Part A

The following is a sequence that needs sorting. In answering the two sub-questions, show enough detail to demonstrate that you understand what you are doing.

6, 3, 4, 9, 2, 3, 7

Answer:

a) Sort the sequence from smallest to largest using insertion sort. Show each step on a new line, underline the sorted part of the array and circle the next element to be inserted.

- Insertion sort iterates, consuming one input element each repetition, and growing a sorted output list. At each iteration, insertion sort removes one element from the input data, finds the location it belongs within the sorted list, and inserts it there.

6 3 4 9 2 3 7 Step-1
3 6 4 9 2 3 7 Step-2
3 4 6 9 2 3 7 Step-3
3 4 6 9 2 3 7 Step-4
2 3 4 6 9 3 7 Step-5
2 3 3 4 6 9 7 Step-6
2 3 3 4 6 7 9 Step-7

b) Sort the sequence from smallest to largest using bubble sort, stopping after any iteration with no swaps. Show each iteration on a new line, underline the sorted part of the array and circle the next pair to be compared.

- Bubble sort, sometimes referred to as sinking sort, is a simple sorting algorithm that repeatedly steps through the list, compares adjacent elements, and swaps them if they are in the wrong order. The algorithm passes through the list repeated until the list is sorted. Each time that all the elements of an array are passed through, it is called an iteration.

6 3 4 9 2 3 7

3 6 4 9 2 3 7

3 4 6 9 2 3 7

3 4 6 9 2 3 7

3 4 6 2 9 3 7

3 4 6 2 3 9 7

1st Iteration

3 4 6 2 3 7 9

3 4 6 2 3 7 9

3 4 6 2 3 7 9

3 4 2 6 3 7 9

3 4 2 3 6 7 9

3 4 2 3 6 7 9

2nd Iteration

3 4 2 3 6 7 9

3 4 2 3 6 7 9

3 2 4 3 6 7 9

3 2 3 4 6 7 9

3rd Iteration

3 2 3 4 6 7 9

2 3 3 4 6 7 9

4th Iteration

<u>Part B</u>

Prove by induction that $1+3+9+\cdots+3^{n-1}=(3^n-1)/2$

<u>Answer</u>

A proof by induction consists of two cases, the base case and the induction step The inductive step proves that if the statement holds for any given case n = k, then it must also hold for the next case n = k + 1. Given n=k, S(k) is shown below:

<u>$S(k) = 1+3+9+\cdots+3^{k-1}=(3^k-1)/2$</u>…………………… Equation (ii)

<u>Base Case: n=1</u>

LHS = 1

RHS = $(3^1-1)/2 = (3-1)/2 = 1$

Therefore, LHS=RHS and S(1) proven.

<u>Inductive Step: n =k+1</u>

LHS = $3^0+3^1+3^2+3^3\ldots\ldots+3^{k-1}+3^k$

LHS = $(3^k-1)/2 + 3^k$ …………………… From Equation (ii)

LHS = $(3^k-1+2.\ 3^k)/2$

LHS = $(3.3^k-1)/2$

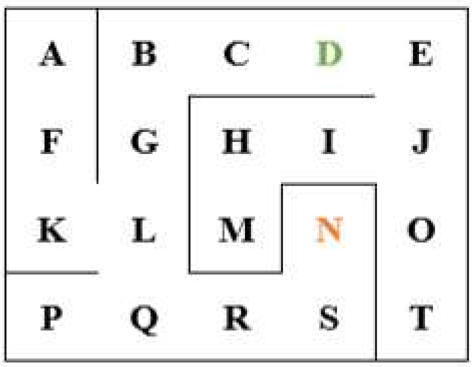
LHS = $(3^{k+1}-1)/2$

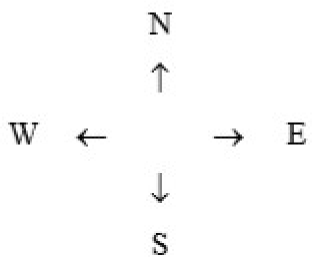
RHS = $(3^{k+1}-1)/2$

Therefore, LHS=RHS and S(k+1) proven.

<u>Part C</u>

Consider the following maze:





Each square in the maze has been labelled with a letter.

The starting square is D and the goal square is N. The arrows show the directions north(N), west(W), south(S) and east(E). All squares are reachable though you cannot go over the wall-lines directly and there are no loops.

