PROPOSITIONAL LOGIC

Introduction:

Propositional logic, also known as sentential logic and statement logic, is a branch of classical logic that deals with propositions, which are statements that are either true or false. Propositional logic does not concern itself with the internal structure or meaning of statements but focuses on their logical relationships and combinations. It forms the basis for more complex logics and is widely used in computer science, mathematics, and philosophy.

Key Concepts in Propositional Logic:

1. Propositions:

- Definition: Statements that are either true or false.
- Examples: "The sky is blue," "2 + 2 = 5," "It is raining."

2. Logical Connectives:

- Definition: Symbols or words used to combine propositions to form more complex statements.
- Examples:
  - Conjunction (∧): Represents "and." e.g., P ∧ Q (P and Q).
  - Disjunction (∨): Represents "or." e.g., P ∨ Q (P or Q).
  - Negation (¬): Represents "not." e.g., ¬P (not P).
  - Implication (→): Represents "implies." e.g., P → Q (if P, then Q).
  - Biconditional (↔): Represents "if and only if." e.g., P ↔ Q (P if and only if Q).

3. Truth Tables:

- Definition: Tables that show the truth values of compound propositions for all possible combinations of truth values of their component propositions.

- Example Truth Table for Conjunction (P ∧ Q):

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

4. Logical Equivalence:

- Definition: Two propositions are logically equivalent if they have the same truth values for all possible combinations of truth values of their component propositions.

- Example: $P\wedge (Q \vee R)$ is logically equivalent to $(P \wedge Q) \vee (P \wedge R)$.

5. Tautology and Contradiction:

- Tautology: A proposition that is always true, regardless of the truth values of its component propositions.

- Contradiction: A proposition that is always false, regardless of the truth values of its component propositions.

- Example: $P \vee \neg P$ is a tautology, and $P \wedge P$ is a contradiction.

Logical Inference:

Logical inference in propositional logic involves drawing conclusions from a set of premises using valid inference rules. Common inference rules include modus ponens, modus tollens, and the law of syllogism.

<u>Example Inference Rule: Modus Ponens:</u>

If $(P \rightarrow Q)$ is true and P is true, then Q must be true.

Example:

-$P \rightarrow Q$

- P

- Therefore, Q

Applications of Propositional Logic:

1. Circuit Design:

- Representing and analysing digital circuits.

2. Programming:

- Boolean conditions in programming languages.

3. Artificial Intelligence:

   - Representing knowledge and reasoning in expert systems.

4. Mathematics:

   - Logical foundations of mathematical reasoning.

5. Philosophy:

   - Analysing and representing logical relationships in philosophical arguments.

Propositional logic provides a foundation for reasoning about truth and falsehood in a structured and systematic way. It forms the basis for more complex logical systems, such as predicate logic and modal logic, which extend its expressive power to handle more intricate aspects of reasoning.