Math Document Template

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***Abstract*—This a simple document explaining a question about the concept of similar triangles.**

Download all python codes from

```
svn co https://github.com/kamalrajnegi/internship/
    trunk/geometry/codes
```

and latex-tikz codes from

```
svn co https://github.com/kamalrajnegi/trunk/
    geometry/figs
```

1 Problem

$ABCE$ is a quadrilateral and D is a point on BC such that, $AC = AE, AB = AD$ and $\angle BAD = \angle EAC$. Show that $BC = DE$

2 Construction

2.1. The figure for A quadrilateral obtained in the question looks like Fig. 2.1. with side a, c, e and d.

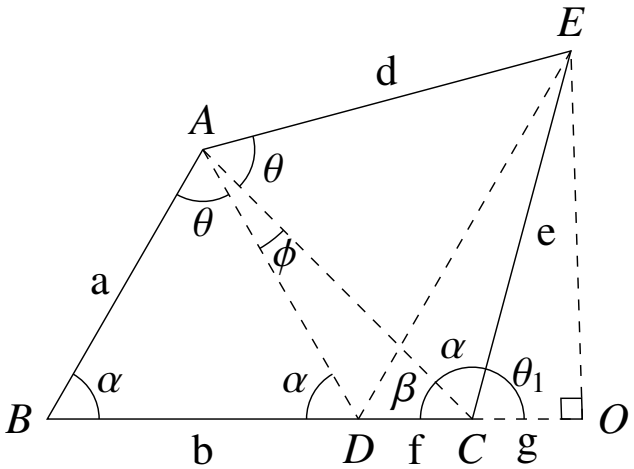

Fig. 2.1: Quadrilateral ABCE by Latex-Tikz

2.2. List the design parameters for construction
Solution: See Table. 2.2.

Parameter	Value
a	3
$\angle BAD$	60°

TABLE 2.2: To construct quadrilaterl $ABCE$

2.3. Find the coordinates of the various points in Fig. 2.1
Solution: From the given information,

$$\mathbf{A} = \begin{pmatrix} p \\ q \end{pmatrix} \tag{2.3.1}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{2.3.2}$$

$$\mathbf{C} = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{2.3.3}$$

$$\mathbf{D} = \begin{pmatrix} b \\ 0 \end{pmatrix} \tag{2.3.4}$$

$$\mathbf{E} = \begin{pmatrix} r \\ s \end{pmatrix} \tag{2.3.5}$$

For Vertices A

$$AB = \|A - B\|^2 = \|A\|^2 = a^2 \tag{2.3.6}$$

$$DB = \|D - B\|^2 = \|D\|^2 = b^2 \tag{2.3.7}$$

$$AD = \|A - D\|^2 = a^2 \tag{2.3.8}$$

Solving equation 2.3.8
$a^2 = A^T A + D^T D - A^T D - D^T A$
$a^2 = \|A\|^2 + \|D\|^2 - 2A^T D$
$a^2 = a^2 + b^2 - 2pb$

$$p = \frac{b^2 + a^2 - a^2}{2b} = \frac{b^2}{2b} = \frac{b}{2} \tag{2.3.9}$$

From equation 2.3.6
$\|A\| = a^2 = p^2 + q^2$

$$q = \pm\sqrt{a^2 - p^2} \tag{2.3.10}$$

For Vertices E

If we draw perpendicular line from E, it meets on x-axis at point O.

Consider $OC = g$ Then,

$$cos\theta_1 = \frac{OC}{EC} = \frac{g}{e} \tag{2.3.11}$$

$$g = ecos\theta_1 \tag{2.3.12}$$

$$tan\theta_1 = \frac{OE}{OC} = \frac{h}{g} \tag{2.3.13}$$

$$h = gtan\theta_1 \tag{2.3.14}$$

$$r = c + g \tag{2.3.15}$$

$$s = h \tag{2.3.16}$$

The values are listed in Table. 2.3

Derived Values.	
A	$\begin{pmatrix}1.5\\2.6\end{pmatrix}$
C	$\begin{pmatrix}4.1\\0\end{pmatrix}$
E	$\begin{pmatrix}5.05\\3.55\end{pmatrix}$

TABLE 2.3: To construct $ABCE$

2.4. Draw Fig. 2.1.

Solution: The following Python code generates Fig. 2.1

codes/quadrilateral.py

and the equivalent latex-tikz code generating Fig. 2.4 is

figs/quadrilateral.tex

The above latex code can be compiled as a standalone document as

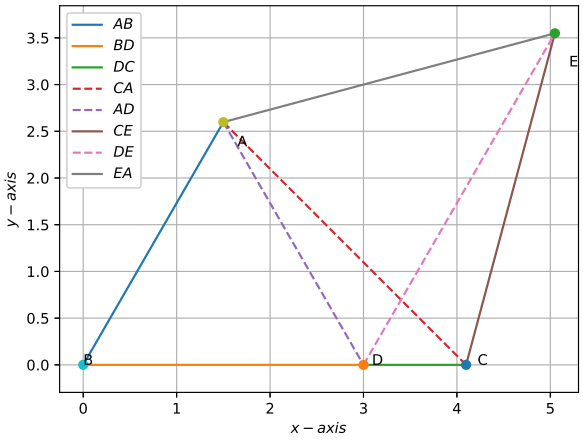

Fig. 2.4: Quadrilateral generated using python

figs/quadrilateral_fig.tex

3 Solution

3.1. Given that:
$AC = AE$ **and** $AB = AD$
In $\triangle ABC$, $\angle BAC = \theta + \phi$
Also, In $\triangle AED$, $\angle DAE = \theta + \phi$

3.2. $\triangle ABC \cong \triangle ADE$ By SAS Congruency Rule
hence, BC = DE