CSIT113
Problem Solving

Workshop - Week 12

Red and Blue Island

- Near the Knights & Knaves island there is Red & Blue island
- 222 ‘perfectly’ logical people inhabit Red & Blue island
  - Each has a Blue or Red spot on the back of their head.
- The Red & Blue islanders have no mirrors and never discuss spot colours
  - Nobody knows the colour of their own spot, but they can see the colour of everyone else's
- Each morning everyone on the island gathers in the public meeting place at 6am.
  - Strict custom, which is always followed, dictates that anyone who has determined the colour of their spot during the last day must declare this and leave for ever.
  - They otherwise never leave, die or reproduce.

Red and Blue Island

- One day a rude tourist visits and announces at the morning gathering: "I can see that at least one of you has a red spot.”

- The tourist leaves and returns a year later.
  - What has happened? (Assume that the Red & Blue islanders believe the tourist)

- What if the tourist had announced “I can see that at least one of you has a blue spot.”?

Solution

- Assume only one person has a red spot. What happens?
- he can see all others are blue and knows he has red he then leaves. Next day everyone else leaves.
- Why?
- Assume there are two reds. What happens?
- on day one nobody leaves, on day two the two reds leave. Next day everyone else leaves.
- Why?

- This follows for any number of red – if there are k reds, the reds leave on day k and the blues on day k+1
- Symmetry means the same happens if blue.
- The Island is empty after at most 222 days

Smørrebrød (sandwiches)

- You have a part-time job in a Danish Restaurant Chain
- Unfortunately you neither speak nor read Danish
- The rest of the staff neither speak nor read English
- Needless to say, the menu is all Danish to you
- To hide your ignorance you conceive a cunning plan…
- …Go to the same restaurant in a nearby town and learn what all the items on the menu are

Smørrebrød

- There are 10 items on the menu (say A,B,C,D,E,F,G,H,I,J)
- You are a poor student and can’t afford to buy all the dishes on the menu
- You can buy a tasting plate with 5 items, in any combination.
- How many plates will you need to buy to learn what every item is?

Solution

- Clearly you can find the solution with 10 plates: Each plate has 5 of a different item.
- Less clearly, 9 plates will do – once you identify 9 items the one left is the last one you don’t know

- With 1 plate I can order 2 of one item and 3 of others/another and identify both.
- How many times do you need to order?
- Five times
- How?
- AABCD => can determine A
- CDEFF => determine F, determine B, E by comparing with AABCD
- DDGHI => determine D, determine C (using AABCD)
- GHHIJ => determine H, determine J (compare with DDGHI)
- It remains to determine G and I.
- So use GGHIJ to determine G, and hence I

- Actually, I can do this with 3 items per plate: 1 of one 2 of another.
- I can use the remaining two items to identify extra items by placing them on multiple plates.
- This allows me to get unique combinations of 9 items on three plates.
- ABBCD => Determine B
- EFFCG => Determine F, and C (compare with ABBCD)
- HIIDG => Determine I, and hence G, and D, and H, and A , and E
- J, the tenth item is the one we don’t get any of.

Tours

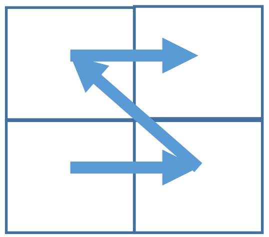
- A tour is a path that visits every available location exactly once, and does not retrace its steps
  - A closed tour is one which returns to the starting location
  - An open tour does not have this restriction
- The travelling salesman problem is to find a closed tour of a set of towns with minimal total path length
- In graph theory, a tour of edges is referred to as an Eulerian tour.
- In graph theory, a closed tour is referred to as a Hamiltonian circuit

Prince’s tour

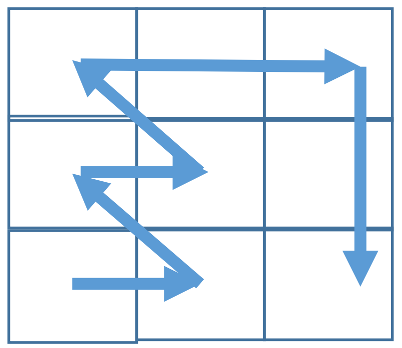
- Consider a special chess piece—to be called here a “prince”
- Prince can move one square to the right, or one square downward, or one square diagonally upward to the left.
- Find all values of n for which a prince can visit all the squares of an $n \times n$ board exactly once on the same tour.

