# Statistics for Data Science

Tutorial 1

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# Digital Images

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- Normalised histogram $p[k] = \dfrac{1}{MN} h[k]$

# Which has ‘Better’ Contrast?

(a) Image 1

(b) Image 2

# Histogram Equalisation

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### Proof.

Since $Y = F_X(X)$, $0 \leq Y \leq 1$.

$$\begin{aligned} \forall y \in [0, 1],\ F_Y(y) &= P[Y \leq y], \\ &= P[F_X(X) \leq y], \\ &= P[X \leq F_X^{-1}(y)], \\ &= F_X(F_X^{-1}(y)) = y. \end{aligned}$$

Therefore, $Y = \mathcal{U}[0, 1]$. $\square$

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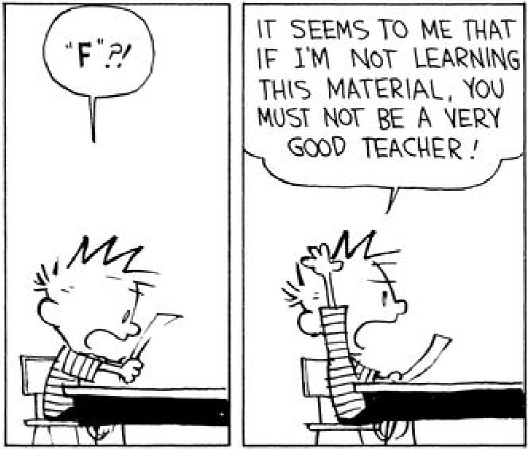
- Warning: Not all distribution functions are invertible; discrete implementations introduce errors; output histogram may not be the uniform distribution.

# *Fin.*


"F"?!
IT SEEMS TO ME THAT IF I'M NOT LEARNING THIS MATERIAL, YOU MUST NOT BE A VERY GOOD TEACHER!