Merge k sorted arrays | Set 1

Given k sorted arrays of size n each, merge them and print the sorted output.

Example:

```
Input:
k = 3, n =  4
arr[][] = { {1, 3, 5, 7},
            {2, 4, 6, 8},
            {0, 9, 10, 11}} ;

Output: 0 1 2 3 4 5 6 7 8 9 10 11
```

A simple solution is to create an output array of size n*k and one by one copy all arrays to it. Finally, sort the output array using any O(nLogn) sorting algorithm. This approach takes O(nkLognk) time.

We can merge arrays in O(nk*Logk) time using Min Heap. Following is detailed algorithm.

1. Create an output array of size n*k.
2. Create a min heap of size k and insert 1st element in all the arrays into a the heap
3. Repeat following steps n*k times.
a) Get minimum element from heap (minimum is always at root) and store it in output array.
b) Replace heap root with next element from the array from which the element is extracted. If the array doesn't have any more elements, then replace root with infinite. After replacing the root, heapify the tree.

Following is C++ implementation of the above algorithm.

```
// C++ program to merge k sorted arrays of size n each.
#include<iostream>
#include<limits.h>
using namespace std;

#define n 4

// A min heap node
struct MinHeapNode
{
    int element; // The element to be stored
    int i; // index of the array from which the element is taken
    int j; // index of the next element to be picked from array
};

// Prototype of a utility function to swap two min heap nodes
void swap(MinHeapNode *x, MinHeapNode *y);

// A class for Min Heap
class MinHeap
{
    MinHeapNode *harr; // pointer to array of elements in heap
    int heap_size; // size of min heap
public:
    // Constructor: creates a min heap of given size
    MinHeap(MinHeapNode a[], int size);

    // to heapify a subtree with root at given index
    void MinHeapify(int );

    // to get index of left child of node at index i
    int left(int i) { return (2*i + 1); }

    // to get index of right child of node at index i
    int right(int i) { return (2*i + 2); }
```

```
    // to get the root
    MinHeapNode getMin() { return harr[0]; }

    // to replace root with new node x and heapify() new root
    void replaceMin(MinHeapNode x) { harr[0] = x;  MinHeapify(0); }
};

// This function takes an array of arrays as an argument and
// All arrays are assumed to be sorted. It merges them together
// and prints the final sorted output.
int *mergeKArrays(int arr[][n], int k)
{
    int *output = new int[n*k];  // To store output array

    // Create a min heap with k heap nodes.  Every heap node
    // has first element of an array
    MinHeapNode *harr = new MinHeapNode[k];
    for (int i = 0; i < k; i++)
    {
        harr[i].element = arr[i][0]; // Store the first element
        harr[i].i = i;  // index of array
        harr[i].j = 1;  // Index of next element to be stored from array
    }
    MinHeap hp(harr, k); // Create the heap

    // Now one by one get the minimum element from min
    // heap and replace it with next element of its array
    for (int count = 0; count < n*k; count++)
    {
        // Get the minimum element and store it in output
        MinHeapNode root = hp.getMin();
        output[count] = root.element;

        // Find the next elelement that will replace current
        // root of heap. The next element belongs to same
        // array as the current root.
        if (root.j < n)
        {
            root.element = arr[root.i][root.j];
            root.j += 1;
        }
        // If root was the last element of its array
        else root.element =  INT_MAX; //INT_MAX is for infinite

        // Replace root with next element of array
        hp.replaceMin(root);
    }

    return output;
}

// FOLLOWING ARE IMPLEMENTATIONS OF STANDARD MIN HEAP METHODS
// FROM CORMEN BOOK
// Constructor: Builds a heap from a given array a[] of given size
MinHeap::MinHeap(MinHeapNode a[], int size)
{
    heap_size = size;
    harr = a;  // store address of array
    int i = (heap_size - 1)/2;
    while (i >= 0)
    {
        MinHeapify(i);
        i--;
    }
}

// A recursive method to heapify a subtree with root at given index
// This method assumes that the subtrees are already heapified
void MinHeap::MinHeapify(int i)
{
    int l = left(i);
    int r = right(i);
    int smallest = i;
```

```
    int smallest = i;
    if (l < heap_size && harr[l].element < harr[i].element)
        smallest = l;
    if (r < heap_size && harr[r].element < harr[smallest].element)
        smallest = r;
    if (smallest != i)
    {
        swap(&harr[i], &harr[smallest]);
        MinHeapify(smallest);
    }
}

// A utility function to swap two elements
void swap(MinHeapNode *x, MinHeapNode *y)
{
    MinHeapNode temp = *x;  *x = *y;  *y = temp;
}

// A utility function to print array elements
void printArray(int arr[], int size)
{
   for (int i=0; i < size; i++)
       cout << arr[i] << " ";
}

// Driver program to test above functions
int main()
{
    // Change n at the top to change number of elements
    // in an array
    int arr[][n] =  {{2, 6, 12, 34},
                     {1, 9, 20, 1000},
                     {23, 34, 90, 2000}};
    int k = sizeof(arr)/sizeof(arr[0]);

    int *output = mergeKArrays(arr, k);

    cout << "Merged array is " << endl;
    printArray(output, n*k);

    return 0;
}
```

Output:

```
Merged array is
1 2 6 9 12 20 23 34 34 90 1000 2000
```

Time Complexity: The main step is 3rd step, the loop runs n*k times. In every iteration of loop, we call heapify which takes O(Logk) time. Therefore, the time complexity is O(nk Logk).

There are other interesting methods to merge k sorted arrays in O(nkLogk), we will sonn be discussing them as separate posts.