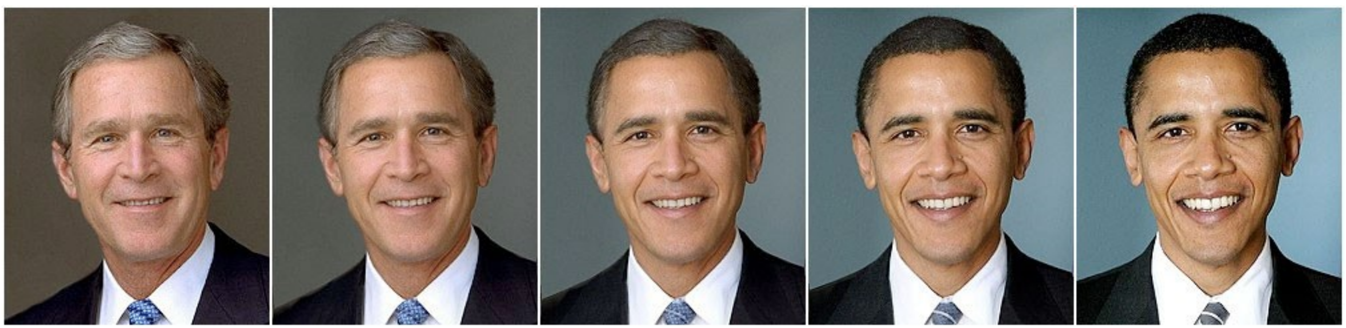
Advanced Topics In Computer Vision And Deep Learning

L4. Digital Image Warping



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Learning Outcomes

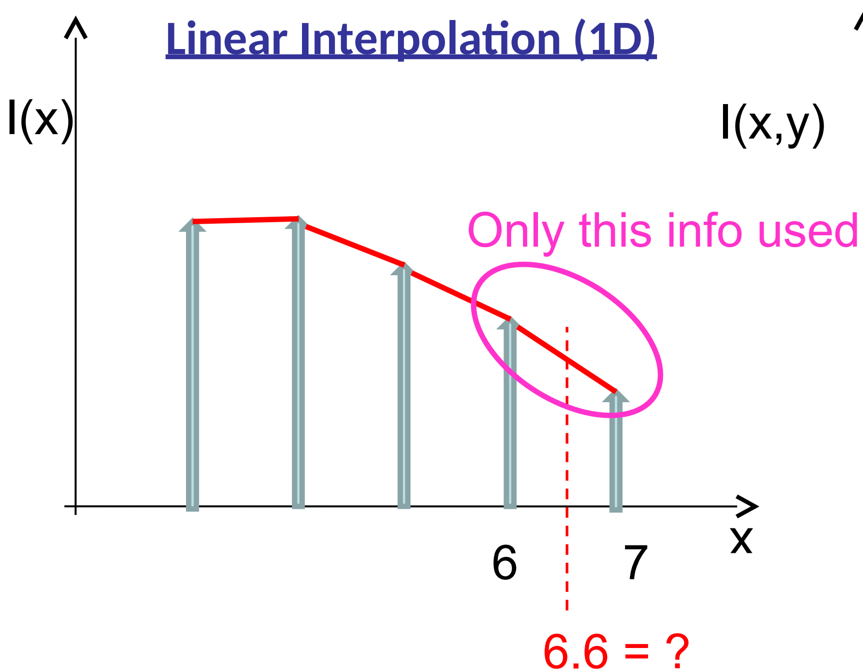
After attending this lecture, and doing the reading you should be able to:

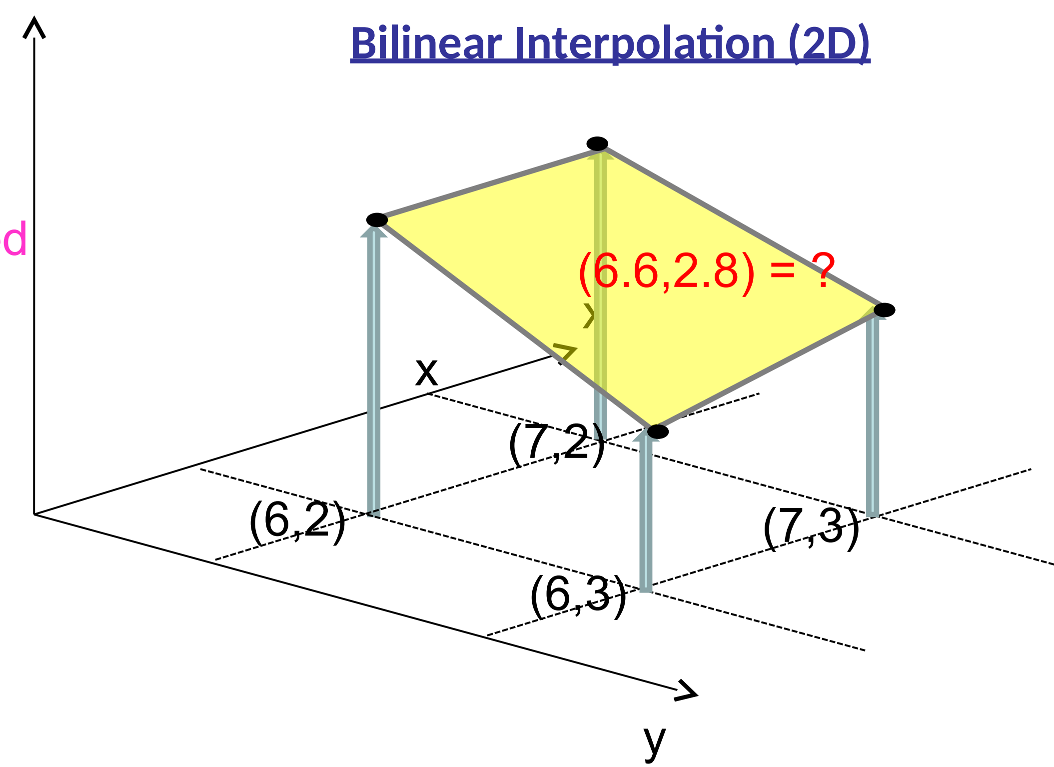
- Describe and apply high quality image interpolation techniques for digital image warping e.g. bi-cubic and Gaussian resampling
- Compare and contrast different image interpolation techniques
- Implement techniques to reduce aliasing within warped images
- Describe the origins of aliasing grounding an explanation in sampling theory (Nyquist limit)
- Describe Fant's algorithm and outline its utility
- Outline the steps necessary to create an image morphing program

Bi-linear interpolation

Bi-linear interpolation uses only immediate neighbours (i.e. closest the pixel either "side")

- ✓ Could we use more information?



