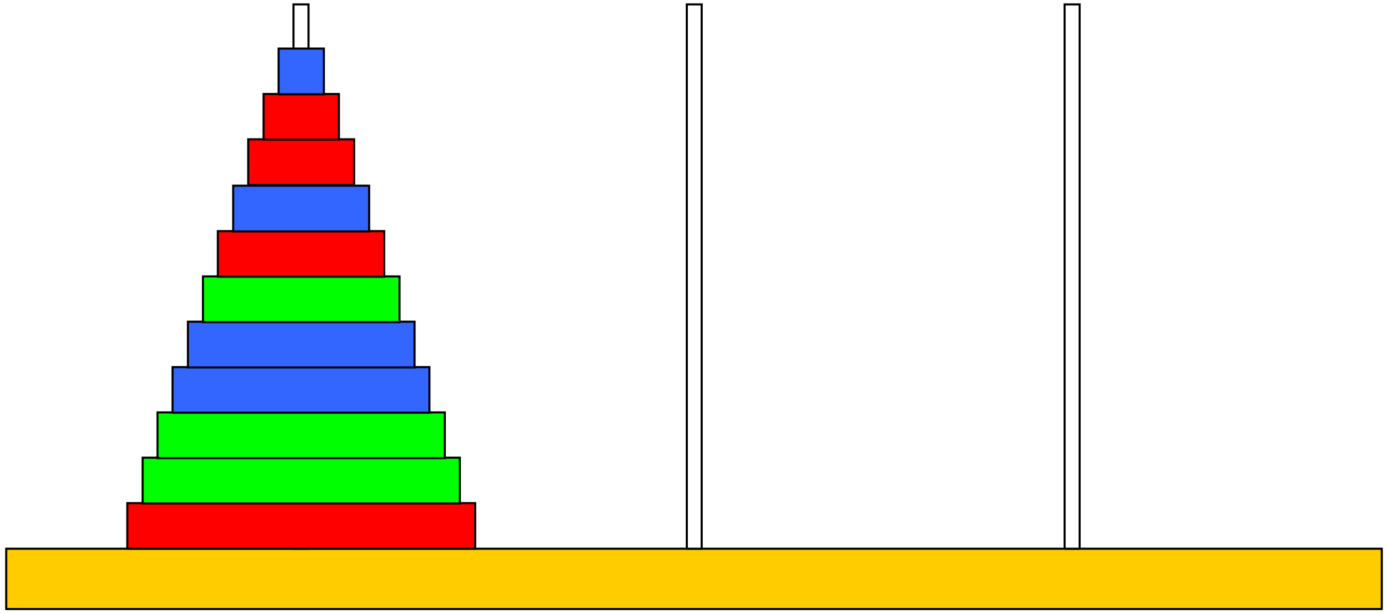
- The above can be also written as:
- $H_{1,\ a}$ = <1, a> 1 step
- $H_{2,\ c}$ = $H_{1,\ a}$; <2, c>; $H_{1,\ a}$ 2x1 + 1 = 3 steps
- $H_{3,\ a}$ = $H_{2,\ c}$; <3, a>; $H_{2,\ c}$ 2x3 + 1 = 7 steps

Number of steps

- From previous slide it can be seen that:
  - $H_{1,\,a}$ takes $2^1 - 1$ steps
  - $H_{2,\,a}$ takes $2^2 - 1$ steps
  - ….
  - $H_{n,\,a}$ takes $2^n - 1$ steps
- It can also be seen using recursion, that:
  - $H_{n,\,a}$ takes $(2 \times H_{n-1,\,c} + 1)$ steps