Solution

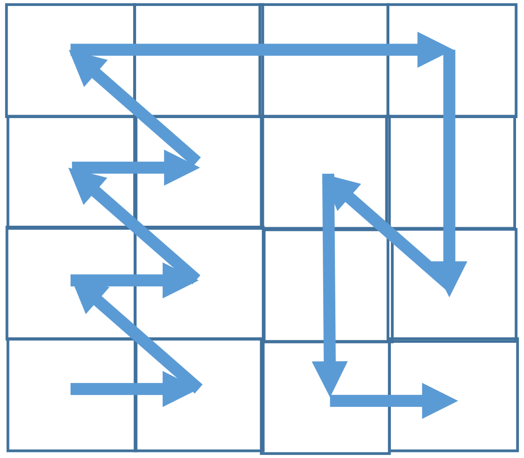
- The problem has a solution for any *n*.
- n = 1 : it is trivial
- How about for n = 2?



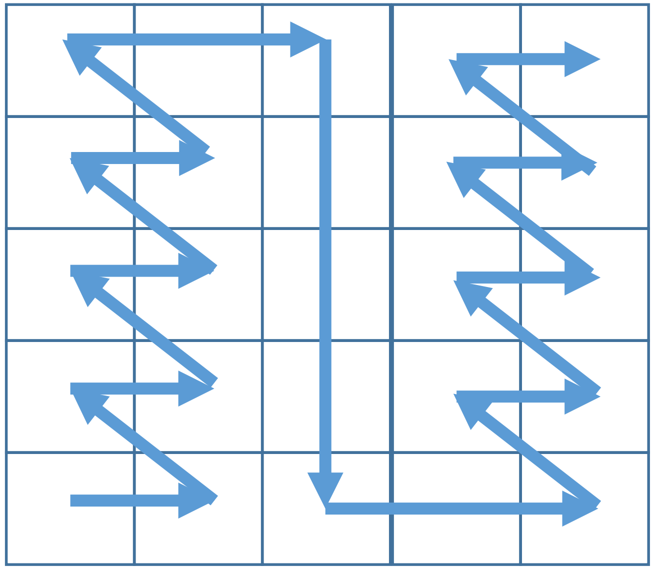
- How about n = 3 ?



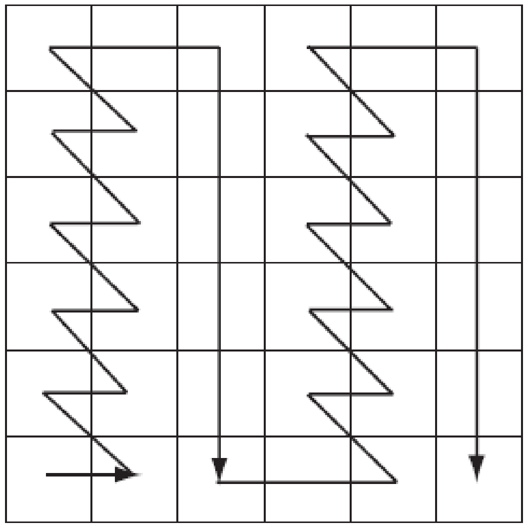
- How about n= 4 ?



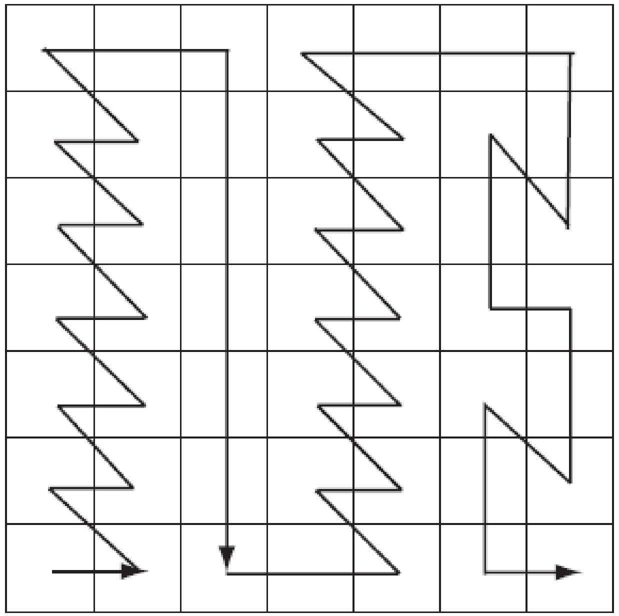
- How about n = 5?