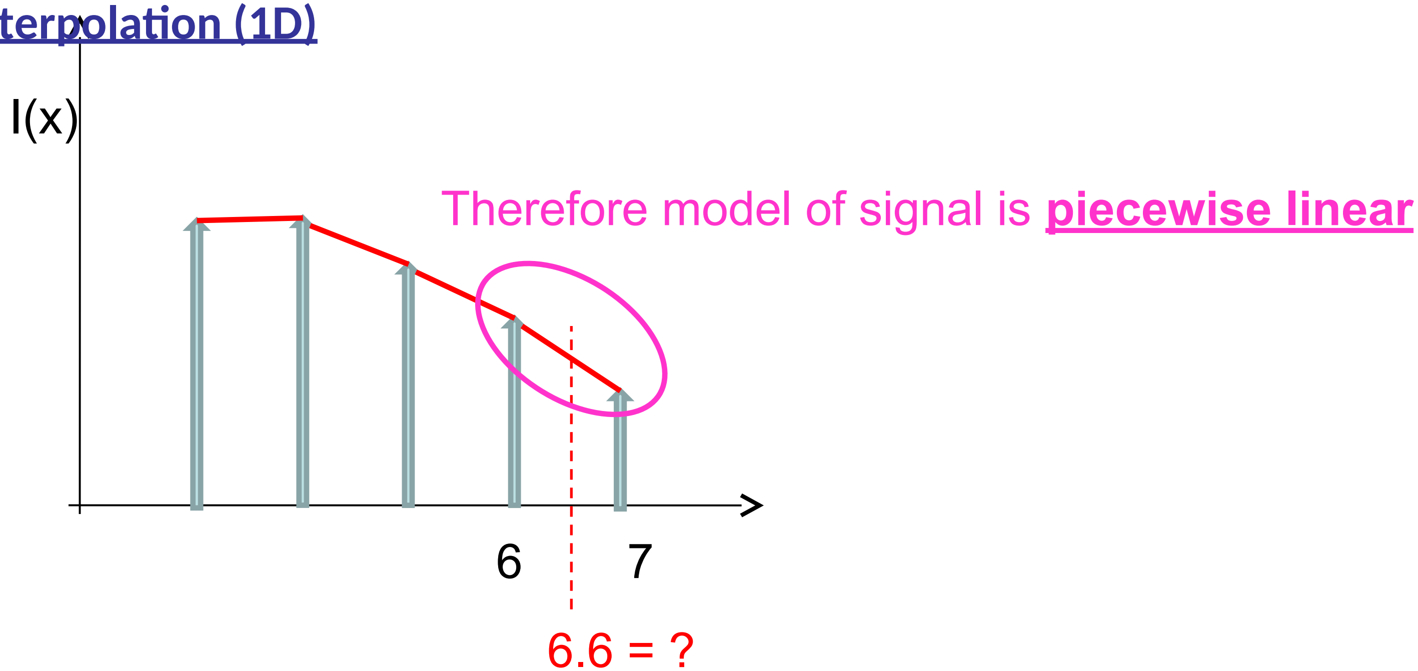


Resampling under a model

When we interpolate, we are “guessing” missing data by **resampling** the known data (signal)

We make a smoothness assumption according to some **model**

Linear Interpolation (1D)

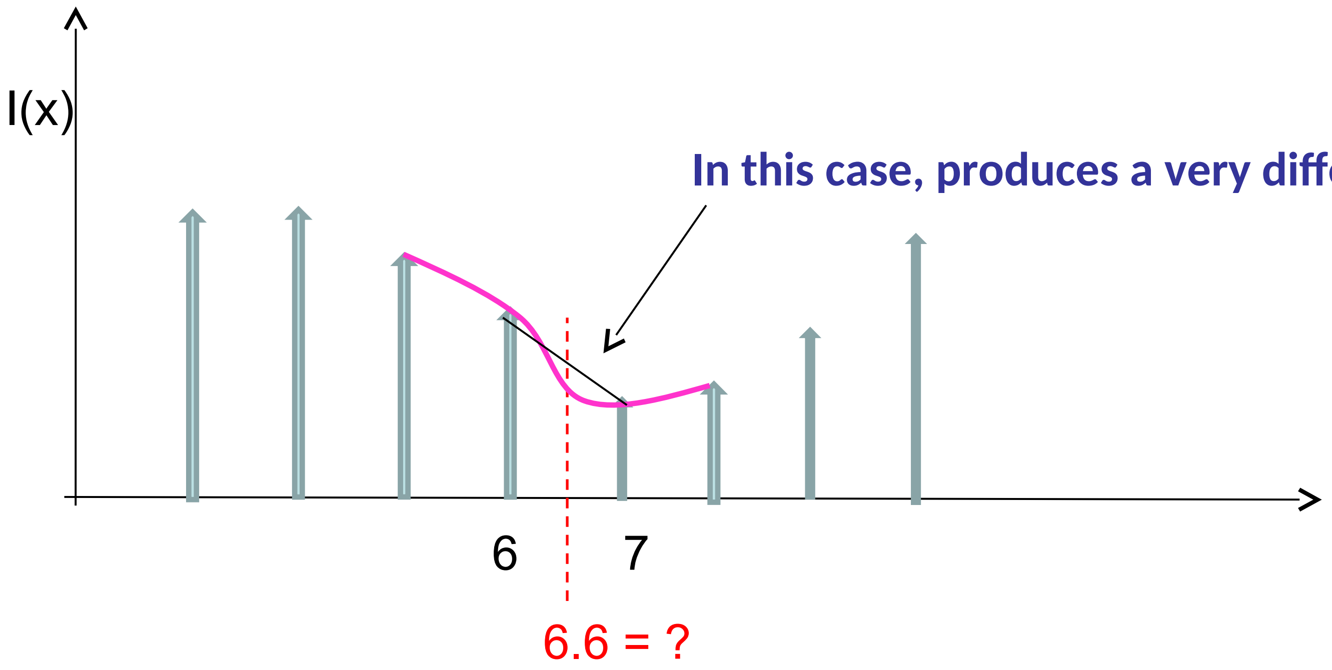


A line requires 2 points to define it (i.e. a pixel either side of point being interpolated)

Bi-cubic interpolation

Consider a 1D signal – we could **model** the signal as a **curve** (e.g. a cubic curve) rather than in piecewise linear fashion.

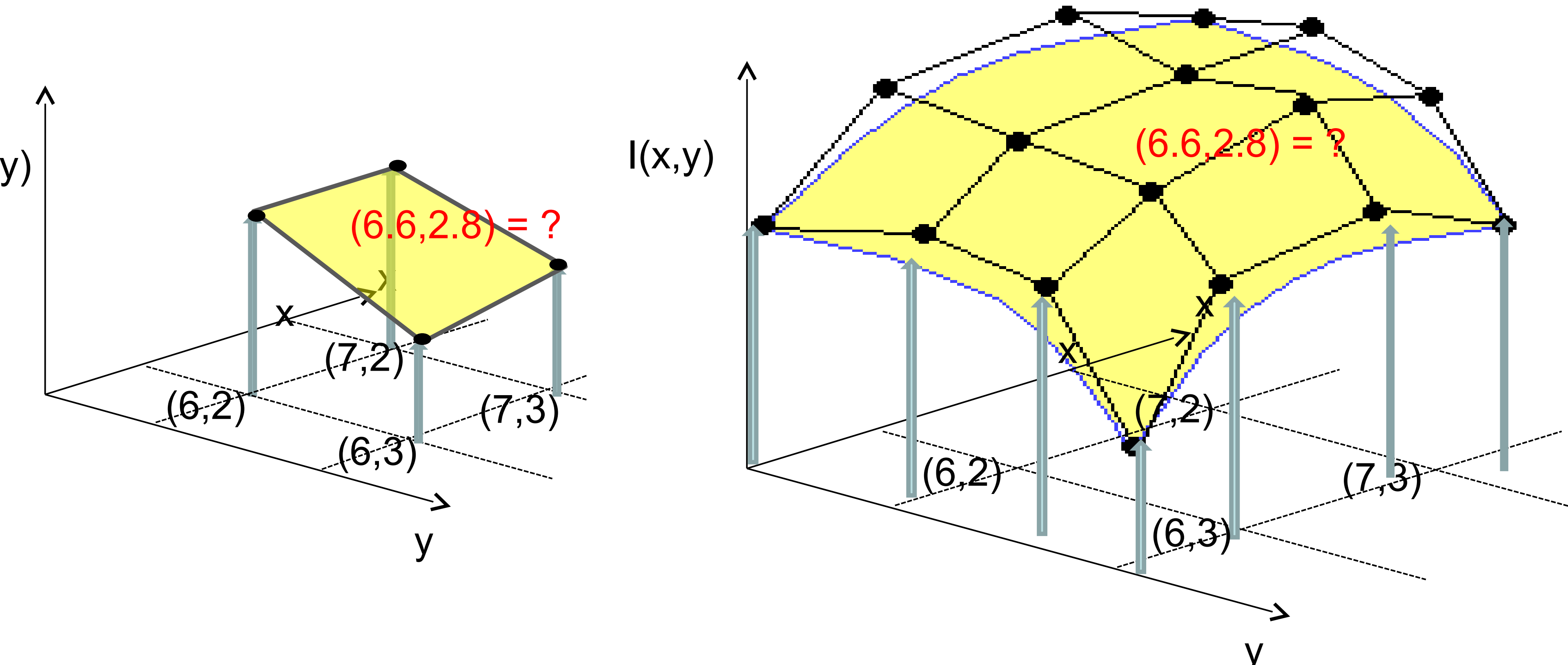
A cubic curve uses the closest 2 neighbours each “side” of the point