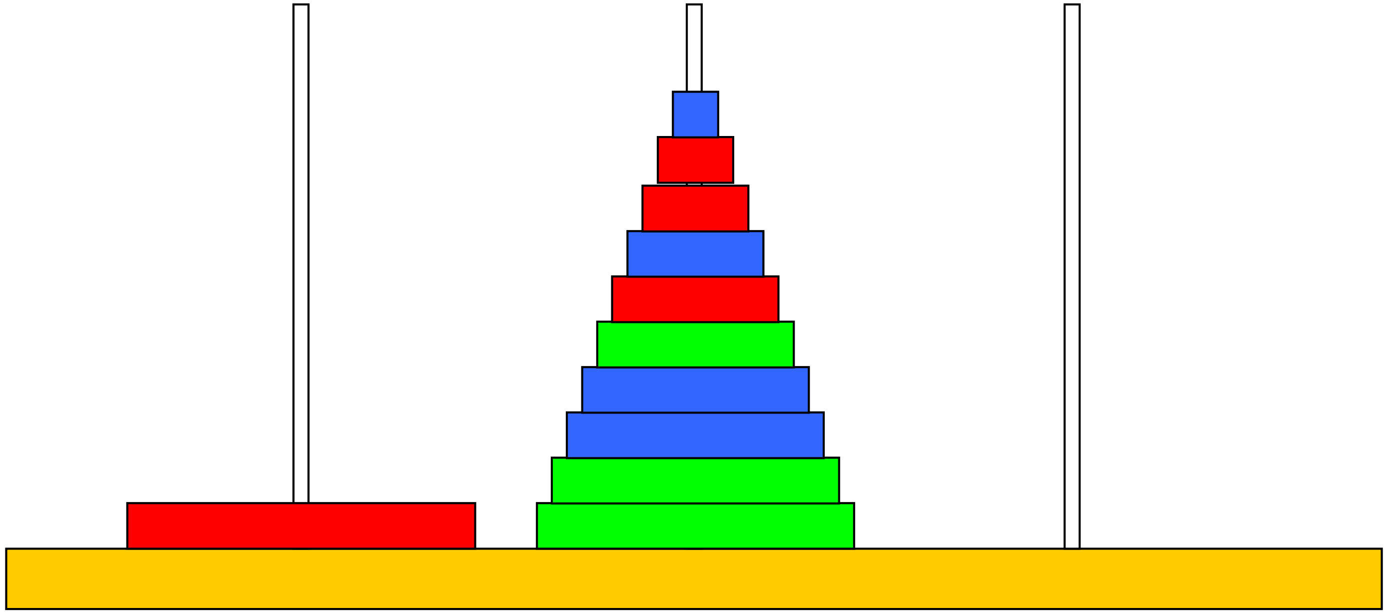
Induction

- Solution



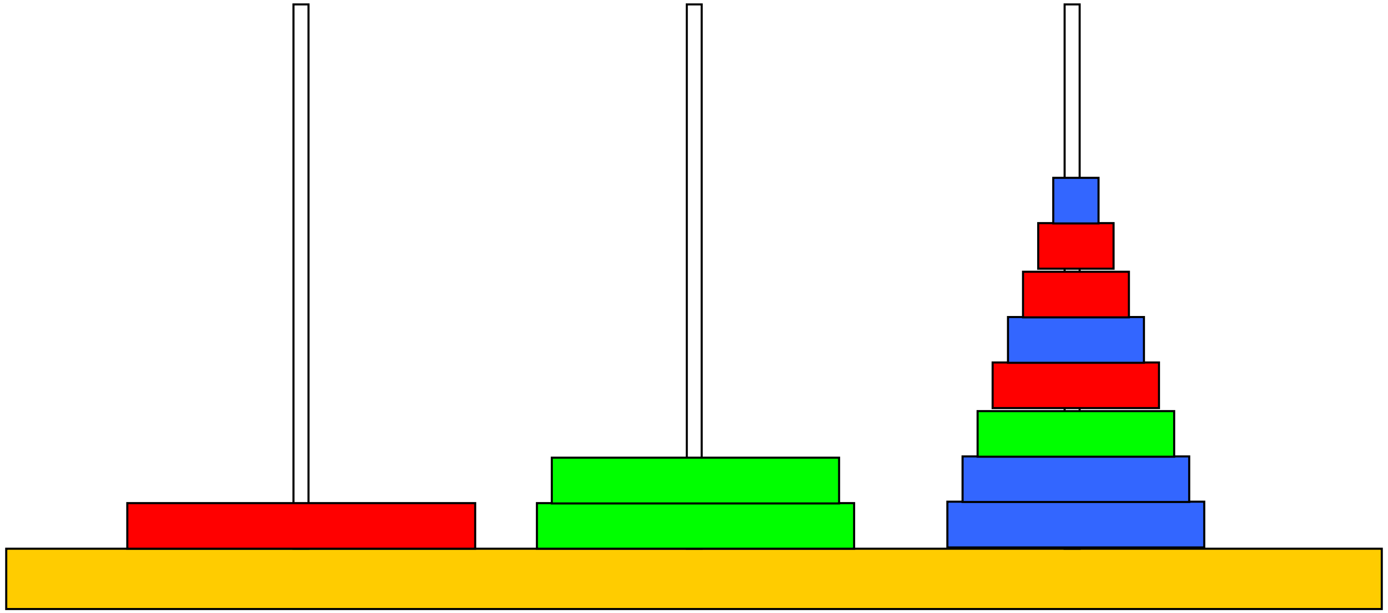
Induction

- $H_{10,C}$



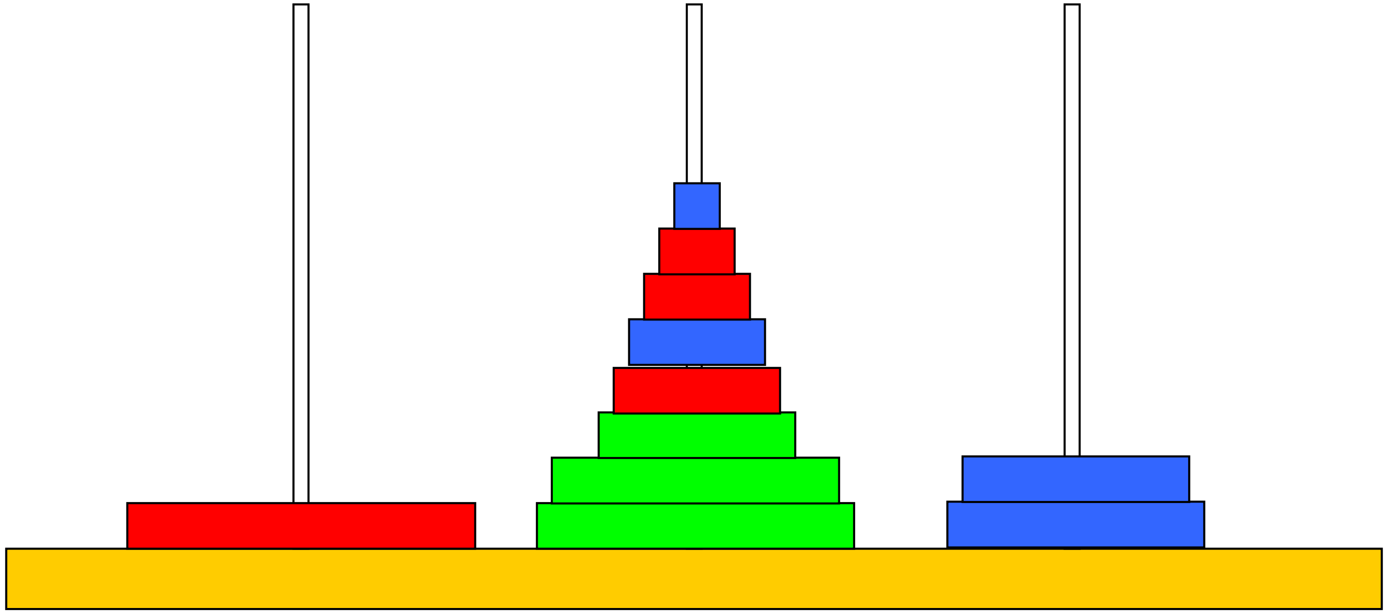
Induction

- $H_{8,C}$



