

Check if an array can be divided into pairs whose sum is divisible by k

Given an array of integers and a number k, write a function that returns true if given array can be divided into pairs such that sum of every pair is divisible by k.

Examples:

```
Input: arr[] = {9, 7, 5, 3}, k = 6
Output: True
We can divide array into (9, 3) and (7, 5).
Sum of both of these pairs is a multiple of 6.

Input: arr[] = {92, 75, 65, 48, 45, 35}, k = 10
Output: True
We can divide array into (92, 48), (75, 65) and
(45, 35). Sum of all these pairs is a multiple of 10.

Input: arr[] = {91, 74, 66, 48}, k = 10
Output: False
```

We strongly recommend you to minimize your browser and try this yourself first.

A **Simple Solution** is to iterate through every element `arr[i]`. Find if there is another not yet visited element that has remainder as $(k - arr[i] \% k)$. If there is no such element, return false. If a pair is found, then mark both elements as visited. Time complexity of this solution is $O(n^2)$ and it requires $O(n)$ extra space.

An **Efficient Solution** is to use Hashing.

- 1) If length of given array is odd, return false. An odd length array cannot be divided in pairs.
- 2) Traverse input array and count occurrences of all remainders.
`freq[arr[i] % k]++`
- 3) Traverse input array again.
 - a) Find remainder of current element.
 - b) If remainder divides k into two halves, then there must be even occurrences of it as it forms pair with itself only.
 - c) If remainder is 0, then there must be even occurrences.
 - c) Else, number of occurrences of current remainder must be equal to number of occurrences of "k - current remainder".

Time complexity of above algorithm is $O(n)$.

Below implementation uses map in C++ STL. The map is typically implemented using Red-Black Tree and takes $O(\log n)$ time for access. Therefore time complexity of below implementation is $O(n \log n)$, but the algorithm can be easily implemented in $O(n)$ time using hash table.

```

// A C++ program to check if arr[0..n-1] can be divided
// in pairs such that every pair is divisible by k.
#include <bits/stdc++.h>
using namespace std;

// Returns true if arr[0..n-1] can be divided into pairs
// with sum divisible by k.
bool canPairs(int arr[], int n, int k)
{
    // An odd length array cannot be divided into pairs
    if (n & 1)
        return false;

    // Create a frequency array to count occurrences
    // of all remainders when divided by k.
    map<int, int> freq;

    // Count occurrences of all remainders
    for (int i = 0; i < n; i++)
        freq[arr[i] % k]++;

    // Traverse input array and use freq[] to decide
    // if given array can be divided in pairs
    for (int i = 0; i < n; i++)
    {
        // Remainder of current element
        int rem = arr[i] % k;

        // If remainder with current element divides
        // k into two halves.
        if (2*rem == k)
        {
            // Then there must be even occurrences of
            // such remainder
            if (freq[rem] % 2 != 0)
                return false;
        }

        // If remainder is 0, then there must be two
        // elements with 0 remainder
        else if (rem == 0)
        {
            if (freq[rem] & 1)
                return false;
        }

        // Else number of occurrences of remainder
        // must be equal to number of occurrences of
        // k - remainder
        else if (freq[rem] != freq[k - rem])
            return false;
    }
    return true;
}

/* Driver program to test above function */
int main()
{
    int arr[] = {92, 75, 65, 48, 45, 35};
    int k = 10;
    int n = sizeof(arr)/sizeof(arr[0]);
    canPairs(arr, n, k)? cout << "True": cout << "False";
    return 0;
}

```

Output:

True