STRATEGY 1. The maze is to be traversed in the following manner:

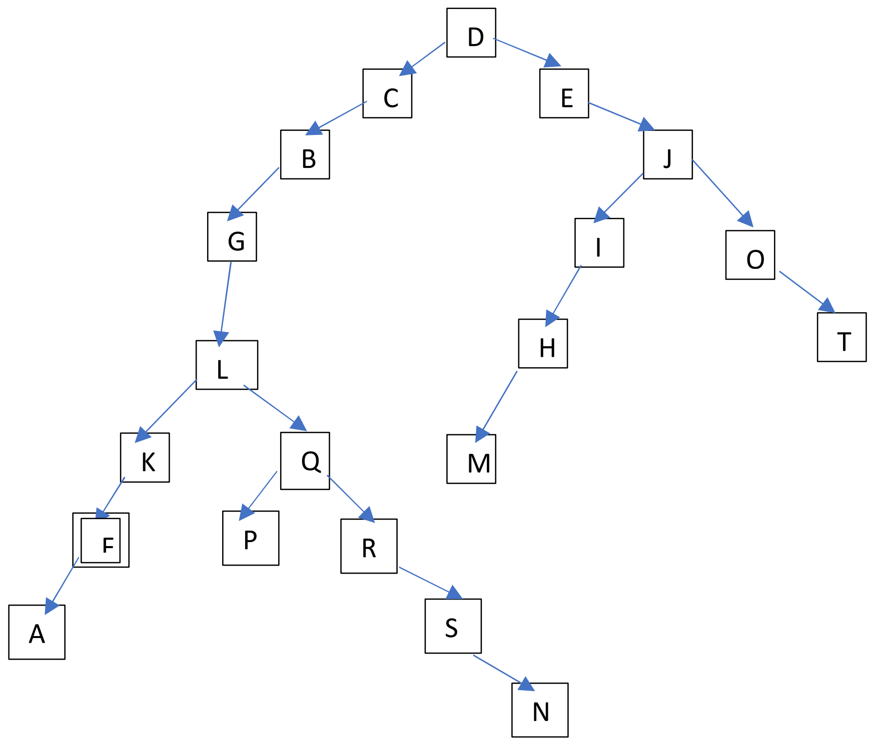
Begin on the starting square
Start with an empty list
While you have not reached the goal square
For each square adjacent to the current square (in the order E, S, W, N)
If the square has not yet been visited, add it to the start of the list.
Visit the square at the start of the list and remove that square from the list.

STRATEGY 2. The maze is to be traversed in the following manner:

Begin on the starting square
Start with an empty list
While you have not reached the goal square
For each square adjacent to the current square (in the order N, W, S, E)
If the square has not yet been visited add it to the end of the list.
Visit the square at the start of the list and remove that square from the list.

<u>Answer</u>

a) Draw a tree representing the maze, rooted at the starting square, with siblings (directly reachable nodes) appearing in the order N, W, S, E.



b) List the squares in the order they are visited. Only list a square the first time it is visited.

I. Using Strategy 1.

- D, C, B, G, L, K, F, A, Q, P, R, S, N, E, J, I, H, M, O, T

II. Using Strategy 2.

- D, C, E, B, J, G, I, O, L, H, T, K, Q, M, F, P, R, A, S, N

c) I. Give the pre-order and post-order traversals of the tree.

- Preorder: D, C, B, G, L, K, F, A, Q, P, R, S, N, E, J, I, H, M, O, T

Postorder: A, F, K, P, N, S, R, Q, L, G, B, C, M, H, I, T, O, J, E, D

II. Which one of these is related to the traversal produced by strategy 1

Explain why this relationship exists.

- Strategy 1 is preorder transversal, this is because the algorithm starts at the root square D and explores as far as possible along each branch before backtracking.

d) What sort of traversal of the tree is produced by strategy 2?

- Strategy 2 is inorder transversal

Part D

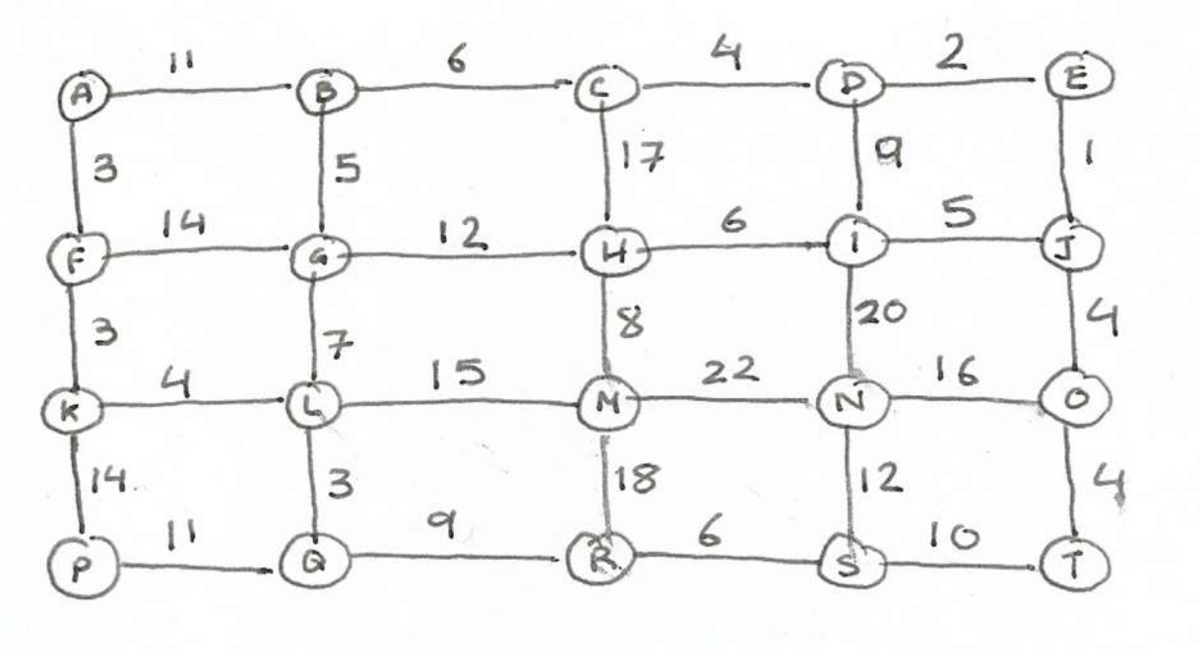
Consider the following weighted graph, represented as an adjacency list, where each node corresponds to a square on the 4 × 5 grid from part C (without maze walls).

