Example about BBG correspondence

Kamal Saleh

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Contents

1 Koszul Syzygies modules

In the following S is the graded polynomial ring $\mathbb{Q}[x_0, \ldots, x_n]$ with $\deg(x_i) = 1, i = 0, \ldots, n$ and A is its dual graded ring, i.e., the exterior algebra generated by $e_i, i = 0, \ldots, n$ with $\deg(e_i) = -1$.

Definition 1. For $i = -1, \ldots, n$, the Koszul syzygy S-module Ω^i_S is given by

$$\Omega^i_S = \operatorname{coker}[S \otimes_K \bigwedge^{i+2} W \to S \otimes_K \bigwedge^{i+1} W],$$

i.e., the generators of Ω^i_S have degree $i+1$. The sheafifications of these modules are the exterior powers of the cotangent bundle on projective space.

For example, $\Omega^{-1}_S = K, \Omega^0_S = W \subset S$ and $\Omega^n_S = S \otimes_K \bigwedge^{n+1} W$.

```
S;
Q[x0,x1,x2,x3]
(weights: [ 1, 1, 1, 1 ])
A;
Q{e0,e1,e2,e3}
(weights: [ -1, -1, -1, -1 ])
omega_m1 := KoszulSyzygyModule(S,-1);
<An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])>
Display( omega_m1 );
-x0,
-x1,
-x2,
-x3
(over a graded ring)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 0 ])
omega_S_0 := KoszulSyzygyModule(S,0);;
Display( omega_S_0 );
0, 0,  x3, -x2,
0, x2, -x1,0,
0, x3, 0,  -x1,
x1,-x0,0,  0,
x2,0,  -x0,0,
x3,0,  0,  -x0
(over a graded ring)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
omega_S_1 := KoszulSyzygyModule(S,1);;
Display( omega_S_1 );
-x1,-x3,x2, 0,  0,  0,
-x0,0,  0,  0,  -x3,x2,
0,  -x0,0,  -x2,x1, 0,
0,  0,  -x0,-x3,0,  x1
```

```
(over a graded ring)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 2, 2, 2, 2, 2, 2 ])
omega_S_2 := KoszulSyzygyModule(S,2);;
Display( omega_S_2 );
-x0,x1,x3,-x2
(over a graded ring)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 3, 3, 3, 3 ])
omega_S_3 := KoszulSyzygyModule(S,3);;
Display( omega_S_3 );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 4 ])
omega_S_4 := KoszulSyzygyModule(S,4);;
Display( omega_S_4 );
(an empty 0 x 0 matrix)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[  ])
```

Let us compute the modules $\Omega^i_S(i), i = -1, \ldots, n = 3$.

```
twist_by_m1 := TwistFunctor(S,-1);;
omega_m1_m1 := ApplyFunctor( twist_by_m1, omega_m1 );;
Display( omega_m1_m1 );
-x0,
-x1,
-x2,
-x3
(over a graded ring)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 1 ])
omega_S_0_0 := omega_S_0;;
twist_by_1 := TwistFunctor(S,1);;
omega_S_1_1 := ApplyFunctor( twist_by_1, omega_S_1 );;
Display( omega_S_1_1 );
-x1,-x3,x2, 0,  0,  0,
-x0,0,  0,  0,  -x3,x2,
0,  -x0,0,  -x2,x1, 0,
0,  0,  -x0,-x3,0,  x1
(over a graded ring)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1, 1, 1 ])
twist_by_2 := TwistFunctor(S,2);;
omega_S_2_2 := ApplyFunctor( twist_by_2, omega_S_2 );;
Display( omega_S_2_2 );
-x0,x1,x3,-x2
(over a graded ring)

An object in The category of graded left f.p. modules
```

```
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
twist_by_3 := TwistFunctor(S,3);;
omega_S_3_3 := ApplyFunctor( twist_by_3, omega_S_3 );;
Display( omega_S_3_3 );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 1 ])
```

