Indian Institute of Science
Department of Electrical Communication Engineering

E1 244: Detection and Estimation

March 2021 - June 2021

Course Project (deadline 18 June 2021 23:59 hrs)

This project consists of two parts related to automating a driving test: (a) detecting the entry and exit of a vehicle on the track (b) estimating its position based on radar measurements using a Kalman filter.

- Make a short report containing the required Matlab/Python files, plots, explanations, and answers, and turn it in by the deadline using Microsoft Teams under your name.
- Create an MS Teams meeting with yourself to record a "5 min presentation" explaining your code, observations, and outputs related to each question. The presentation video needs to be uploaded on MS Teams. Projects for which the presentation video duration is more than 5 minutes will not be evaluated.

Part A: Vehicle detection

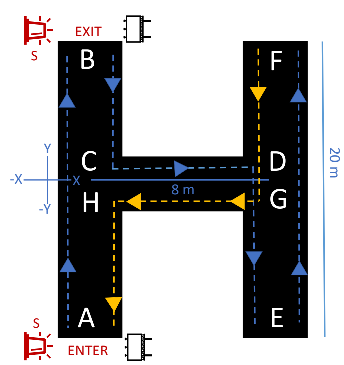

Figure 1: 'H' Track for the driving test.

Consider a track of the shape 'H' for the driving test as shown in Fig. 1. Vehicles enter the track at A, move along the track via B, C, D, E, F, G, H, A (in this order) and exit at B. We are interested in automating the driving test by tracking the path of vehicles undergoing the test using observations of the positions and velocities of the vehicles from a radar combined with an assumed state space-model for the vehicles. We do so by using a Kalman filter (Part B). Furthermore, we would like to know if a vehicle has reached the start of the track, A, and is ready for the test so that we switch the radar on only when the presence of a vehicle is detected, to save power. We also wish to know when the track is ready for the next vehicle and hence would want to detect the presence of a vehicle also at the end of the track, B, to avoid multiple vehicles on the track.

For detection, we employ light dependent resistors (LDRs). LDRs are light sensitive passive components whose resistance decreases as the amount of light incident on it increases. LDRs usually have an LED and a resistor connected in series. The voltage across these elements in series increases with an increase in the incident light. A source of light S and LDR sensors are placed on either side of the track at entry and exit as shown in Fig. 1. The light from S falls on the sensors till a vehicle comes in the path of the light, preventing it from falling on the sensors. In the presence of a vehicle, the resistance of each of the LDRs increases, resulting in the corresponding LEDs connected in series to turn off. However, it should be noted that the ambient light (from the Sun) affects the LDR measurements. Also, this light varies as the day progresses. We assume that we have M identical LDRs and that the voltage measured across the associated elements connected in series, $x_m[n]$, at different time instances $n = 1, \dots, N$, in the absence of any vehicle is given by

$$x_m[n] = A + B_t + w_m[n], \; n = 1, \dots, N, m = 1, \dots, M, \tag{1}$$

where $B_t + w_m[n]$ corresponds to the voltage drop across the elements due to the ambient light at nth time instant at a particular time, t, of the day, and A is the voltage drop due to the source S. B_t ranges from $B_{\min}$ in the morning to $B_{\max}$ in the noon. $w_m[n]$ is white Gaussian noise with zero mean and variance σ^2. Based on the observed voltage $x_m[n]$, we infer the presence or absence of a vehicle.

Derivation

We derive two Neyman-Pearson detectors for detecting vehicles at the entry and the exit. Note that the requirement of the two detectors is different. While designing the detector at the start of the track, we aim for a detector with the probability of missing a detection to be below a certain threshold, as missing to detect a vehicle hinders our later experiment of tracking it. In contrast, at the end of the track, we want the probability of false alarms to be not more than a certain threshold, as a false alarm would wrongly indicate that the track is ready for the next vehicle while it is not.

1. For detecting vehicles at the entry, design a detector that has a probability of false alarm less than any other detector satisfying the constraint that the probability of detection P_D is equal to β, assuming we know B_t at the time of the day when we want the detector.