A: (B, 11), (F, 3)
B: (A, 11), (C, 6), (G, 5)
C: (B, 6), (D, 4), (H, 17)
D: (C, 4), (E, 2), (I, 9)
E: (D, 2), (J, 1)
F: (A, 3), (G, 14), (K, 3)
G: (B, 5), (F, 14), (H, 12), (L, 7)
H: (C, 17), (G, 12), (I, 6), (M, 8)
I: (D, 9), (H, 6), (J, 5), (N, 20)
J: (E, 1), (I, 5), (O, 4)
K: (F, 3), (L, 4), (P, 14)
L: (G, 7), (K, 4), (M, 15), (Q, 3)
M: (H, 8), (L, 15), (N, 22), (R, 18)
N: (I, 20), (M, 22), (O, 16), (S, 12)
O: (J, 4), (N, 16), (T, 4)
P: (K, 14), (Q, 11)
Q: (L, 3), (P, 11), (R, 9)
R: (M, 18), (Q, 9), (S, 6)
S: (N, 12), (R, 6), (T, 10)

T: (O, 4), (S 10)

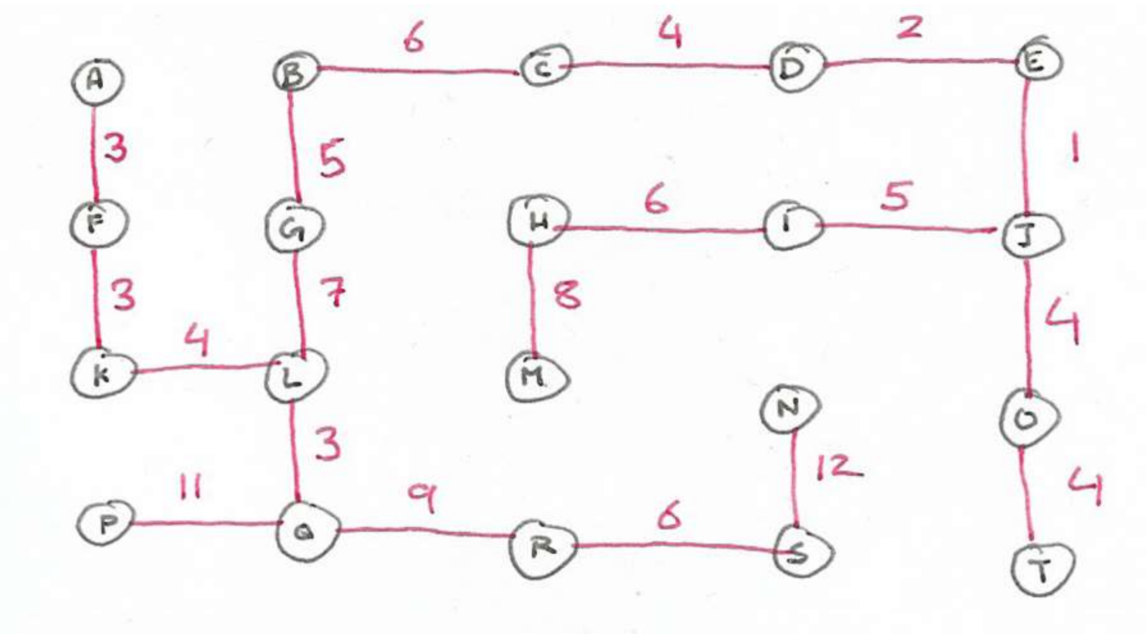
<u>Answer</u>

a) Draw the graph:



b) Find its minimal spanning tree.

- The minimum spanning tree was drawn using Krushals algorithm which finds an edge of the least possible weight that connects any two nodes in the forest. The edges in the minimum spanning tree are drawn in red.



c) What is the relationship between this spanning tree and the maze of part C?

- The minimum spanning tree is the only possible path in the maze of part C.