<u>**A cubic curve requires 4 points to define it i.e. 2 pixels either side of interpolated point**</u>

Bi-cubic interpolation

In 2D this implies fitting piecewise cubic surface patches to the signal, rather than piecewise planar approximations.



Bilinear Interpolation

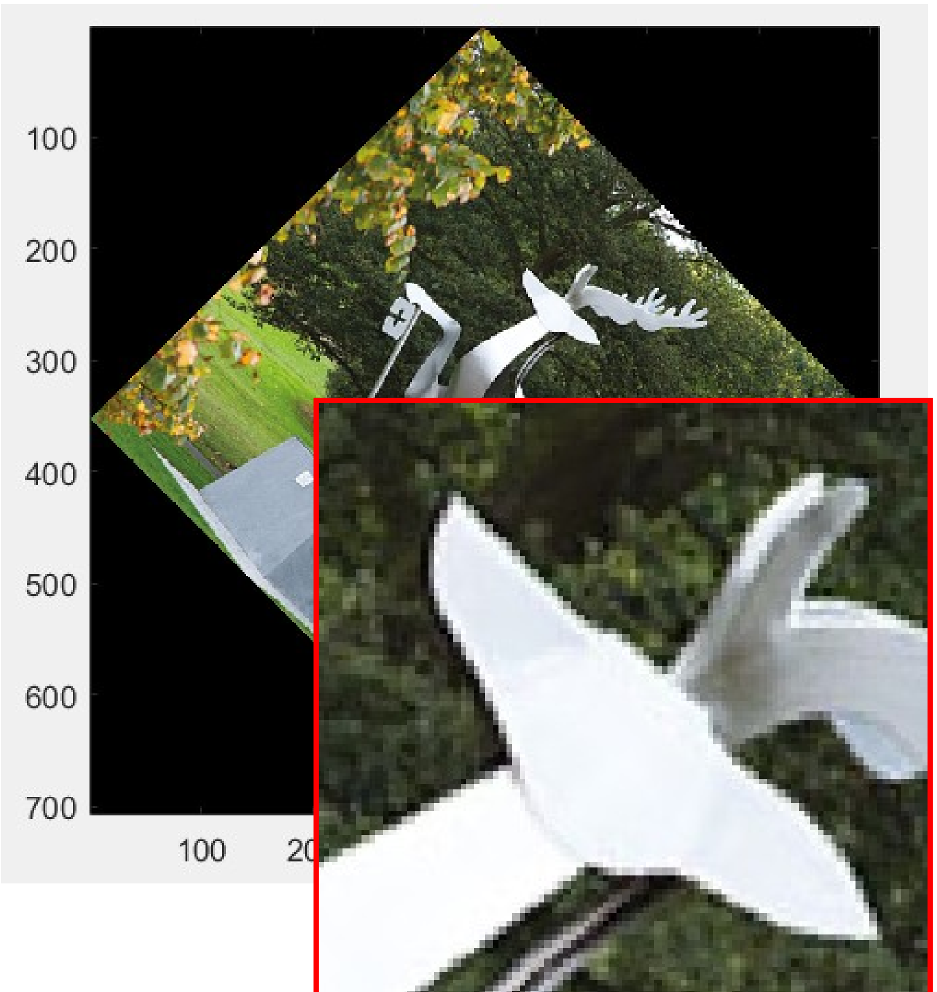
Bicubic Interpolation

Fits surface (plane) using 4 points

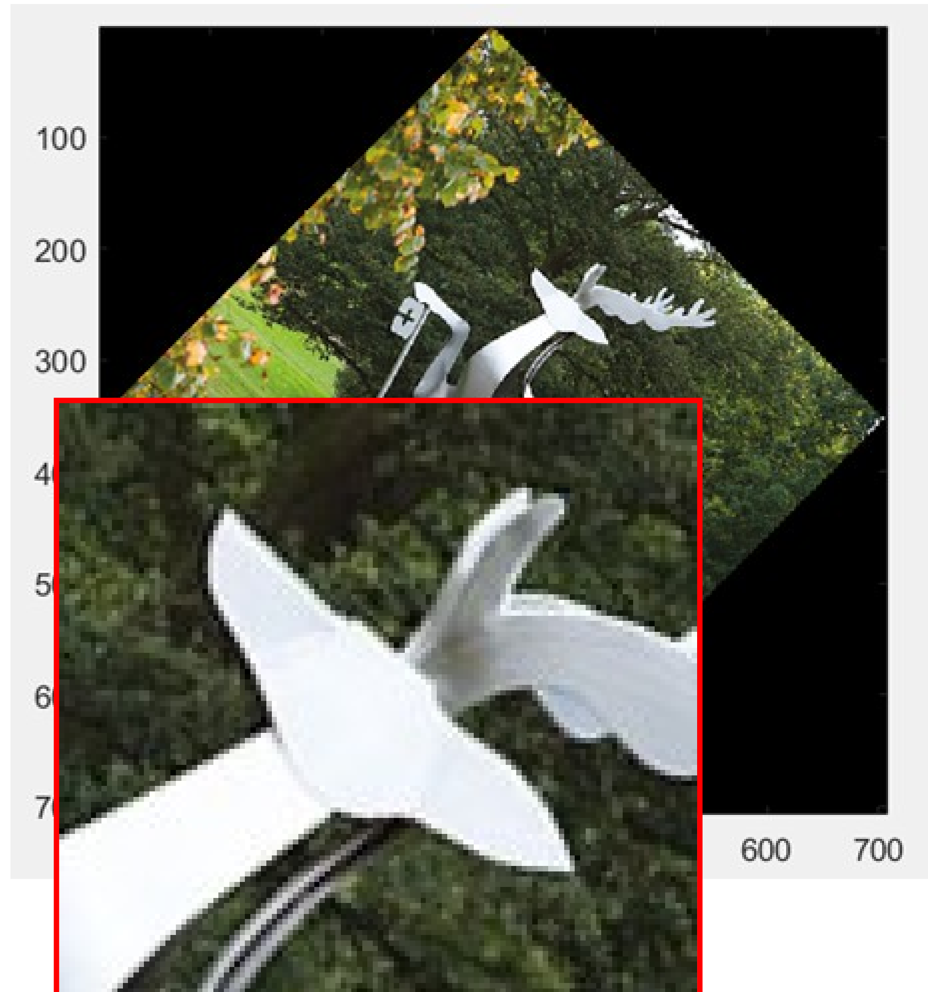
Fits surface (bicubic patch) using 16 points