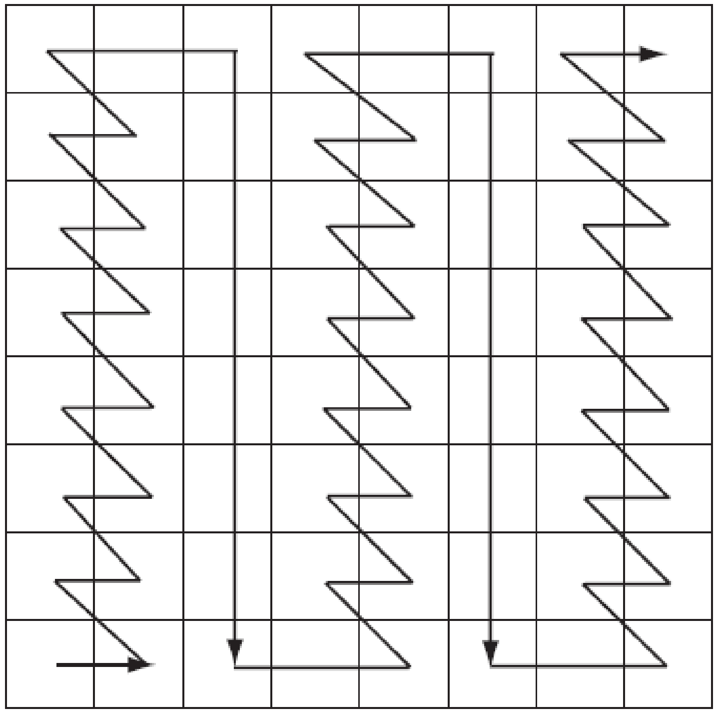
- In general, you have the following solutions for the case n = 6 ,7 and 8, representing for the general cases n = 3k, n = 3k+1, n = 3k+2