2. Modify the detector in the previous question such that our requirement on the probability of detection P_D being not less than β is met at any time of the day. We refer to this detector as $\mathrm{D}_{\text{entry}}$.

3. For detecting vehicles at the end of the track, design a detector that has a probability of detection P_D more than any other detector satisfying the condition that the probability of false alarm P_{FA} is α. Relate this to the detector in question 1. What do you observe and why?

4. Modify the detector in the previous question such that our requirement of the probability of false alarm P_{FA} being not more than α is met at any time of the day. We refer to this detector as D_{exit}.

5. Reformulate the problem in Question 3 as a one-sided parameter test with no nuisance parameters, where only B_t is known. Derive the Locally Most Powerful (LMP) detector and relate it to the Neyman-Pearson detector.

Implementation

For implementing the detectors, take $A = 1\,V, B_{\min} = 0.1\,V, B_{\max} = 0.6\,V$ and noise variance $\sigma^2 = 1$. We consider B_t at different times, denoted by $t = 1, \dots, 6$, of the day from morning to evening and are given by $B_1 = B_{\min} = 0.1\,V, B_2 = 0.3\,V, B_3 = 0.5\,V, B_4 = B_{\max} = 0.6\,V, B_5 = 0.4\,V, B_6 = 0.2\,V$. Assume that we have $N = 5$ observations each from M different sensors at each time t of the day considered. Vary M from 1 to 3 to see how the results change in each of the following questions.

1. Implement $\mathrm{D_{entry}}$. Verify that $\mathrm{D_{entry}}$ is valid at any time of the day by plotting the experimental P_D against 100 different values of P_D at equal intervals from 0.01 to 0.99 for the values of B_t considered.

2. Plot the ROC curve (i.e., P_{FA} vs P_D) of the detector D_{entry} for 100 different values of P_D at equal intervals from 0.01 to 0.99 and compare it to the theoretical one.

3. Implement $\mathrm{D_{exit}}$. Verify that $\mathrm{D_{exit}}$ is valid at any time of the day by plotting the experimental P_{FA} against 100 different values of P_{FA} at equal intervals from 0.01 to 0.99 for the values of B_t considered.

4. Plot the ROC curve (i.e., P_D vs P_{FA}) of the detector D_{exit} for 100 different values of P_{FA} at equal intervals from 0.01 to 0.99 and compare it to the theoretical one.

Part B: Vehicle tracking

In this section, we proceed to track the vehicles, detected by the detectors designed in Part A, using a Kalman filter to find out if a vehicle passed the test using the estimates from the filter.

We assume that we receive noise corrupted radar observations of the positions $\mathbf{p}[n] = [p_x[n], p_y[n]]^{\mathrm{T}}$ and the velocities $\dot{\mathbf{p}}[n] = [\dot{p}_x[n], \dot{p}_y[n]]^{\mathrm{T}}$ of the vehicles during the entire duration of their driving test at every Δ seconds. Here, $p_x[n]$ and $p_y[n]$ denote the x-coordinate and y-coordinate of the vehicle's position at time instant n, respectively, and $\dot{p}_x[n]$ and $\dot{p}_y[n]$ denote the x and y components of the vehicle's velocity at nth time instant, respectively. The reference axes are as shown in Fig. 1. Our measurement model is given by

$$\mathbf{y}[n] = \mathbf{x}[n] + \mathbf{v}[n],$$

where $\mathbf{x}[n] = [\mathbf{p}^{\mathrm{T}}[n], \dot{\mathbf{p}}^{\mathrm{T}}[n]]^{\mathrm{T}}$ and $\mathbf{v}[n] = [v_1[n], v_2[n], v_3[n], v_4[n]]$ is a zero mean Gaussian random vector, referred to as the observation noise. $E\{\mathbf{v}[n]\mathbf{v}^{\mathrm{T}}[k]\} = \mathbf{Q}_v[n]\delta(n-k)$, where $\mathbf{Q}_v = \mathrm{diag}([\sigma^2_{\mathrm{v}_1}, \sigma^2_{\mathrm{v}_2}, \sigma^2_{\mathrm{v}_3}, \sigma^2_{\mathrm{v}_4}])$.

