MATH221 Mathematics for Computer Science

Tutorial Sheet Week 11 - Autumn 2021

1. (i) Let $f : [0, 1) \to \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Show that f is one-to-one but not onto. How can the range of f be changed to allow the inverse function f^{-1} to be defined?

(ii) Let $f : \mathbb{R} \to [0, \infty)$ be defined by $f(x) = x^4$. Show that f is onto but not one-to-one. How can the domain of f be changed to allow the inverse function f^{-1} to be defined?

2. (i) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) := x^3$ for $x \in \mathbb{R}$. Can the inverse function be defined? If so, what is it?

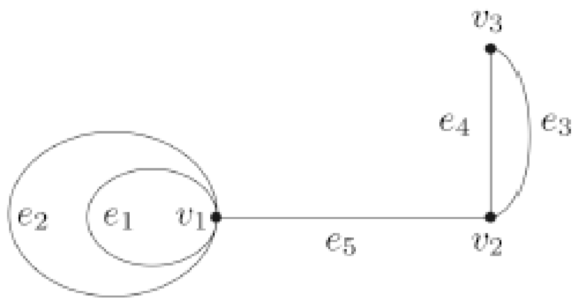
(ii) Let $f : (0, 1) \to (0, \infty)$ be defined by $f(x) := \dfrac{x}{1-x}$ for $x \in (0, 1)$. The function f is one-to-one and onto; show that $g : (0, \infty) \to (0, 1)$, defined by $g(x) := \dfrac{x}{x+1}$ for $x > 0$ is the inverse of f.

(iii) The function $f(x) = \cos x : \mathbb{R} \to \mathbb{R}$ is not one-to-one nor onto. How can the domain of f be changed to allow the inverse function $f^{-1}(x) = \arccos x$ to be defined?

(iv)The function $f(x) = \tan x : \mathbb{R} \to \mathbb{R}$ is not one-to-one but is onto. How can the domain of f be changed to allow the inverse function $f^{-1}(x) = \arctan x$ to be defined?

(v) The function $f(x) = e^x : \mathbb{R} \to \mathbb{R}$ is not onto. How can the range of f be changed to allow the inverse function $f^{-1}(x) = \ln x$ to be defined?

3. Let $G = (V, E)$ be the graph given below.



Draw at least two subgraphs $H = (V_H, E_H)$ of G for which $\sum_{v \in V_H} \delta(v) = 4$.

4. In each case below, draw a graph with the specified properties (several answers may be possible).

(i) A graph with four vertices of respective degrees 1, 2, 3 and 4.

(ii) A graph without loops or parallel edges in which each vertex has degree 3 and which has exactly 6 edges.

(iii) A graph without loops or parallel edges with four vertices of respective degrees 1, 1, 2 and 2.

(iv) A simple graph with five vertices of respective degrees 2, 3, 3, 3 and 5.

(v) A simple graph with five edges and with four vertices of respective degrees 1, 1, 3 and 3.

(vi) A graph with four vertices of respective degrees 1, 1, 2 and 6.

5. By suitably labelling the vertices (and, if necessary, the edges) of the two graphs below, show that the graphs are isomorphic.