Bi-cubic interpolation

Bi-cubic interpolation **typically** produces superior results vs. NN / BL



Bilinear Interpolation

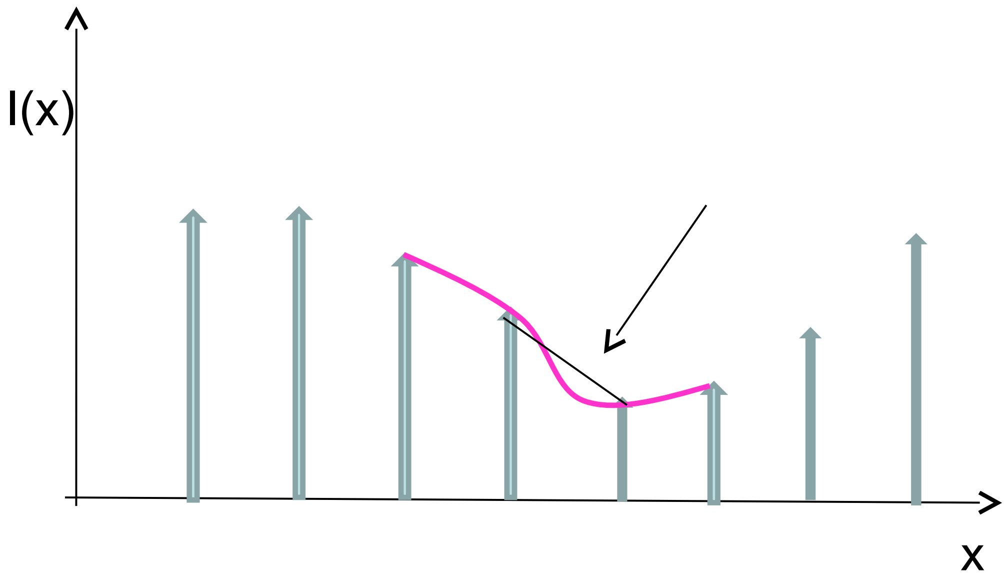


Bicubic Interpolation

Over to you

When do you think it is undesirable to use bi-cubic interpolation?

Think about the shape of a 1D signal (e.g. pixel row) near edges



A. Bicubic interpolation over-smooths step edges. It would blur very sharp edges and so images containing many of these would degrade.

Problems with Bi-cubic

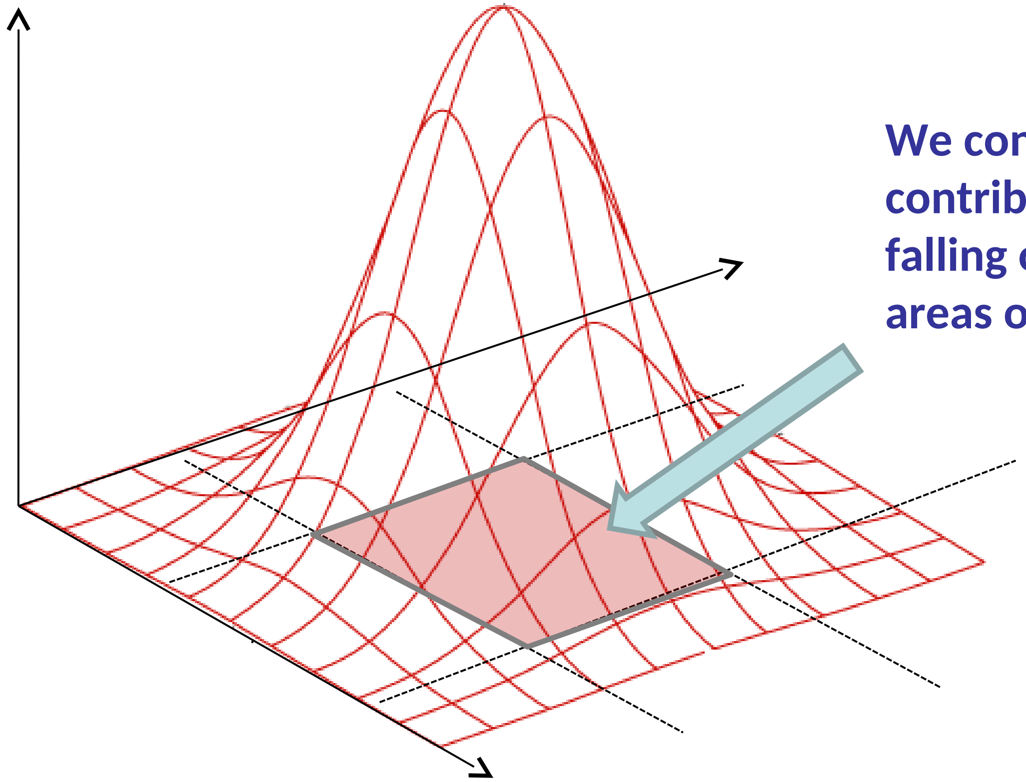
Some modern hardware e.g. HD Televisions upsample i.e. (warps via a scaling transform) legacy signals from low resolution hardware.

Original **Bicubic up-sample**

Gaussian interpolation

Pixels are digitally represented as **single points** of light…

…but they encode **real areas of light** e.g. average colour of light falling on a photosensor in a camera



We consider this pixel (red) to be contributed to by light falling on neighbouring pixels / areas of the sensor

We can model this area relationship as a 2D Gaussian (bell curve)

2D Gaussian Distribution

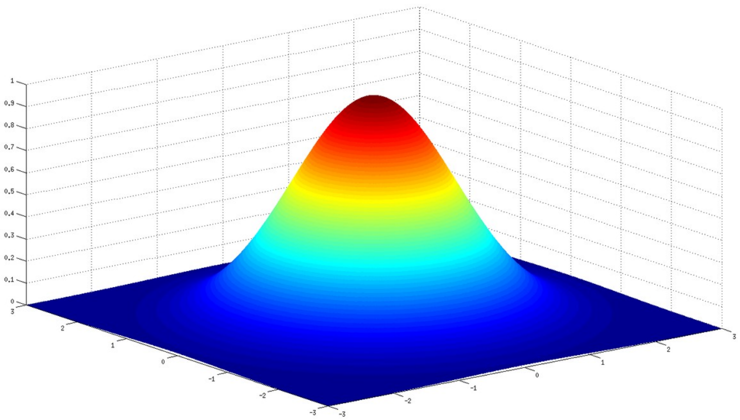
The 2D generalisation of a Gaussian distribution is

$$f(x,y) = A\exp\left(-\left(\frac{(x-x_o)^2}{2\sigma_x^2} + \frac{(y-y_o)^2}{2\sigma_y^2}\right)\right).$$

