

De Polignac's formula

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In number theory, **de Polignac's formula**, named after Alphonse de Polignac, gives the prime decomposition of the factorial $n!$, where $n \geq 1$ is an integer. L. E. Dickson attributes the formula to Legendre.^[1]

The formula

Let $n \geq 1$ be an integer. The prime decomposition of $n!$ is given by

$$n! = \prod_{\text{prime } p \leq n} p^{s_p(n)},$$

where

$$s_p(n) = \sum_{j=1}^{\infty} \left\lfloor \frac{n}{p^j} \right\rfloor,$$

and the brackets represent the floor function. Note that the former product can equally well be taken only over primes less than or equal to n , and the latter sum can equally well be taken for j ranging from 1 to $\log_p(n)$, i.e.:

$$s_p(n) = \sum_{j=1}^{\lfloor \log_p(n) \rfloor} \left\lfloor \frac{n}{p^j} \right\rfloor$$

Note that, for any real number x , and any integer n , we have:

$$\left\lfloor \frac{x}{n} \right\rfloor = \left\lfloor \frac{\lfloor x \rfloor}{n} \right\rfloor$$

which allows one to more easily compute the terms $s_p(n)$.

The small disadvantage of the De Polignac's formula is that **we need to know all the primes up to n** . In fact,

$$n! = \prod_{i=1}^{\pi(n)} p_i^{s_{p_i}(n)} = \prod_{i=1}^{\pi(n)} p_i^{\sum_{j=1}^{\lfloor \log_{p_i}(n) \rfloor} \left\lfloor \frac{n}{p_i^j} \right\rfloor}$$

where $\pi(n)$ is a prime-counting function counting the number of prime numbers less than or equal to n

Notes and references

- [^] Leonard Eugene Dickson, *History of the Theory of Numbers*, Volume 1, Carnegie Institution of Washington, 1919, page 263.

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