Homework 6

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Question:

Q1. (6 points) A full-adder is a combinational circuit that forms the arithmetic sum of three input bits. It consists of three inputs, x, y, z, and two outputs, C and S. Two of the input, that is, x and y, represent the two significant bits to be added. The third input, z, represents the carry from the previous lower significant position. The output S denotes the sum of two bits and C denotes carry. Answer the following sub-questions.
a. Construct a truth table for the Full-Adder

Answer:

Truth Table:

x	y	z	C	S
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Question:

b. Based on the truth table, construct a K-map for the output S and derive a Boolean equation using K-map. Make the equation as simple as possible.

Answer:

S K-map:

	00	01	11	10
0	0	1	0	1
1	1	0	1	0

S = $\neg xyz + \neg xy\neg z + x\neg y\neg z + xyz$

Question:

c. Based on the truth table, construct a K-map for the output C and derive a Boolean equation using K-map. Make the equation as simple as possible.

Answer:

C K-map:

	00	01	11	10
0	0	0	1	0
1	1	1	1	1

C = $yz + x$

Question:

d. By algebraic manipulation, show that S can be expressed as the exclusive-OR of the three input variables. That is, show that,
$S = x \oplus y \oplus z$

Answer:

$$S = x \oplus y \oplus z = (x \oplus y) \oplus z$$

$$x \oplus y = \neg xy + x\neg y, \quad (x \oplus y) \oplus z = (\neg xy + x\neg y) \oplus z$$

$$= \neg(\neg xy + x\neg y)z + (\neg xy + x\neg y)\neg z$$

$$= \neg(x\neg y + \neg x\neg y)z + (\neg xy + x\neg y)\neg z$$

$$= (\neg xy + xy)z + (\neg xy + x\neg y)\neg z$$

$$= \neg xyz + xyz + \neg xy\neg z + x\neg y\neg z$$

$$= \neg x\neg yz + \neg xy\neg z + x\neg y\neg z + xyz$$

Question:

e. By algebraic manipulation, show that C can be expressed as the following term.
$C = xy + (x \oplus y)z$

Answer:

$$= xy + (\neg xy + x\neg y)z = xy + \neg xyz + x\neg yz$$

$$xy + \neg xyz + x\neg yz = xy + \neg xyz + x\neg yz$$

$$= xy + yz(\neg x) + xz(\neg y)$$

$$= xy + yz - xyz + xz - xyz$$

$$= xy + yz + xz - 2xyz$$

$$= xy + yz + xz - xyz - xyz$$

$$= xy(1 - z) + yz + xz - xyz$$

$$= xy - xyz + yz + xz - xyz$$

$$= xy + yz + xz - 2xyz$$

$$= xy + yz + xz$$

$$= yz + xz + xy$$
$$= yz + x$$

Question:

Based on d and e, draw a circuit for the full-adder in Logisim simulator, attach the image file and submit the circuit file!

Answer:

sup

Question:

Q2. (4 points) A sequential circuit has one D flip-flop and one JK flip-flop, two inputs x and y, and one output z. A is the output of D flip-flop, and B is the output of JK-flip-flop; A and B together form the "output state" of the circuit. The flip-flop input equations and the circuit output are as follows. Here DA is the D input of the D-flip flop of A, and JB, KB is the J and K input of the JK-flip flop of B.

$$DA = \neg xy + yB$$

$$JB = \neg yB + xy$$

$$KB = xB + \neg yA$$

$$z = x + \neg xy$$

a. Draw the logic diagram of the circuit and test it with Logisim.
Please attach the circuit image and the generated table!

Answer:

Generated Table:

x	y	z
0	0	1
0	1	1
1	0	0
1	1	0

Question:

b. Construct a state diagram of this circuit.

Answer:

$$A_{n+1} = D_A = \sim xy + yB_n$$

$$B_{n+1} = J_B \sim B_n + K_B B_n = (\sim yB_n + xy) \sim B_n + (xB_n + (\sim y)A_n)B_n$$