2 Twisted cotangent bundles

Definition 2. For $i = 0, \dots, n$ we define the twisted cotangent graded A-module $\Omega^i_A(i)$ to be the submodule of $\omega_A(i)$ that is generated by all homogeneous elements of degree ≤ 0.

```
omega_A_0_0 := TwistedCotangentBundle(A,0);;
Display( omega_A_0_0 );
e3,
e2,
e1,
e0
(over a graded ring)

An object in The category of graded left f.p. modules over
Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degree of generator:[ 0 ])
omega_A_1_1 := TwistedCotangentBundle(A,1);;
Display( omega_A_1_1 );
0, 0, e3,0,
0, e3,0, 0,
e3,0, 0, 0,
0, 0, 0, e2,
0, 0, e2,e3,
0, e2,0, 0,
e2,0, 0, 0,
0, 0, 0, e1,
0, 0, e1,0,
0, e1,0, -e3,
e1,0, 0, 0,
0, 0, 0, e0,
0, 0, e0,0,
0, e0,0, 0,
e0,0, 0, e3
(over a graded ring)

An object in The category of graded left f.p. modules over
Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degrees of generators:[ 0, 0, 0, 0 ])
omega_A_2_2 := TwistedCotangentBundle(A,2);;
Display( omega_A_2_2 );
0, 0, 0,  e3,0,  0,
0, e3,0,  0, 0,  0,
e3,0, 0,  0, 0,  0,
0, 0, 0,  0, e2, 0,
0, 0, 0,  e2,e3, 0,
0, 0, e2, 0, 0,  0,
0, e2,e3, 0, 0,  0,
e2,0, 0,  0, 0,  0,
0, 0, 0,  0, 0,  e1,
0, 0, 0,  0, e1, e2,
0, 0, 0,  e1,0,  e3,
0, 0, e1, 0, 0,  0,
```

```
0, e1,0,  0, 0,  0,
e1,0, -e3,0, 0,  0,
0, 0, 0,  0, 0,  e0,
0, 0, 0,  0, e0, 0,
0, 0, 0,  e0,0,  0,
0, 0, e0, 0, 0,  -e2,
0, e0,0,  0, 0,  -e3,
e0,0, 0,  0, -e3,0
(over a graded ring)

An object in The category of graded left f.p. modules
over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degrees of generators:[ 0, 0, 0, 0, 0, 0 ])
omega_A_3_3 := TwistedCotangentBundle(A,3);;
Display( omega_A_3_3 );
e3,0, 0, 0,
0, e2,0, 0,
e2,e3,0, 0,
0, 0, e1,0,
0, e1,e2,0,
e1,0, e3,0,
0, 0, 0, e0,
0, 0, e0,e1,
0, e0,0, e2,
e0,0, 0, e3
(over a graded ring)

An object in The category of graded left f.p. modules
over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degrees of generators:[ 0, 0, 0, 0 ])
```

3 Relation between $\Omega^i_S(i)$ and $\Omega^i_A(i)$

We have the following relations

$$\underline{\Omega^i_A(i)} \cong \underline{\mathbf{syz}}^0(\mathbf{Tate}(\Omega^i_S(i))),$$

$$\Omega^i_A(i) \cong \mathbf{syz}^0(\mathbf{Tate}^{\mathbf{min}}(\Omega^i_S(i))),$$

```
T := TateFunctor(S);
Tate 'functor' from The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ]) to Cochain complexes category over the category of
graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])
syz0_omega_S_0_0 := Source( CyclesAt( ApplyFunctor( T, omega_S_0_0 ), 0 ) );;
g := graded_generators_of_external_hom( syz0_omega_S_0_0, omega_A_0_0 );
[ <A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3}
(with weights [ -1, -1, -1, -1 ])> ]
IsIsomorphism( g[1] );
true
syz0_omega_S_1_1 := Source( CyclesAt( ApplyFunctor( T, omega_S_1_1 ), 0 ) );;
g := graded_generators_of_external_hom( syz0_omega_S_1_1, omega_A_1_1 );
[ <A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3}
(with weights [ -1, -1, -1, -1 ])> ]
IsIsomorphism( g[1] );
true
syz0_omega_S_2_2 := Source( CyclesAt( ApplyFunctor( T, omega_S_2_2 ), 0 ) );;
g := graded_generators_of_external_hom( syz0_omega_S_2_2, omega_A_2_2 );
[ <A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3}
(with weights [ -1, -1, -1, -1 ])> ]
IsIsomorphism( g[1] );
true
syz0_omega_S_3_3 := Source( CyclesAt( ApplyFunctor( T, omega_S_3_3 ), 0 ) );;
g := graded_generators_of_external_hom( syz0_omega_S_3_3, omega_A_3_3 );
[ <A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3}
(with weights [ -1, -1, -1, -1 ])> ]
IsIsomorphism( g[1] );
true
```

