

# **COMPGI14 : Machine Vision Coursework 2**

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# Homographies Part 1

(a)

Code: `07_Practical_Homographies/Part1/practical1.m`

The projective transformation (also known as a collinearity or homography) can map any four points in the plane to any other four points. It is a linear transformation in homogenous coordinates but is nonlinear in Cartesian coordinates. It subsumes the Euclidean, similarity, and affine transformations as special cases. It exactly describes the mapping between the 2D coordinates of points on a plane in the real world and their positions in an image of that plane. A 2D point  $(x, y)$  in an image can be represented as a 3D vector  $x = (x_1, x_2, x_3)$  where

$$x = \frac{x_1}{x_3}$$

and

$$y = \frac{x_2}{x_3}$$

This is called the homogeneous representation of a point and it lies on the projective plane. A homography matrix is a 3x3 transformation matrix that relates to planar image transformations. It is used for image alignment such as motion compensation or panorama stitching discussed in the next part.

Blue points are the `pts1cart` and the red points are the `pts2cart` which we get after applying homography using the homography matrix as :

$$H = \begin{pmatrix} 0.6 & 0.7 & -100 \\ 1.0 & 0.6 & 50 \\ 0.001 & 0.002 & 1.0 \end{pmatrix} \quad (1)$$

and converting back to cartesian and then adding noise.

Then we call the function `calcBestHomography` which uses direct linear transform (DLT) algorithm to calculate best homography that maps the points in `pts1Cart` to their corresponding matching in `pts2Cart` to transform blue points to pink using the following equation:

$$\begin{bmatrix} 0 & 0 & 0 & -u_1 & -v_1 & -1 & y_1 u_1 & y_1 v_1 & y_1 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -x_1 u_1 & -x_1 v_1 & -x_1 \\ 0 & 0 & 0 & -u_2 & -v_2 & -1 & y_2 u_2 & y_2 v_2 & y_2 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -x_2 u_2 & -x_2 v_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -u_I & -v_I & -1 & y_I u_I & y_I v_I & y_I \\ u_I & v_I & 1 & 0 & 0 & 0 & -x_I u_I & -x_I v_I & -x_I \end{bmatrix} \begin{bmatrix} \phi_{11} \\ \phi_{12} \\ \phi_{13} \\ \phi_{21} \\ \phi_{22} \\ \phi_{23} \\ \phi_{31} \\ \phi_{32} \\ \phi_{33} \end{bmatrix} = 0 \quad (2)$$

and

$$SVD(A) = ULV^T \quad (3)$$

[1]

## OUTPUTS

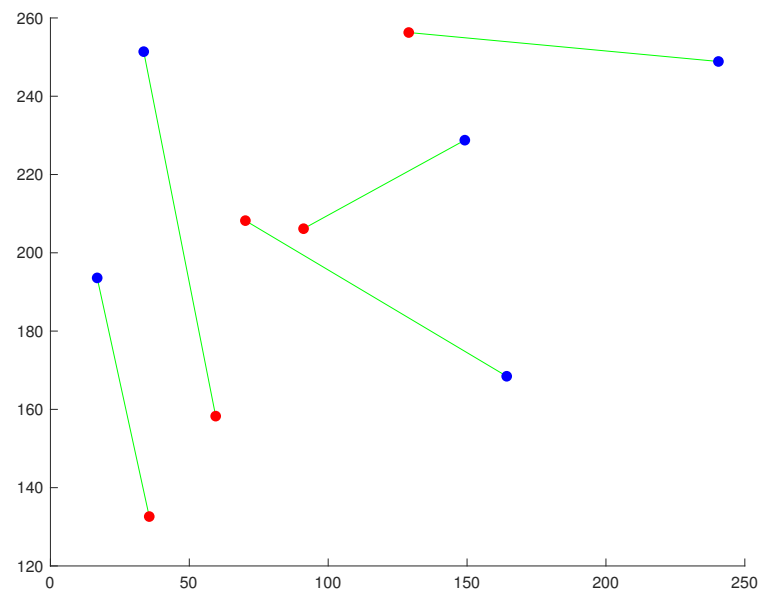


Figure 1: Image before Transformation

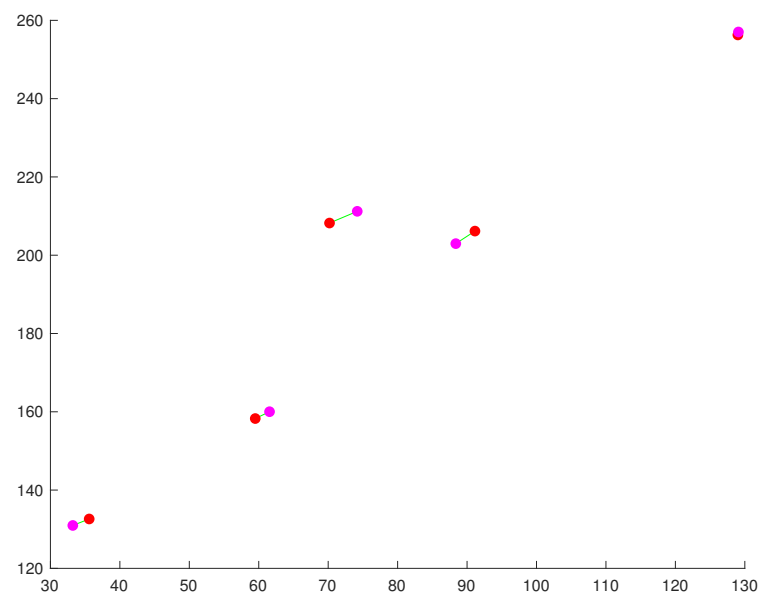


Figure 2: Image after transformation

The above figures shows that after transformation, the points came closer to each other due to homography.

