

# MATH221 Mathematics for Computer Science

## Tutorial Sheet Week 12 – Autumn 2021

1. By suitably labelling the vertices and edges of the two graphs below, show that the graphs are isomorphic.

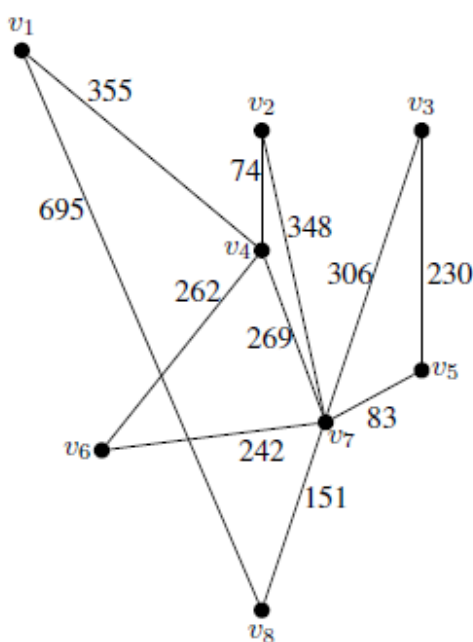


2.

(i) How many subgraphs of  $K_n$  that have all  $n$  vertices are there? (Hint: ideas about power sets are relevant.) Note that the question is not asking about the number of *non-isomorphic* subgraphs of  $K_n$ ; that is a harder problem.

(ii) How many non-isomorphic subgraphs of  $K_3$  that have all 3 vertices are there?

3. Use Kruskal's and Prim's Algorithms to find a minimum spanning tree for the following weighted graph. What is the total weight of the minimum spanning tree? Did you get the same tree in both instances? Under what circumstances will this happen and when will it not happen?



4. Let  $f : \mathbb{N} \rightarrow \mathbb{N}$  be given by  $f(n) = 1 + n/2$  if  $n$  is even and  $f(n) = 1 + (n - 1)/2$  if  $n$  is odd. Calculate the range of  $f$  and determine whether  $f$  is one-to-one.

5. Prove the following statements.

(i)  $f : [0, \infty) \rightarrow \mathbb{R}$ , defined by  $f(x) := x^2 + 1$  is one-to-one but not onto.

(ii)  $f : \mathbb{R} \rightarrow (0, \infty)$ , defined by  $f(x) := x^2$  is onto but not one-to-one.

(iii)  $f : (0, 1) \rightarrow (0, \infty)$ , defined by  $f(x) := \frac{x}{1-x}$  is bijective.