$$\widetilde{\Omega^i_S(i)} \cong \mathbf{H}^0(\widetilde{\mathbf{L}(\Omega^i_A(i))})$$

```
L := LFunctor( S );
L functor from The category of graded left f.p. modules over Q{e0,e1,e2,e3}
(with weights [ -1, -1, -1, -1 ]) to Cochain complexes category over the category
of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])
L_omega_A_0_0 := ApplyFunctor( L, omega_A_0_0 );
<A bounded object in cochain complexes category over the category of graded left
f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ]) with active lower
bound -1 and active upper bound 5>
CohomologySupport( L_omega_A_0_0, -1, 5 );
[ 0 ]
HO_L_omega_A_0_0 := CohomologyAt( L_omega_A_0_0, 0 );;
Display( HO_L_omega_A_0_0 );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 0 ])
Display( omega_S_0_0 );
0, 0,  x3, -x2,
0, x2, -x1,0,
0, x3, 0,  -x1,
x1,-x0,0,  0,
x2,0,  -x0,0,
x3,0,  0,  -x0
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
L_omega_A_1_1 := ApplyFunctor( L, omega_A_1_1 );
<A bounded object in cochain complexes category over the category of graded left
 f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ]) with active lower
bound -1 and active upper bound 5>
CohomologySupport( L_omega_A_1_1, -1, 5 );
[ 0, 1 ]
Remark: The cohomology in cohomological index 1 sheafifies to 0
HO_L_omega_A_1_1 := CohomologyAt( L_omega_A_1_1, 0 );;
Display( HO_L_omega_A_1_1 );
x1,-x3,-x2,0,  0,  0,
x0,0,  0,  0,  -x3,-x2,
0, x0, 0,  -x2,-x1,0,
0, 0,  x0, x3, 0,  -x1
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1, 1, 1 ])
Display( omega_S_1_1 );
-x1,-x3,x2, 0,  0,  0,
-x0,0,  0,  0,  -x3,x2,
0,  -x0,0,  -x2,x1, 0,
0,  0,  -x0,-x3,0,  x1
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1, 1, 1 ])
g := graded_generators_of_external_hom( HO_L_omega_A_1_1, omega_S_1_1 );
[ <A morphism in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])> ]
IsIsomorphism( g[1] );
```

```
true
L_omega_A_2_2 := ApplyFunctor( L, omega_A_2_2 );
<A bounded object in cochain complexes category over the category of graded left
f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ]) with active lower
bound -1 and active upper bound 5>
CohomologySupport( L_omega_A_2_2, -1, 5 );
[ 0, 2 ]
Remark: The cohomology in cohomological index 2 sheafifies to 0
H0_L_omega_A_2_2 := CohomologyAt( L_omega_A_2_2, 0 );;
Display( H0_L_omega_A_2_2 );
x0,-x3,-x2,-x1
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
Display( omega_S_2_2 );
-x0,x1,x3,-x2
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with  weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
g := graded_generators_of_external_hom( H0_L_omega_A_2_2, omega_S_2_2 );
[ <A morphism in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])> ]
IsIsomorphism( g[1] );
true
L_omega_A_3_3 := ApplyFunctor( L, omega_A_3_3 );
<A bounded object in cochain complexes category over the category of graded left
 f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ]) with active lower
bound -1 and active upper bound 5>
CohomologySupport( L_omega_A_3_3, -1, 5 );
[ 0, 3 ]
Remark: The cohomology in cohomological index 3 sheafifies to 0
H0_L_omega_A_3_3 := CohomologyAt( L_omega_A_3_3, 0 );;
Display( H0_L_omega_A_3_3 );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with  weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 1 ])
Display( omega_S_3_3 );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with  weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 1 ])
```