Output

pts1Hom =

240.5000	16.8351	33.5890	164.2696	149.1911
248.8770	193.5890	251.3901	168.4581	228.7723
1.0000	1.0000	1.0000	1.0000	1.0000

H =

0.6000	0.7000	-100.0000
1.0000	0.6000	50.0000
0.0010	0.0020	1.0000

pts2Hom =

218.5139	45.6134	96.1265	116.4824	149.6553
439.8262	182.9885	234.4231	315.3445	336.4545
1.7383	1.4040	1.5364	1.5012	1.6067

pts2Cart =

125.7088	32.4878	62.5673	77.5936	93.1424
253.0276	130.3325	152.5825	210.0636	209.4025

pts2EstHom =

0.9270	0.1765	0.3551	0.5307	0.6262
1.8310	0.7490	0.7500	1.5540	1.3856
0.0072	0.0056	0.0052	0.0073	0.0065

pts2EstCart =

129.0560	31.2471	68.0889	72.8293	95.8878
254.8991	132.6275	143.8165	213.2420	212.1558

sqDiff =

2.1381

**Scale Ambiguity:** In homography, a constant rescaling of all the nine values produces the same transformations as the scaling factor cancels out the numerator and denominator in equation. Homography contains eight degrees of freedom despite of having nine entries in the matrix; the entries are redundant with respect to scale.[1]

**Exact mapping for pairs of four points:** If we remove the elements of any of the one column out of the 5 columns, then the exact mapping of the four points can be seen from the output below:.

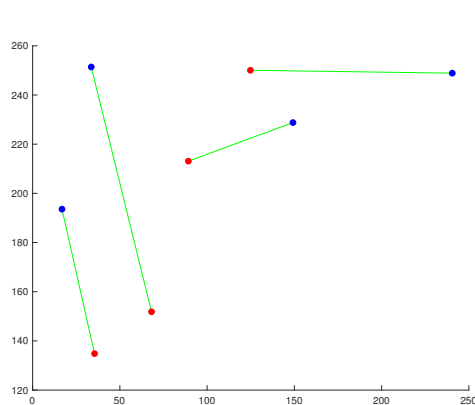


Figure 3: Image before Transformation

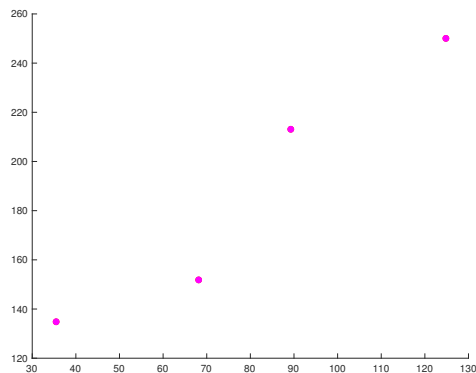


Figure 4: Image after transformation showing the exact mapping of four points



(b)

**Code:** 07\_Practical\_Homographies/Part1/practical2.m**Description:**

Here we are making a panorama of several images that are related by a homography. We are provide 3 images (centre image, left image and the right image). We match a set of points between these images using homography. Transformation of pixel position is done for every pixel in first image (centre image) using homography and if the transformed position is within the boundary of second image( left image or right image), then the pixel colour from second image pixel is copied to current position of the first image given the output above.

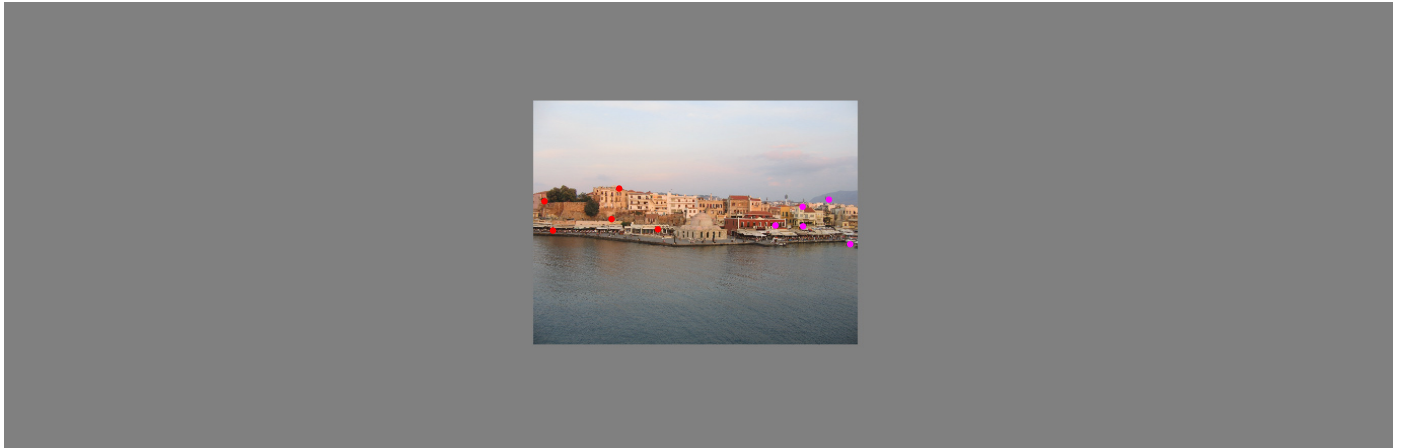


Figure 5: centre Image



Figure 6: Left Image



Figure 7: Right Image



Figure 8: Image stitching of the left image to the centre image after applying homography