(x,y) – The point at which we wish to evaluate 2D Gaussian

(x_0,y_0) – The mean i.e. point at which Gaussian is centered

(σ_x,σ_y) – The spread i.e. standard deviation in x and y directions

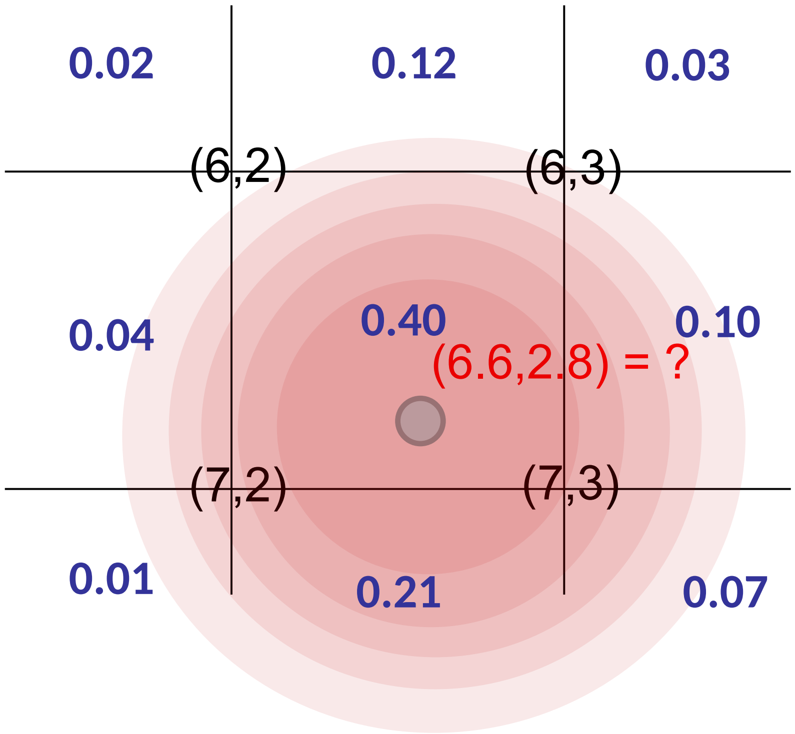


Gaussian is infinite in extent (but tails off quickly)

Area under Gaussian sums to 1.

Gaussian interpolation

- ✓ Centre a Gaussian on the point to be interpolated
- ✓ Sum up area of influence under Gaussian for each pixel
- ✓ Interpolated colour is a weighted sum of neighbouring pixels

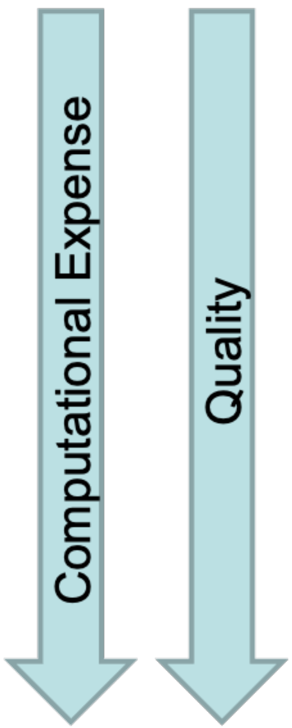


Gaussian influence

Source Image

Summary of interpolation

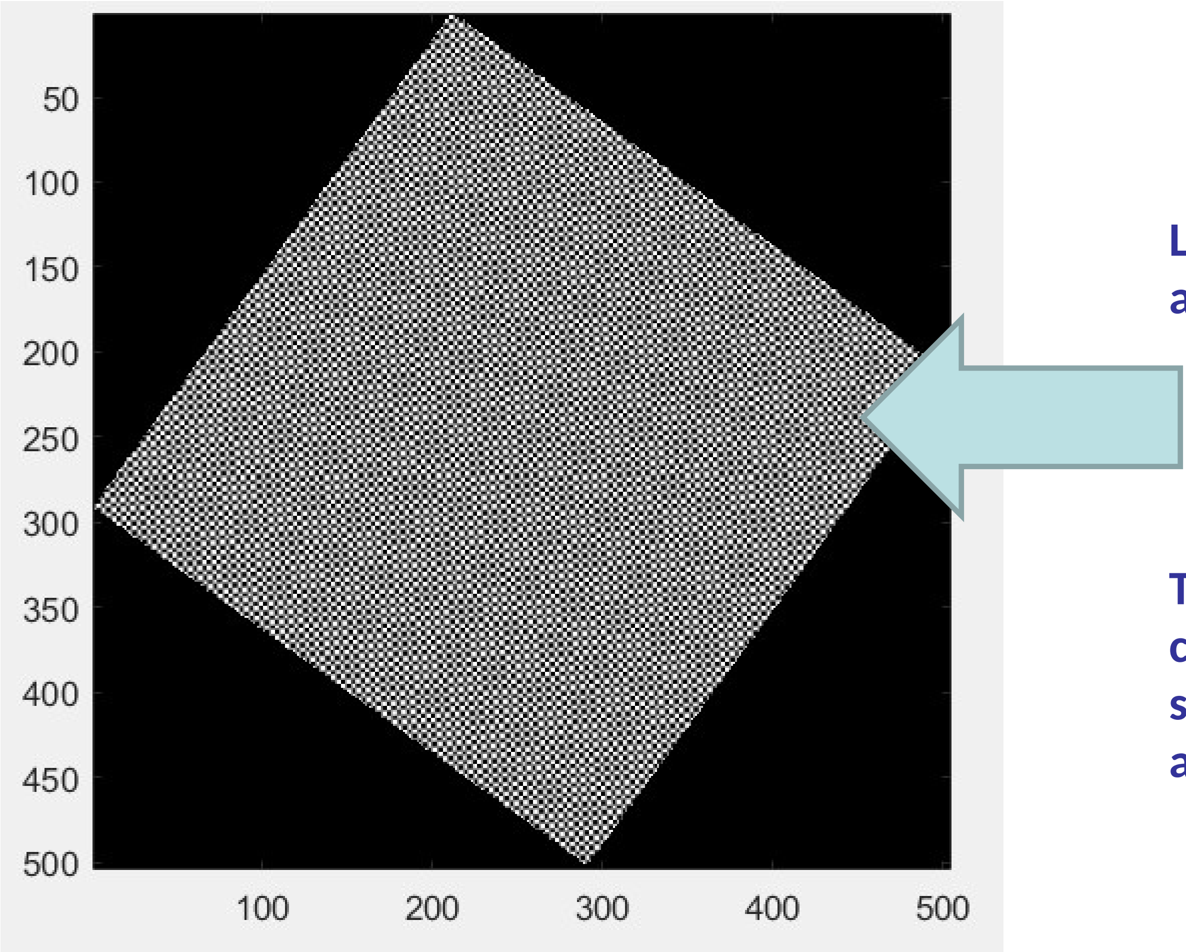
We have looked at 4 types of image interpolation:



	Pixel Neighbourhood
Nearest Neighbour (not really interpolation)	**1 (nearest) pixel**
Bi-linear interpolation	**4 pixels**
Bi-cubic interpolation	**16 pixels**
Gaussian interpolation	**Many pixels (depends on σ)**

Aliasing Artifacts

We will often see wave-like artifacts in images when warping, especially when rotating or reducing detail e.g. a scale reduction



Low frequency waves appear in a high frequency signal

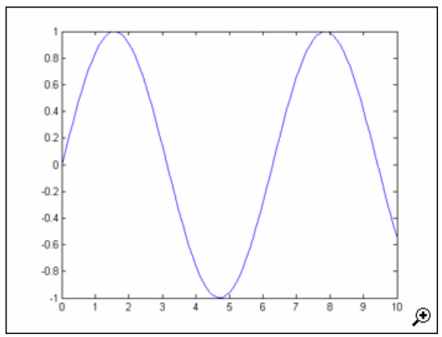
This is a 1200x1200 chequerboard with squares of size 2 pixels scaled by factor 0.3 and rotated 110 degrees

Fourier's Theorem

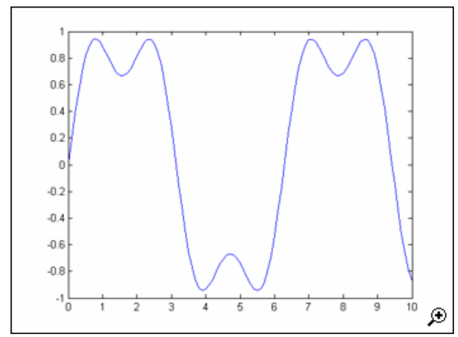
All signals can be considered to be made up of a (possibly infinite) sum of sine and cosine waves, all different amplitudes and phases.