$$\mathrm{Hom}_K(\widetilde{\Omega^i_S(i)}, \widetilde{\Omega^j_S(j)}) \sim \mathrm{Hom}_K(\Omega^i_A(i), \Omega^j_A(j)) \sim \mathrm{Hom}_K(\omega_A(i), \omega_A(j))$$

```
Length( graded_generators_of_external_hom(w_E(2), w_E(0)));
6
Length( graded_generators_of_external_hom(omega_A_2_2, omega_A_0_0));
6
Display( w_E( 0 ) );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3}
(with weights [ -1, -1, -1, -1 ])

(graded, degree of generator:[ 4 ])
```

For every $f : \omega_A(i) \to \omega_A(j)$, there exists a morphism $h : \Omega^i_A(i) \to \Omega^j_A(j)$ that makes the following diagram commutative.

$$\begin{tikzcd}
\mathbf{H}^0(\mathbf{L}(\Omega^i_A(i))) \arrow[d, dashed, "\mathbf{H}^0(\mathbf{L}(h))"'] & \mathbf{L}(\Omega^i_A(i)) \arrow[d, dashed, Rightarrow, "\mathbf{L}(h)"'] & \Omega^i_A(i) \arrow[r, tail] \arrow[d, dashed, "\exists\, h"'] & \omega_A(i) \arrow[d, "f"] \\
\mathbf{H}^0(\mathbf{L}(\Omega^j_A(j))) & \mathbf{L}(\Omega^j_A(j)) & \Omega^j_A(j) \arrow[r, tail] & \omega_A(j)
\end{tikzcd}$$

Let $U : A\text{-gmod} \to A\text{-gmod}$ be the endomorphism defined by $U(\omega_A(i) \to \omega_A(j)) = \Omega^i_A(i) \to \Omega^i_A(j)$.

```
w_2 := w_A(2);
<An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])>
w_0 := w_A(0);
<An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])>
Display( w_2 );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degree of generator:[ 2 ])
Display( w_0 );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degree of generator:[ 4 ])
g := graded_generators_of_external_hom(w_2,w_0);;
f := Sum(g);
<A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])>
Display(f);
-e0*e1-e0*e2-e1*e2-e0*e3-e1*e3-e2*e3
(over a graded ring)

A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])
U := ToMorphismBetweenCotangentBundlesFunctor;;
h := ApplyFunctor( U, f );
<A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])>
Display(h);
1,
-1,
1,
1,
-1,
1
(over a graded ring)

A morphism in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])
IsWellDefined( h );
true
Lh := ApplyFunctor( L, h );
<A bounded morphism in cochain complexes category over the category of graded left f.p. modules over
Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ]) with active lower bound -1 and active upper bound 5>
H0 := CohomologyFunctorAt( cochains_graded_lp_cat_sym, graded_lp_cat_sym, 0 );
0-th cohomology functor in the category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])
H0_Lh := ApplyFunctor( H0, Lh );
<A morphism in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])>
Display( Source( H0_Lh ) );
x0,-x3,-x2,-x1
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
Display( omega_S_2_2 );
-x0,x1,x3,-x2
(over a graded ring)
```

```
An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
Display( Range( H0_Lh ) );
(an empty 0 x 1 matrix)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degree of generator:[ 0 ])
Display( omega_S_0 );
0, 0,  x3, -x2,
0, x2, -x1,0,
0, x3, 0,  -x1,
x1,-x0,0,  0,
x2,0,  -x0,0,
x3,0,  0,  -x0
(over a graded ring)

An object in The category of graded left f.p. modules
over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, 1, 1, 1 ])
```