Figure 9: Image stitching of both the left image and right image to the centre image after applying homography

## Homographies Part2

(c)

Code: 07\_Practical\_Homographies/Part2/practical2.m

```
%% Output
T =

    0.9851    -0.0492    0.1619   46.0000
   -0.1623    -0.5520    0.8181   70.0000
    0.0490    -0.8324   -0.5518  500.8900
         0         0         0    1.0000

TEst =

    0.9856    -0.0557    0.1599   46.9718
   -0.1640    -0.5502    0.8188   71.9582
    0.0424    -0.8332   -0.5514  513.5154
         0         0         0    1.0000
```

Here we are exploring the geometry of a single camera. The aim is to take several points on a plane, and predict where they will appear in the camera image. We are then re-estimating the Euclidean transformation based on these observed points relating the plane and the camera.

We first estimate the four points on the plane that will appear in the image using the function "projectiveCamera". Then noise is added to the pixel positions to simulate having to find these points in a noisy image and the results are stored in xImCart.

Now using the image points (xImCart) and the known positions on the plane, we estimate the extrinsic matrix in the function "estimate plane pose" using the following equations:

We eliminate the effect of the intrinsic parameters by pre-multiplying the estimated homography by the inverse of the intrinsic matrix. This gives a new homography as :

$$\begin{bmatrix} \phi'_{11} & \phi'_{12} & \phi'_{13} \\ \phi'_{21} & \phi'_{22} & \phi'_{23} \\ \phi'_{31} & \phi'_{32} & \phi'_{33} \end{bmatrix} = \lambda' \begin{bmatrix} \omega_{11} & \omega_{12} & \tau_x \\ \omega_{21} & \omega_{22} & \tau_y \\ \omega_{31} & \omega_{32} & \tau_z \end{bmatrix}$$

$$\begin{bmatrix} \phi'_{11} & \phi'_{12} \\ \phi'_{21} & \phi'_{22} \\ \phi'_{31} & \phi'_{32} \end{bmatrix} = \mathbf{U}\mathbf{L}\mathbf{V}^T,$$

[1]

$$\begin{bmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \\ \omega_{31} & \omega_{32} \end{bmatrix} = \mathbf{U} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \mathbf{V}^T.$$

(d)

Code: 07\_Practical\_Homographies/Part2/practical2b.m

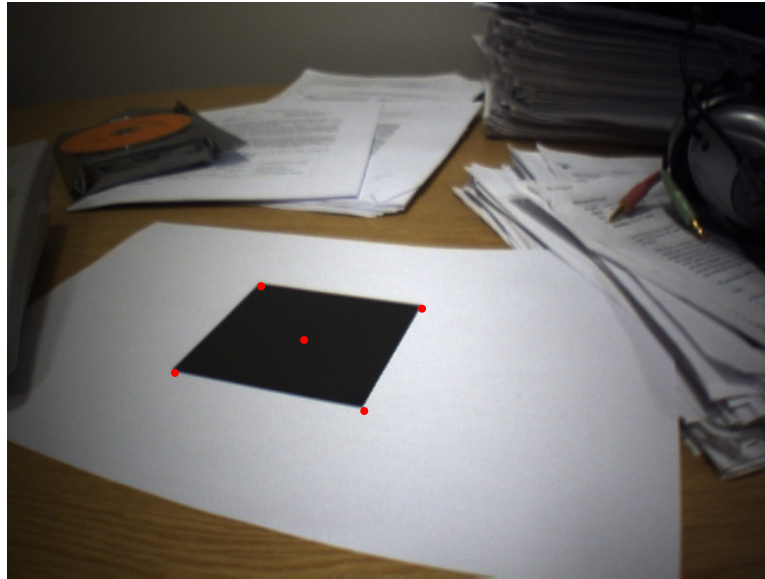


Figure 10: planar black square with augmented corners and centre

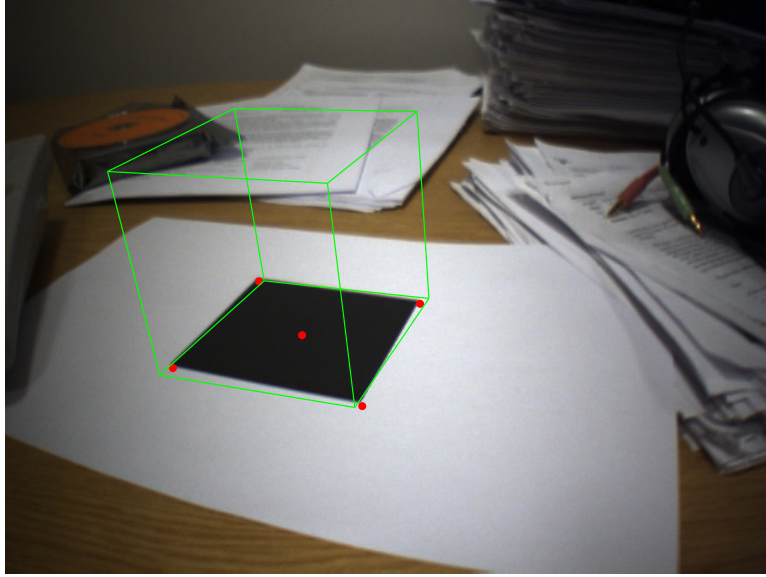


Figure 11: A wire frame cube superimposed over the planar black square as if it is rigidly attached to the surface of the cube