This is "Fourier's Theorem"

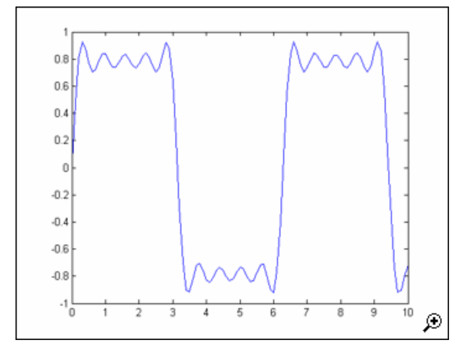
For example, a square wave such as across the chequer pattern.



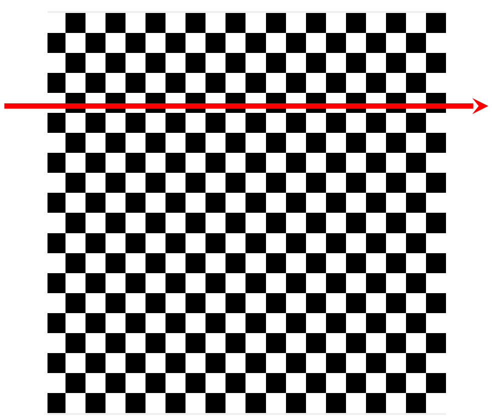
y = sin(t);

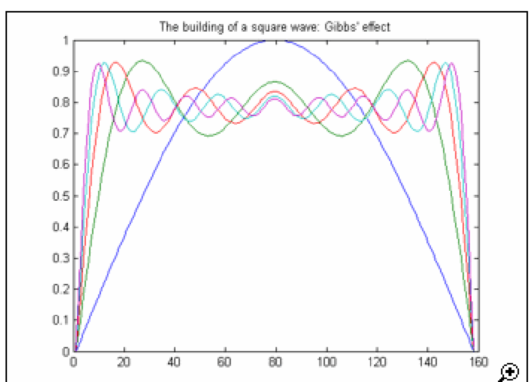


y = sin(t) + sin(3*t)/3;



y = sin(t) + sin(3*t)/3 + sin(5*t)/5 + sin(7*t)/7 + sin(9*t)/9;





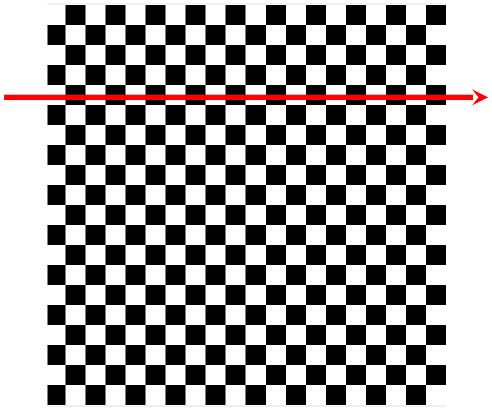
Images are 2D signals

Nyquist Limit

The Nyquist Limit states that in order to sample all the frequencies in a signal correctly, **you must sample with at least double the sampling rate of the highest frequency in that signal.**

If you don't, you will observe spurious low frequencies "added in" to your sampled signal… these are the higher frequencies masquerading ("aliasing") as lower frequencies.

This is why music CDs / MP3s are sampled at 44Khz – at least twice the max frequency an human can hear (20-22Khz)



This is a 200x200 chequerboard with 20 squares per row = 10 cycles per row.

To resample this signal without aliasing we need to sample it at 20 samples per pixel row i.e. once every 10 pixels.

Anti-Aliasing

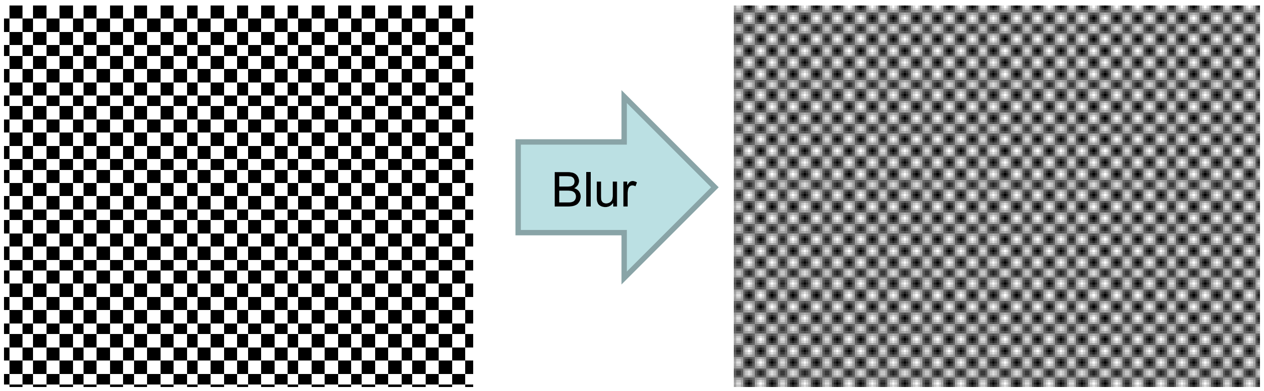
When warping we are **re-sampling images**, so must obey **Nyquist limit**

Often, the warp transform will end up sampling the source image at a lower rate than Nyquist – and aliasing will occur.

So, **we must filter out high frequencies from the source image** prior to warping (pre-filtering)

