

Homework 4

1. (50ps) For the RC circuit shown in Fig. 2, compute the Elmore delays of from node 1 to node 5 and from node 1 to node 6 of this circuit in terms of R and C component values. (20)

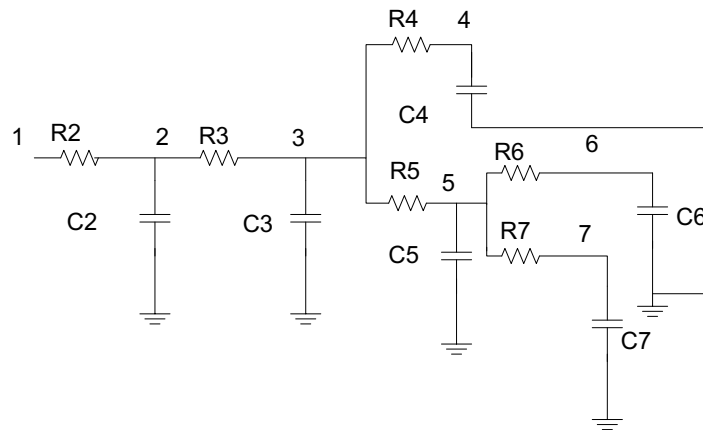


Figure 1 A RC Tree Circuit

2. Given a linear dynamic system in state space form $\dot{x} = Ax + Bu(t)$, where A is a $n \times n$ matrix and B is a $n \times 1$ matrix, $u(t)$ is 1×1 input vector, which changes with time. Assume that the initial condition is $x(0)$ and the input $u(t)$ is a step input i.e. $u(t) = 1$, when $t > 0$. Please derive the recursive moment matching formula for response $x(s)$ for this problem. (30)
3. (50ps) A plane wall of 2cm x 2cm with thickness 8cm has internal heat generation of $2 \times 10^4 \text{ W/m}^3$ with thermal properties of $k=20 \text{ W/mC}$, $\rho=80 \text{ kg/m}^3$ and specific heat $c_p=60 \text{ J/kgC}$. It is initially at the uniform temperature of 50C and is suddenly subject to the heat generation and a convective boundary condition as shown below. Assume time step $\Delta t = 0.1 \text{ s}$ and use the **two elements** to solve the problem.
- Write the finite element differential equation
 - Write the finite element equation using Back Euler method
 - Compute the temperature at the middle node (node 2) at $t = 0.1$ and $t = 0.2$ seconds.

