FIRST ORDER LOGIC, INFERENCE IN FIRST ORDER LOGIC

Introduction:

First-Order Logic (FOL):

First-Order Logic, also known as Predicate Logic or First-Order Predicate Calculus, extends propositional logic by introducing the concept of variables, quantifiers, and predicates.

- Variables: Represent objects or elements whose identity is not specified.
- Quantifiers: Specify the scope of variables.
  - Existential Quantifier (∃): "There exists," e.g., ∃x P(x) ("There exists an x such that P(x)").
  - Universal Quantifier (∀): "For all" or "For every," e.g., ∀x P(x) ("For every x, P(x)").

- Predicates: Express relationships between objects.
  - (P(x)): "x has property P."
  - (Q(x, y)): "x is related to y by Q."

- Logical Connectives: Include conjunction (∧), disjunction (∨), negation (¬), implication (→), and biconditional (↔).

Example Statements in FOL:

1. ∀ x (P(x) →Q(x)): "For every x, if x has property P, then x has property Q."
2. ∃ y (Q(y) ∧ R(y, z))): "There exists a y such that y has property Q and is related to z by R."

Inference in First-Order Logic:

Inference in FOL involves deriving new information from existing knowledge represented in the form of logical statements. Common inference rules include:

1. Modus Ponens:
   - If (P → Q) is true, and (P) is true, then (Q) is true.

$$\frac{P \to Q,\ P}{Q}$$

2. Universal Instantiation:

- If a statement is universally quantified, you can substitute any specific value for the variable.

$$\frac{\forall \mathrm{x}\ \mathrm{P}(\mathrm{x})}{P(a)}$$

3. Existential Instantiation:

- If a statement is existentially quantified, you can introduce a new constant or variable.

$$\frac{\forall \mathrm{x}\ \mathrm{P}(\mathrm{x})}{p(a)}$$

4. Universal Generalization:

- If a statement holds for a specific object, it holds universally.

$$\frac{P(a)}{\forall \mathrm{x}\ \mathrm{P}(\mathrm{x})}$$

5. Existential Generalization:

- If a statement holds for a specific object, there exists an object for which the statement holds.

$$\frac{P(a)}{\exists \mathrm{x}\ \mathrm{P}(\mathrm{x})}$$

Example:

Consider the following statements:

1. ∀x Human(x) → Mortal(x))

2. Human (Socrates)

Using universal instantiation and modus ponens, we can infer (Mortal (Socrates):

$$\frac{Human(socrates)}{\forall \mathrm{x}(\mathrm{Human}(\mathrm{x}) \rightarrow \mathrm{Mortal}(\mathrm{x})}$$

These rules allow us to draw logical conclusions and extend our knowledge within the framework of First-Order Logic. Automated reasoning systems and theorem provers often use these rules to perform logical inference.