1)

Finite Element Method

Consider the tent function

Compute the inner product

We have two integrals to do: and

For

For without loss of generality

All other spacings will result in zero.

2)

Consider the tent function from Problem 1. Compute the inner product

So,

wolog

3)

The only inner products that exist will be the elements:

With evaluations:

4)

Consider the basis functions

As a basis for the spectral Galerkin method for solving

where

a)

show that if the coefficient matrix

Is diagonal

As these will evaluate to some multiple of and at which both terms go to zero

Which evaluates as above to

Thus, all non-diagonal elements will be zero, and all diagonal elements will be some real value

b)

The matrix may no longer be diagonal: for example, the well known for the infinite square well is non-zero

Which is potentially non zero for or

5)

Discretize the following equation:

Using 5 grid points (including the boundary) in each dimension.

Write down the linear system for the solution at the 9 interior points in matrix vector form.

Quick definition:

From boundary conditions:

For interior points:

Using a second order accurate approximation:

Define

6)

Consider approximating the Laplacian in 2D using the fourth order accurate stencil

In both and

Write down the block matrix using this approximation for the 2D Laplacian

7)

Consider approximating the Laplacian in2D using the fourth order approximation

Write down the block matrix using this approximating for the 2D Laplacian. Write down the elements for each block that you introduce.

8)

One way to show that an approximation for the Laplacian in 2D is second order accurate is to show that it is exact when applying it to the polynomials

Use this method to show that the Laplacian approximation:

Without loss of generality, set

9)

Assume that is the eigenvector of and such that:

The commutator

Using the eigenvector as a test function, we see:

Since the eigenvalues are just complex constants:

Optional:  
Show that the matrix

Cannot be diagonalized

Algebraic multiplicity 2

There is only one eigenvector, and the matrix has geometric multiplicity 1

A matrix is diagonalizable if and only if the geometric multiplicity is equal to the algebraic multiplicity, and thus the matrix is non-diagonalizable.