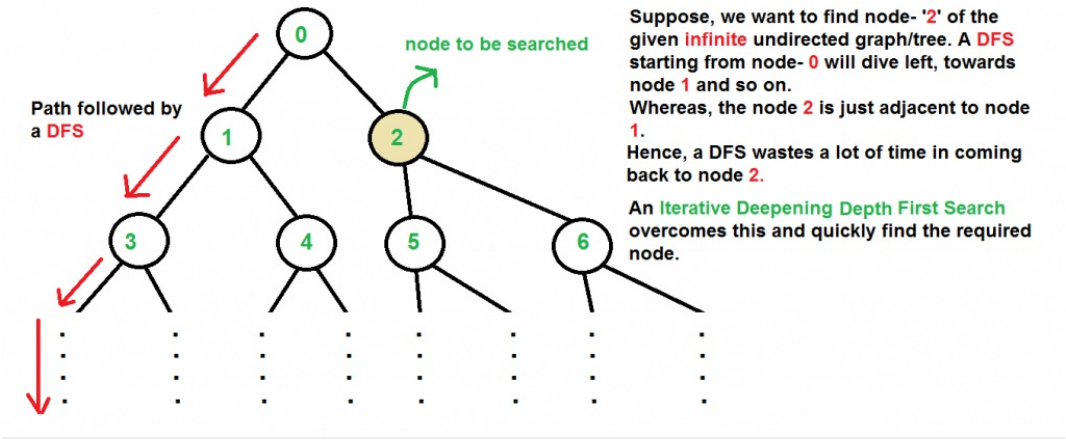
Iterative Deepening Search(IDS) or Iterative Deepening Depth First Search(IDDFS)

There are two common ways to traverse a graph, BFS and DFS. Considering a Tree (or Graph) of huge height and width, both BFS and DFS are not very efficient due to following reasons.

1. **DFS** first traverses nodes going through one adjacent of root, then next adjacent. The problem with this approach is, if there is a node close to root, but not in first few subtrees explored by DFS, then DFS reaches that node very late. Also, DFS may not find shortest path to a node (in terms of number of edges).



2. **BFS** goes level by level, but requires more space. The space required by DFS is O(d) where d is depth of tree, but space required by BFS is O(n) where n is number of nodes in tree (Why? Note that the last level of tree can have around n/2 nodes and second last level n/4 nodes and in BFS we need to have every level one by one in queue).

IDDFS combines depth-first search's space-efficiency and breadth-first search's fast search (for nodes closer to root).

How does IDDFS work?

IDDFS calls DFS for different depths starting from an initial value. In every call, DFS is restricted from going beyond given depth. So basically we do DFS in a BFS fashion.

Algorithm:

```
// Returns true if target is reachable from
// src within max_depth
bool IDDFS(src, target, max_depth)
    for limit from 0 to max_depth
       if DLS(src, target, limit) == true
           return true
    return false

bool DLS(src, target, limit)
    if (src == target)
        return true;

    // If reached the maximum depth,
    // stop recursing.
    if (limit <= 0)
        return false;

    foreach adjacent i of src
        if DLS(i, target, limit?1)
            return true

    return false
```

An important thing to note is, we visit top level nodes multiple times. The last (or max depth) level is visited once, second last level is visited twice, and so on. It may seem expensive, but it turns out to be not so costly, since in a tree most of the nodes are in the bottom level. So it does not matter much if the upper levels are visited multiple times.

