How to determine if a binary tree is height-balanced?

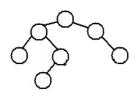
A tree where no leaf is much farther away from the root than any other leaf. Different balancing schemes allow different definitions of "much farther" and different amounts of work to keep them balanced.

Consider a height-balancing scheme where following conditions should be checked to determine if a binary tree is balanced.

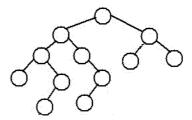
An empty tree is height-balanced. A non-empty binary tree T is balanced if:

- 1) Left subtree of T is balanced
- 2) Right subtree of T is balanced
- 3) The difference between heights of left subtree and right subtree is not more than 1.

The above height-balancing scheme is used in AVL trees. The diagram below shows two trees, one of them is height-balanced and other is not. The second tree is not height-balanced because height of left subtree is 2 more than height of right subtree.



A height-balanced Tree



Not a height-balanced tree

To check if a tree is height-balanced, get the height of left and right subtrees. Return true if difference between heights is not more than 1 and left and right subtrees are balanced, otherwise return false.

C

```
/* C program to check if a tree is height-balanced or not */
#include<stdio.h>
#include<stdlib.h>
#define bool int
/* A binary tree node has data, pointer to left child
  and a pointer to right child */
struct node
{
   int data;
   struct node* left;
   struct node* right;
};
/* Returns the height of a binary tree */
int height(struct node* node);
/* Returns true if binary tree with root as root is height-balanced */
bool isBalanced(struct node *root)
  int lh; /* for height of left subtree */
  int rh; /* for height of right subtree */
  /* If tree is empty then return true */
  if(root == NULL)
   return 1;
  /* Get the height of left and right sub trees */
  lh = height(root->left);
  rh = height(root->right);
   if( ahs(lh-rh) <= 1 &&
```

```
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       isBalanced(root->left) &&
      isBalanced(root->right))
   /* If we reach here then tree is not height-balanced */
}
/* UTILITY FUNCTIONS TO TEST isBalanced() FUNCTION */
/* returns maximum of two integers */
int max(int a, int b)
{
  return (a >= b)? a: b;
/* The function Compute the "height" of a tree. Height is the
   number of nodes along the longest path from the root node
    down to the farthest leaf node.*/
int height(struct node* node)
   /* base case tree is empty */
  if(node == NULL)
       return 0;
   /* If tree is not empty then height = 1 + max of left
     height and right heights */
   return 1 + max(height(node->left), height(node->right));
}
/* Helper function that allocates a new node with the
   given data and NULL left and right pointers. */
struct node* newNode(int data)
   struct node* node = (struct node*)
                               malloc(sizeof(struct node));
   node->data = data;
   node->left = NULL;
   node->right = NULL;
    return(node);
}
int main()
   struct node *root = newNode(1);
   root->left = newNode(2);
   root->right = newNode(3);
   root->left->left = newNode(4);
   root->left->right = newNode(5);
   root->left->left->left = newNode(8);
   if(isBalanced(root))
     printf("Tree is balanced");
    else
     printf("Tree is not balanced");
   getchar();
    return 0;
}
```

Java

```
/* Java program to determine if binary tree is
   height balanced or not */

/* A binary tree node has data, pointer to left child,
   and a pointer to right child */
class Node
{
```

```
int data;
   Node left, right;
   Node(int d)
       data = d;
       left = right = null;
   }
}
class BinaryTree
   Node root:
   /* Returns true if binary tree with root as root is height-balanced */
   boolean isBalanced(Node node)
       int lh; /* for height of left subtree */
       int rh; /* for height of right subtree */
       /* If tree is empty then return true */
       if (node == null)
           return true;
       /* Get the height of left and right sub trees */
       lh = height(node.left);
       rh = height(node.right);
       if (Math.abs(lh - rh) <= 1
               && isBalanced(node.left)
               && isBalanced(node.right))
           return true;
       /* If we reach here then tree is not height-balanced */
       return false;
   /* UTILITY FUNCTIONS TO TEST isBalanced() FUNCTION */
   /* returns maximum of two integers */
   int max(int a, int b)
       return (a >= b) ? a : b;
   }
   /* The function Compute the "height" of a tree. Height is the
       number of nodes along the longest path from the root node
       down to the farthest leaf node.*/
   int height(Node node)
       /* base case tree is empty */
       if (node == null)
           return 0;
       /* If tree is not empty then height = 1 + max of left
        height and right heights */
       return 1 + max(height(node.left), height(node.right));
   }
   public static void main(String args[])
       BinaryTree tree = new BinaryTree();
       tree.root = new Node(1);
       tree.root.left = new Node(2);
       tree.root.right = new Node(3);
       tree.root.left.left = new Node(4);
       tree.root.left.right = new Node(5);
       tree.root.left.left.left = new Node(8);
       if(tree.isBalanced(tree.root))
           System.out.println("Tree is balanced");
       else
```

```
System.out.printin( Tree is not Daianced );
}

// This code has been contributed by Mayank Jaiswal(mayank_24)
```

Output:

```
Tree is not balanced
```

Time Complexity: O(n^2) Worst case occurs in case of skewed tree.