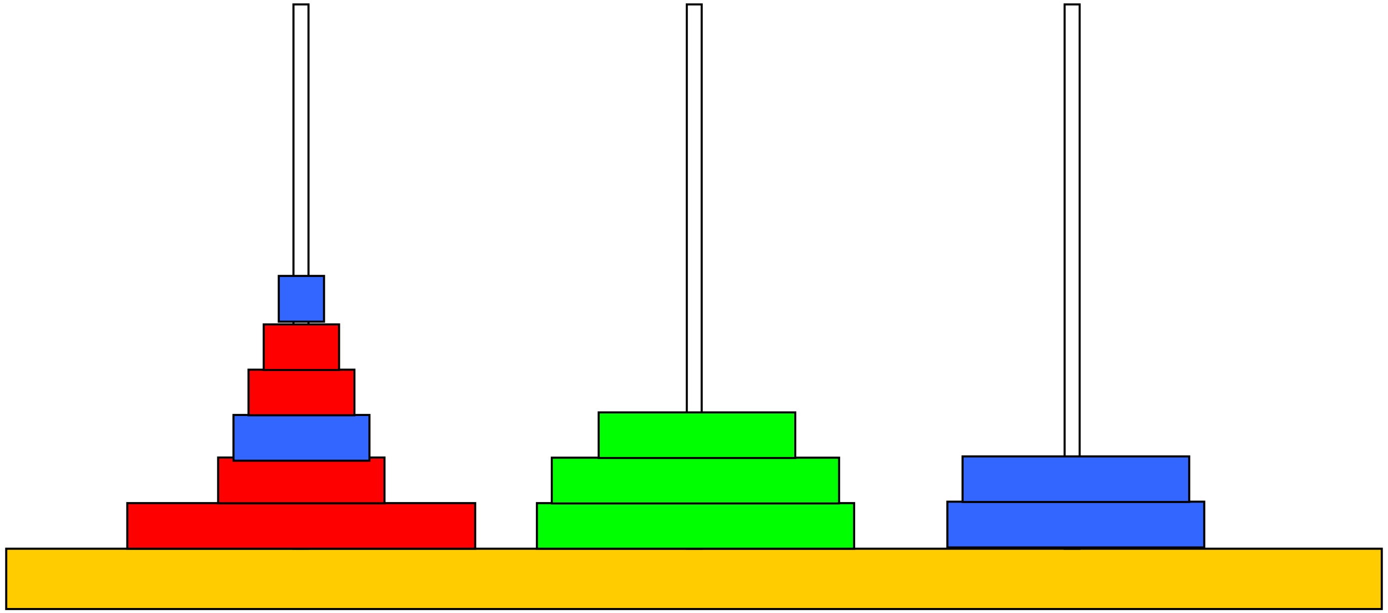
Induction

- $H_{6,A}$



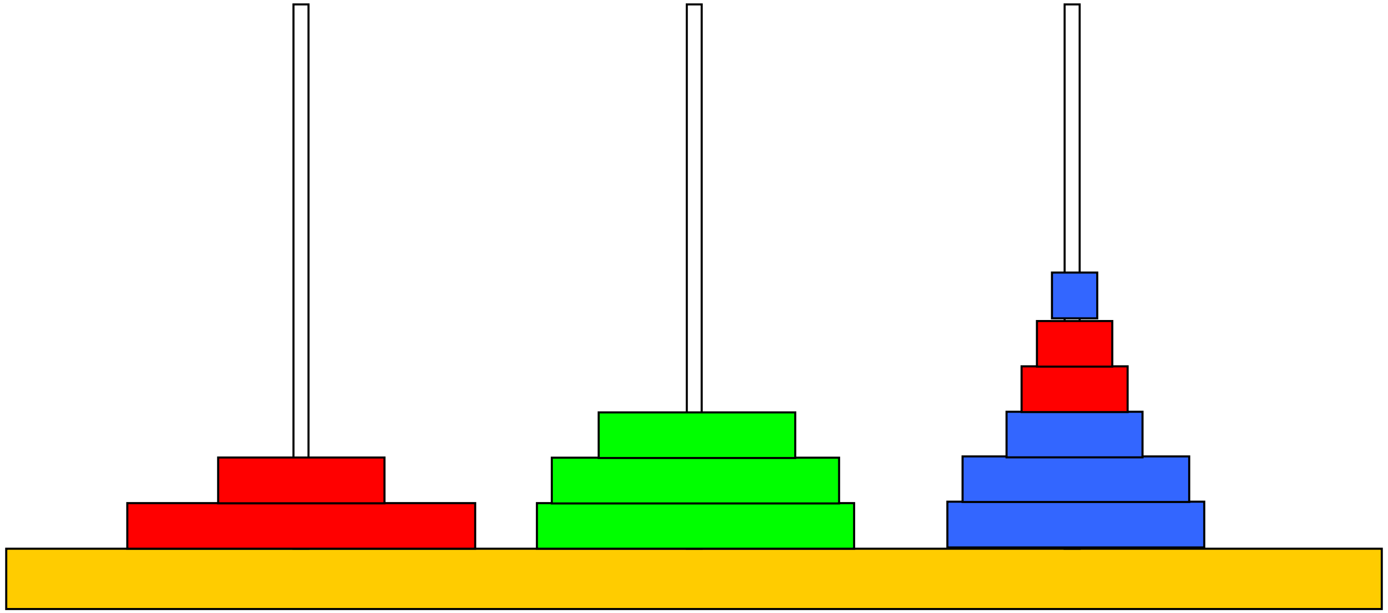
Induction

- $H_{5,A}$



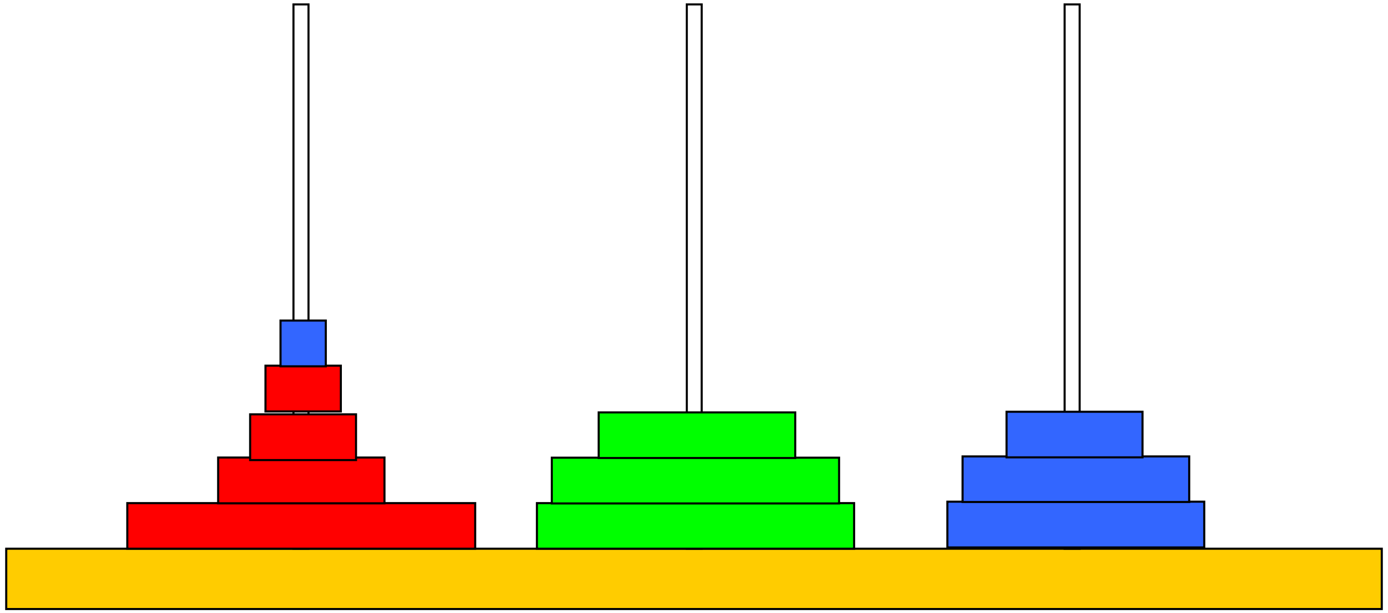
Induction

- $H_{4,A}$



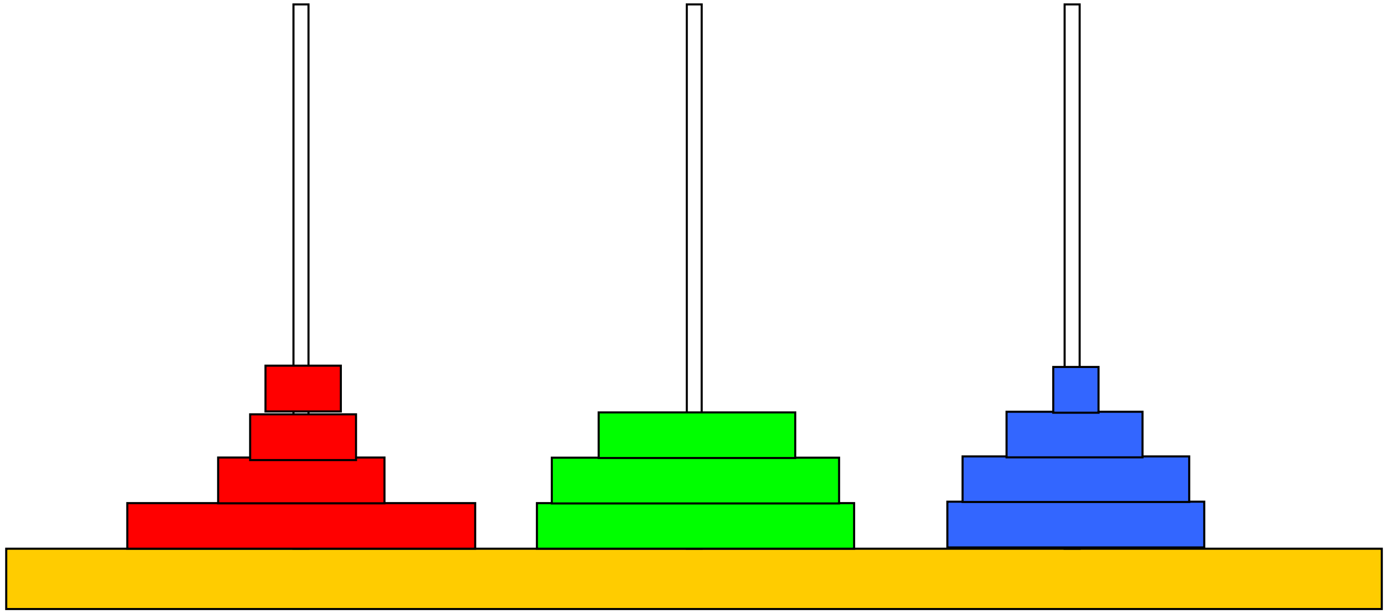
