Game 1 (Easy)

1) **Devise Strategy:** Player that makes the first move (i.e. Player 1) must always name the last day of each month. (example: January 31st, February 28th). When Player 1 names the last day of each month, this forces the player that makes the second move (i.e. Player 2) to choose the first day of the next month. As players must always increase the day of the month by an arbitrary amount or chose the first day of the next month, choosing the end of the month makes the successive player choose the 1st of the following month regardless.

   Furthermore, this allows Player 1 to continue with this pattern till both players reach the month of December, in which case Player 2 is forced to choose the 1st of December and Player 1 then chooses the 31st of December thereby winning the game. The leap year in this case does not affect the strategy, as Player 1 chooses the end of February so he/she has a choice between the 29th or the 28th of February.

2) **Optimal Play:**
   Player 1 – January 31st
   Player 2 – February 1st
   Player 1 – February 28th/29th
   .
   .
   .
   Player 2 – November 1st
   Player 1 – November 30th
   Player 2 – December 1st
   Player 1 – December 31st – Goal State

3) **Pseudocode:**

```
Initialize Player 1 date D1 to 1st January
Initialize Player 2 date D2 to 1st January
Initialize Current Month to January

While (D1 != 31st December or D2 != 31st December)
    If (D1 != End of the Current Month)
        D1 = End of Current Month;
        D2 = First of Next Month;
    End
    Current Month = Next Month;
End While
```

4) **Observations**
   Winning Positions: February 1st, March 1st, April 1st ……. December 1st
   Losing Positions: January 31st, February 28th/29th, April 31st ……. December 31st
   Player that plays first and makes the first move wins the game.

Game 2 (Hard)

1) **Devise Strategy:** Player that makes the first move (i.e. Player 1) can either name the next day after the current date or name the first of the next month. However, given that both players start from the 1st of January, the person who names the last day of the month depends on the parity of the month. If the month has odd parity (i,e odd number of days) then the player who plays second (i.e. Player 2) names the last day of the month, if both players optimally name dates in succession from the 1st of the month. Thereby, Player 1 is eventually forced to choose the 1st of February, as he/she can only name even dates. We can conclude that if a player is positioned at the first of the current month, then he/she is forced to name the 1st of the next month, if the current month has odd parity. The parity of the months of a year are given below:

   January – Odd, February – Odd/Even, March – Odd, April – Even, May – Odd, June – Even, July – Odd, August – Odd, September -Even, October – Odd, November -Even, December-Odd

   For the player who names the last day of January (i.e. Player 2), to preserve this pattern they must make sure the opposing player is in the position of the first day of a month with odd parity. Therefore, we can conclude if a player is positioned at the first day of the current month, then he/she optimally must name the 1st of the next month, if the current month has even parity or if the month is Febuary so that the opposing player is in the 1st day of a month with odd parity.

   June is an exception to the conclusions drawn earlier, as it is a month with even parity however it is followed by two months of odd parity. So, Player 2 must name all the even dates in succession, thus forcing Player 1 to name the first day of the month with odd parity (i.e. July). This would force Player 2 to name the 1st of August which is also a month with odd parity and Player 1 would be forced to name 1st September which is a month with even party. This would reinstate the pattern seen before June for Player 2.

   Following this pattern, Player 2 will eventually name the 1st of December, as Player 1 can only name even dates when it is in the position of the first day of the month, it won't name December 31st which is an odd date and Player 2 will win the game.

   The leap year in this case does not affect the strategy, as Player 2 optimally chooses to bypass February and move to the 1st of March.

2) **Optimal Play:**

   Player 1 – February 1st
   Player 2 – March 1st
   Player 1 – April 1st
   Player 2 – May 1st
   Player 1 – June1st
   Player 2 – June 2nd
   .
   .

.
Player 1 – July 1st
Player 2 – August 1st
Player 1 – September 1st
Player 2 – October 1st
Player 1 – November 1st
Player 2 – December 1st
Player 1 – December 2nd
.
.
Player 2 – December 31st

3) Pseudocode:

Initialize Player 1 date D1 to 1st January
Initialize Player 2 date D2 to 1st January
Initialize Current Month to January

While (D1 != 31st December or D2 != 31st December)
    If (Current Month has Odd Parity & Current Month != February)
        D1 = First of Next Month;
        D2 = End of Current Month;
    End
    Else If (Current Month has Even Parity Or Current Month = February & Current Month != June)
        D2 = First of the Next Month;
    End
    Else If (Current Month has Even Parity & Current Month == June)
        D1 = First of the Next Month;
        D2 = End of the Current Month;
    End
    Current Month = Next Month;
End While

4) Observations:

Losing Positions: March 1st, May 1st ……. December 31st
Winning Positions: February 1st, April 1st ……. November 1st
<u>Player that plays second and makes the second move wins the game</u>