In addition to the measurement model, we have a state model of the vehicles taking the test to design the Kalman filter. To develop the state model, we assume that the vehicles move at a constant speed of v m/s, with their velocities changing only at the points B, C, D, E, F, G and H. We have two sets of general state equations, one for the horizontal motion, i.e., when the vehicle moves along the x-axis and the other for the vertical motion i.e., when the vehicle moves along the y-axis. For the vertical motion, the state equations are

$$\begin{aligned} p_x[n] &= p_x[n-1] + w_1[n] \\ p_y[n] &= p_y[n-1] + \dot{p}_y[n-1]\Delta + w_2[n] \\ \dot{p}_x[n] &= w_3[n] \\ \dot{p}_y[n] &= \dot{p}_y[n-1] + w_4[n]. \end{aligned} \tag{2}$$

Notice that the x-component of velocity is 0. The state noise can be compactly represented as $\mathbf{w}[n] = [w_1[n], w_2[n], w_3[n], w_4[n]]^{\mathrm{T}}$, which is assumed to be a zero mean Gaussian random vector with covariance matrix $E\{\mathbf{w}[n]\mathbf{w}^{\mathrm{T}}[k]\} = \mathbf{Q}_w[n]\delta(n-k)$, where $\mathbf{Q}_w = \mathrm{diag}([\sigma^2_{\mathrm{w}_1}, \sigma^2_{\mathrm{w}_2}, \sigma^2_{\mathrm{w}_3}, \sigma^2_{\mathrm{w}_4}])$.

Similarly, we can obtain the state transition matrix for the horizontal motion. However, it should be noted that the transition matrix remains unchanged only till the velocity is constant (i.e., till the vehicle's direction does not change). Therefore, the transition matrix changes at the points B, C, D, E, F, G and H on the track. For example, when a vehicle takes a turn at C, the velocity vector changes from $[0, -v]^{\mathrm{T}}$ in the $(n-1)$th time instant to $[v, 0]$ in the next time instant. Formally, we have

$$\begin{aligned} p_x[n] &= p_x[n-1] + w_1[n] \\ p_y[n] &= p_y[n-1] + \dot{p}_y[n-1]\Delta + w_2[n] \\ \dot{p}_x[n] &= -\dot{p}_y[n-1] + w_3[n] \\ \dot{p}_y[n] &= w_4[n]. \end{aligned} \tag{3}$$

Derivation

1. Develop the state model to simulate the vehicle's path along A-B-C-D-E-F-G-H-A.

2. Assume that we observe the radar measurements only of the velocities, i.e., $\mathbf{y}[n] = \dot{\mathbf{p}}[n] + [v_3[n], v_4[n]]^{\mathrm{T}}$. Design a Kalman filter to track a vehicle in the driving test track using the entire state model so that the Kalman filter produce both position and velocity estimates.

3. Using the derived state-space model, design a Kalman filter to track a vehicle, given its radar measurements of both positions and velocities. Here, we use both the position and velocity observations from the radar.

Implementation

Assume that the dimensions of the track are as shown in Fig. 1. Use the sample spacing $\Delta =$ 0.1s. Assume that the vehicle moves at a constant speed of $v = 2.5$ m/s. For the state model, assume that state transition matrix changes at points $n = \{80, 120, 152, 192, 272, 312, 344\}$, where $n = 0$ is the initiaization.

Unzip the file `dataset.zip`. It contains 8 `mat` files containing true positions and radar measurements of the path traced by the vehicle under different circumstances, and a `readme.txt` text file. Each trace is a 4×384 matrix whose columns corresponds to the instantaneous position and velocity of the car at different time instants. Please refer `readme.txt` for more details about the dataset. Ideal test track is contained in the file `trace_ideal.mat`. You may visualize the ideal track and measurements using the following (Matlab) script after loading trace_ideal.mat

```
figure
plot(true_trace(1,:), true_trace(2,:),'bo-')
```

