

Find maximum value of $\text{Sum}(i \cdot \text{arr}[i])$ with only rotations on given array allowed

Given an array, only rotation operation is allowed on array. We can rotate the array as many times as we want. Return the maximum possible of summation of $i \cdot \text{arr}[i]$.

Example:

```
Input: arr[] = {1, 20, 2, 10}
Output: 72
We can 72 by rotating array twice.
{2, 10, 1, 20}
 $20 \cdot 3 + 1 \cdot 2 + 10 \cdot 1 + 2 \cdot 0 = 72$ 

Input: arr[] = {10, 1, 2, 3, 4, 5, 6, 7, 8, 9};
Output: 330
We can 330 by rotating array 9 times.
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
 $0 \cdot 1 + 1 \cdot 2 + 2 \cdot 3 + \dots + 9 \cdot 10 = 330$ 
```

We strongly recommend you to minimize your browser and try this yourself first.

A **Simple Solution** is to find all rotations one by one, check sum of every rotation and return the maximum sum. Time complexity of this solution is $O(n^2)$.

We can solve this problem in $O(n)$ time using an **Efficient Solution**.

Let R_j be value of $i \cdot \text{arr}[i]$ with j rotations. The idea is to calculate next rotation value from previous rotation, i.e., calculate R_j from R_{j-1} . We can calculate initial value of result as R_0 , then keep calculating next rotation values.

How to efficiently calculate R_j from R_{j-1} ?

This can be done in $O(1)$ time. Below are details.

```
Let us calculate initial value of  $i \cdot \text{arr}[i]$  with no rotation
 $R_0 = 0 \cdot \text{arr}[0] + 1 \cdot \text{arr}[1] + \dots + (n-1) \cdot \text{arr}[n-1]$ 

After 1 rotation  $\text{arr}[n-1]$ , becomes first element of array,
 $\text{arr}[0]$  becomes second element,  $\text{arr}[1]$  becomes third element
and so on.
 $R_1 = 0 \cdot \text{arr}[n-1] + 1 \cdot \text{arr}[0] + \dots + (n-1) \cdot \text{arr}[n-2]$ 

 $R_1 - R_0 = \text{arr}[0] + \text{arr}[1] + \dots + \text{arr}[n-2] - (n-1) \cdot \text{arr}[n-1]$ 

After 2 rotations  $\text{arr}[n-2]$ , becomes first element of array,
 $\text{arr}[n-1]$  becomes second element,  $\text{arr}[0]$  becomes third element
and so on.
 $R_2 = 0 \cdot \text{arr}[n-2] + 1 \cdot \text{arr}[n-1] + \dots + (n-1) \cdot \text{arr}[n-3]$ 

 $R_2 - R_1 = \text{arr}[0] + \text{arr}[1] + \dots + \text{arr}[n-3] - (n-1) \cdot \text{arr}[n-2] + \text{arr}[n-1]$ 

If we take a closer look at above values, we can observe
below pattern

 $R_j - R_{j-1} = \text{arrSum} - n \cdot \text{arr}[n-j]$ 

Where  $\text{arrSum}$  is sum of all array elements, i.e.,

 $\text{arrSum} = \sum_{i=0}^{n-1} \text{arr}[i]$ 
```

Below is complete algorithm:

```

1) Compute sum of all array elements. Let this sum be 'arrSum'.

2) Compute  $R_0$  by doing  $i \cdot arr[i]$  for given array.
   Let this value be currVal.

3) Initialize result: maxVal = currVal // maxVal is result.

// This loop computes  $R_j$  from  $R_{j-1}$ 
4) Do following for  $j = 1$  to  $n-1$ 
.....a) currVal = currVal + arrSum -  $n \cdot arr[n-j]$ ;
.....b) If (currVal > maxVal)
           maxVal = currVal

5) Return maxVal

```

Below are C++ and Python implementations of above idea.

C++

```

// C++ program to find max value of  $i \cdot arr[i]$ 
#include <iostream>
using namespace std;

// Returns max possible value of  $i \cdot arr[i]$ 
int maxSum(int arr[], int n)
{
    // Find array sum and  $i \cdot arr[i]$  with no rotation
    int arrSum = 0; // Stores sum of arr[i]
    int currVal = 0; // Stores sum of  $i \cdot arr[i]$ 
    for (int i=0; i<n; i++)
    {
        arrSum = arrSum + arr[i];
        currVal = currVal + (i * arr[i]);
    }

    // Initialize result as 0 rotation sum
    int maxVal = currVal;

    // Try all rotations one by one and find
    // the maximum rotation sum.
    for (int j=1; j<n; j++)
    {
        currVal = currVal + arrSum - n * arr[n-j];
        if (currVal > maxVal)
            maxVal = currVal;
    }

    // Return result
    return maxVal;
}

// Driver program
int main(void)
{
    int arr[] = {10, 1, 2, 3, 4, 5, 6, 7, 8, 9};
    int n = sizeof(arr)/sizeof(arr[0]);
    cout << "\nMax sum is " << maxSum(arr, n);
    return 0;
}

```

Python

```

'''Python program to find maximum value of Sum(i*arr[i])'''

# returns max possible value of Sum(i*arr[i])
def maxSum(arr):

    # stores sum of arr[i]
    arrSum = 0

    # stores sum of i*arr[i]
    currVal = 0

    n = len(arr)

    for i in range(0, n):
        arrSum = arrSum + arr[i]
        currVal = currVal + (i*arr[i])

    # initialize result
    maxVal = currVal

    # try all rotations one by one and find the maximum
    # rotation sum
    for j in range(1, n):
        currVal = currVal + arrSum-n*arr[n-j]
        if currVal > maxVal:
            maxVal = currVal

    # return result
    return maxVal

# test maxsum(arr) function
arr = [10, 1, 2, 3, 4, 5, 6, 7, 8, 9]
print "Max sum is: ", maxSum(arr)

```

Output:

```

Max sum is 330

```

Time Complexity: $O(n)$

Auxiliary Space: $O(1)$