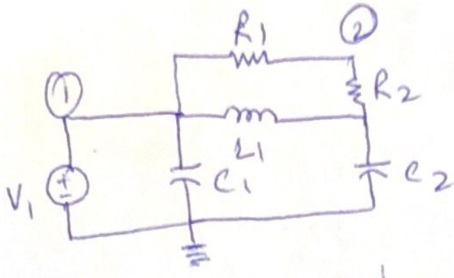
10/11/18 KAMALIKA PODDAR $\boxed{\text{HW1}}$ SID- 862002289 ①

$\boxed{\text{RR-213}}$

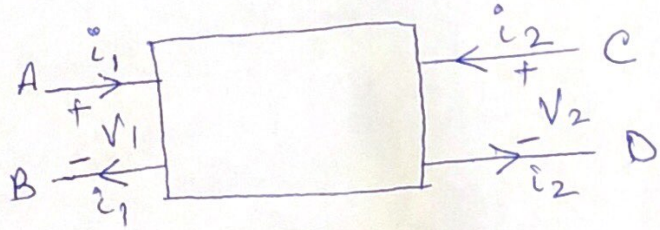
Question 1



$$
\begin{array}{c|ccc|cc}
 & 1 & 2 & 3 & i_s & i_{L_1} \\
\hline
1 & \frac{1}{R_1}+sC_1 & -\frac{1}{R_1} & 0 & 1 & 1 \\
2 & -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & 0 & 0 \\
3 & 0 & -\frac{1}{R_2} & \frac{1}{R_2}+sC_2 & 0 & -1 \\
\hline
\text{branch}_1 & 1 & 0 & 0 & 0 & 0 \\
\text{branch } 2 & 1 & 0 & -1 & 0 & -sL_1
\end{array}
$$

The above is the MNA equation for the RLC circuit.

Question 2



$V_1 = e_A - e_B$

$V_2 = e_C - e_D$

$e_A - e_B = h_{11} i_1 + h_{12}(e_C - e_D)$

$i_2 = h_{21} i_1 + h_{22}(e_C - e_D)$

P.T.O →

2.a MNA stamp for this element.

$$
\begin{array}{c|cccc|cc|}
 & e_A & e_B & e_C & e_D & i_1 & i_2 \\
\hline
e_A & 0 & 0 & 0 & 0 & 1 & 0 \\
e_B & 0 & 0 & 0 & 0 & -1 & 0 \\
e_C & 0 & 0 & 0 & 0 & 0 & 1 \\
e_D & 0 & 0 & 0 & 0 & 0 & -1 \\
\hline
\text{branch1} & -1 & 1 & h_{12} & -h_{12} & h_{11} & 0 \\
\text{branch2} & 0 & 0 & h_{22} & -h_{22} & h_{21} & -1
\end{array}
\qquad
\begin{array}{c}
\text{RHS} \\
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{array}
$$

2.b

$$v_1 = h_{11} i_1 + h_{12} v_2$$

$$h_{11} i_1 = v_1 - h_{12} v_2$$

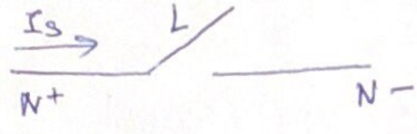
$$i_1 = \frac{v_1}{h_{11}} - \frac{h_{12}}{h_{11}} v_2 \quad \text{—(i)}$$

$$i_2 = \frac{h_{21}}{h_{11}} v_1 + \left(h_{22} - \frac{h_{12}}{h_{11}} \cdot h_{21}\right) v_2$$

NA stamp

$$
\begin{array}{c}
 \\ e_A \\ e_B \\ e_C \\ e_D
\end{array}
\begin{array}{c}
\begin{array}{cccc} e_A & e_B & e_C & e_D \end{array} \\
\left[\begin{array}{cccc}
\frac{1}{h_{11}} & -\frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} & \frac{h_{12}}{h_{11}} \\
-\frac{1}{h_{11}} & \frac{1}{h_{11}} & \frac{h_{12}}{h_{11}} & -\frac{h_{12}}{h_{11}} \\
\frac{h_{21}}{h_{11}} & -\frac{h_{21}}{h_{11}} & \left(h_{22} - \frac{h_{21} h_{12}}{h_{11}}\right) & -\left(h_{22} - \frac{h_{21} h_{12}}{h_{11}}\right) \\
-\frac{h_{21}}{h_{11}} & \frac{h_{21}}{h_{11}} & -\left(h_{22} - \frac{h_{21} h_{12}}{h_{11}}\right) & \left(h_{22} - \frac{h_{21} h_{12}}{h_{11}}\right)
\end{array}\right]
\end{array}
\qquad
\begin{array}{c}
\text{RHS} \\
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{array}
$$