1. Implement a Kalman filter to obtain the position of the car, $\hat{\mathbf{x}}$, using only the velocity measurements provided in Radar_med.mat and comment on the observations. Use $\mathbf{Q}_w = 0.1\mathbf{I}$ and initialize the Kalman filter with the all zero vector. Now change the initialization to $\mathbf{x}_{\text{init}} = [1\ 0\ 0\ 0]^{\mathrm{T}}$. Will anything change? Also compute the error of the Kalman filter and radar measurements as $E_{\mathrm{KF}}(n) = \|[\hat{x}(1,n)\ \hat{x}(2,n)]^{\mathrm{T}} - [x(1,n)\ x(2,n)]^{\mathrm{T}}\|^2$, and $E_{\mathrm{R}}(n) = \|[y(1,n)\ y(2,n)]^{\mathrm{T}} - [x(1,n)\ x(2,n)]^{\mathrm{T}}\|^2$, respectively, and comment on the observations. Consider that the radar velocity measurements are perfect and the path traced by the car strictly follows the state model (i.e., $\sigma^2_{w_3} = \sigma^2_{w_4} = 0$, and $\mathbf{Q}_w = \mathbf{0}$). Is it possible to track the position of the car using just the velocity measurements and Kalman filtering? Why?

2. Implement the Kalman filter that make use of both position and velocity measurements in Radar_med.mat. Use $\mathbf{Q}_w \in \{0.01\mathbf{I}, 0.1\mathbf{I}, \mathbf{I}\}$ and comment on the observations. Compare the error of the Kalman filter with that of using only the radar measurements in all cases.

3. To appreciate the advantage of Kalman filter in tracking, we will now consider a scenario where the measurement noise is large.Implement the Kalman filter to track the position of the car given by the radar measurement in Radar_high.mat. Use $\mathbf{Q}_v = \mathbf{I}$ and $\mathbf{Q}_w \in \{0.0001\mathbf{I}, 0.01\mathbf{I}, 0.1\mathbf{I}, \mathbf{I}\}$ and comment on the observations. Which choice is better and why? Compute and compare the errors for Kalman filter and radar for all cases. Do you observe any particular pattern on the estimation error of the Kalman filter, especially for low values of model error? *Hint: Observe the error when the car makes the turns.*

4. Suppose that we use an incorrect value of $\mathbf{Q}_v$ while using the Kalman Filter. Consider the dataset in the previous question. Re-run the Kalman filter iterations by choosing $\mathbf{Q}_v \in \{0.01\mathbf{I}, 0.1\mathbf{I}\}$. Use $\mathbf{Q}_w = 0.1$ and compare the estimation errors of the Kalman filter to those obtained in the previous question. Why is this happening? Does this mean that the Kalman filter is not useful when the radar measurement is noisy?

5. Now, we will use the designed Kalman filter to automate the testing process. Load the file Test_{x}.mat where x = 1,2,3,4, each of which contains a single variable radar_measurement. Use $\mathbf{Q}_v = 0.1\mathbf{I}$, $\mathbf{Q}_w = 0.1\mathbf{I}$ and apply the Kalman filter to estimate the position of the car. For the different datasets provided, report the driving test results by checking whether the instantaneous error is beyond a certain threshold, τ at any time instant. Plot the estimated location using Kalman filter and overlay the plot on top of the ideal track. Plot the instantaneous error $E_{\mathrm{KF}}(n)$ and report that a vehicle passed the driving test if the Kalman filter's instantaneous error is never greater than a threshold τ during the entire duration of the test, or say it failed otherwise. Note that we does not know the true position of the car and the error should be computed with respect to the ideal track given in trace_ideal.mat.

To choose the threshold, let us allow a maximum deviation of 1.25 metres on either side of the ideal track. This means that $|\hat{x}(1,n) - x(1,n)| \leq 1.25$ and $|\hat{x}(2,n) - x(2,n)| \leq 1.25$, where $x(:,n)$ is the ideal track. Then we have $E_{\mathrm{KF}}(n) = \|[\hat{x}(1,n)\ \hat{x}(2,n)]^{\mathrm{T}} - [x(1,n)\ x(2,n)]^{\mathrm{T}}\|^2 \leq 2 \times 1.25^2 = 3.125$. Hence choose $\tau = 3.125$.