Optimized implementation: Above implementation can be optimized by calculating the height in the same recursion rather than calling a height() function separately. Thanks to Amar for suggesting this optimized version. This optimization reduces time complexity to O(n).

C

```
/* program to check if a tree is height-balanced or not */
#include<stdio.h>
#include<stdlib.h>
#define bool int
/* A binary tree node has data, pointer to left child
   and a pointer to right child */
struct node
{
 int data;
  struct node* left;
  struct node* right;
};
/* The function returns true if root is balanced else false
  The second parameter is to store the height of tree.
  Initially, we need to pass a pointer to a location with value
  as 0. We can also write a wrapper over this function */
bool isBalanced(struct node *root, int* height)
  /* lh --> Height of left subtree
    rh --> Height of right subtree */
  int 1h = 0, rh = 0;
  /* l will be true if left subtree is balanced
   and r will be true if right subtree is balanced ^{*}/
  int 1 = 0, r = 0;
  if(root == NULL)
    *height = 0;
    return 1;
  /* Get the heights of left and right subtrees in lh and rh
   And store the returned values in 1 and r */
  l = isBalanced(root->left, &lh);
  r = isBalanced(root->right,&rh);
  /* Height of current node is max of heights of left and
     right subtrees plus 1*/
  *height = (lh > rh? lh: rh) + 1;
  /* If difference between heights of left and right
     subtrees is more than 2 then this node is not balanced
     so return 0 */
  if((1h - rh >= 2) || (rh - 1h >= 2))
    return 0;
```

```
/* If this node is balanced and left and right subtrees
    are balanced then return true */
  else return 1&&r;
}
/* UTILITY FUNCTIONS TO TEST isBalanced() FUNCTION */
/st Helper function that allocates a new node with the
   given data and NULL left and right pointers. */
struct node* newNode(int data)
   struct node* node = (struct node*)
                               malloc(sizeof(struct node));
   node->data = data;
    node->left = NULL;
   node->right = NULL;
    return(node);
}
int main()
 int height = 0;
  /* Constructed binary tree is
           1
      3
          5 6
  7
  */
  struct node *root = newNode(1);
  root->left = newNode(2);
  root->right = newNode(3);
  root->left->left = newNode(4);
  root->left->right = newNode(5);
  root->right->left = newNode(6);
  root->left->left->left = newNode(7);
  if(isBalanced(root, &height))
   printf("Tree is balanced");
  else
   printf("Tree is not balanced");
  getchar();
  return 0;
}
```

Java

```
/* Java program to determine if binary tree is
height balanced or not */

/* A binary tree node has data, pointer to left child,
    and a pointer to right child */
class Node {

    int data;
    Node left, right;

    Node(int d) {
        data = d;
        left = right = null;
    }
}
```

```
class Height {
    int height = 0;
    int lh = 0; // Height of left subtree
   int rh = 0; // Height of right subtree
}
class BinaryTree {
    Node root;
    /* Returns true if binary tree with root as root is height-balanced */
    boolean isBalanced(Node root, Height height)
        /* 1 will be true if left subtree is balanced
          and r will be true if right subtree is balanced */
        boolean 1 = false, r = false;
        /st If tree is empty then return true st/
       if (root == null) {
            height.height = 0;
            return true;
        /* Get the height of left and right sub trees */
       1 = isBalanced(root.left, height);
        r = isBalanced(root.right, height);
        /* If difference between heights of left and right
           subtrees is more than 2 then this node is not balanced
           so return 0 */
        if ((height.lh - height.rh >= 2) || (height.rh - height.lh >= 2))
            return false;
        /* If this node is balanced and left and right subtrees
           are balanced then return true */
            return 1 && r;
   }
    /* UTILITY FUNCTIONS TO TEST isBalanced() FUNCTION */
    /* returns maximum of two integers */
   int max(int a, int b)
   {
        return (a >= b) ? a : b;
   }
    /* The function Compute the "height" of a tree. Height is the
        number of nodes along the longest path from the root node
        down to the farthest leaf node.*/
   int height(Node node)
        /* base case tree is empty */
       if (node == null)
            return 0;
        /* If tree is not empty then height = 1 + max of left
        height and right heights */
        return 1 + max(height(node.left), height(node.right));
   }
    public static void main(String args[])
       Height height = new Height();
        /* Constructed binary tree is
                 1
               2
                     3
              / \
                 5 6
```

```
*/
       BinaryTree tree = new BinaryTree();
       tree.root = new Node(1);
        tree.root.left = new Node(2);
        tree.root.right = new Node(3);
       tree.root.left.left = new Node(4);
       tree.root.left.right = new Node(5);
       tree.root.right.right = new Node(6);
       tree.root.left.left.left = new Node(7);
       if (tree.isBalanced(tree.root, height))
           System.out.println("Tree is balanced");
        else
           System.out.println("Tree is not balanced");
   }
}
// This code has been contributed by Mayank Jaiswal(mayank_24)
```

Time Complexity: O(n)