MATH221 Mathematics for Computer Science

Tutorial Sheet Week 13 - Autumn 2021

1. The two-way table below gives the thousands of commuters in Massachusetts in 2015 by transportation method and one-way commute length.

	Less than 15 minutes	15-29 minutes	30-44 minutes	45-59 minutes	60 or more minutes	Total
Private vehicle	636	908	590	257	256	2647
Public Transportation	9	54	96	62	108	329
Other	115	70	23	7	7	222
Total	760	1032	709	326	371	3198

(a) Given that a randomly selected commuter used public transportation, find the probability that he had a commute of 60 minutes or more.

(b) Given that a randomly selected commuter had a commute of 60 minutes or more, find the probability that he used public transportation.

2. You toss a fair coin three times.

(a) What is the probability that you observe exactly two tails?

(b) Given that you have observed at least one tail, what is the probability that you observed at least two tails?

3. Business owners and partners Yousef and Olivia sold me a product two years ago, the Yousef–Olivia Super–Happy–Important–Thing. This product is known to function properly until it breaks down after t years with probability

$$P(T \geq t) = e^{-\frac{t}{5}}.$$

For example, the probability that the product lasts 1 year or more is

$$P(T \geq 1) = e^{-\frac{1}{5}} \approx 0.82.$$

Since I have used my Super–Happy–Important–Thing for the past two years without incident, how likely is it that it will stop working in the coming year?

4. You are a respectable basketball player with a 70% chance of making a free throw at any time. You are at a charity event that involves a competition; the most free throws out of 5 attempts wins. You know that the other competitors are average players and will probably score less than your most likely score.

(a) Graph a histogram of all your possible outcomes and another of all cumulative outcomes.

(b) Determine your most probable minimum performance necessary to win the competition and the probability that this will happen.

5. Yousef produces 20 Super–Happy–Important–Things per day, with a 2% defective rate. Olivia is 50% faster than her partner, but she produces double the rate of defective items. At the end of every day, the products are packed randomly into one box of 20 items and one box of 30 items.

Today, the owner of the umbrella corporation Super–Happy–Important–Things–R–Us, Kumuthu, is on site for a surprise inspection. He chooses the box of 30 and observes how many items are defective. What is the probability that no more than 2 of the 30 products do not work? **Hint:** a tree will be helpful.