Below is C++ implementation of above algorithm

```
// C++ program to search if a target node is reachable from
// a source with given max depth.
#include<bits/stdc++.h>
using namespace std;

// Graph class represents a directed graph using adjacency
// list representation.
class Graph
{
    int V;    // No. of vertices

    // Pointer to an array containing
    // adjacency lists
    list<int> *adj;

    // A function used by IDDFS
    bool DLS(int v, int target, int limit);

public:
    Graph(int V);   // Constructor
    void addEdge(int v, int w);

    // IDDFS traversal of the vertices reachable from v
    bool IDDFS(int v, int target, int max_depth);
};

Graph::Graph(int V)
{
    this->V = V;
    adj = new list<int>[V];
}

void Graph::addEdge(int v, int w)
{
    adj[v].push_back(w); // Add w to v’s list.
}

// A function to perform a Depth-Limited search
// from given source 'src'
bool Graph::DLS(int src, int target, int limit)
{
    if (src == target)
        return true;

    // If reached the maximum depth, stop recursing.
    if (limit <= 0)
        return false;

    // Recur for all the vertices adjacent to source vertex
    for (auto i = adj[src].begin(); i != adj[src].end(); ++i)
       if (DLS(*i, target, limit-1) == true)
          return true;

     return false;
}

// IDDFS to search if target is reachable from v.
// It uses recursive DFSUtil().
bool Graph::IDDFS(int src, int target, int max_depth)
{
    // Repeatedly depth-limit search till the
    // maximum depth.
    for (int i = 0; i <= max_depth; i++)
       if (DLS(src, target, i) == true)
          return true;

    return false;
}

// Driver code
```

```
int main()
{
    // Let us create a Directed graph with 7 nodes
    Graph g(7);
    g.addEdge(0, 1);
    g.addEdge(0, 2);
    g.addEdge(1, 3);
    g.addEdge(1, 4);
    g.addEdge(2, 5);
    g.addEdge(2, 6);

    int target = 6, maxDepth = 3, src = 0;
    if (g.IDDFS(src, target, maxDepth) == true)
        cout << "Target is reachable from source "
                "within max depth";
    else
        cout << "Target is NOT reachable from source "
                "within max depth";
    return 0;
}
```

Output :

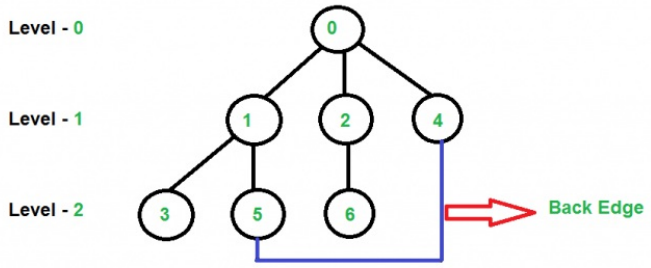
```
Target is reachable from source within max depth
```

Illustration:

There can be two cases-

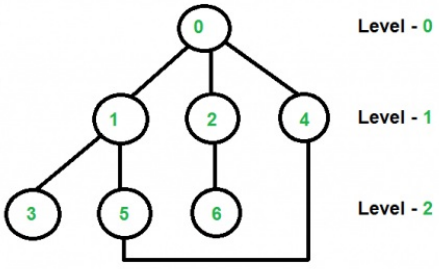
a) When the graph has no cycle: This case is simple. We can DFS multiple times with different height limits.

b) When the graph has cycles. This is interesting as there is no visited flag in IDDFS.



Although, at first sight, it may seem that since there are only 3 levels, so we might think that Iterative Deepening Depth First Search of level 3, 4, 5,...and so on will remain same. But, this is not the case. You can see that there is a cycle in the above graph, hence IDDFS will change for level-3,4,5..and so on.

Depth	Iterative Deepening Depth First Search
0	0
1	0 1 2 4
2	0 1 3 5 2 6 4 5
3	0 1 3 5 4 2 6 4 5 1



The explanation of the above pattern is left to the readers.

Time Complexity: Suppose we have a tree having branching factor 'b' (number of children of each node), and its depth 'd', i.e., there are b^d nodes.

In an iterative deepening search, the nodes on the bottom level are expanded once, those on the next to bottom level are expanded twice, and so on, up to the root of the search tree, which is expanded d+1 times. So the total number of expansions in an iterative deepening search is-

$$(d)b + (d-1)b^2 + ... + 3b^{d-2} + 2b^{d-1} + b^d$$

That is,

$$\text{Summation}[(d + 1 - i)\, b^i], \text{ from } i = 0 \text{ to } i = d$$

Which is same a

s $O(b^d)$

After evaluating the above expression, we find that asymptotically IDDFS takes the same time as that of DFS and BFS, but it is indeed slower than both of them as it has a higher constant factor in its time complexity expression.

IDDFS is best suited for a complete infinite tree

References:

A comparison table between DFS, BFS and IDDFS

	Time Complexity	Space Complexity	When to Use ?
DFS	$O(b^d)$	O (d)	=> Don't care if the answer is closest to the starting vertex/root. => When graph/tree is not very big/infinite.
BFS	$O(b^d)$	$O(b^d)$	=> When space is not an issue => When we do care/want the closest answer to the root.
IDDFS	$O(b^d)$	O(bd)	=> You want a BFS, you don't have enough memory, and somewhat slower performance is accepted. In short, you want a BFS + DFS.

https://en.wikipedia.org/wiki/Iterative_deepening_depth-first_search