E9 241: Digital Image Processing (2020)
Assignment 3
Due Date: 10th December 2020 (Late Submissions will be penalized)

Note: Submit a pdf file consisting of answers to all the reasoning questions.

Total Marks: 80

*** 10 marks is allotted for Viva**

1. Spatial domain filtering: **(20)**

Note: You can use matlab built in function imfilter (or corresponding function in python) for filtering. However, read its documentation carefully and explore its different options (especially boundary options).

a. Mitigate the noise in the image noisy.tif by filtering it with a square averaging mask of sizes 5,10 and 15. What do you notice with increasing mask size.

b. Use high boost filtering to sharpen the denoised image from part a. Choose the scaling constant for the high pass component that minimizes the mean squared error between the sharpened image and the image characters.tif.

2. Filtering in frequency domain: **(30)**

a. Generate a $M \times N$ sinusoidal image $\sin(2\pi u_0 m/M + 2\pi v_0 n/N)$ for $M=N=1001$, $u_0=100$ and $v_0=200$ and compute its DFT. To visualize the DFT of an image take logarithm of the magnitude spectrum.

Note: Fast Fourier Transform (FFT) is an algorithm which is used for efficient computation of DFT of discrete signals. You can use matlab (or python) built in function for computing the FFT.

b. Filter the image characters.tif in the frequency domain using an ideal low pass filter (ILPF). The expression for the ILPF is

$$H(u,v) = \begin{cases} 1 & D(u,v) \leq D_0 \\ 0 & if\ D(u,v) > D_0, \end{cases}$$

where D_0 is a positive constant referred to as the cut-off frequency and $D(u,v)$ is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle, i.e., $(u,v)=\sqrt{(u-P/2)^2 + (v\,Q/2)^2}$,

where P and Q are the number of rows and columns in the image. What artefacts do you notice in the image obtained by computing the inverse DFT of the filtered image?

Note: The filter given above is centred in frequency domain. To use such centred filters, you will either need to shift the filter to (0,0) (by using fftshift in matlab or corresponding function in python) or center the DFT of the image. To center the DFT of the image, you can either shift the DFT of the image, or scale each image pixel $I(x,y)$ by -1^{x+y} before computing its DFT. If you center the DFT of the image, then you will need to compensate for it by multiplying the image obtained from inverse DFT of the filtered image by -1^{x+y}.

c. Filter the image characters.tif in the frequency domain using the Gaussian low pass filter given by

$$H\,(u, v) = \exp\,(-D^2(u, v)/2D_0^2),$$

where all the terms are as explained in part b. For $D_0 = 100$, compare the result with that of the ILPF.

3. Homomorphic filtering: **(20)**

Use homomorphic filtering to enhance the contrast of the image PET_image.tif. Use the following filter to perform the high pass filtering

$$D\,(u, v) = (\gamma H - \gamma L)\,[1 - \exp\,(-D^2(u, v)/2D_0^2)] + \gamma L,$$

where γ_H, γ_L and D_0 are the parameters that you need to adjust through experimentation.

4. Bonus Question : **(20)**

a) Consider the 5x5 discrete Laplacian filter given by:

$$G = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 \\ 1 & 2 & -16 & 2 & 1 \\ 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Suppose we design the Laplacian in frequency domain as

H (u, v) = K*(u^2 + v^2) for u, v = -2,-1,0,1,2

Let h(m,n) be the IDFT of H(u, v). Find **K** such that $\sum_m \sum_n \left(h(m, n) - g(m,\ n)\right)^2$ is minimized.

b) Consider another 5x5 discrete Laplacian filter given by:

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -24 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Compute the value of '**K**' in this case. Comment on the value of **K** that you obtained by step (a) and (b).