Induction

- $H_{3,C}$



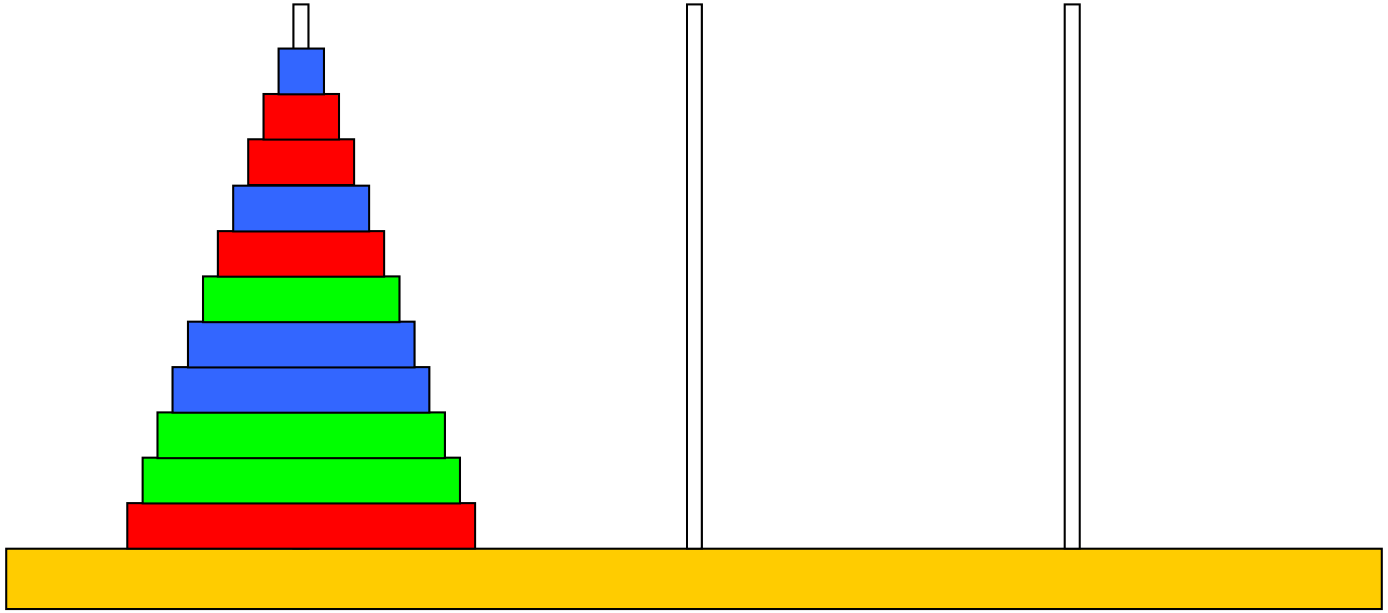
Induction

- $H_{1,A}$



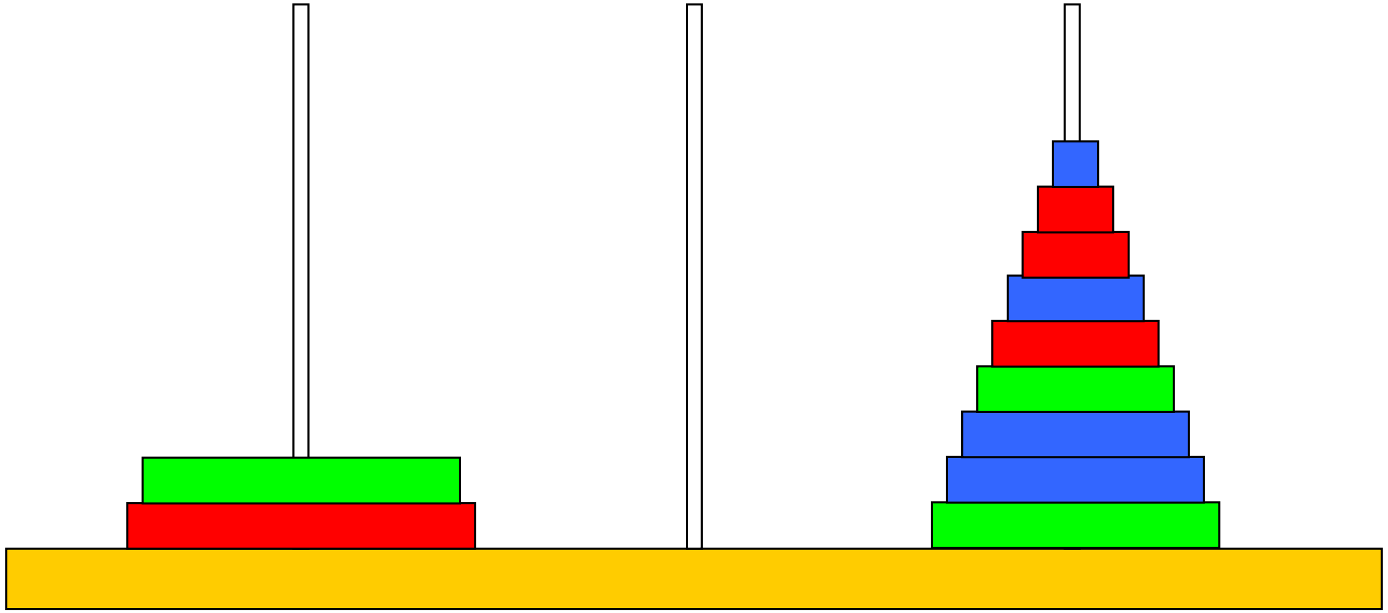
Induction

- Improved Solution



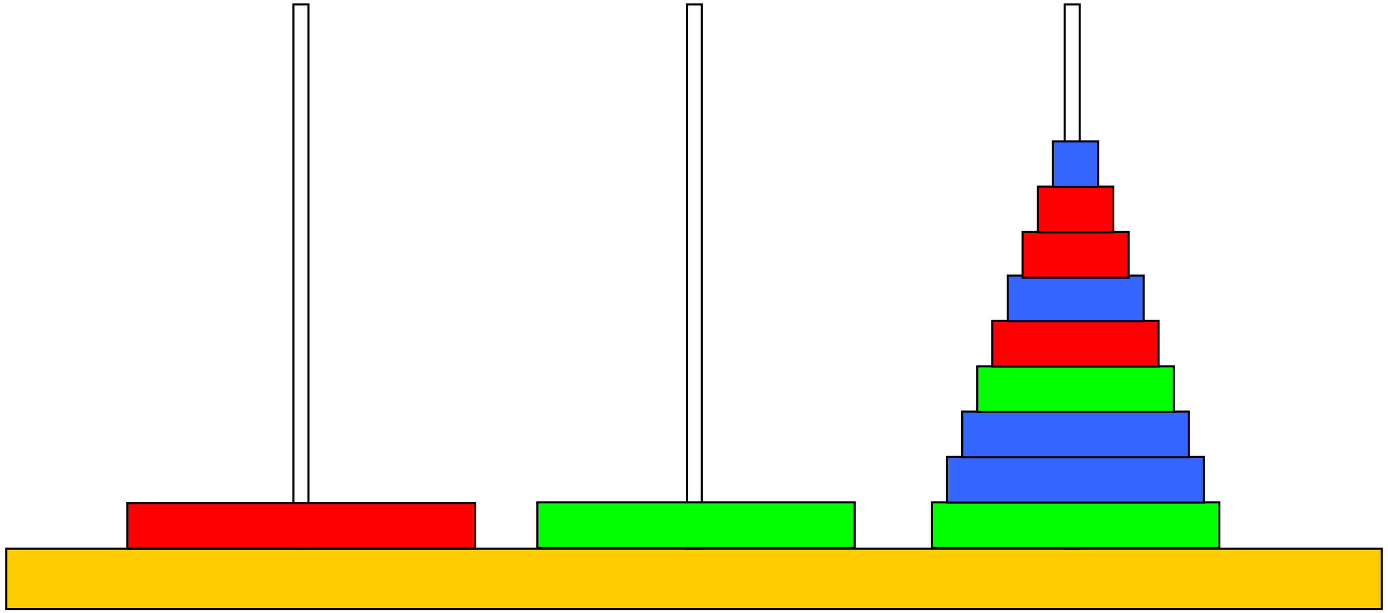