Here first we calculated the extrinsic matrix relating the plane position to the camera position. Then the 2D positions of the corners on the marker surface and the corresponding positions in the image is computed using transformations. This transformation is analyzed to find the rotation and translation of the camera relative to the marker. This allows us to superimpose a 3D object as if it were rigidly attached to the surface of the image. The following equations are used :[1]

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \phi_x & \gamma & \delta_x & 0 \\ 0 & \phi_y & \delta_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} & \tau_x \\ \omega_{21} & \omega_{22} & \omega_{23} & \tau_y \\ \omega_{31} & \omega_{32} & \omega_{33} & \tau_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ 1 \end{bmatrix}$$

## Condensation 3

(e)

Code: 09\_PracticalCondensation/Practical9a.m

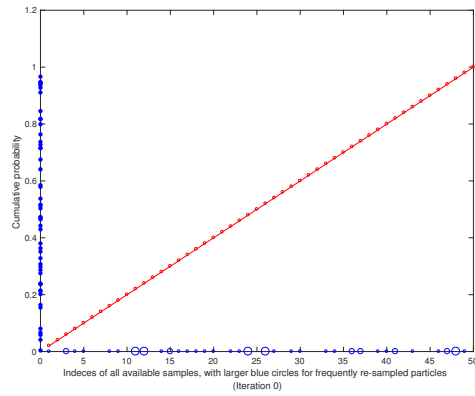


Figure 12: Histogram for the first iteration

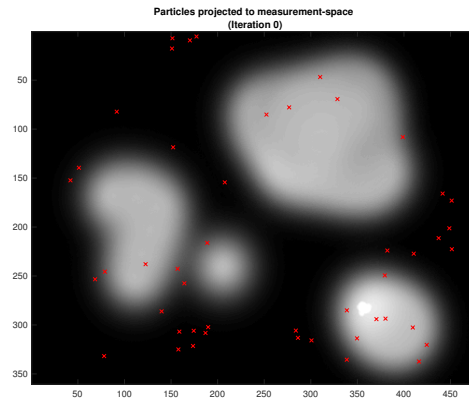


Figure 13: output for the first iteration

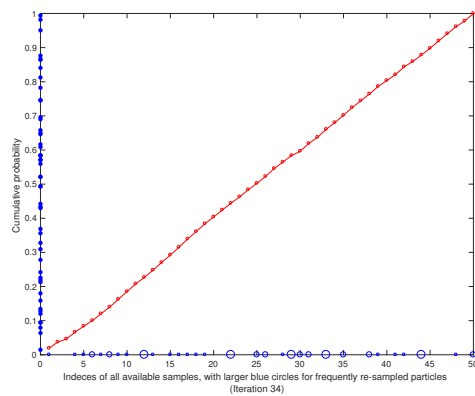


Figure 14: Histogram for the iteration no. 34

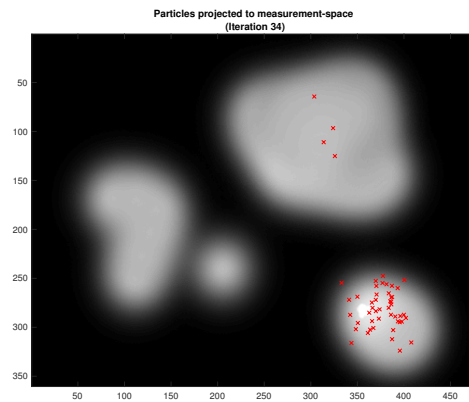


Figure 15: output for the iteration no. 34

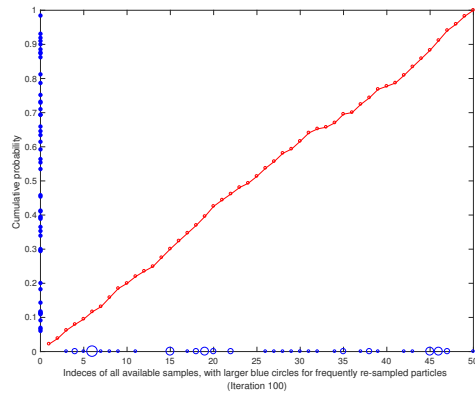


Figure 16: Histogram for the iteration no. 100

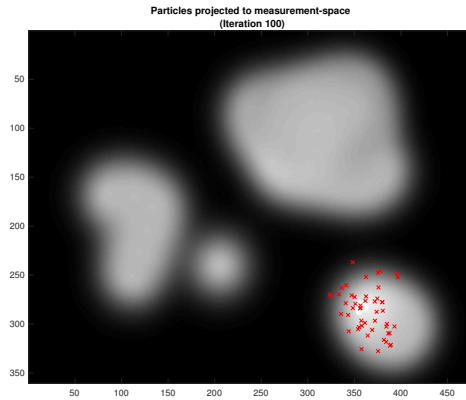


Figure 17: output for the iteration no. 100

Here we are using Condensation algorithm (factored sampling) which incorporates a Brownian motion model when applied iteratively to a sequence of observations. Here we are sampling 50 particles on the the red channel of an abstract image (provided). After adding some gaussian noise with standard deviation of 10 and after doing some predictions and measurements, resampling of old distribution is done based on the higher posterior probabilities because of which more of the particles land near the peaks in the distribution.