Question 3



MNA stamp for the switch

$$\begin{array}{c|c} & \begin{array}{ccc} N^+ & N^- & I \end{array} \\ \begin{array}{c} N^+ \\ N^- \\ I \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ L & -L & -(1-L) \end{bmatrix} \end{array} \quad \begin{array}{c} RHS \\ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{array}$$

where we have $L = 1$ for switch on and $L = 0$ for off.

Question 5

$$\begin{bmatrix} G+SC & A_e^T \\ -A_e & S_L \end{bmatrix} \begin{bmatrix} V_n \\ i_l \end{bmatrix} = \begin{bmatrix} i_n(s) \\ 0 \end{bmatrix}$$

Schur Decomposition

$$\begin{bmatrix} G+SC & A_e^T \\ 0 & S_L + \frac{1}{G+SC} \end{bmatrix} \begin{bmatrix} V_n \\ i_e \end{bmatrix} = \begin{bmatrix} i_n(s) \\ \frac{A_e}{G+SC} \cdot i_n(s) \end{bmatrix}$$

$$R_2 \leftarrow R_2 + \frac{A_e}{G+SC} R_1$$
$$R_2 \leftarrow S_L + \frac{A_e A_e^T}{G+S_C}$$

Node Analysis

5(a)
$$\left(S_L + \frac{1}{G+S_C}\right) i_e = \frac{A_e}{G+S_C} \cdot i_n(s)$$

$$(G+S_C) S_L i_e + i_e = A_e i_n(s)$$

Above is the nodal Analysis formation of the RLCM circuit based on the node reduction formula.

5.(b) The NA formulation is not equivalent to the MNA formulation when the RLCM circuit has $[G+S_C = 0]$. Therefore the capacitive and resistive components can't be equal to zero and the inductive branches (A_e) cannot be zero.

Question 4

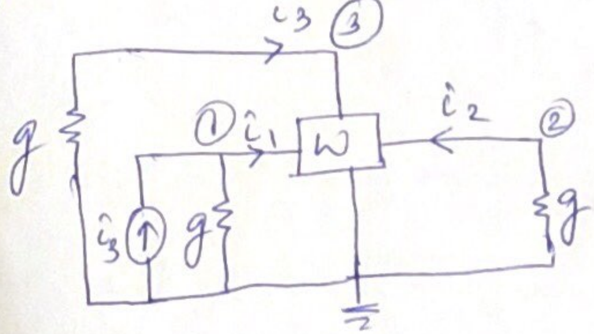
4.a The constitutive relation for any circuit is
$$k_i i + k_u u = s$$
where k_i & k_u is dependent on the nature of the element and s is a vector of source terms.

Normally for the equation $Gv = i$, can be solved by solving i on one side and the k_i matrix can be inverted. In this situation nodal analysis can be applied.

P.T.O →

4.a For the element W, the ki matrix is singular i.e, first column equals the sum of the second plus third. Therefore we cannot solve the equation for i, and thus cannot use NA for the circuit which features W-element.

4.b We are going to use modified Nodal Analysis for this problem :



∴ We have
$$g v_1 = i_S - i_1$$
$$g v_2 = -\hat{i}_2$$
$$g v_3 = -\hat{i}_3$$

∴ we have Constitutive relation of W element.
$$v_1 + \hat{i}_1 + \hat{i}_2 = 0$$
$$v_1 + v_2 + \hat{i}_2 - \hat{i}_3 = 0$$
$$v_1 + v_2 + v_3 + \hat{i}_1 + \hat{i}_3 = 0$$

P.T.O →

4.b Therefore we have MNA as.

$$\begin{bmatrix} g & 0 & 0 & 1 & 0 & 0 \\ 0 & g & 0 & 0 & 1 & 0 \\ 0 & 0 & g & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$