1)

A ratio of 700 time steps to 100 spatial steps( 0.1402). Any larger ratio and the solution explodes:

650 time steps to 100 spatial steps (0.151)

2)

The solution for the advection equation is

Let , so that it covers both the positive and negative:

Set

3)

See Attached

4)

See Attached

5)

a)

See Attached

b)

The spatial step size, and the error that arises from too large a step; changing the temporal step size only changes the temporal resolution. The use of the exponential approximation in time is responsible for the accuracy of our time step. The spatial approximation requires a certain number of time steps so that the frequency dependence does not stray from the actual value. Interestingly temporal resolution seems to change the group velocity while the spatial changes the individual wave’s velocities.

6)

a)

See attached

b)

Max epsilon

7)

We do not observe numerical dispersion when using the pseudo-spectral method as it gives a more exact approximation of the spatial derivative, forcing the frequency dependence to stay close to the exact value.

Optional

Assume is a diagonalizable such that

Show that if is a polynomial, then

But,

And since is a scalar, it we can switch the order of it and

But this is just

As desired.