(f)

Code: 09\_PracticalCondensation/Practical9b.m

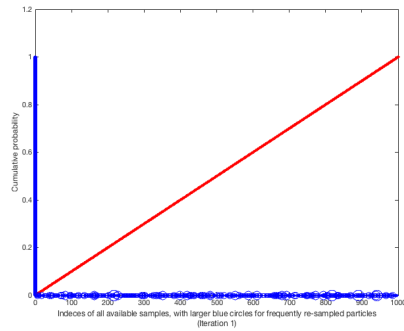


Figure 18: output for the first iteration

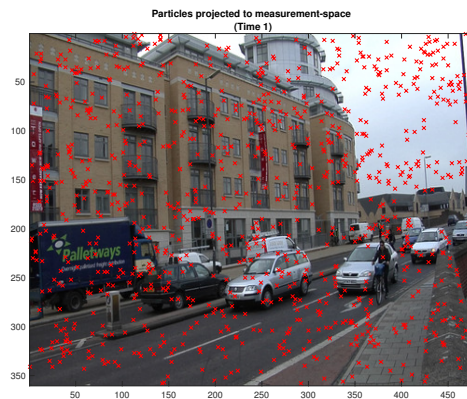


Figure 19: output for the first iteration

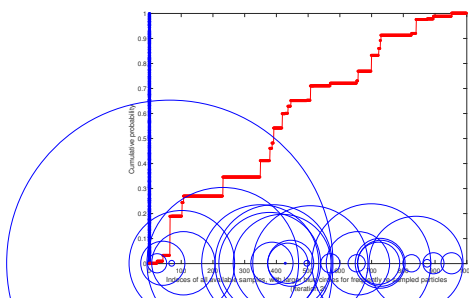


Figure 20: output for the second iteration



Figure 21: output for the second iteration



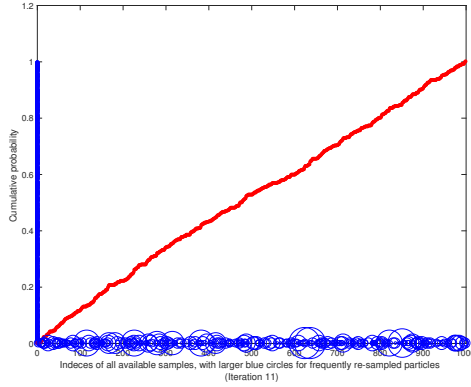


Figure 22: output for the iteration no. 11



Figure 23: output for the iteration no. 11

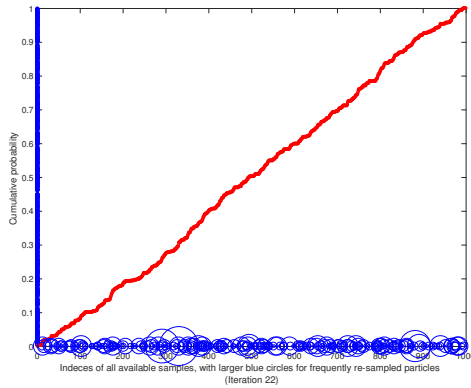


Figure 24: output for the iteration no. 22



Figure 25: output for the iteration no. 22

Here we are using the same code from part "e" where we used Condensation algorithm (factored sampling) to incorporate a Brownian motion model when applied iteratively to a sequence of observations. Here we are sampling 1000 particles to track the wheel of the car (template) as it moves in a sequence of frames. Initially, all the particles are scattered and then each pixel location is matched with the pixels in the template, when found similar it increases its weightage in the next iteration and gets aggregated based on the higher posterior probabilities, this keeps on repeating until the wheel of the car is tracked.

## Combining Tracking and Homographies 4

(g)

Code: HW2/HW2\_Practical9c.m

Condensation algorithm uses a single object state as the basic state representation. Using this algorithm, a peak corresponding to the dominant likelihood value will increasingly dominate over all other peaks when the estimation progresses over the time. In other words, a dominant peak is established if some objects obtain larger likelihood values more frequently. If the posterior is propagated with fixed number of samples, eventually, all samples will be around the dominant peak.

In this practical we are detecting all the four corners of the squares using point based approach using the same code from part "e" and "f". Under this approach, an object of interest is represented by a set of feature points such as corners of the object. It does a one-to-one correspondence matching of pixels from the template provided and finally tracks the corners as desired. However, several problems such as object drifting and significant appearance changes during tracking can be seen which can be solved using contour-based approach which provide localization of the target object with its boundaries (e.g. edges and lines).