4 Beilinson Monad

Let us now compute the Beilinson Monad of a coherent sheaf given by the sheafification of graded f.p. S-module.

```
m := RandomMatrixBetweenGradedFreeLeftModules([2,1],[1,-1,2],S);
<A 2 x 3 matrix over a graded ring>
M := AsGradedLeftPresentation(m,[1,-1,2]);
<An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])>
Display( M );
-2*x0+2*x1+4*x2,_[1,2],                                        -2,
-1,             2*x0*x1+x0*x2-2*x1*x2-5*x2^2-x0*x3-4*x1*x3-x2*x3,0
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, -1, 2 ])
Tate := TateFunctor( S );
Tate 'functor' from The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ]) to Cochain complexes category over the category of graded left f.p. modules
over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])
TM := ApplyFunctor( Tate, M );;
U := ToMorphismBetweenCotangentBundlesFunctor;;
ChU := ExtendFunctorToCochainComplexCategoryFunctor(U);;
ChU_TM := ApplyFunctor( ChU, TM );;
L := LFunctor( S );
L functor from The category of graded left f.p. modules over Q{e0,e1,e2,e3}
(with weights [ -1, -1, -1, -1 ]) to Cochain complexes category over the category of
graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])
ChL := ExtendFunctorToCochainComplexCategoryFunctor(L);;
ChL_ChU_TM := ApplyFunctor( ChL, ChU_TM );;
B := CohomologicalBicomplex( ChL_ChU_TM );
<A cohomological bicomplex in The category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ]) concentrated in window
[ -inf.. inf ] x [ -inf .. inf ]>
SupportInWindow( B, -4, 4, -4, 4 );
. . . . . . . . .   |4
. . . . . . . . .   |3
. . . . . . . . .   |2
. . . * . . . . .   |1
. . . * * . . . .   |0
. . . . . . . . .   |-1
. . . . . . . . .   |-2
. . . . . . . . .   |-3
```

```
. . . . . . . . . .   |-4
SetRight_Bound( B, 1 );;
SetLeft_Bound( B, -2 );;
T := TotalComplex( B );
<An object in Cochain complexes category over the category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])>
CohomologySupport( T, -4, 4 );
[ 0 ]
H0 := CohomologyFunctorAt( cochains_graded_lp_cat_sym, graded_lp_cat_sym, 0 );
0-th cohomology functor in the category of graded left f.p. modules over Q[x0,x1,x2,x3]
(with weights [ 1, 1, 1, 1 ])
H0_T := ApplyFunctor( H0, T );
<An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])>
Display( H0_T );
2,0,0, 0, x0,
0,2,-1,-1,-x1,
0,4,0, 0, x2,
0,0,-2,0, -x3
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 0, 0, 0, 0, -1 ])
Display( M );
-2*x0+2*x1+4*x2,_[1,2],                                     -2,
-1,             2*x0*x1+x0*x2-2*x1*x2-5*x2^2-x0*x3-4*x1*x3-x2*x3,0
(over a graded ring)

An object in The category of graded left f.p. modules over Q[x0,x1,x2,x3] (with weights [ 1, 1, 1, 1 ])

(graded, degrees of generators:[ 1, -1, 2 ])
syz0_M := Source( CyclesAt( ApplyFunctor( Tate, M ), 0 ) );
<An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])>
syz0_H0_T := Source( CyclesAt( ApplyFunctor( Tate, H0_T ), 0 ) );
<An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])>
Display( syz0_M );
e0*e1*e2*e3
(over a graded ring)

An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degree of generator:[ 3 ])
Display( syz0_H0_T );
e0*e1*e2*e3
(over a graded ring)

An object in The category of graded left f.p. modules over Q{e0,e1,e2,e3} (with weights [ -1, -1, -1, -1 ])

(graded, degree of generator:[ 3 ])
```