

RESOLUTION

Introduction:

Resolution in First-Order Logic:

Resolution is a method for proving the validity of a first-order logic statement. Given a set of clauses in CNF, resolution involves applying the resolution rule iteratively until a resolution is reached or it is determined that the set of clauses is unsatisfiable.

The resolution rule states that if there are two clauses that contain complementary literals (one positive and the other negated), then a new clause can be generated by resolving (combining) the two clauses and eliminating the complementary literals.

Resolution Rule:

$$\frac{P \vee Q, \neg Q \vee R}{P \vee R}$$

Here's a step-by-step outline of the resolution process:

1. Convert the statements to CNF:

- Express the logical statements in CNF (clause form).

2. Apply Resolution:

- Identify pairs of clauses that contain complementary literals.
- Resolve the pairs to generate new clauses.
- Continue until a resolution is reached or it is determined that the set of clauses is unsatisfiable.

3. Repeat:

- Iterate the resolution process until a resolution is reached or no further resolutions can be applied.

Example:

Consider the following statements in CNF:

1. $(P \vee Q)$
2. $(\neg P \vee R)$
3. $(\neg Q \vee S)$
4. $(\neg R \vee \neg S)$

Applying resolution:

$$\frac{P \vee Q, \neg Q \vee R}{P \vee R}$$

$$\frac{Q \vee R, \neg Q \vee S}{R \vee S}$$

$$\frac{R \vee S, \neg R \vee \neg S}{(R \vee S) \vee (\neg R \vee \neg S)}$$

The result is a resolved clause, and if it simplifies to an empty clause (\blacksquare), then the original set of clauses is unsatisfiable.

Resolution is a fundamental technique in automated theorem proving and is used in various areas, including knowledge representation and reasoning in artificial intelligence.