Induction

- $H_{9,A}$



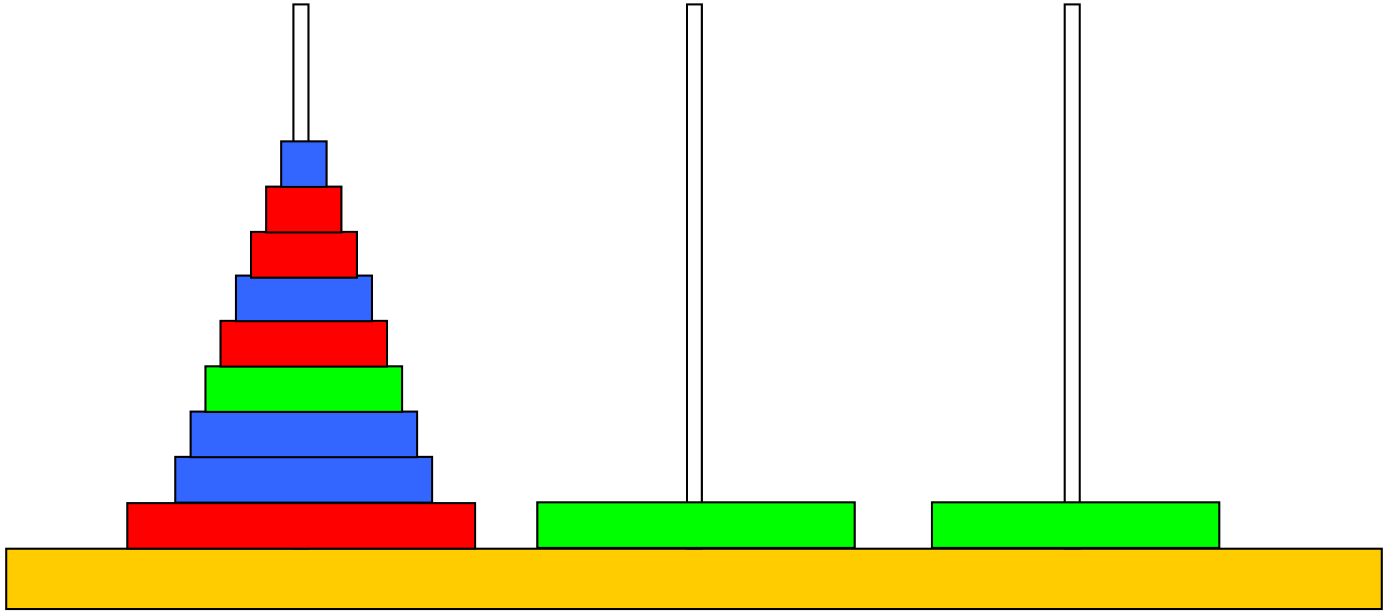
Induction

- <10,C>



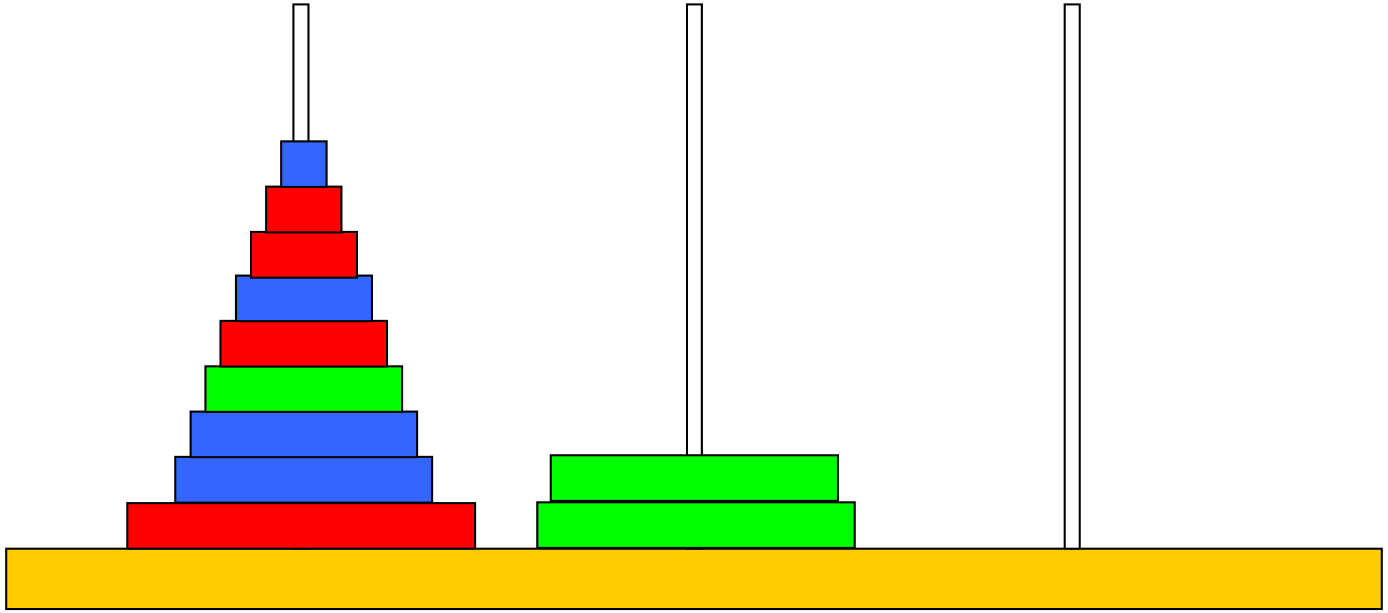
Induction

- $H_{8,C}$



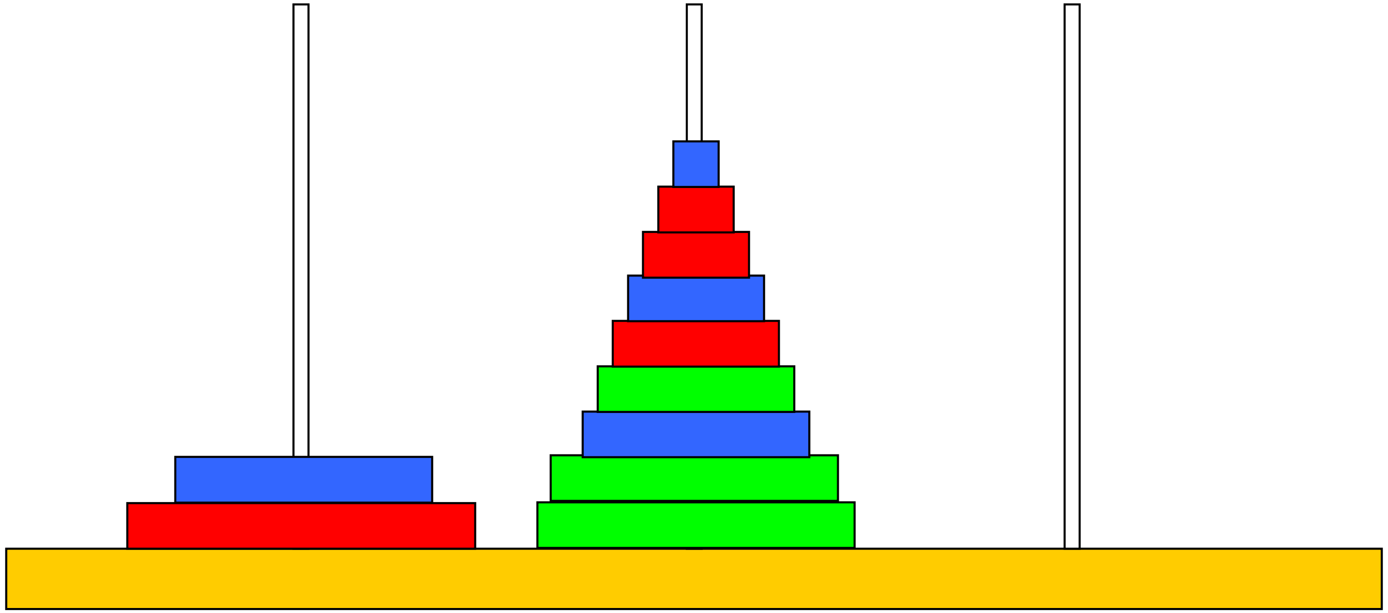
Induction

