Statistics for Data Science

Tutorial 2

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Digital Images

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- Each *pixel* (i, j) takes values in $\{0, 1, \ldots, 255\}$

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- Normalised histogram $p[k] = \dfrac{1}{MN} h[k]$

Which has ‘Better’ Contrast?

(a) Image 1

(b) Image 2

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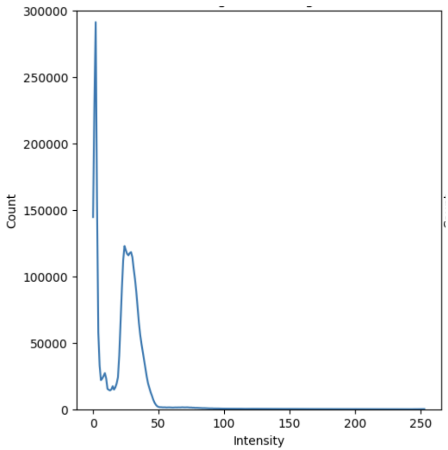
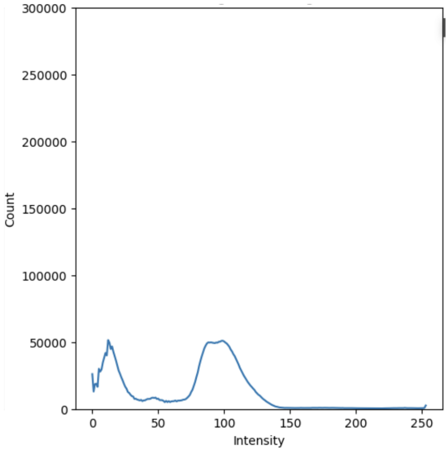


(c) Histogram of Image 1



(d) Histogram of Image 2

Figure: Flat histogram $\implies$ better contrast.

Histogram Equalisation

Lemma

Let X be a continuous random variable with an invertible cumulative distribution function F_X, and let $Y = F_X(X)$. Then, $Y = \mathcal{U}[0, 1]$.

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Proof.

Since $Y = F_X(X)$, $0 \leq Y \leq 1$.

$$\begin{aligned}\forall y \in [0,1],\ F_Y(y) &= P[Y \leq y],\\ &= P[F_X(X) \leq y],\\ &= P[X \leq F_X^{-1}(y)],\\ &= F_X(F_X^{-1}(y)) = y.\end{aligned}$$

Therefore, $Y = \mathcal{U}[0,1]$. $\square$

Discrete Implementation of Histogram Equalisation

- Given image $I[i, j]$
- Compute the normalized histogram $p[k]$
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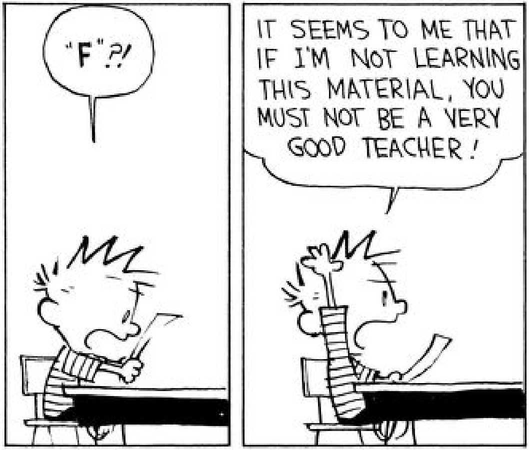
- Warning: Not all distribution functions are invertible; discrete implementations introduce errors; output histogram may not be the uniform distribution.

Fin.


"F"?!
IT SEEMS TO ME THAT IF I'M NOT LEARNING THIS MATERIAL, YOU MUST NOT BE A VERY GOOD TEACHER!