This is done by blurring the image, which removes high frequencies

Warping then proceeds as normal



Anti-Aliasing

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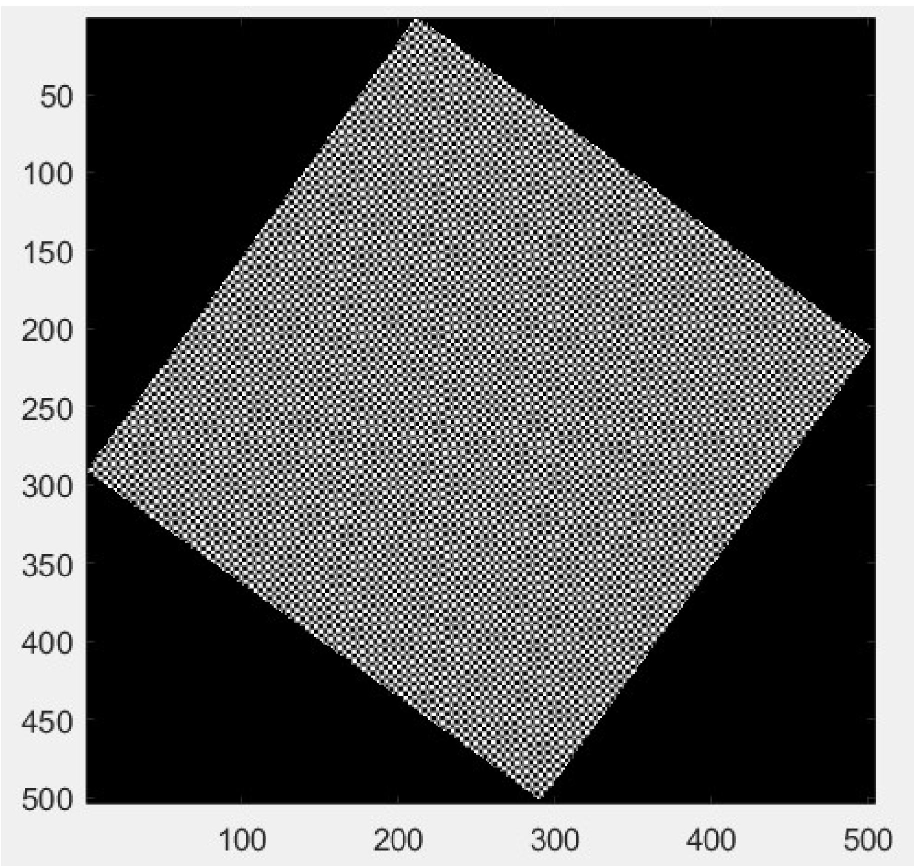
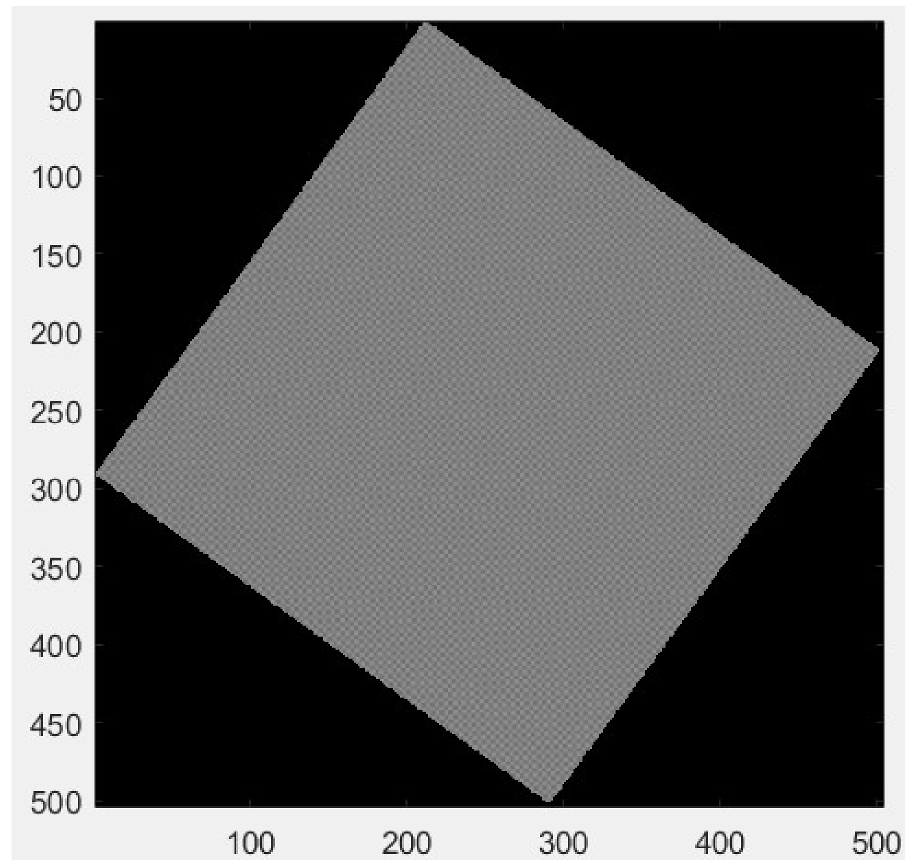


Without pre-filtering



With pre-filtering

Non-Invertable F

Recall: Warping is digital transformation of a **source** image into a **target** image, under some **mathematical function** $F : \Re^2 \rightarrow \Re^2$

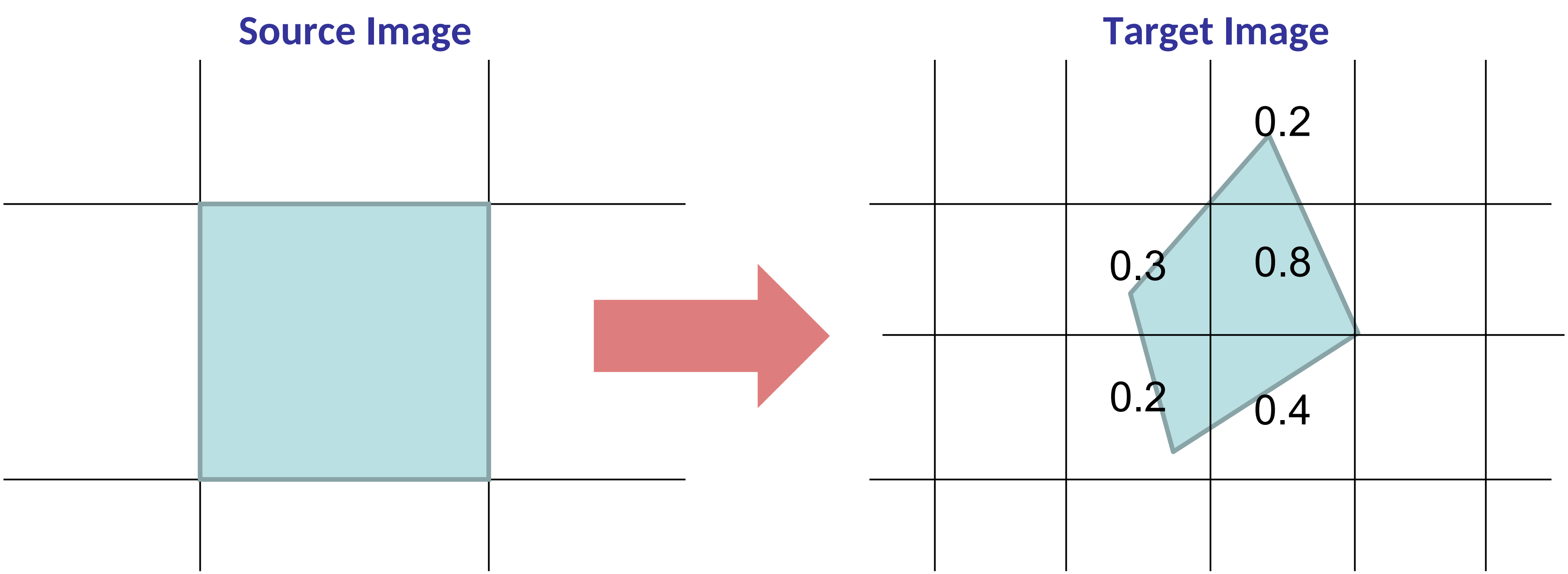
Sometimes we may want to warp with a **F** that is **not invertable**

This means we will need to use forward mapping, and deal with holes

Non-Invertable F

Naive approaches to high quality forward mapping are expensive.

