Optimisation Basics 1 — Coding Practice Questions

CM52054: Foundational Machine Learning
Practice set with fully worked answers

1 The Generic Iterative Loop

1.1 Steepest Descent with Constant Step Size

Task. Implement $x_{t+1} = x_t - \alpha \nabla f(x_t)$ for $f(x, y) = x^2 + y^2$, start at $(2, 2)$. Stop when $\|\nabla f\| \leq 10^{-6}$ or 1000 iterations.

Listing 1: Steepest Descent

```python
import numpy as np

# ---- Objective and Gradient ---------------------------------------------------
# f(x, y) = x^2 + y^2 has a minimum at (0,0).
def f(v):
    return v[0]**2 + v[1]**2

# Analytic gradient: [2x, 2y].
def grad(v):
    return np.array([2*v[0], 2*v[1]])

# ---- Steepest Descent Loop ----------------------------------------------------
x = np.array([2.0, 2.0])  # initial point
alpha = 0.1               # constant step size (learning rate)
maxit = 1000
tol = 1e-6

for t in range(maxit):
    g = grad(x)                       # compute gradient at current x
    if np.linalg.norm(g) <= tol:      # termination check on gradient norm
        break
    x = x - alpha * g                 # steepest-descent update

# Report the final point, number of iterations, and final objective value
print("x:",x,"g:",g,"t:",t,"f(x)",f(x))
# https://programiz.pro/ide/python
```