MATH221 Mathematics for Computer Science

Tutorial Sheet Week 8 – Autumn 2021

1. Let $X = \{a, b, c, d, e, f\}$. Determine whether the following statements are true or false.

(i) $X \in \mathcal{P}(X)$ (ii) $\{\emptyset\} \in \mathcal{P}(X)$ (iii) $a \in \mathcal{P}(X)$ (iv) $\{a\} \in X$
(v) $a \in X$ (vi) $X \subseteq \mathcal{P}(X)$ (vii) $a \subseteq \mathcal{P}(X)$ (viii) $\{X\} \subseteq \mathcal{P}(X)$

2. Which of the following sets are equal? In some cases, you can list the elements of the sets explicitly.

(i) $A = \{0, 1, 2\}$ (ii) $B = \{x \in \mathbb{R} : -1 \leq x < 3\}$(iii) $C = \{x \in \mathbb{R} : -1 < x < 3\}$
(iv) $D = \{x \in \mathbb{Z} : -1 < x < 3\}$ (v) $E = \{x \in \mathbb{N} : -1 < x < 3\}$

3. Let $U = \mathbb{R}$ and let $A = \{1\}$, $B = (0, 1) = \{x \in \mathbb{R} : 0 < x < 1\}$ and $C = [0, 1] = \{x \in \mathbb{R} : 0 \leq x \leq 1\}$. Find the sets below.

$$A \cup B \qquad A \cap B \qquad B \cap C \qquad A \cup C \qquad A \cap C$$

4. Prove or disprove the statement $\{0, 1\} = \left\{ n \in \mathbb{Z} : \exists k \in \mathbb{Z} \text{ s.t. } n = \dfrac{1 - (-1)^k}{2} \right\}$.

5. Let $U = \mathbb{N}$ and let $A = \big\{x \in \mathbb{N} : x \text{ is odd}\big\}$, $B = \big\{x \in \mathbb{N} : x \text{ is even}\big\}$, and $P = \big\{x \in \mathbb{N} : x \text{ is a prime number}\big\}$. Find the sets below. Are A and B disjoint? Is $P \subseteq A$?

$$\overline{A} \qquad \overline{P} \qquad P - A \qquad B - P \qquad A - B$$

6. Let U be the universal set and let A, B and C be subsets of U. By using the properties of $\{\cup, \cap, \overline{}\}$ and any results from lectures, simplify the following.

(i)$(C \cap U) \cup \overline{C}$ (ii) $\overline{(A \cap U)} \cup \overline{A}$ (iii)$\overline{\overline{(C \cup \emptyset)} \cup C}$ (iv) $(A \cap B) \cap \overline{A}$

7. Let U be a non-empty universal set, and let A, B and C be subsets of U. Prove or disprove each of the following statements.

(i) $\overline{A} - \overline{B} = B - A$ (ii) $A - (B - C) = (A - B) - C$

You may find the relation $A - B = A \cap \overline{B}$, the Distributive Laws and DeMorgan's Laws helpful.