n = 3k

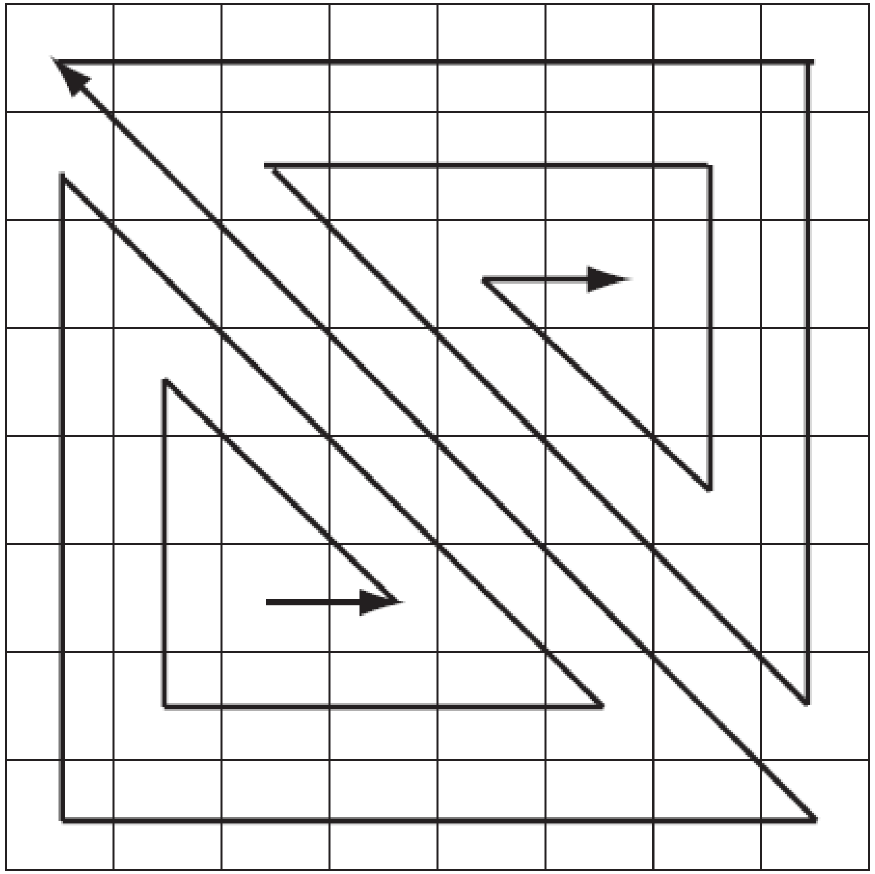


n = 3k+1



n = 3k+2

- The solution is not unique!
- Look at the following example for n = 8



Knight’s Tour

- A chess knight moves to a square that is two squares away horizontally and one square vertically, or two squares vertically and one square horizontally.
- Is It possible for a chess knight to visit all the cells of an 8x8 chessboard exactly once, ending at a cell one knight’s move away from the starting cell? (see figure in next slide)

- For example, the knight starts at square 1 and ends at square 64.
- We can assume this since the board is symmetric.

1							
	64						

Idea

- Always move to an unvisited square that is as close as possible to the nearest edge of the board. More precisely:
- we will try first visiting squares in the two outer layers of the board
- jumping to one of the 16 central squares only as the last resort
- In addition, we will always visit a corner square as soon as it becomes feasible to do so.

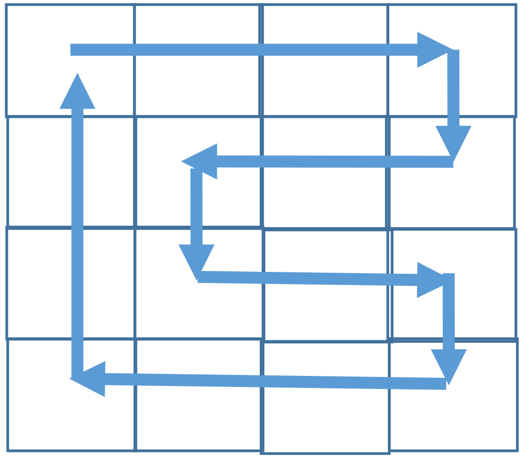
1	38	17	34	3	48	19	32
16	35	2	49	18	33	4	47
39	64	37	54	59	50	31	20
36	15	56	51	62	53	46	5
11	40	63	60	55	58	21	30
14	25	12	57	52	61	6	45
41	10	27	24	43	8	29	22
26	13	42	9	28	23	44	7

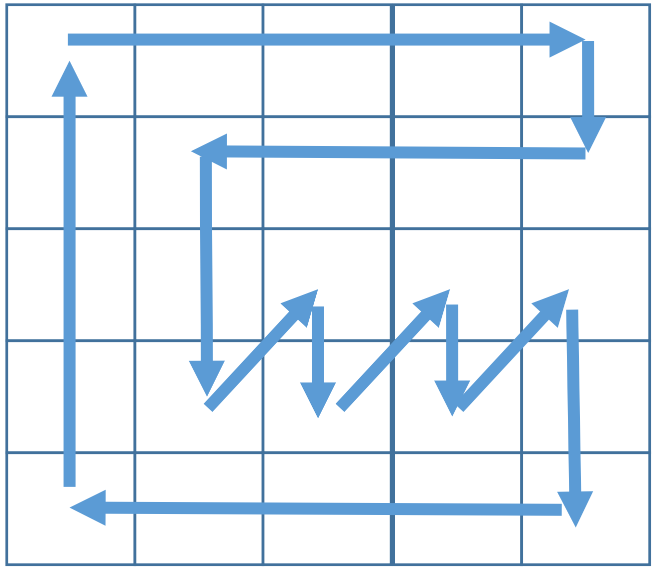
King’s Tour

- A King can move one square in any direction
- Can we construct a closed king’s tour of a standard 8x8 chessboard?
  - If so, find it.
  - If not, prove it.
- What is the largest square board for which we can construct a king’s tour?

Hint

- You can construct a closed King's tour for every n. We can use the following strategy for even and odd cases.





Bishop’s Tour

- A Bishop can only move diagonally, on its own colour
- Assuming the bishop’s tour only requires visiting all squares of the same colour…
- can we construct a closed bishop’s tour of a standard 8x8 chessboard?
  - If so, find it.
  - If not, prove it.
- What is the largest square board for which we can construct a bishop’s tour?

Hint

- Any square board with an even number along the edge is impossible (corner squares must be start and end – two of each colour)
- Boards with sides of length 3 and 5 are possible for one colour but not the other.
- 7 and up is impossible – you have a hole in the middle.