

```

import math
from typing import TypeVar, Generic, List, Optional

T = TypeVar('T')

class Heap(Generic[T]):
    """
    A generic heap implementation that supports arbitrary number of children per node.
    The heap maintains the max-heap property where parent nodes are greater than their
    children.
    """

    def __init__(self, child_count: int):
        """
        Initialize the heap with specified number of children per node.

        Args:
            child_count: Number of children each node can have (must be a power of 2)

        Raises:
            ValueError: If child_count is not valid (<=0 or not a power of 2)
        """
        self._validate_child_count(child_count)
        self.child_count = child_count
        self.data: List[T] = []

    def _validate_child_count(self, child_count: int) -> None:
        """
        Validate that the child count is positive and a power of 2.

        Args:
            child_count: Number to validate

        Raises:
            ValueError: If child_count is invalid
        """
        # Ensure child_count is greater than zero
        if child_count <= 0:
            raise ValueError("child_count must be greater than zero")

        # Ensure child_count is a power of 2
        log_child_count = math.log2(child_count)
        if math.ceil(log_child_count) != math.floor(log_child_count):
            raise ValueError("child_count must be a power of 2")

```

```
def insert(self, item: T) -> None:
```

```
    """
```

```
    Insert an item into the heap and maintain the heap property.
```

```
    Args:
```

```
        item: Item to insert into the heap
```

```
    """
```

```
    self.data.append(item)
```

```
    item_index = len(self.data) - 1
```

```
    # Keep swapping up until we reach the root or heap property is satisfied
```

```
    while item_index > 0:
```

```
        item_index = self._swap_up(item_index)
```

```
        if item_index == -1:
```

```
            break
```

```
def _swap_up(self, child_index: int) -> int:
```

```
    """
```

```
    Check a child against its parent and swap if necessary to maintain heap property.
```

```
    Args:
```

```
        child_index: Index of the child to potentially swap up
```

```
    Returns:
```

```
        New index of the item after swap, or -1 if no swap occurred
```

```
    """
```

```
    child_value = self.data[child_index]
```

```
    parent_index = math.floor((child_index - 1) / self.child_count)
```

```
    if parent_index >= 0:
```

```
        parent_value = self.data[parent_index]
```

```
        if child_value > parent_value: # type: ignore
```

```
            # Swap the values
```

```
            self.data[parent_index] = child_value
```

```
            self.data[child_index] = parent_value
```

```
            return parent_index
```

```
    return -1
```

```
def pop_max(self) -> Optional[T]:
```

```
    """
```

```
    Remove and return the maximum value from the heap.
```

Returns:

The maximum value in the heap, or None if the heap is empty

"""

if not self.data:

return None

max_item = self.data[0]

if len(self.data) > 1:

Move last item to root and maintain heap property

self.data[0] = self.data.pop()

item_index = 0

Keep swapping down until leaf node or heap property is satisfied

while item_index >= 0:

item_index = self._swap_down(item_index)

else:

self.data.pop()

return max_item

def _swap_down(self, parent_index: int) -> int:

"""

Check a parent against all children and swap with the largest if necessary.

Args:

parent_index: Index of the parent to potentially swap down

Returns:

New index of the item after swap, or -1 if no swap occurred

"""

parent_value = self.data[parent_index]

largest_child_index = -1

largest_child_value = None

Find the largest child

for i in range(self.child_count):

child_index = self.child_count * parent_index + i + 1

if child_index < len(self.data):

child_value = self.data[child_index]

if largest_child_value is None or child_value > largest_child_value: # type: ignore

largest_child_index = child_index

largest_child_value = child_value

```
# Perform swap if necessary
if largest_child_value is not None and parent_value < largest_child_value: # type: ignore
    self.data[parent_index] = largest_child_value
    self.data[largest_child_index] = parent_value
    return largest_child_index
```

```
return -1
```

```
# Example usage:
```

```
if __name__ == "__main__":
```

```
    # Create a heap with 2 children per node
```

```
    heap = Heap[int](2)
```

```
    # Insert some numbers
```

```
    numbers = [4, 10, 3, 5, 1]
```

```
    for num in numbers:
```

```
        heap.insert(num)
```

```
    # Pop all numbers (will come out in descending order)
```

```
    while True:
```

```
        max_value = heap.pop_max()
```

```
        if max_value is None:
```

```
            break
```

```
        print(max_value)
```