$$= \sim yB_n \sim B_n + xy \sim B_n + xB_nB_n + \sim yA_nB_n$$

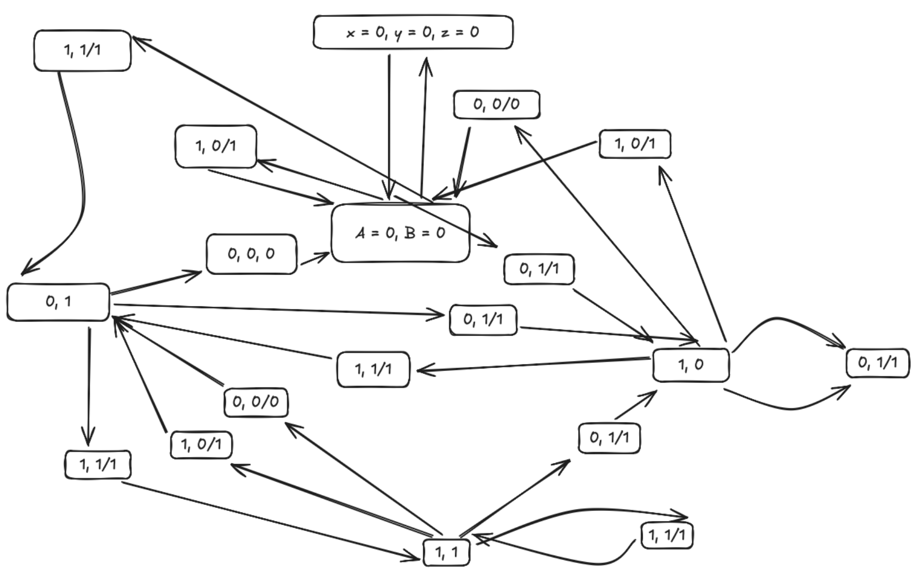
$$= 0 + xy \sim B_n + xB_n + \sim yA_nB_n$$

$$B_{n+1} = xy \sim B_n + xB_n + \sim yA_nB_n$$

$$z = x + \sim xy$$

State Table:

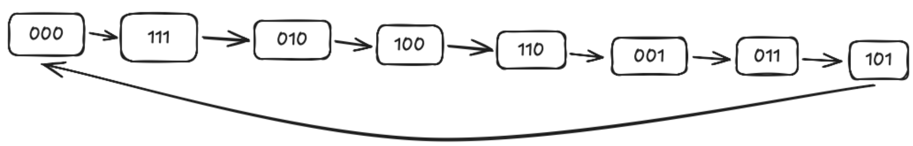
Input	Present State	Next State	Output
xy	A_nB_n	$A_{n+1}B_{n+1}$	z
00	00	00	0
00	01	00	0
00	10	00	0
00	11	01	0
01	00	10	1
01	01	10	1
01	10	10	1
01	11	10	1
10	00	00	1
10	01	01	1
10	10	00	1
10	11	01	1
11	00	01	1
11	01	11	1
11	10	01	1
11	11	11	1



Question:

Q3. (10 points) Design a system with the following state changes: This is a sequential circuit with three flip-flops. The state sequence is changed with a clock as in the order of, 111, 010, 100,110, 001, 011, 101, 000, 111 and repeat. Use JK flip-flops.
a. Draw a state diagram.

Answer:



Question:

b. Construct an excitation table.

Answer:

Present State	Next State	Flip Flop Inputs					
Q2 Q1 Q0	Q2+ Q1+ Q0+	J2	K2	J1	K1	J0	K0
111	010	0	1	0	0	0	1
010	100	1	X	0	1	0	X
100	110	0	0	1	X	0	X
110	001	0	1	0	1	1	X
001	011	0	X	1	X	0	0
011	101	1	X	0	1	0	0
101	000	0	1	0	X	0	1
000	111	1	X	1	X	1	X

Question:

c. Draw K-maps and derive Boolean equations using K-maps. Make the equations as simple as possible.

Answer:

J0 K-map:

	00	01	11	10
0	1	0	X	X
1	X	X	0	1

J1 K-map:

	00	01	11	10
0	1	1	0	X
1	X	X	0	1

J2 K-map:

	00	01	11	10
0	1	1	0	1
1	X	X	0	0

K0 K-map:

	00	01	11	10
0	1	0	X	X
1	X	X	0	1

K1 K-map:

	00	01	11	10
0	X	X	X	1
1	X	X	1	1

K2 K-map:

	00	01	11	10
0	X	X	X	X
1	X	1	1	1

$J_0 = \neg Q_2 \neg Q_1 \neg Q_0 + Q_2 Q_1 \neg Q_0$
$= \neg Q_0(\neg Q_2 \neg Q_1 + Q_2 Q_1)$
$J_1 = \neg Q_2 \neg Q_1 \neg Q_0 + \neg Q_2 \neg Q_1 Q_0 + Q_2 \neg Q_1 \neg Q_0$
$= \neg Q_1(\neg Q_0 + \neg Q_2 Q_0) + Q_2 \neg Q_1 \neg Q_0 = \neg Q_1 \neg Q_0 + \neg Q_1 \neg Q_2 Q_0$
$J_2 = \neg Q_2 \neg Q_1 \neg Q_0 + \neg Q_2 \neg Q_1 Q_0 + \neg Q_2 Q_1 Q_0$
$= \neg Q_2(\neg Q_1 + Q_1 Q_0)$
$K_0 = Q_2 Q_0 + Q_2 Q_1$
$K_1 = Q_1(Q_2 + Q_0)$
$K_2 = Q_2$

Question:

e. Draw the system in Logisim simulator, attach the circuit image and submit the circuit file.
f. Test the system and attach the generated table.

Answer:

sup