- <9,A>



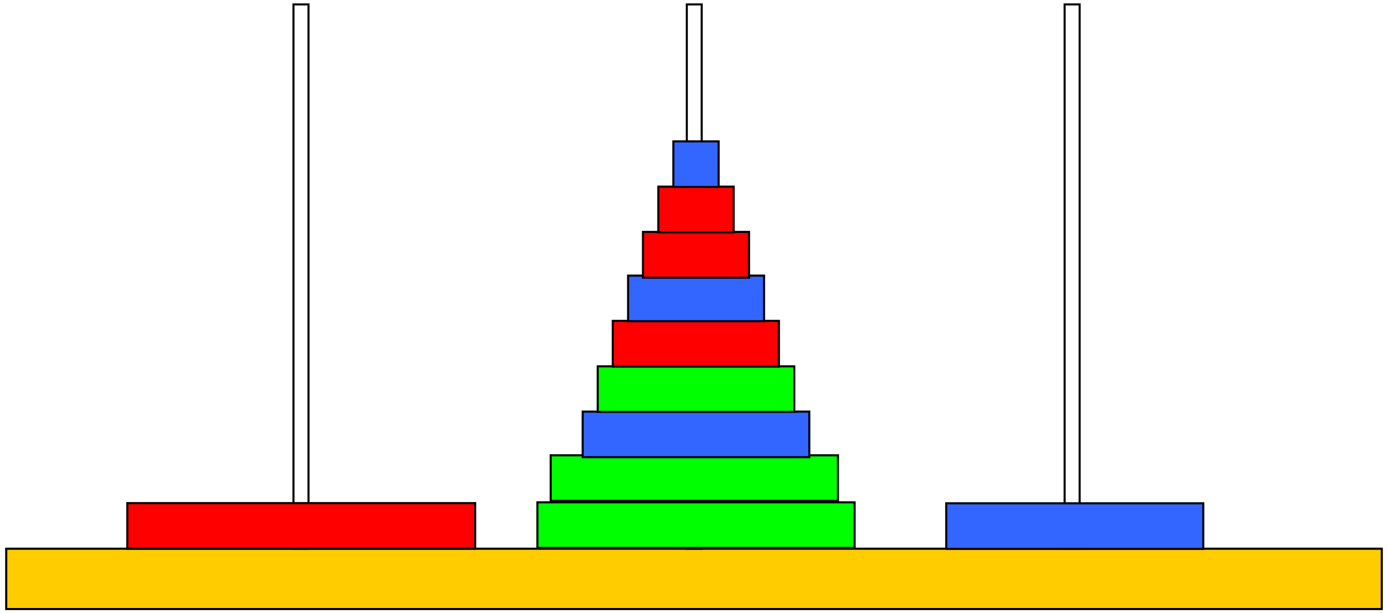
Induction

- $H_{7,C}$



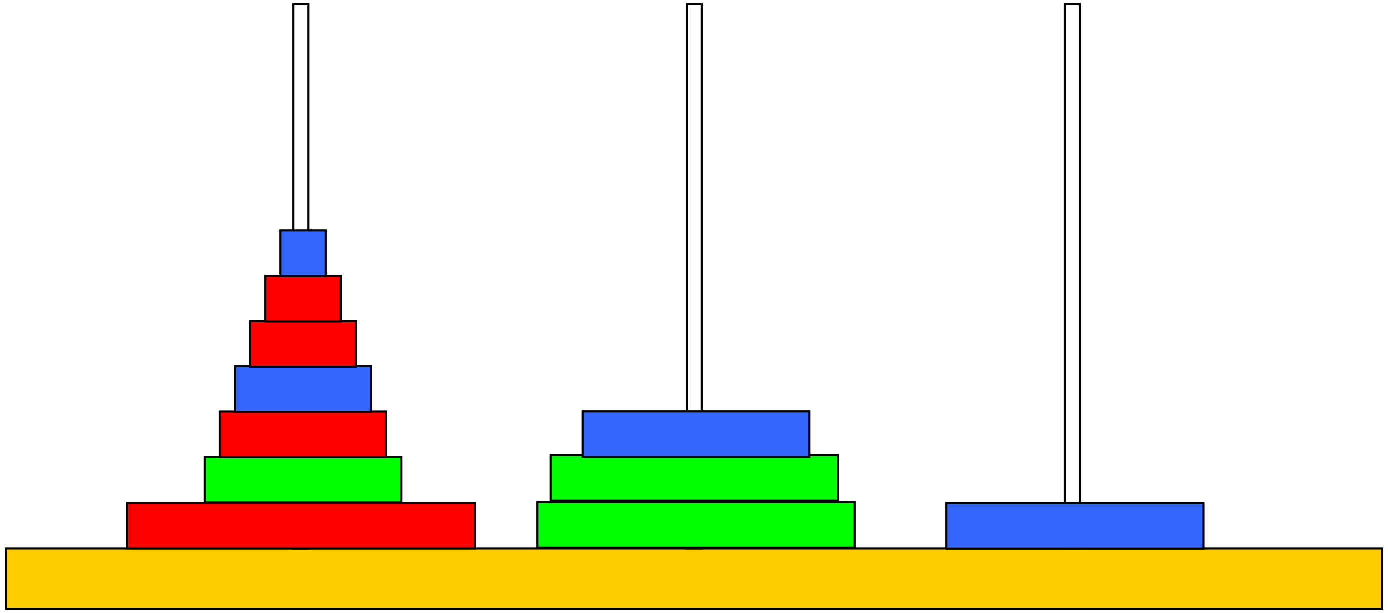
Induction

- <8,A>



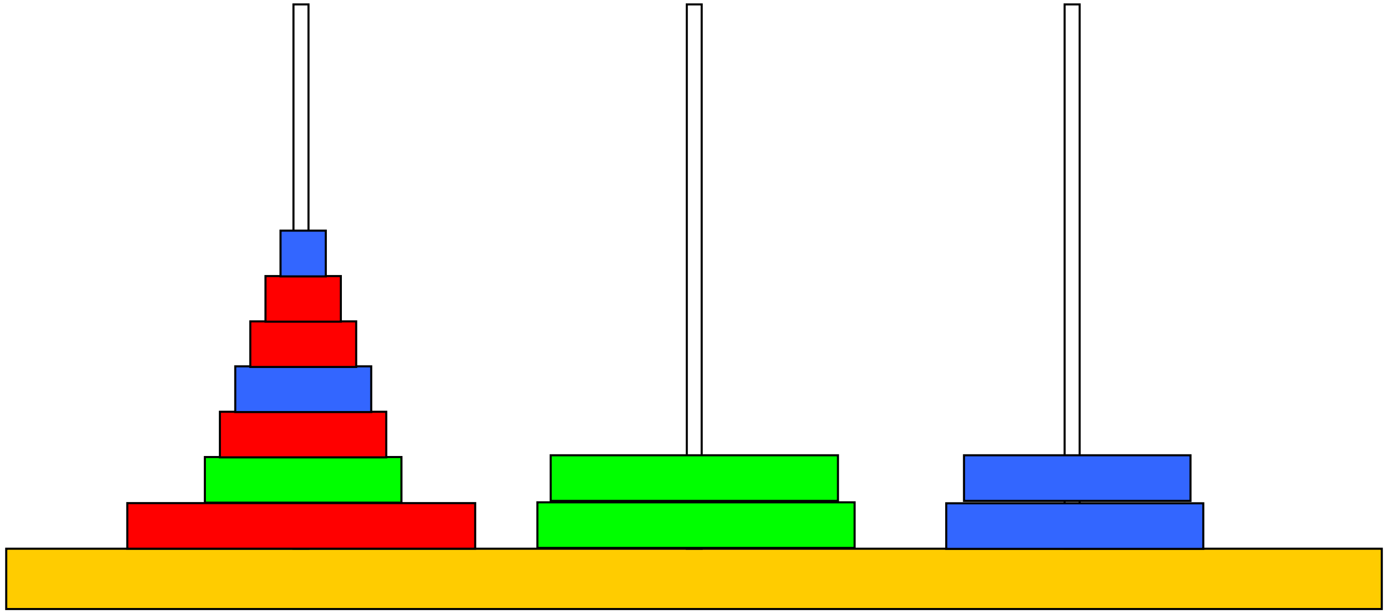
Induction

- $H_{6,A}$