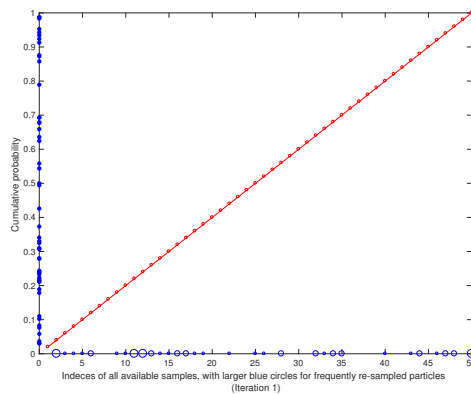


Figure 26: Histogram for the first iteration to detect lower left corner

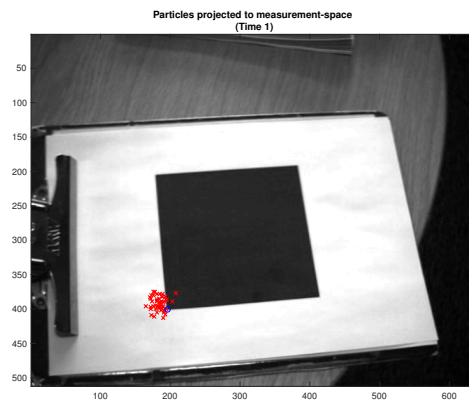


Figure 27: output for the first iteration

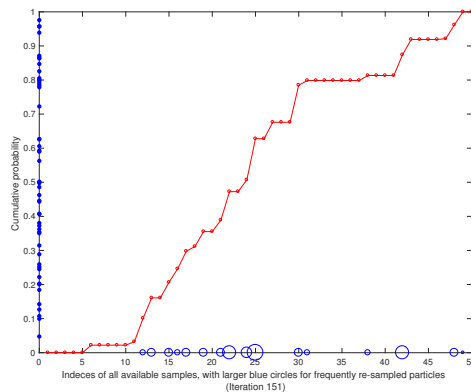


Figure 28: Histogram for the iteration no. 151

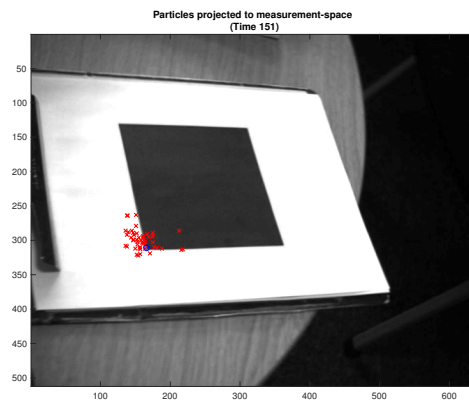


Figure 29: output for the iteration no. 151

Here, we are tracking the lower left corner using the similar approach we used in part e and f.

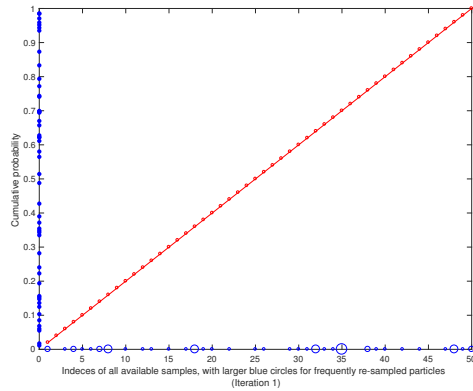


Figure 30: Histogram for the first Iteration to detect the lower right corner

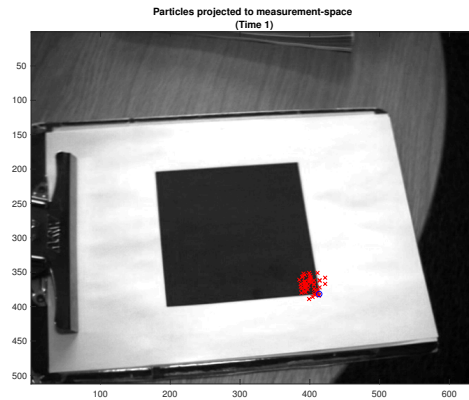


Figure 31: output for the first iteration

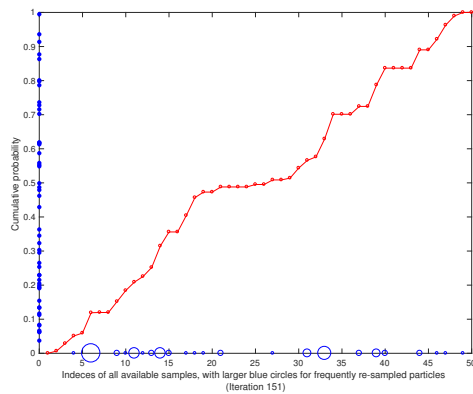


Figure 32: Histogram for the iteration no. 151

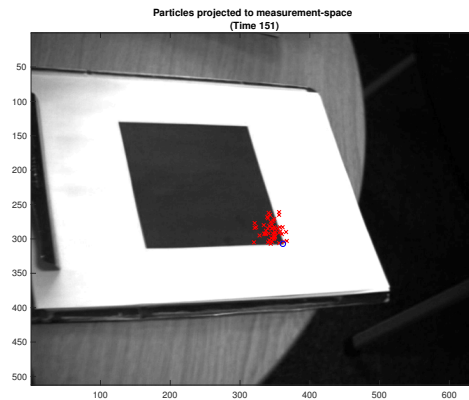


Figure 33: output for the iteration no. 151

Here, we are tracking the lower right corner using the similar approach we used in part e and f.

