

Homework 3

1) For following equation $Ax=b$

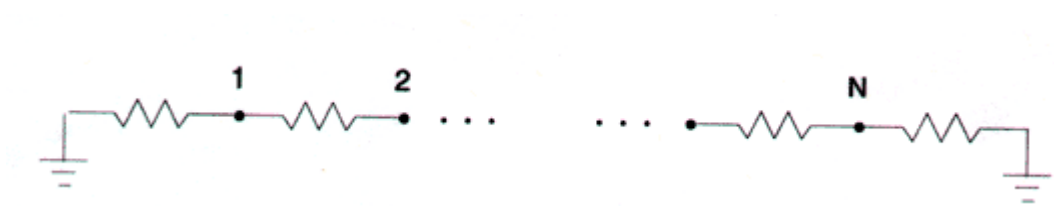
$$A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 10 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

- (a) Use Gaussian elimination to solve $Ax = b$
 - (b) Perform the LU factorization of the matrix $A = LU$
(show all the steps)
- 2) (a) Find the LU decomposition of A using left-looking method
 (b) Check the solution you got in 1) by solving x again using the LU you obtained.
 (c) Solve x with the same A (LU) and

$$b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

(show all the step)

3) Consider the following line of N resistors



- a) Determine the number of nonzero entries in the $N \times N$ conductance matrix associated with a N -long line of identical resistors of value R .
- b) Determine the number of nonzero entries in the $N \times N$ resistance matrix associated with the N -long line of resistors. Recall that the resistance matrix is the inverse of the conductance matrix.
- c) Determine the number of nonzero entries in the L and U factors of the conductance matrix associated with a N -long line of resistors. Compare your result from part (b) for $N = 1000$
- d) What is the moral of this exercise?

4) Compute the Markowitz's product value for all the matrix elements shown below and show which one is the best element for the pivot

$$A = \begin{bmatrix} x & 0 & x & 0 & x \\ 0 & x & x & x & 0 \\ x & 0 & 0 & x & 0 \\ 0 & 0 & x & 0 & 0 \\ 0 & x & 0 & 0 & x \end{bmatrix}$$

5) Using the compressed Sparse Row (CSR) format to represent the following matrix (using Row_Prt[], Col_Idx[], Values[] as the three vectors)

$$A = \begin{bmatrix} 2.1 & 3.4 & 0 & 0 & 3.4 \\ 0 & 5.2 & 5.6 & 0 & 0 \\ 0 & 3.4 & 0 & 4.5 & 0 \\ 0 & 0 & 6.7 & 0 & 0 \\ 0 & 4.6 & 0 & 0 & 5.6 \end{bmatrix}$$

6) For 2 x 2 matrix $A = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$, derive the expressions for the singular value as the functions of w, x, y, z.