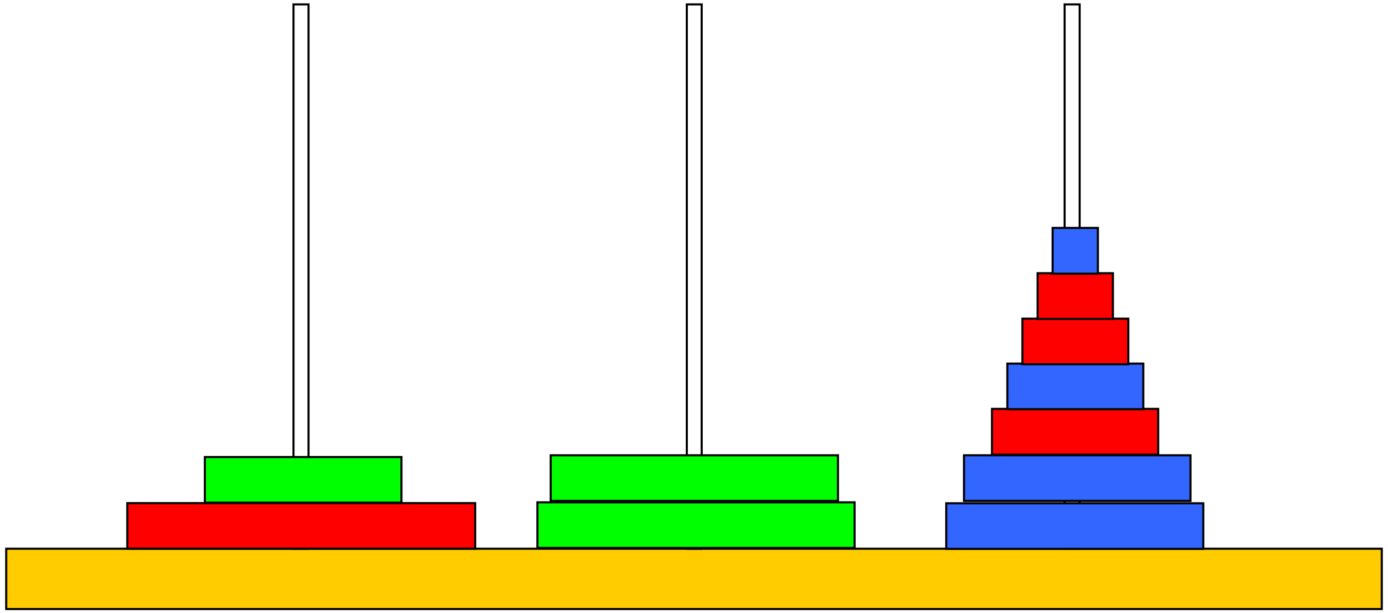
Induction

- <7,C>



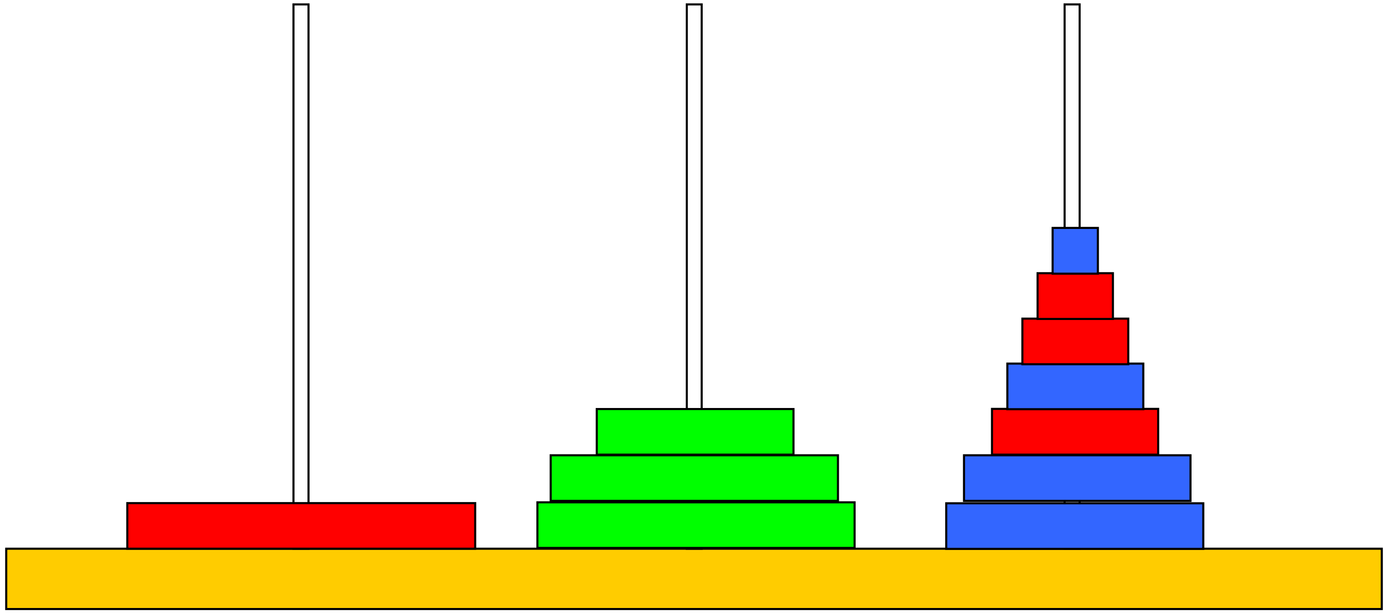
Induction

- $H_{5,A}$



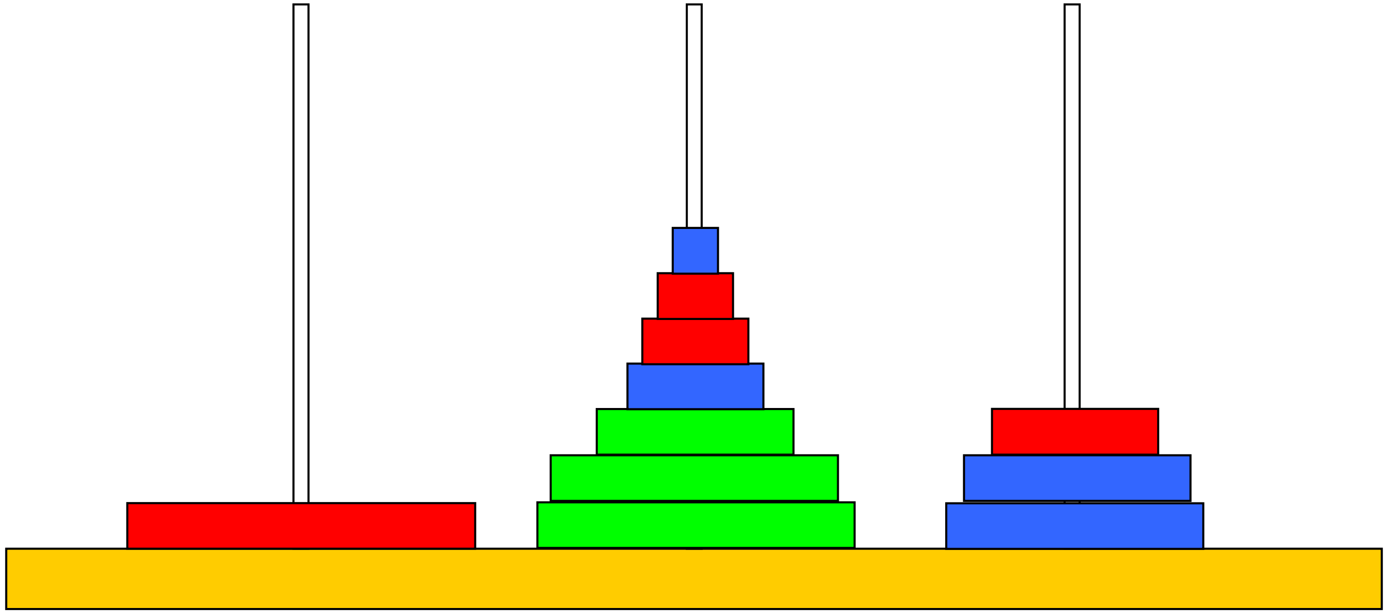
Induction

- <6,C>



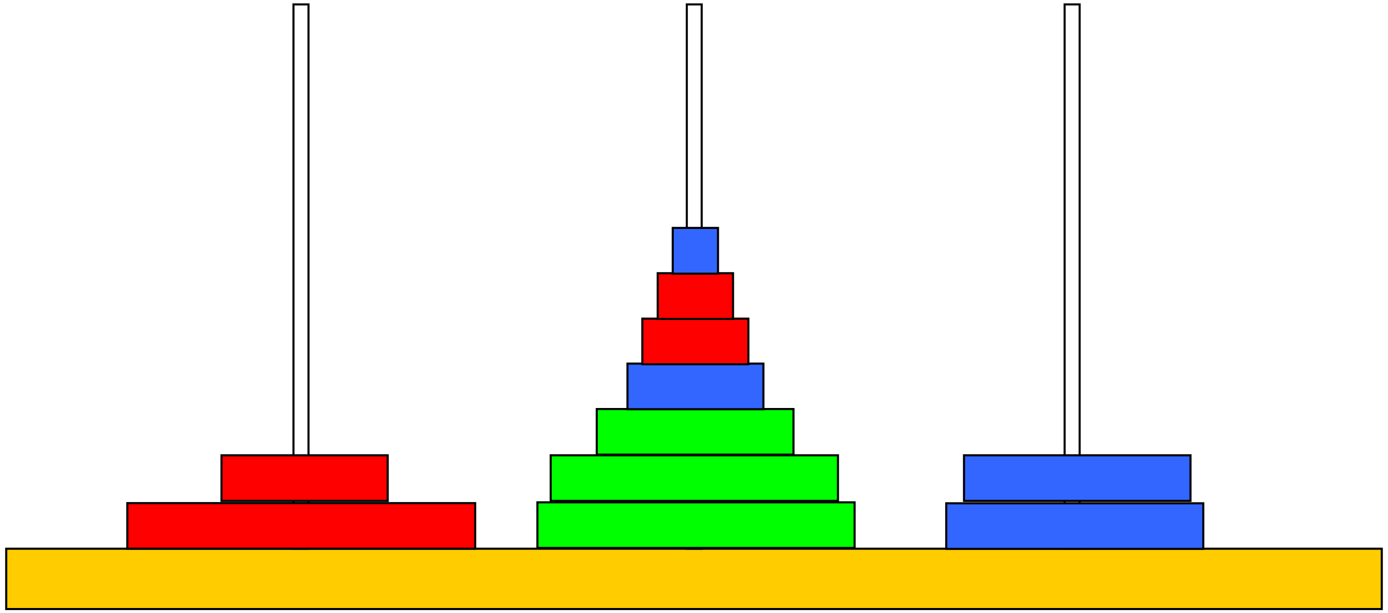