Each pixel treated as a quadrilateral and its corners warped to target



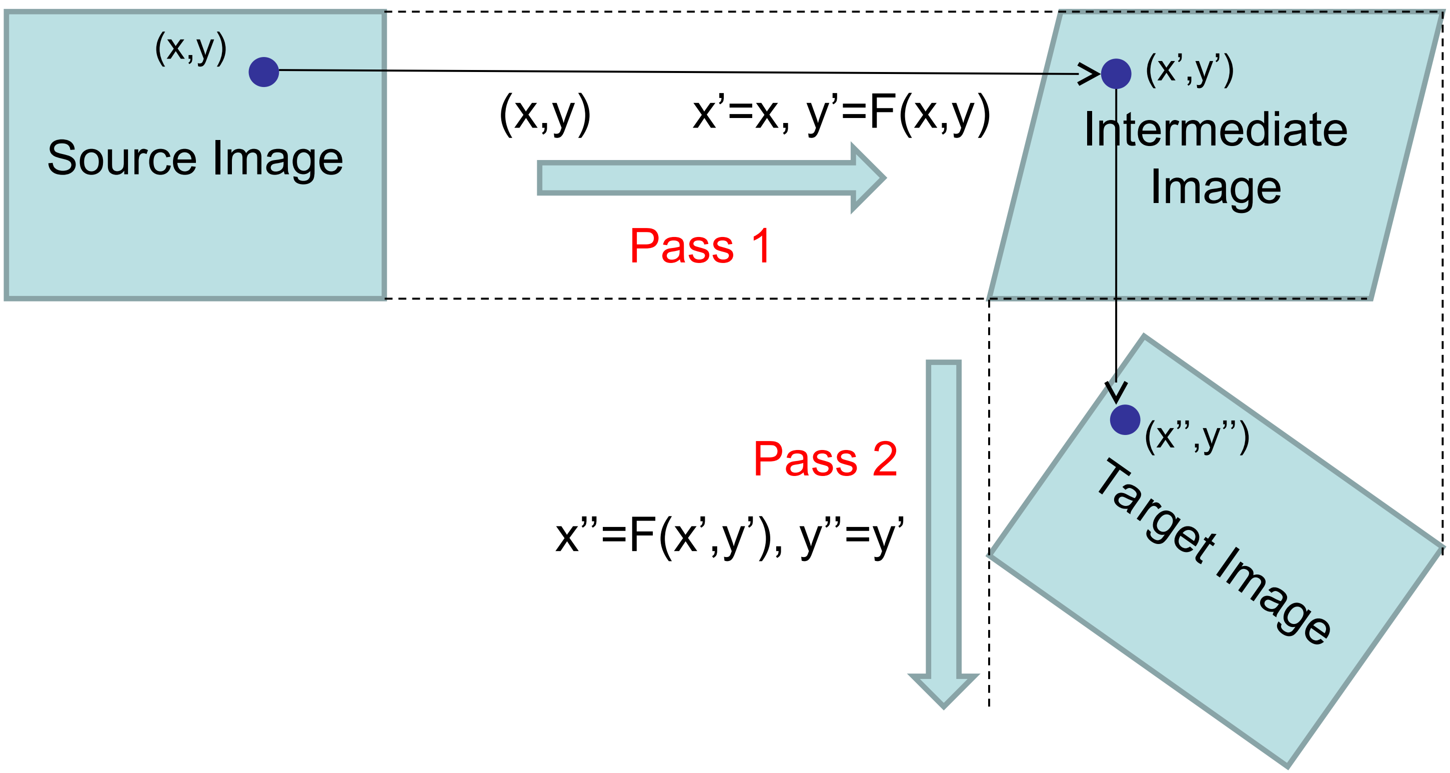
For each pixel covered by the quadrilateral, the area of overlap is computed. A proportionate amount of source colour is added to pixel.

Fants Algorithm (2-pass warping)

Fant developed a **2-pass approach** to efficiently perform forward warping, whilst work at ILM on The Abyss (c. 1990s).

An **intermediate image** is produced by **warping only the x dimension**

Then **intermediate image is warped** in the y dimension to yield target



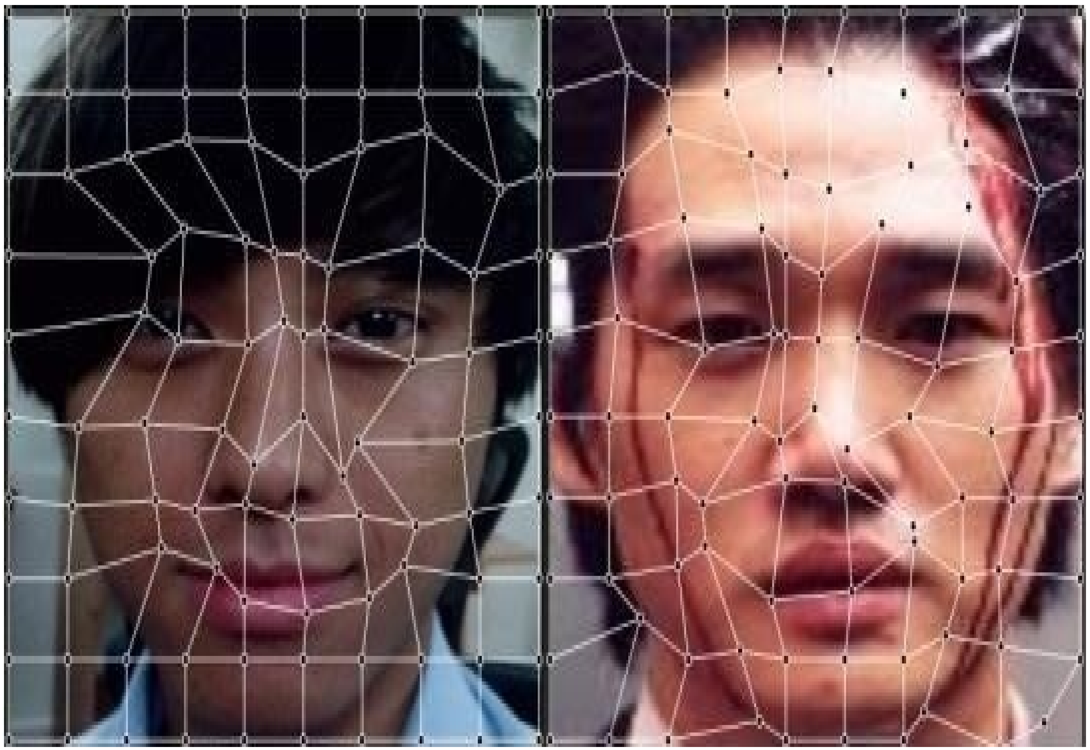
Fants Algorithm (2-pass warping)

Many non-invertable warps possible!

Application: Facial Morphing

Morphing is the warping of one image into another, visualising this process as a gradual change (in-betweening)



Correspondences are established (manually) between two images

A mesh with consistent topology is used

From the differences in mesh vertex positions, a per-pixel (dense) vector field can be interpolated. This vector field defines **F**.

Fants algorithm is applied. Pixel colours are also blended to enable change of skin tone / background / clothing.

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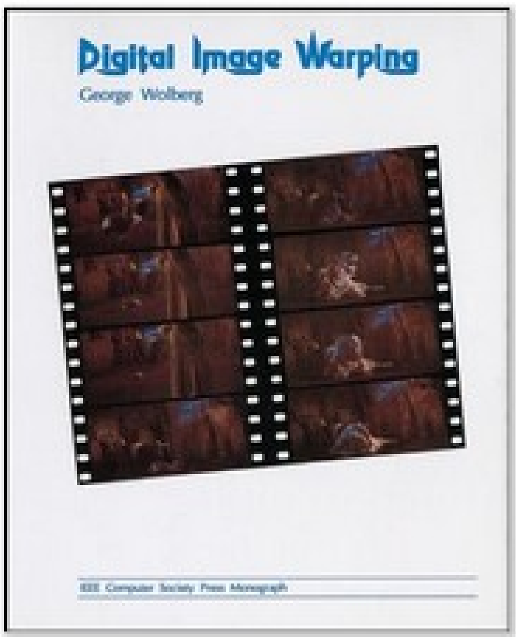
Further Reading

George Wolberg's "Digital Image Warping" (optional)



Digital Image Warping

George Wolberg

ISBN: 978-0-8186-8944-4

344 pages
July 1990, Wiley-IEEE Computer Society Press