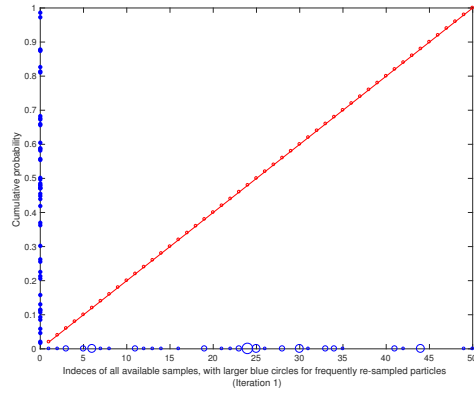


Figure 34: Histogram for the first iteration to detect the upper left corner

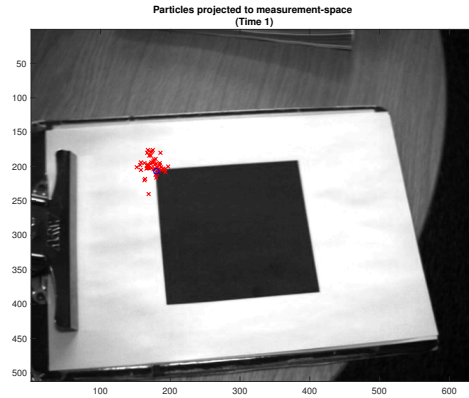


Figure 35: output for the first iteration

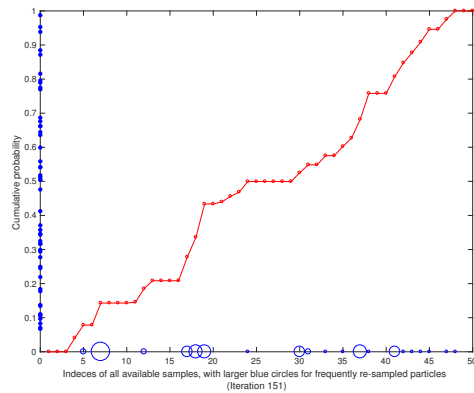


Figure 36: Histogram for the iteration no. 151

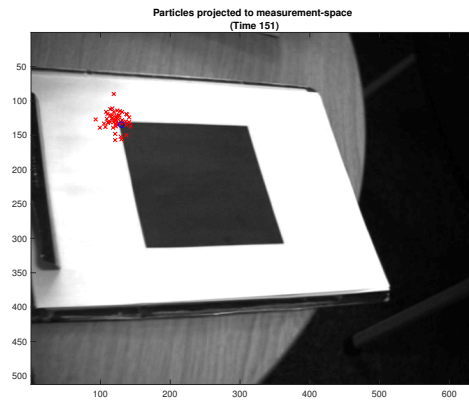


Figure 37: output for the iteration no. 151

Here, we are tracking the upper left corner using the similar approach we used in part e and f.

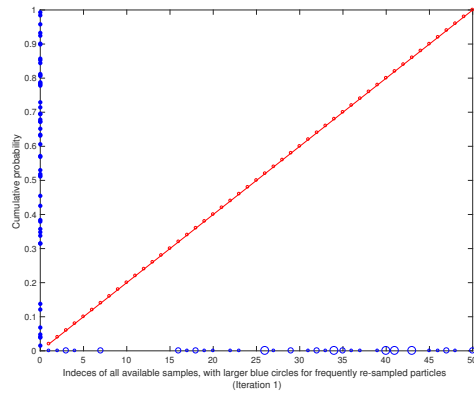


Figure 38: Histogram for the first iteration to detect the upper right corner

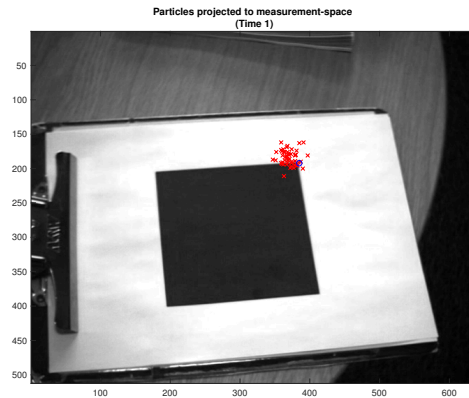


Figure 39: output for the first iteration

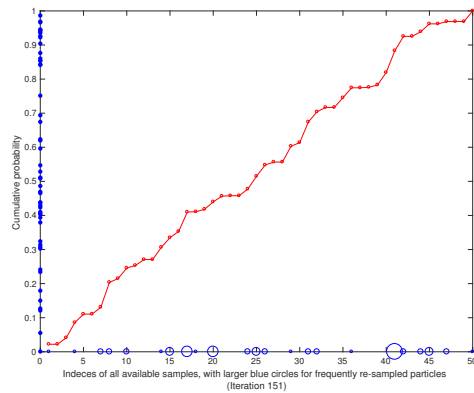


Figure 40: Histogram for the iteration no. 151

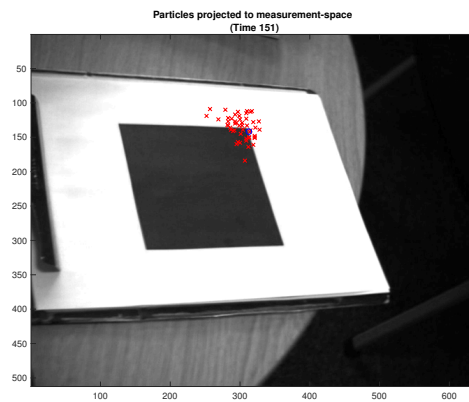


Figure 41: output for the iteration no. 151

Here, we are tracking the upper right corner using the similar approach we used in part e and f.

The Histogram plot shows cumulative probability distribution of the current time frame for each particle which is being propagated.

(h)