Induction

- $H_{4,A}$



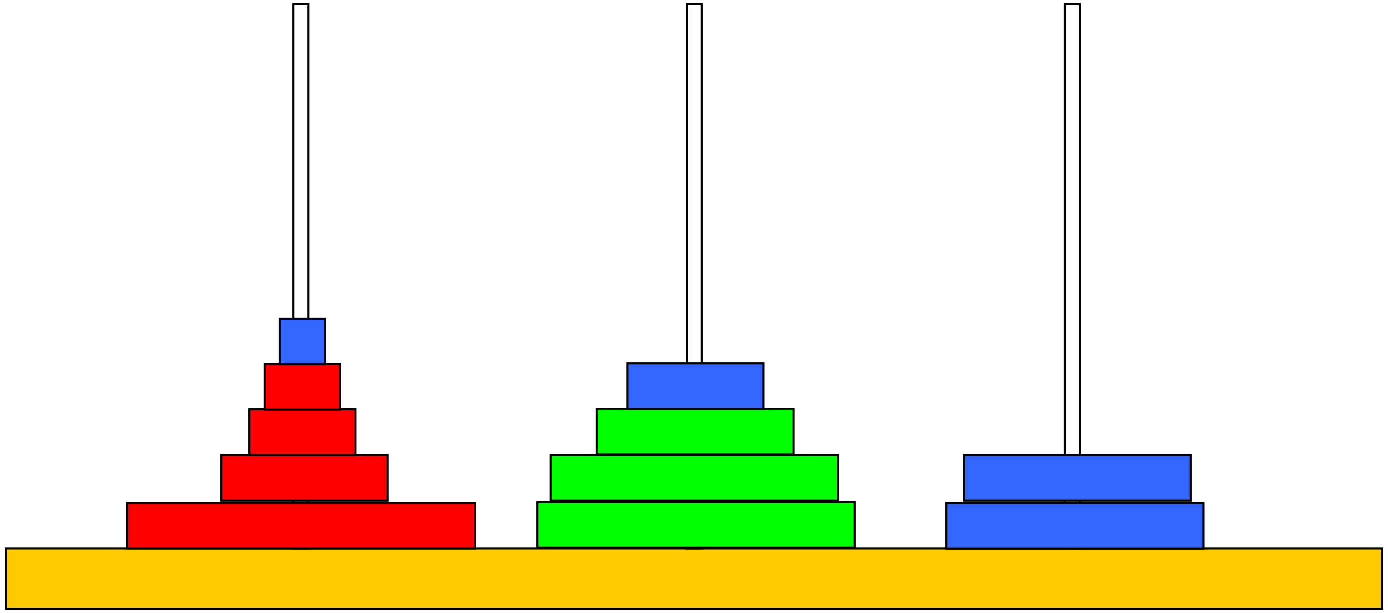
Induction

- <5,C>



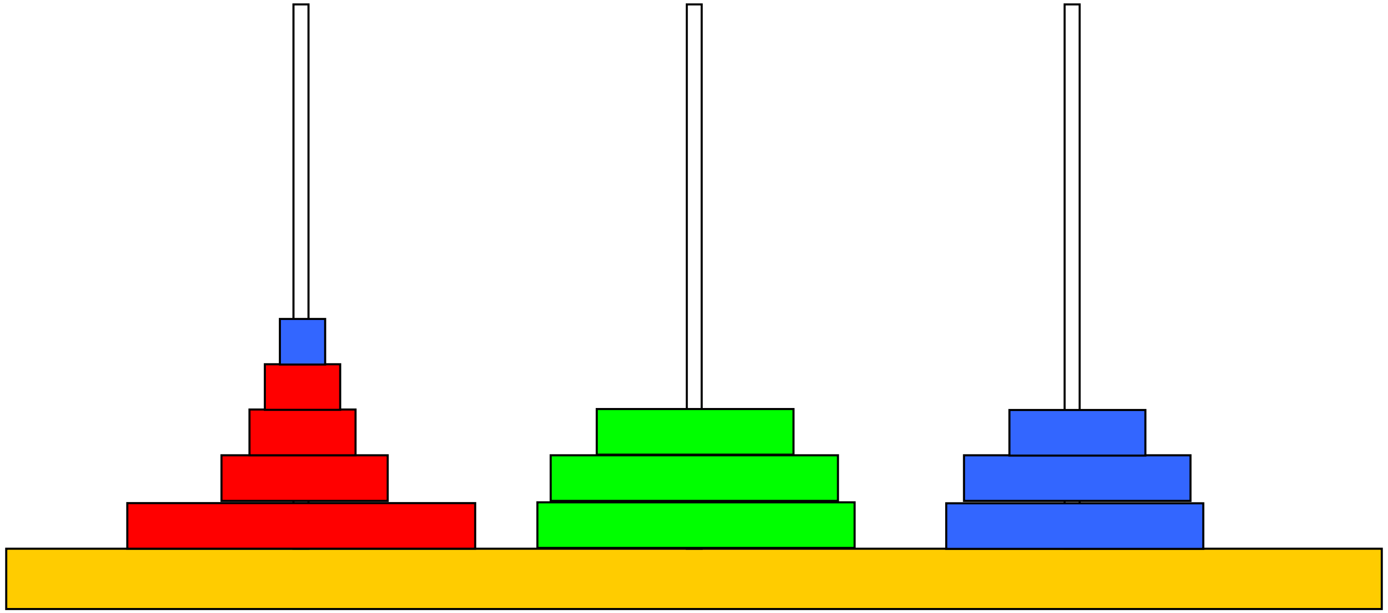
Induction

- $H_{3,A}$



Induction

- <4,C>



Induction

- <1,A>