

MATH221: Mathematics for Computer Science

Tutorial Sheet Week 2

Autumn 2021

Logic - Lectures 1 & 2

Name: _____

If each person in this class has at least 3 eyes, then write the subjects you are currently enrolled in on the back of this paper. If each person in this class has at most 3 eyes, then circle the number of eyes you have: 0, 1, 2, 3, 4, 5, 6, or more. If no person in this class has exactly 3 eyes, then draw a small triangle at the top of this paper beside your name.

If you are neither a fish nor a fowl, then you must omit the next sentence. Fold this paper into two equal parts and draw a happy face on the largest of the two parts.

If a prime number must be greater than 1, draw a circle (approximately) around your surname at the top of this paper. However, if the statement that a prime number must be greater than 1 is false, draw a rectangle (approximately) around your surname at the top of this paper.

If a square is not round, you must omit the next sentence. In the margin of this paper draw a square approximately 1cm on a side. If a square has two right angles, underline the longest word in this sentence. However, if the underlined word begins with a vowel, then draw a rectangle around the second word having not less than 5 letters which follows the underlined word.

Now, if you are less than 40 years of age, then put a tick in the following box only if you are more than 2 metres tall.

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If it is not true that all triangles are isosceles, then underline all words of more than 2 letters beginning with the letter “i” in this sentence.

A student who walks 8 kilometres to attend MATH221 lectures walks the first 4 km at a rate of 8km/h and the next 4km at a rate of 6km/h. If the student’s average rate of speed is 7km/h, then draw a star in the margin beside this paragraph. If the student’s average speed is greater than 7km/h, then draw a heart instead. If neither of these answers is correct, write the correct answer instead.

If an integer that has exactly two different prime factors less than 20 can be greater than 320, stop working on this page and begin working on the next page. If not, write out the multiplication table for the number 19 below, then begin working on the next page.

1. For each of the following collections of words:

- (a) Determine if it is a statement.
- (b) If it is a statement, determine if it is true or false.
- (c) Where possible, translate the statement into symbols, using the connectives presented in lectures.
 - (i) If $x = 3$, then $x < 2$.
 - (ii) If $x = 0$ or $x = 1$, then $x^2 = x$.
 - (iii) $x^2 = x$ only if $x = 0$ or $x = 1$.
 - (iv) There exists a natural number x for which $x^2 = \frac{x}{2}$.
 - (v) If $x \in \mathbb{N}$ and $x > 0$, then $\sqrt{x} > 1 \implies x > 1$.
 - (vi) $xy = 5 \implies x = 1$ and $y = 5$ or $x = 5$ and $y = 1$.
 - (vii) $xy = 0 \implies x = 0$ or $y = 0$.
 - (viii) $xy = yx$.
 - (ix) There is a unique even prime number.

2. Translate into symbols the following compound statements. In each case list the statements $p, q, r \dots$ and give the form of the compound statement.

- (i) If x is odd and y is odd then $x + y$ is even.
- (ii) It is not both raining and hot.
- (iii) It is raining but it is hot.
- (iv) It is neither raining nor hot.
- (v) $-1 \leq x \leq 2$.

3. Let p be the statement "Mathematics is easy", and q be the statement "I do not need to study". Write down in words the following statements, and simplify if possible.

$$p \vee q, \quad \sim q, \quad \sim\sim q, \quad \sim p, \quad \sim p \wedge q, \quad p \Rightarrow q.$$

4. Let p and q be statements.

- (i) Write down the truth tables for $(\sim p \vee q) \wedge q$, and $(\sim p \wedge q) \vee q$. What do you notice about the truth tables? Based on this result, a creative MATH221 student concludes that one can always interchange \vee and \wedge without changing the truth table.
- (ii) Write down the truth tables for $(\sim p \vee q) \wedge p$, and $(\sim p \wedge q) \vee p$. What do you think of the rule formulated by the student in 4(i)?

5. Construct truth tables for the compound statements $p \vee \sim p$ and $p \wedge \sim p$. What do you notice about each of the statements? Determine the truth value of the compound statements $(p \vee \sim p) \vee q$ and $(p \wedge \sim p) \wedge q$. What do you notice?

6. Construct truth tables for the compound statements $(p \vee \sim p) \wedge (q \vee r)$ and $q \vee r$. What do you notice? Construct truth tables for the compound statements $(p \wedge \sim p) \vee (q \wedge r)$ and $q \wedge r$. What do you notice?