Code: HW2/HW2\_TrackingAndHomographies.m

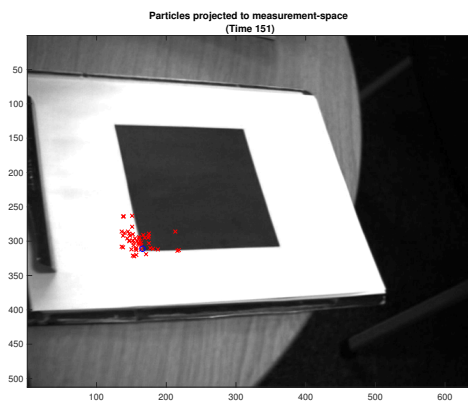


Figure 42: Particles tracking the lower left corner

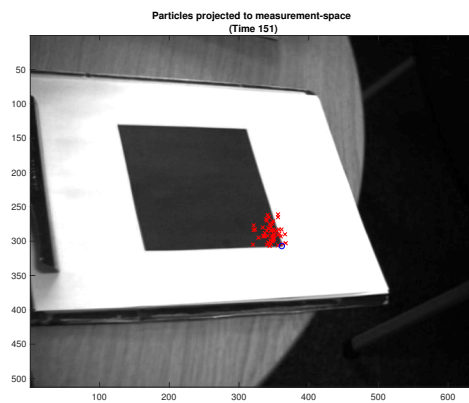


Figure 43: Particles tracking the lower right corner

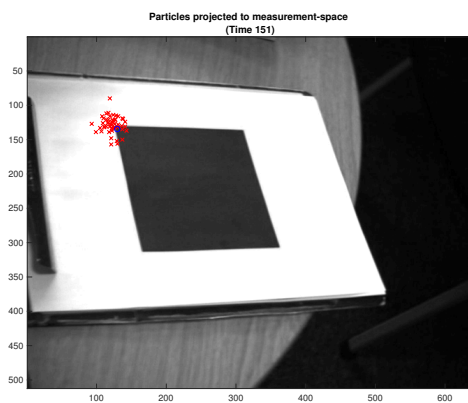


Figure 44: Particles tracking the upper left corner

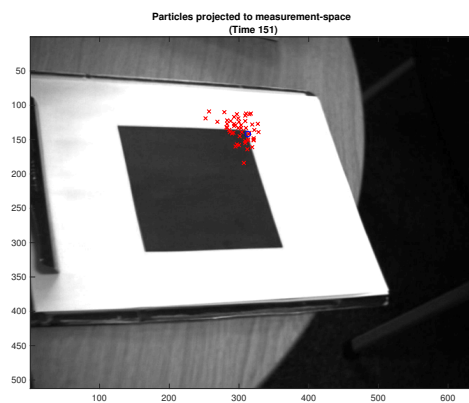


Figure 45: Particles tracking the upper right corner

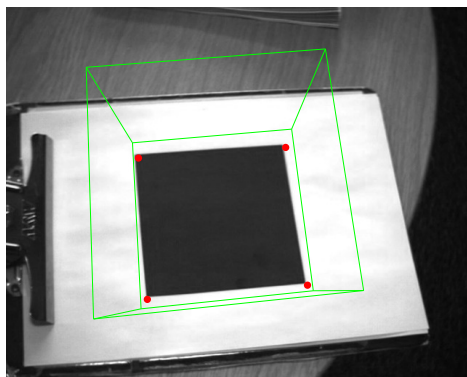


Figure 46: output for the iteration no. 2

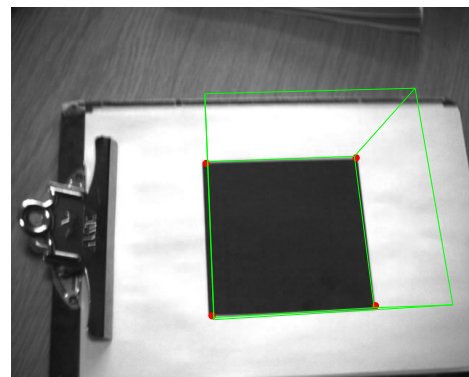


Figure 47: output for the iteration no. 34

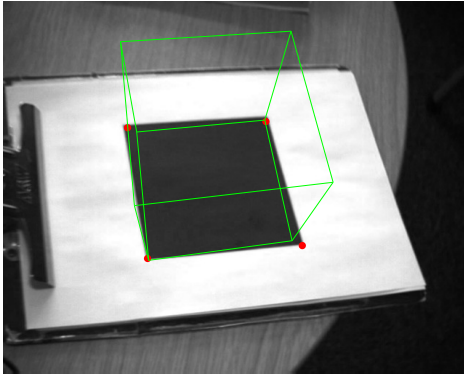


Figure 48: output for the iteration no. 129

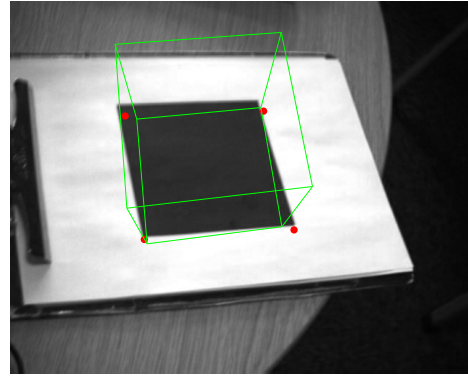


Figure 49: output for the iteration no. 136

In this practical we are combining both factor sampling and homography and performing augment reality. After tracking all the four corners of the black square using condensation algorithm using the same code from part "g" we store the coordinates and apply homography transformations over these coordinates to superimpose a wire frame cube over the black planar square which looks as if it were rigidly attached to the surface of the square.

Since, for few frames the corner tracking is not perfect i.e displaced. Hence for those frames we get the distorted cube.

**Two actions or changes we could make which could improve the result are.**

1) We can calculate the homography matrix using DLT algorithm with RANSAC. In RANSAC the intermediate approximations of the homography matrices from 4 or more corresponding points is computed using Direct linear transformation (DLT). When using DLT directly without RANSAC (which we are doing in our practical), it becomes more prone to outliers thus RANSAC eliminates outlier influence and thus is a more robust option.

2) Feature extraction of image mosaicing can be improved using fast extracting algorithm based on ORB (Oriented FAST and Rotated BRIEF). It uses median filter method to detect more accurate feature points. The speed of detection is increased by using this algorithm, it also uses RANSAC algorithm and homography matrix to eliminate false matching points. Finally, it uses perspective transformation to map points for obtaining the target coordinates to correct images and achieve two images matching in the scene. [2]

## References

- [1] S. J. Prince, “Computer vision: models, learning and inference,” July 2012.
- [2] Z. Y. Lei Yu and Y. Gong, *An Improved ORB Algorithm of Extracting and Matching Features*.